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Virtuous Circles and Contested Identities: on Collective Identification Procedures with Independent Qualified Certification

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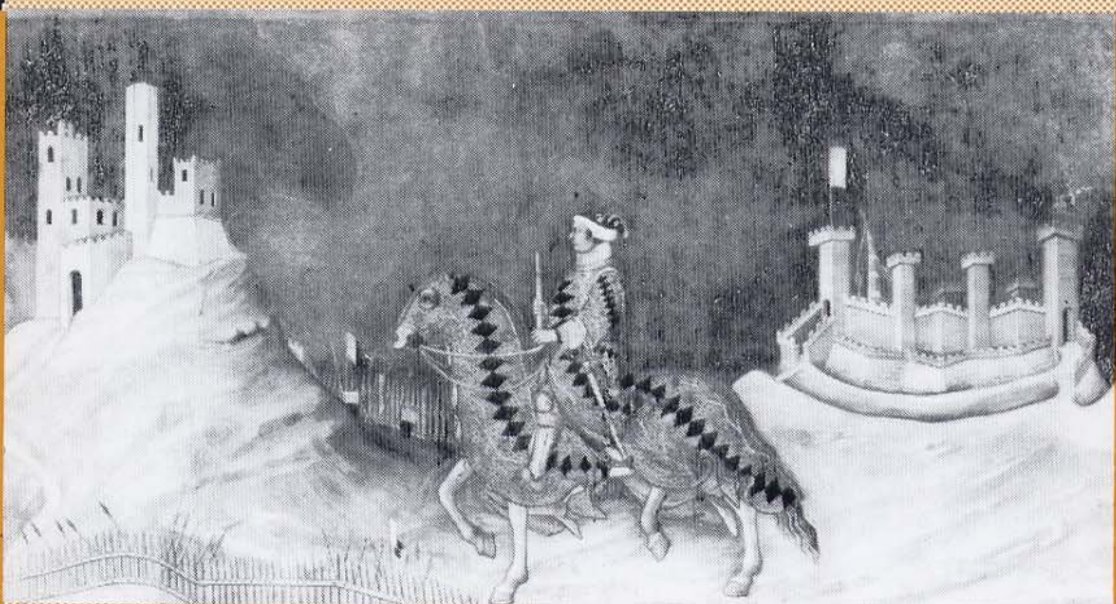

Abstract - This paper studies Collective Identification Procedures in a finite lattice when the standard Independence axiom is dropped and replaced with an Independent Qualified Certification requirement.

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1 Introduction

Collective identification procedures (CIPs) are meant to model the wide class of more or less formal protocols that are used in order to identify the legitimate members of certain associations, clubs, or communities. In the last decade, some work has been devoted to the formal social-choice-theoretic study of CIPs. The bulk of the extant literature has focussed on two essentially disjoint classes of procedures namely those which satisfy a counterpart of the arrowian social-choice-theoretic *independence* property including of course the self-certification-based 'libertarian' rule, and those which rely on some *cooptation* principle (see e.g. Samet and Schmeidler (2003), Çengelci and Sanver (2005), Sung and Dimitrov (2003), Kasher and Rubinstein (1996), Dimitrov, Sung and Xu (2003)). In the present collective identification setting, Independence establishes that membership of each population unit does only depend on the assessment of her qualifications on the part of all population units to the effect of disregarding the qualifications of units to assess each other. In turn, cooptation principles amount to allowing some asymmetries among more or less active and passive members in the nomination process, including possibly the distinction between 'founding' members and 'others'.

This paper is mainly devoted to those CIPs that rely on the principle of Independent Qualified Certification (IQC): *membership requires certification/approval by another qualified unit namely by another member.* Clearly enough, the IQC principle is not consistent with Independence. However, it does not rule out cooptation altogether. Therefore, a further Collective Self-Determination property (a generalization of a condition due to Samet and Schmeidler (2003)) is introduced to the effect of ruling out cooptation. Moreover, a Participatory Certification condition ensuring that membership is voluntary is also considered. Several CIPs satisfying various combinations of those requirements are identified and analyzed. It should be remarked that while the extant literature is typically concerned with the boolean lattice of subsets of the universal (finite) set of agents, our analysis is pursued -along the lines of Monjardet (1990)[1]- in the considerably more general framework

[1]Monjardet (1990) is in fact concerned with arrowian i.e. 'independent' aggregation rules in semi-lattices. Our IQC-consistent CIPs may be regarded as specialized 'non-independent' aggregation rules in lattices. In a more specialized setting Miller (2006) does also heavily rely on semi-latticial-theoretic properties ('join-separability' and 'meet-separability'). In contrast, Ballester and García-Lapresta (2005) is focussed on sequential

of an arbitrary finite lattice. The choice of such a general environment allows one to accommodate the cases of abstention and of many-valued memberships[2]. The present paper is organized as follows: Section 2 is devoted to a presentation of the model, and the results. Section 3 provides some short remarks about possible extensions of the analysis.

2 Collective Identification Procedures with Independent Qualified Certification

2.1 Notation and Characterizations

Let $\mathbf{L} = (L, \leqslant)$ be a finite lattice namely a finite partially ordered set[3] such that for any $x, y \in L$ both the greatest lower bound $x \wedge y$ and the least upper bound $x \vee y$ of $\{x, y\}$ do exist[4]. A *join irreducible* element of $\mathbf{L}$ is any $j \in L$ such that for any $x, y \in L$ if $j = x \vee y$ then $j \in \{x, y\}$. The set of all join irreducible elements of L is denoted J^*: it is also assumed that $\#J^* \geq 2$ in order to avoid tedious qualifications or trivialities. The following analysis refers to an arbitrary but fixed $J_L \subseteq J^*$such that $\#J_L \geq 2$[5].

A *Collective Identification Procedure (CIP)* on J_L is a function $F : L^{J_L} \to L$. For any pair F, F' of CIPs on J_L, it will be written $F' \leqq F$ whenever $F'(x) \leqslant F(x)$ for all $x \in L^{J_L}$.

The present work will be mainly focussed on the following properties of CIPs:

Independent Qualified Certification (IQC): For any $j \in J_L$ and

identification procedures with several degrees of membership.

[2] See note 5 below.

[3] Thus, by definition, $\leqslant$ is a transitive, reflexive and antisymmetric binary relation on L.

[4] For any $A \subseteq L$, $\wedge_{x \in A} x$ and $\vee_{x \in A} x$ are defined in the obvious way.

[5] Notice that the extant literature on collective identification procedures is typically focussed on the special boolean lattice $(\mathcal{P}(N), \subseteq)$, where N is the finite population of agents. In that lattice, the join-irriducible elements are the atoms i.e. the singletons. Thus the standard case with set of agents N reduces to a special instance of our model with $L = 2^N$, $\leqslant = \subseteq$ and $J_L = J^* \simeq N$.

When agents are allowed to abstain one has $L = 3^N$, while $\leqslant$ reduces to the componentwise partial order, and $J_L \subset J^*$, $J_L \simeq N$.

A similar approach can be applied to the case of several degrees of memberships.

$x \in L^{J_L}$ such that $j \leqslant F(x)$ there exists $i \in J_L, i \neq j$ such that $i \leqslant F(x)$ and $j \leqslant x_i$.

Participatory Certification (PC): For any $x \in L^{J_L}$ and any $j \in J_L$, if $j \leqslant F(x)$ then $j \leqslant x_j$.

Collective Self-Determination (CSD): For any $x, x' \in L^{J_L}$ if $[j \leqslant x_i$ iff $i \leqslant x'_j$ for any $i, j \in J_L]$ then $F(x) = F(x')$.

Clearly, IQC establishes that membership requires certification of eligibility by another member, while PC simply requires voluntariness of membership. CSD is a no-cooptation property which amounts to imposing identity of the set of members under reversal of roles between certificators and nominees.

We shall then proceed to introduce and characterize some CIPs, relying on the foregoing axioms.

To begin with, we consider two CIPs which satisfy IQC while not disallowing cooptation.

Definition 1 *The **Extended Qualified Nomination (EQN)** procedure*[6]*: for any* $x \in L^{J_L}$

$$F^E(x) = \bigvee \left\{ \begin{array}{c} j \in J_L : \textit{there exist } k \in \mathbb{Z},\ k \geq 2\ ,\ \{i_1, .., i_k\} \subseteq J_L \\ \textit{and } i \in \{i_1, .., i_k\} \setminus \{j\} \textit{ such that} \\ h \leqslant x_{h+1 (\operatorname{mod} k)} \textit{ for any } h \in \{i_1, .., i_k\} \textit{ and } j \leqslant x_i \end{array} \right\}$$

In plain words, EQN identifies as members the nominees of some agent[7] in some circle of nominees where each agent is nominated by her successor/neighbour.

Definition 2 *The **Participatory Extended Qualified Nomination (PEQN)** procedure: for any* $x \in L^{J_L}$

$$F^{PE}(x) = \bigvee \left\{ \begin{array}{c} j \in J_L : \textit{there exist } k \in \mathbb{Z},\ k \geq 2\ ,\ \{i_1, .., i_k\} \subseteq J_L \\ \textit{and } i \in \{i_1, .., i_k\} \setminus \{j\} \textit{ such that} \\ h \leqslant x_h \wedge x_{h+1 (\operatorname{mod} k)} \textit{ for any } h \in \{i_1, .., i_k\} \textit{ and} \\ j \leqslant x_j \wedge x_i \end{array} \right\}$$

[6]The suffix $(\operatorname{mod} k)$ (following an addition) which is introduced below denotes k-modular sum namely sum in the finite group $\mathbb{Z}/k$ (where k is any positive integer).

[7]The term ‘agent’ shall be henceforth used to denote any relevant population unit namely any element of J^L.

Thus, PEQN identifies as members the *consenting* nominees of some agent who belong to some circle of nominees where each agent is nominated by her successor/neighbour and *declares herself a member.*

EQN and PEQN can be readily characterized as follows:

Proposition 3 *F^E satisfies IQC. Moreover, for any CIP $F : L^{J_L} \rightarrow L$, if F satisfies IQC then $F \leqq F^E$.*

Proof. Let us consider $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F^E(x)$. Then there exist $k \in \mathbb{Z}$, $k \geq 2$, $\{i_1, .., i_k\} \subseteq J_L$ and $i \in \{i_1, .., i_k\} \setminus \{j\}$ such that $\#\{i_1, .., i_k\} = k$, $i_h \leqslant x_{i_{h+1(\mathrm{mod}\, k)}}$ for any $h \in \{1, .., k\}$, and $j \leqslant x_j \wedge x_i$. Moreover, by definition, $i_h \leqslant F^E(x)$ for any $h \in \{1, .., k\}$ and $j \neq i$ hence F^E does indeed satisfy IQC.

Now, let F be a CIP that satisfies IQC.

For any $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F(x)$, there exists $i_1 \in J_L$, $i_1 \neq j$ such that $i_1 \leqslant F(x)$ and $j \leqslant x_{i_1}$, by IQC. But then, by IQC again, there exists $i_2 \in J_L$, $i_2 \neq i_1$ such that $i_2 \leqslant F(x)$ and $i_1 \leqslant x_{i_2}$. By repeating the argument, and in view of finiteness of J_L, we may conclude that there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\#\{i_1, .., i_k\} = k$, $\{i_1, .., i_k\} \subseteq J_L$ such that $h \leqslant x_{h+1(\mathrm{mod}\, k)}$ and $h \leqslant F(x)$ for any $h \in \{i_1, .., i_k\}$. Thus, by definition, $j \leqslant F^E(x)$ as required. ■

Proposition 4 *F^{PE} satisfies IQC and PC. Moreover, for any CIP F : $L^{J_L} \rightarrow L$, if F satisfies IQC and PC then $F \leqq F^{PE}$.*

Proof. Let us consider $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F^{PE}(x)$. Then there exist $k \in \mathbb{Z}$, $k \geq 2$, $\{i_1, .., i_k\} \subseteq J_L$ and $i \in \{i_1, .., i_k\} \setminus \{j\}$ such that $\#\{i_1, .., i_k\} = k$, $i_h \leqslant x_{i_h} \wedge x_{i_{h+1(\mathrm{mod}\, k)}}$ for any $h \in \{1, .., k\}$, and $j \leqslant x_i$. Moreover, by definition, $i_h \leqslant F^{PE}(x)$ for any $h \in \{1, .., k\}$ and $j \neq i$ hence F^{PE} does indeed satisfy IQC. It is straightforward to check that, by definition, F^{PE}satisfies PC as well.

Let F be a CIP that satisfies IQC and PC. For any $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F(x)$, there exists $i_1 \in J_L$, $i_1 \neq j$ such that $i_1 \leqslant F(x)$ and $j \leqslant x_j \wedge x_{i_1}$, by IQC and PC. But then, by IQC again, there exists $i_2 \in J_L$, $i_2 \neq i_1$ such that $i_2 \leqslant F(x)$ and $i_1 \leqslant x_{i_2}$. By repeating the argument, and in view of finiteness of J_L, we may conclude that there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\#\{i_1, .., i_k\} = k$, $\{i_1, .., i_k\} \subseteq J_L$ such that $h \leqslant x_h \wedge x_{h+1(\mathrm{mod}\, k)}$ and $h \leqslant F(x)$ for any $h \in \{i_1, .., i_k\}$. Thus, by definition, $j \leqslant F^{PE}(x)$. ■

Clearly enough, both EQN and PEQN explicitly allow cooptation of members (namely, nominees who are not members of the basic 'circle' are coopted by some member within such 'circle').

Let us then turn to some CIPs that satisfy IQC *and* disallow cooptation practices.

Definition 5 *The **Restricted Qualified Nomination (RQN)** procedure: for any* $x \in L^{J_L}$

$$F^R(x) = \bigvee \left\{ \begin{array}{c} j \in J_L : \text{there exist } k \in \mathbb{Z},\ k \geq 2 \text{ and } \{i_1, .., i_k\} \subseteq J_L \\ \text{such that } j \in \{i_1, .., i_k\} \\ \text{and } h \leqslant x_{h+1(\operatorname{mod} k)} \text{ for any } h \in \{i_1, .., i_k\} \end{array} \right\}$$

Hence, by definition, the RQN procedure identifies as members those agents who belong to some circle of nominees where each agent is nominated by her successor/neighbour.

Definition 6 *The **Participatory Restricted Qualified Nomination (PRQN)** procedure: for any* $x \in L^{J_L}$

$$F^{PR}(x) = \bigvee \left\{ \begin{array}{c} j \in J_L : \text{there exist } k \in \mathbb{Z},\ k \geq 2 \text{ and } \{i_1, .., i_k\} \subseteq J_L \\ \text{such that } j \in \{i_1, .., i_k\} \\ \text{and } h \leqslant x_h \wedge x_{h+1(\operatorname{mod} k)} \text{ for any } h \in \{i_1, .., i_k\} \end{array} \right\}$$

Again, the PRQN procedure is a voluntary version of RQN, namely it identifies as members those agents who belong to some circle of nominees where each agent is nominated by her successor/neighbour and *declares herself to qualify as a member.*

RQN and PRQN are also amenable to an easy characterization in terms of our axioms, as presented by the following results.

Proposition 7 F^R *satisfies IQC and CSD. Moreover, for any CIP* $F : L^{J_L} \to L$*, if* F *satisfies IQC and CSD then* $F \leqq F^R$.

Proof. Let us consider $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F^R(x)$. Then there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\{i_1, .., i_k\} \subseteq J_L$ such that $\# \{i_1, .., i_k\} = k$, $j = i_{h^*} \in \{i_1, .., i_k\}$ for some $h^* \in \{1, .., k\}$, and $i_h \leqslant x_{i_{h+1(\operatorname{mod} k)}}$ for any $h \in \{1, .., k\}$. Moreover, by definition, $i_h \leqslant F^R(x)$ for any $h \in \{1, .., k\}$. Since $k \geq 2$ it must be the case that in particular $j \neq i_{h^*+1(\operatorname{mod} k)}$ hence F^R does indeed satisfy IQC.

Moreover, let $x, x' \in L^{J_L}$ be such that [$j \leqslant x_i$ iff $i \leqslant x'_j$ for any $i, j \in J_L$]. Next, take any $j \in J_L$ such that $j \leqslant F^R(x)$. Then, by definition, there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\{i_1, .., i_k\} \subseteq J_L$ such that $\#\{i_1, .., i_k\} = k$, $j \in \{i_1, .., i_k\}$ and $h \leqslant x_{h+1(\mathrm{mod}\, k)}$ for any $h \in \{i_1, .., i_k\}$. But then, $h + 1(\mathrm{mod}\, k) \leqslant x'_h$ for any $h+1(\mathrm{mod}\, k) \in \{i_1, .., i_k\} = \{i'_1, .., i'_k\}$ where $i'_h = i_{k-h+1(\mathrm{mod}\, k)}$, $h = 1, .., k$. Therefore, equivalently, $h \leqslant x'_{h+1(\mathrm{mod}\, k)}$ for any $h \in \{i'_1, .., i'_k\}$. It follows that, by definition, $j \leqslant F^R(x')$ whence $F^R(x) \leqslant F^R(x')$. Since $(x')' = x$, by a similar argument $F^R(x') \leqslant F^R(x)$. Thus, $F^R(x) = F^R(x')$ i.e. F^R satisfies CSD.

Now, let F be a CIP that satisfies IQC and CSD.

For any $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F(x)$, there exists $i_1 \in J_L$, $i_1 \neq j$ such that $i_1 \leqslant F(x)$ and $j \leqslant x_{i_1}$, by IQC. But then, by IQC again, there exists $i_2 \in J_L$, $i_2 \neq i_1$ such that $i_2 \leqslant F(x)$ and $i_1 \leqslant x_{i_2}$. By repeating the argument, and in view of finiteness of J_L, we may conclude that there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\#\{i_1, .., i_k\} = k$, $\{i_1, .., i_k\} \subseteq J_L$ such that $h \leqslant x_{h+1(\mathrm{mod}\, k)}$ and $h \leqslant F(x)$ for any $h \in \{i_1, .., i_k\}$, and $j \leqslant F(x)$. If $j \in \{i_1, .., i_k\}$ then $j \leqslant F^R(x)$ and we are done. Let us then assume that $j \notin \{i_1, .., i_k\}$ for any such set $\{i_1, .., i_k\} \subseteq J_L$. Hence in particular $i_h \nleqslant x_j$ for any $i_h \in \{i_1, .., i_k\}$ (otherwise, one might consider $\{i_1, .., i_h, j\} \subseteq J_L$, which would violate our previous assumption). Next, take $x' \in L^{J_L}$ such that [$i \leqslant x'_l$ iff $l \leqslant x_i$ for any $i, l \in J_L$]. Therefore, $j \nleqslant x'_{i_h}$ for any such $\{i_1, .., i_k\} \subseteq J_L$ and any $i_h \in \{i_1, .., i_k\}$. By IQC it follows that $j \nleqslant F(x')$, while CSD entails $j \leqslant F(x')$, a contradiction. Thus, $j \in \{i_1, .., i_k\}$ whence $j \leqslant F^R(x)$. ∎

Proposition 8 *F^{PR} satisfies IQC, CSD and PC. Moreover, for any CIP $F : L^{J_L} \to L$, if F satisfies IQC, CSD and PC then $F \leqq F^{PR}$.*

Proof. Let us consider $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F^{PR}(x)$. Then there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\{i_1, .., i_k\} \subseteq J_L$ such that $\#\{i_1, .., i_k\} = k$, $j = i_h \in \{i_1, .., i_k\}$, $j \leqslant x_{i_h} \wedge x_{i_{h+1(\mathrm{mod}\, k)}}$ and $i_{h+1(\mathrm{mod}\, k)} \leqslant x_{i_{h+2(\mathrm{mod}\, k)}}$. Clearly, by definition, $i_h \leqslant F^{PR}(x)$ for any $h \in \{1, .., k\}$. Since $k \geq 2$ it must be the case that $j \neq i_{h+1(\mathrm{mod}\, k)} \neq i_{h+2(\mathrm{mod}\, k)}$ hence by definition again, F^{PR} satisfies IQC.

Moreover, let $x, x' \in L^{J_L}$ be such that [$j \leqslant x_i$ iff $i \leqslant x'_j$ for any $i, j \in J_L$]. Next, take any $j \in J_L$ such that $j \leqslant F^{PR}(x)$. Then, by definition, there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\{i_1, .., i_k\} \subseteq J_L$ such that $\#\{i_1, .., i_k\} = k$, $j \in \{i_1, .., i_k\}$ and $h \leqslant x_h \wedge x_{h+1(\mathrm{mod}\, k)}$ for any $h \in \{i_1, .., i_k\}$. But then,

$h+1(\mathrm{mod}\, k) \leqslant x'_{h+1} \wedge x'_h$ for any $h+1(\mathrm{mod}\, k) \in \{i_1, .., i_k\} = \{i'_1, .., i'_k\}$ where $i'_h = i_{k-h+1(\mathrm{mod}\, k)}$, $h = 1, .., k$. Therefore, equivalently, $h \leqslant x'_h \wedge x'_{h+1(\mathrm{mod}\, k)}$ for any $h \in \{i'_1, .., i'_k\}$. It follows that, by definition, $j \leqslant F^{PR}(x')$ whence $F^{PR}(x) \leqslant F^{PR}(x')$. Since $(x')' = x$, by a similar argument $F^{PR}(x') \leqslant F^{PR}(x)$. Thus, $F^{PR}(x) = F^{PR}(x')$ i.e. F^{PR} also satisfies CSD.

Finally, F^{PR} clearly satisfies PC, by definition.

Now, let F be a CIP that satisfies IQC, CSD and PC.

For any $x \in L^{J_L}$ and $j \in J_L$ such that $j \leqslant F(x)$, there exists $i_1 \in J_L$, $i_1 \neq j$ such that $i_1 \leqslant F(x)$ and $j \leqslant x_{i_1}$, by IQC. Moreover, $j \leqslant x_j$ by PC. But then, by IQC and PC again, there exists $i_2 \in J_L$, $i_2 \neq i_1$ such that $i_2 \leqslant F(x)$, $i_2 \leqslant x_{i_2}$ and $i_1 \leqslant x_{i_2}$. By repeating the argument, and in view of finiteness of J_L, we may conclude that there exist $k \in \mathbb{Z}$, $k \geq 2$ and $\#\{i_1, .., i_k\} = k$, $\{i_1, .., i_k\} \subseteq J_L$ such that $h \leqslant x_h \wedge x_{h+1(\mathrm{mod}\, k)}$ for any $h \in \{i_1, .., i_k\}$, and $j \leqslant F(x)$. But then, take $x' \in L^{J_L}$ such that $[j \leqslant x_i$ iff $i \leqslant x'_j$ for any $i, j \in J_L]$. Therefore, by CSD, $j \leqslant F(x')$ which violates IQC. Thus, $j \in \{i_1, .., i_k\}$ whence $j \leqslant F^{PR}(x)$.

If $j \in \{i_1, .., i_k\}$ then $j \leqslant F^{PR}(x)$ and we are done. Let us then assume that $j \notin \{i_1, .., i_k\}$ for any such set $\{i_1, .., i_k\} \subseteq J_L$. Hence in particular $i_h \nleqslant x_j$ for any $i_h \in \{i_1, .., i_k\}$ (otherwise, one might consider $\{i_1, .., i_h, j\} \subseteq J_L$, which would violate our previous assumption). Next, take $x' \in L^{J_L}$ such that $[i \leqslant x'_l$ iff $l \leqslant x_i$ for any $i, l \in J_L]$. Therefore, $j \nleqslant x'_{i_h}$ for any such $\{i_1, .., i_k\} \subseteq J_L$ and any $i_h \in \{i_1, .., i_k\}$. By IQC it follows that $j \nleqslant F(x')$, while $j \leqslant F(x)$ and CSD entail $j \leqslant F(x')$, a contradiction. Thus, $j \in \{i_1, .., i_k\}$ whence $j \leqslant F^{PR}(x)$. ■

2.2 Axiom Independence

It is easily checked that F^{PE}satisfies IQC and PC but violates CSD, while F^{R} satisfies IQC and CSD but violates PC. Next, consider the following well-known identification rule

Definition 9 ***Libertarian (L^*)*** *procedure: for any $x \in L^{J_L}$,*
$F^{L^*}(x) = \bigvee \{j \in J_L : \ j \leqslant x_j \ \}$[8]

[8]Hence, the libertarian rule identifies as members precisely those agents who declare themselves to qualify as members.

It is straightforward to check that F^{L^*} satisfies PC[9] and CSD but fails to satisfy IQC. Thus, IQC, PC and CSD are mutually independent axioms.

Of course, F^{L^*} also satisfies the arrowian independence condition, namely

Independence (IND): For any $x, x' \in L^{J_L}$ and any $j \in J_L$ if [for all $i \in J_L$: $j \leqslant x_i$ iff $j \leqslant x'_j$] then [$j \leqslant F(x)$ iff $j \leqslant F(x')$].

Another well-known identification rule is the following

Definition 10 ***Unanimous Consent (UC)*** *procedure: for any* $x \in L^{J_L}$,
$F^{L^*}(x) = \bigvee \{j \in J_L : \ j \leqslant x_i \ \ \textit{for all } i \in J_L \ \}$[10]

Now, UC also satisfies IND (and therefore violates IQC): moreover, it satisfies PC but also violates CSD (to check the last claim, just consider the case of a profile that unanimously declares exactly one agent as the only qualified one).

As mentioned in the Introduction some CIPs which do not satisfy IND have also been introduced and studied in the relevant literature (see e.g. Kasher and Rubinstein (1996), Dimitrov, Sung and Xu (2003))[11], namely

Definition 11 *The* ***Liberal Multi-level Qualified Nomination (LMQN)*** *procedure: for any* $x \in L^{J_L}$

$$F^{LM}(x) = \bigvee \left\{ \begin{array}{c} j \in J_L : \textit{there exist } k \in \mathbb{Z},\, k \geq 1, \textit{ and } \{i_1, .., i_k\} \subseteq J_L \\ \textit{such that} \\ i_1 \leqslant x_{i_1} \\ h \leqslant x_{h+1(\mathrm{mod}\, k)} \textit{ for any } h \in \{i_1, .., i_k\} \ \textit{ and } j \leqslant i_k \end{array} \right\}$$

Thus, LMQN may be regarded as a two-stage procedure which identifies as members both the agents who declare themselves to qualify as members *and* their nominees.

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[9] Indeed, $F \leqq F^{L^*}$ for any CIP $F : L^{J_L} \to L$ that satisfies PC.

[10] Hence, the unanimous consent procedure identifies as members precisely those agents that are unanimously declared to qualify for membership. That is far from being an irrelevant, far-fetched procedure: indeed, it may be regarded as a convenient model of those voluntary affiliations where qualifications consist of essentially verifiable information (e.g. professional and related associations).

[11] To be sure, those authors use ‘Independence’ as a label for a much weaker conditional version of the arrowian-like Independence defined above. Indeed, such a Kasher-Rubinstein weakened independence is satisfied by the UCQN and LMQN procedures defined below as well as by the IQC-consistent procedures previously considered in the text.

Definition 12 *The* ***Consensual Multi-level Qualified Nomination (CMQN)*** *procedure is defined as follows: for any $x \in L^{J_L}$*

$$F^{CM}(x) = \bigvee \left\{ \begin{array}{c} j \in J_L : \textit{there exist } k \in \mathbb{Z}, k \geq 1, \textit{ and } \{i_1, .., i_k\} \subseteq J_L \\ \textit{such that} \\ i_1 \leqslant x_i \textit{ for each } i \in J_L \\ h \leqslant x_{h+1(\operatorname{mod} k)} \textit{ for any } h \in \{i_1, .., i_k\} \textit{ and } j \leqslant i_k \end{array} \right\}$$

Clearly, CMQN may also be regarded as a two-stage procedure which identifies as members precisely all the agents who are *unanimously* declared to qualify *and* their nominees.

It is worth considering, for the sake of comparisons, the behavior of the foregoing procedures in terms of the axioms considered in the present work. Indeed, it is easily checked that F^{LM} fails to satisfy IQC, PC or CSD. In contrast, it can be easily shown that F^{CM} does satisfy IQC but violates both PC and CSD. One might easily devise a 'participatory' version of F^{CM} (to be defined in the obvious way) which would satisfy IQC and PC while violating CSD.

3 Concluding Remarks

The main point of the present work is to show by example that there is a rich variety of collective identification procedures worth considering that do not satisfy the classic arrowian independence condition. In particular, I have focussed on the Independent Qualified Certification requirement, but other possibilities might be considered. The procedures discussed in the current paper are probably best regarded as stylized 'ideal' paradigms, mostly useful as reference models for classificatory purposes. However, this is not to say they are unrelated to 'real' collective identification procedures. On the contrary, it seems to me that virtually all the procedures considered in the former sections may claim a rather close similarity to some classes of historically relevant examples. For instance, while affiliations to most political parties in contemporary democracies essentially rely on libertarian procedures such as L^*, admissions to some of their former counterparts operating under nazi-fascist regimes relied on versions of the PEQN procedure as introduced above in Section 2.1. Moreover, one might perhaps claim that 16th century's vicious conflict between Catholics and mainstream Protestant denominations on one side and Anabaptists on the other concerning the validity of early (i.e. infant) baptism is at least to some extent captured by the contrast between

EQN and PEQN. On a more frivolous tone, the same PEQN (as opposed to, say, L^*) is arguably a rather good stylized version of the typical admission procedures used by the best tennis clubs.

Be it as it may, this is certainly not the place to dwell on a serious discussion of those putative historical or common life examples. Rather, a few specific comments on the IQC-consistent procedures introduced and studied in the present paper are in order here.

First, it should be clear at this point that the IQC principle entails the existence of some 'virtuous circles' of mutually sustaining certifications, which may be regarded as a social analogue of certain autocatalic chemical reactions as nicely epitomized by Eigen's well-known 'hypercycles' (see e.g. Hofbauer and Sigmund (1988)). In particular, such 'virtuous circles' may be particularly appropriate as an idealized model of the 'constitutional' phase of an association.

It should also be remarked that CIPs that satisfy IQC may well be consistent with the reality of 'contested identities' namely with the existence of several (possibly disjoint) subcommunities claiming the same identity. This can be prevented by introducing appropriate supplementary axioms.

Finally, it should be noticed that CIPs are nothing but strategic game forms of a highly specialized sort. Therefore, their structural properties are amenable to further analysis through the study of their concept lattices along the lines of Vannucci (1999).

Those topics, however, are both best left as the subjects of some further research.

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