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QUADERNI DELL'ISTITUTO DI ECONOMIA

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SOME NOTES ON THE SECOND VERSION OF KALECKI'S BUSINESS CYCLE THEORY

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1. Introduction

The study of the forces determining investment process has been one of the "leitmotifs" of Kalecki's work. There we can find a continuing attempt to improve the analysis of investment decisions "the centrale 'pièce de résistence' of economics" (Kalecki, M., 1968, p. 263).

The purpose of these Notes is to discuss some problems related to Kalecki's approach to the analysis of investment decisions. In doing this, particular attention will be concentrated on the second version of Kalecki's trade cycle theory which, being the most developed, allows one to analyse and to understand the basic relation between investment decisions and investment realization better than is the case with the other versions.

Many arguments could be advanced as to why these problems merit further examination.

Firstly, recent developments in Macrodynamics -e.g., Malinvaud in "Profitability and Unemployment" (1980) and Lucas, particularly in "An Equilibrium Model of the Business Cycle" (1975)- have called forth fresh looks at the investment equation in such models. In the light of this, it may be worthwhile to look at a classical macroeconomist and his struggle with the same problem, i.e. with the problem of incorporating the investment equation in a macrodynamic model.

Secondly, given the difference between successive versions of Kalecki's trade cycle theory so well stated by Steindl (1981), it may be useful, in order to understand better this difference, to attempt to perform a mathematical analysis of the *difference equation* at the base of the second version of Kalecki's trade cycle theory along the lines of what has been done by Frisch and Holme for the *differential-difference equation* at the base of the first version.

Finally, attention has usually been concentrated on the first, and "most famous", version of Kalecki's theory. However, it seems worth turning attention to the second version where Kalecki devotes two whole Chapters to the analysis of the determination of investment decisions and in consequence the problem of a certain "micro-macro" contrast between the analysis of the determination of investment decisions on the one hand and the rest of the model on the other is more evident.

So that we can develop these points a little further, the next Section is devoted to a summary of Steindl's discussion, while in the Matematical Appendix the difference equation at the base of the second version of Kalecki's theory is analysed with regard to both stability and cyclical properties. Then, in the last two Sections and in the Statistical Appendix attention is concentrated on an analysis of (1) the second version of the investment decisions equation and of (2) the estimation procedure utilized by Kalecki for the latter.

2. Two different versions of Kalecki's theory of the trade cycle

The difference between successive versions of Kalecki's trade cycle theory has been shown by Steindl in an article entitled "Some Comments on the Three Versions of Kalecki's Theory of the Trade Cycle" (1981).

Given that our purpose is to concentrate attention on the second version of Kalecki's theory, it is sufficient here to summarize Steindl's discussion with regard to the first two versions of the theory. Indeed, the first version being the "most famous" version of Kalecki's theory, it is important to compare the second version with it; moreover, although the third version presents further new elements in the analysis of investment decisions, the resulting equation of the dynamics of investment is a difference equation not fundamentally different from the one of the second version.

At first sight, the first two versions of Kalecki's business cycle theory (1935, a, b; 1954) seem very similar. As shown by Steindl, however, a deeper analysis allows one to individualize crucial differences.

Indeed, we have:

(1) *First Version* (1935)

In "A Macrodynamic Theory of Business Cycles", whenever an investment is made *three stages* are discerned by Kalecki:

(a) investment orders or decisions (D):

$$D_t/K_t = f(P_t/K_t) ,$$

with $f' > 0$, where K is the stock of fixed capital and P, the total profits;

(b) production of investment goods (A):

$$A_t = (1/\tau)\int_{t-\tau}^{t} D_t dt,$$

where τ is the gestation period of any investment project so that $\int_{t-\tau}^{t} D_t dt$ is the amount of unfilled orders at time t;

(c) deliveries (L):

$$L_t = D_{t-\tau} ;$$

(II) *Second Version* (1954)

In "Theory of Economic Dynamics", whenever an investment is made, only *two stages* are discerned by Kalecki:

(a') investment decisions (D):

$$D_t = aS_t + b(\Delta P_t/\Delta t) - c(\Delta K_t/\Delta t) + d, \quad a,b,c > 0 ,$$

where S is the total saving; $\Delta P/\Delta t$ and $\Delta K/\Delta t$, the change per unit of time in profits and the stock of fixed capital respectevely; and d, a constant;

(b') investment realization (I):

$$I_t = D_{t-\tau} .$$

Apart from the equations describing the investment process, however, the other equations are the same in both versions of the model. Indeed, considering the simplified case in which it is assumed that the economy is closed with no role for government, workes do not save and inventories are stable all through the cycle, we can write the following definitional relationships:

$$Y_t = C_t + I_t ,$$

$$S_{kt} = S_t ,$$

$$P_t = C_{kt} + I_t ,$$

where Y is the nationale income; C, the total consumption; S_k, capitalists' savings; and C_k, capitalists' consumption.

Moreover:

$$C_{kt} = qP_{t-\lambda} + C_c ,$$

with

$$0 < q < 1 \quad ; \quad C_c > 0 \quad , \quad \lambda \geq 0 .$$

In the first version, the time-lag λ between profits and capitalists' consumption is taken equal to zero. Moreover, given the assumptions, investment coincides with the production of investment goods so that we have:

$$P_t = (C_c + A_t) / (1 - q).$$

Thus:

$$\text{(a)} \quad D_t/K_t = f(P_t/K_t) =$$

$$= f\left[(1/K_t)(C_c+A_t)/(1-q) \right] =$$

$$= g\left[(C_c+A_t)/K_t \right] ,$$

from which, taking a linear approximation,

$$D_t/K_t = m\left[(C_c+A_t)/K_t \right] - n ,$$

we obtain the investment decisions equation:

$$D_t = m(C_c+A_t) - nK_t ,$$

where m and n are both positive constants(1).

Differentiating with respect to time, we obtain:

$$\dot{D}_t = m\dot{A}_t - n\dot{K}_t ,$$

where $\dot{x} = dx/dt$.

In both versions, it is assumed that the total depreciation is constant and equal to U so that, for the first version, we can write:

$$\dot{K}_t = L_t - U = D_{t-\tau} - U .$$

Then, inserting in the investment decisions equation this expression for K and the one for

$$\dot{A} = (1/\tau)(D_t - D_{t-\tau}) ,$$

we obtain the business cycle formula analysed by Kalecki in the 1935 article:

$$(1) \qquad \dot{D}_t = (m/\tau)D_t - [(m/\tau)+n]\, D_{t-\tau} + nU$$

In the second version, where the time-lag between profits and capitalists' consumption is assumed to be positive, we have:

$$P_t = qP_{t-\lambda} + C_c + I_t \,.$$

Thus:

$$P_t = q(C_{kt-\lambda} + I_{t-\lambda}) + C_c + I_t =$$

$$= q(qP_{t-2\lambda} + C_c) + qI_{t-\lambda} + C_c + I_t =$$

$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots =$$

$$= I_t + qI_{t-\lambda} + q^2 I_{t-2\lambda} + \ldots + C_c + qC_c + \ldots$$

The series of coefficients $1, q, q^2, \ldots,$ is quickly decreasing so that after a time the influence of investment on profits becomes negligible. For this reason, it is possibile, as an approximation, to say that profits follow investment with a time-lag and to write:

$$P_t = f(I_{t-w}) \,,$$

where w is the time lag involved. As a consequence, profits are determined fully by lagged investment by the following relation[(2)]:

$$P_t = (C_c + I_{t-w}) / (1 - q) \,.$$

Moreover:

$$\Delta K_t / \Delta t = I_t - U \,,$$

so that we can write the equation of the dynamics of investment analysed by Kalecki in the 1954 book:

$$(2) \qquad I_{t+\tau} = (a-c)I_t + [b/(1-q)]\,(\Delta I_{t-w}/\Delta t) +$$

$$+ cU + d \,.$$

Writing now equation (1) in terms of deliveries of investment goods and taking, both in eq. (1) and in eq. (2), deviations from equilibrium levels, we obtain:

$$(3) \qquad \dot{L}_t = (m/\tau)L_t - [(m/\tau) + n]\, L_{t-\tau} \,,$$

$$(4) \qquad I_{t+\tau} = (a-c)I_t + [b/(1-q)]\,(\Delta I_{t-w}/\Delta t)$$

Usually, the change in notation made by Kalecki from d/dt to $\Delta/\Delta t$*, has been considered as a change of little importance*, and, in consequence of this consideration, the equation (4) has been studied with the term $\Delta I/\Delta t$ replaced by $\dot{I}$, i.e., for the case in which $\Delta t \rightarrow 0$[3]. As underlined by Steindl (1981, pp. 128-130), it is however important to understand that even in this case a crucial difference between the two equations is still present. Indeed, even if we consider the case in which, in (4), the time-lag between capitalists' consumption and profits is equal to zero and in which $\Delta t \rightarrow 0$, we can rewrite the two equations as:

$$(5) \qquad L_{t-\tau} = AL_t + B\dot{L}_t \quad ,$$

$$(6) \qquad I_{t+\tau} = A'I_t + B'\dot{I}_t \quad ,$$

where

$$A = (m/\tau)\,/\,(n+m/\tau),$$

$$B = -1/(n+m/\tau),$$

$$A' = (a-c) \quad ,$$

$$B' = b/(1-q) \quad .$$

Although in this case both equations are of mixed difference-differential type, a crucial difference is self-evident (Steindl, J., 1981, p. 129): in (5) there is a backward argument wheras in (6) a forward one.

This change in the time-lag structure has surely important consequences, for example, with regard to the stability properties of the two models. Thus, following Steindl[4], it seems more correct not to make any direct analogy between the two versions of the business cycle equation, to maintain the difference operator which appears in equation (4), and to try to understand what the economic rationale is for the dissimilarity of the two equations.

As we have seen, the only aspect under which the two versions of the model differ is the analysis of the determination of investment decisions. Thus, it is necessary to look there, i.e. in the two investment decisions equations (a) and (a'), for reasons that can explain the crucial dissimilarity between the two business cycle equations.

On the basis of the analysis developed by Steindl, it is not difficult to understand that behind the two equations (a) and (a') there are two different, alternative approaches to the analysis of the determination of investment decisions. Indeed, whereas in (a') the investment decisions depend only on recent changes of the explanatory variables, in (a) investment decisions are assumed to depend on all investment decisions undertaken in a period of the past of lenght equal to the gestation period. Indeed, we can write:

$$D_t = mC_c + (m/\tau)\int_{t-\tau}^{t} D_t\, dt - nK_t \; .$$

The integral on the right-hand side of the equation accounts for the presence of the differential operator in the first version of the business cycle equation whereas no reasons can be found for justifying the presence of this operator in the second version of the equation. On the contrary, it does not seem to be satisfactory at all to replace the finite difference operator with a differential in this equation because this would mean, for example, that changes in profits from one day to the next influence investment decisions.

An important implication of this conclusion drawn by Steindl is that, in order to study the cyclical and stability properties of the second version of Kalecki's business cycle model, it is not correct to apply Frisch-Holme's analysis to the difference-differential equation with the forward argument, but it is necessary to "readapt" that type of analysis to the case of a difference equation.

In view of this, an indication of how such a readaptation might be possible is given in the Mathematical Appendix.

However, given the crucial role played by the investment decisions equation in Kalecki's trade cycle theory, it seems more important now to

turn our attention to this equation rather than to the equation of the dynamics of investment.

3. *The investment decisions equation (1954)*

Kalecki's aim, in the successive refinements of his theory, has been that of bringing the model closer to reality and, as appears evident from equations (a) and (a'), the result of this effort is reflected in a more realistic analysis of the determination of investment decisions: the profit rate as the unique determinant of investment decisions is replaced in (a') by more factors which, using Steindl's terminology, represent the influence on investment decisions both of "available financial resources" and "marketing prospects"(5).

To understand in which way these two sets of determinants are represented in the investment decisions equation

$$D = aS + b\Delta P/\Delta t - c\Delta K/\Delta t + d \;,$$

it is useful to follow Kalecki's line of argument which runs as follows. Suppose that at the beginning of the period firms have pushed their investment decisions up to a point where they cease to be profitable either because of limited markets for their products or because of "increasing risk" and limitation of the capital market. In this situation, new investment decisions will be made only if there occur changes in the economic situation which extend the boundaries set to investment plans by these factors. In this way, the problem of explaining the determination of investment decisions becomes that of individualizing the changes which can serve this purpose.

In Kalecki's opinion, three categories of such changes:

i) savings out of profits,

ii) change in profits,

iii) change in the capital stock,

can serve this purpose so that the presence of the three terms in the right-hand side of the investment decisions equation is easily explained.

In order to motivate his choice, Kalecki develops all his argument with reference to a single firm. In his opinion, changes in categories ii) and iii) extend the boundaries set to investment by the limitation of the market for the firm's product because they represent an increase in sales and, with a negative influence, an increase in competition. On the other hand, savings out of profits extend the boundaries set to investment plans by the limitation of the capital market and "increasing risk" because, firstly, the amount of the entrepreneurial capital held by the firm determines to a large extent its possibility of gaining access to the capital market, and, secondly, reduces the "risk" involved in investing with borrowed capital.

Once the investment decisions are determined on the basis of these three factors, they are followed by investment with a time-lag equal to

τ. Thus, the investment in fixed capital equation can be written:

$$(b') \quad I_{t+\tau} = aS_t + b(\Delta P_t/\Delta t) - c(\Delta K_t/\Delta t) + d \ ,$$

where I is the investment in fixed capital.

Kalecki's purpose, in "Theory of Economic Dynamics", is not only to construct a cycle theory but also to show that the theory explains the "known facts"(6). For this reason, Kalecki never limits his attention to a theoretical analysis of the equations of the model, but the theoretical analysis is always supported by an empirical application. Although the purpose of the statistical analysis, as underlined by Kalecki in the "Foreword", is only "illustrative", the empirical evidence obtained from the estimation of the equations is always interpreted ad reinforcing the theory. Thus, we also cannot limit the attention to Kalecki's theoretical analysis of the determination of investment decisions but it is crucial to analyse also Kalecki's estimation procedure which, in particular for the investment equation, presents rather peculiar aspects and allows one to focus on a problem that does not appear evident from a theoretical analysis of the equation.

4. Kalecki's estimation procedure

As we have seen, the investment equation to be estimated is of the form:

$$I_t = aS_{t-\tau} + b(\Delta P_{t-\tau}/\Delta t) - c(\Delta K_{t-\tau}/\Delta t) + d,$$

i.e.:

$$(b')\ I_t = aS_{t-\tau} + b(\Delta P_{t-\tau}/\Delta t) - cI_{t-\tau} + d' ,$$

where $d' = d+cU$, and where the coefficients a, b, and c are all positive.

Although this is the investment equation which issues from Kalecki's theoretical analysis and which has been briefly analysed in the previous Section, it is not the equation utilized by Kalecki for empirical application. In fact, before estimating the equation, Kalecki prefers "to alter it somewhat" and all the empirical analysis is based on the "altered" equation.

To obtain the "altered" equation, we write (b') as:

$$I_{t+\tau} + cI_t = aS_t + b(\Delta P_t/\Delta t) + d' ,$$

or

$$(I_{t+\tau} + cI_t)/(1+c) = [a/(1+c)]\ S_t + [b/(1+c)]\ (\Delta P_t/\Delta t) + d'' ,$$

where

$$d'' = d'/(1+c) .$$

The left-hand side of the equation we have obtained is a weighted average of investment in fixed capital at time t and $t+\tau$ which, in Kalecki's opinion, can be assumed to be equal to an intermediate value $I_{t+\theta}$, where

θ is a time-lag such that:

$$0 < \theta \leq \tau^{(7)} ,$$

with $\theta = \tau$ for $c = 0$.

Thus, we obtain:

$$I_{t+\tau} = [a/(1+c)]\ S_t + [b/(1+c)]\ (\Delta P_t/\Delta t) + d'' ,$$

which is the *"altered equation"* estimated by Kalecki.

To analyse Kalecki's estimation procedure, we consider the case in which the time-lag θ is equal to one, i.e., the case in which the equation to be estimated is:

$$(b'')\quad I_t = [a/(1+c)]\ S_{t-1} + [b/(1+c)]\ (\Delta P_{t-1}/\Delta t) + d'' .$$

Applying equation (b'') to the same American data for the period 1929-40 used by Kalecki, and which are given by him in the Statistical Appendix, by means of ordinary least squares, we obtain the following regression equation (Statistical Appendix):

$$I_t = \underset{(5.43)}{0.590}\ S_{t-1} + \underset{(2.67)}{0.277}\ \Delta P_{t-1} + \underset{(2.01)}{2.146} ,$$

where all coefficients are significant and more than eighty percent of the variance of investment is accounted for by the use of total savings and change in profits as explanatory variables.

It is important to notice that the evidence given by this regression equation supports Kalecki's investment theory. Indeed, firstly, all explanatory variables prove to have a significant influence on investment; secondly, the estimates of both coefficients are positive and that of a/(1+c) is less than one[8]; and, thirdly, the estimated coefficients, when inserted in the business cycle equation, are such as to give rise to a damped cyclical solution.

However, the problem is now to understand the usefulness of the alteration of the equation made by Kalecki; usefulness that, at first sight, appears to be doubtful. Indeed, estimating the "altered" equation, we cannot identify the coefficients a, b, and c of the original equation, and, therefore, we cannot get an idea of the relative importance of the factors which, according to Kalecki's theory, determine investment behavior.

To this end, it is useful to estimate, making use of the same data, the equation in its original form.

The result of this estimation (Statistical Appendix) is a regression equation where the coefficients of total savings and of the lagged investment in fixed capital are not significant at all, i.e., the empirical evidence given by the estimation of the investment equation in its original form gives rise to important problems for Kalecki's investment theory. As none of these problems arise if we estimate the "altered" equation, at first sight the role

of Kalecki's estimation procedure might seem to be, to a large extent, that of ensuring support for his brilliant theoretical intuition rather than of submitting the theory to an empirical investigation. In this regard, therefore, it would be fair to compare Kalecki's attitude with respect to econometrics with that of Keynes as the latter is revealed, for example, by Keynes reactions to Colin Clark's findings[9].

However, even if Kalecki does not explain the reasons that induced him to perform the alteration, it is not difficult to distinguish the most important problem which makes it difficult to employ the original equation for an empirical application.

As we have seen, on the basis of the analysis of the behavior of a single firm, three factors are singled out by Kalecki as determining investment decisions: savings out of profits, change in profits, and change in the capital stock held by the firm. The latter, given the assumption of a constant depreciation, independent of the level of the capital stock, makes the investment in fixed capital of the firm depend on past investment.

However, the model being a macro-model, the investment equation is written by Kalecki in aggregate terms with total savings, change in total profits, and total investment in fixed capital as explanatory variables.

The total savings series used for the estimation of the equation is obtained from the national accounting identities as the sum of private investment, export surplus, and budget deficit[10]. As a consequence, two of the explanatory variables -total savings and investment in fixed capital-are highly collinear and this suffices to account for the two coefficients not significantly different from zero in the regression equation.

5. *Concluding remarks*

The purpose of this paper has been to analyse Kalecki's approach to the determination of investment decisions.

The investment decisions equation plays a crucial role in all versions of Kalecki's theory of the trade cycle and accounts for dissimilarities between successive versions of the business cycle equation. For example, the presence of a finite difference operator in place of a differential operator in the second version of Kalecki's equation of the business cycle is not casual. On the contrary, it is the consequence of a different, alternative approach to the analysis of the determination of investment decisions. For this reason, one conclusion of this paper is that, in order to study the cyclical and stability properties of the second version of Kalecki's model of the business cycle on Frisch-Holme lines, it is necessary to "readapt" that kind of analysis to the case of a difference equation.

However, it seems possible to draw a more important conclusion from the analysis we have developed.

Kalecki's model, in the first as in the second version, is a *macro*dynamic model where, as we have seen, in order to obtain the business cycle equation, accounting identities are employed. This is in contrast with the analysis of the determination of investment decisions which refers, explicity, to single "firms". For example, this is self-evident in Chapter 8 - "Entrepreneurial Capital and Investment"- of Kalecki's 1954 book where the author analyses the influence on investment decisions of the "accumulation of firm's capital out of profits". Thus, one possible conclusion of this paper may well be that in Kalecki's model we have analysed there is a "micro-macro" contrast between the analysis of the determination of investment decisions on the one hand and the rest of the model on the other. For example, this contrast may well account for the problems which arise in the estimation of the equation in its original form.

However, given the crucial role played by the investment decisions equation in all versions of Kalecki's *macro*dynamic theory of the business cycle, this is not an encouraging conclusion.

Footnotes

(1) See Kalecki, M., 1935a, p. 331; 1935b, pp. 291-292. In the latter article it is shown that also the coefficient n of the linearized equation must be positive.

(2) See Kalecki, M., 1954, pp. 53-55.

(3) See, for example, Allen, R.G.D., 1959, pp. 259-261.

(4) In noting this crucial dissimilarity, Steindl makes reference to an unpublished paper by Gomulka where the author shows that the difference-differential equation with the forward argument yields explosive minor cycles for all positive A', B', and τ . See Steindl, J., 1981, p. 129.

(5) According to Steindl, in the second version of Kalecki's theory, there are two separate sets of determinants of investment decisions: "... financial resources available to the firm on the one hand, and its marketing prospects on the other. Financial resources are represented by the current saving of the business. This will seek an outlet and therefore it will normally induce investment decisions ... The marketing prospects of the firms are adversely affected by the increase in capital which means more competition, and more claims for the available volume of profits" (Steindl, J., 1981, pp.

126-127).

(6) This is stressed by Goodwin in his review of Kalecki's book. See Goodwin, R.M., 1956, p. 508.

(7) In Kalecki's opinion, since "... c is likely to be a small fraction, θ is of the same order as τ" (Kalecki, M., 1954, p. 104).

(8) Also this is required by Kalecki's theory. See Kalecki, M., 1954, pp. 105-106.

(9) Keynes' attitude with regard to Colin Clark's findings is described by Patinkin, D., 1976, pp. 1102-1104. According to Patinkin, Keynes' use of data in the General Theory reveals important characteristics of his attitude with regard to econometrics. Indeed, this use "... shows, first of all, Keynes' basic concern with integrating his theoretical analysis with the data of the real world. Furthermore, it shows him as a person with strong intuitive feelings for the proper order of magnitude of the various data -indeed, so strong and so confident that he did not hesitate to pit these feelings against the systematic estimates made by specialists in the field. Not unrelatedly, it also shows him as a person who was not too meticulous in his handling of data, and who sometimes succumbed to the temptation to bend the data to fit his preconceptions" (Patinkin, D., 1976, p. 1103). Although the problem

merits further examination, it would seem possible to say that the same characteristics are revealed by Kalecki's use of data in "Theory of Economic Dynamics".

(10) For the case in which the economy is not closed and government expenditure and taxation are not negligible, we have: P(net of direct taxes) + WS(net of direct taxes) + T(direct and indirect) = GNP and I + ExS + G + C_k + C_W = GNP, where WS are total wages and salaries; T, taxes; GNP, gross national product; I, private investment; ExS, export surplus; G, government expenditure on goods and services; and C_W, workers' consumption. Subtracting from both sides of both identities taxes minus transfers (Tr), we obtain: P(net of taxes) + WS(net of taxes) + Tr = GNP - (T-Tr) and I + ExS + Bud + C_k + C_W = GNP - (T-Tr), where Bud is the budget deficit. Subtracting now from both sides the term WS (net of taxes) + Tr, we obtain: P(net of taxes) = I + ExS + Bud - S_W + C_K, where S_W is workers' saving. Finally, subtracting from both sides capitalists' consumption and adding workers' saving, we obtain: $S_W + S_K = S = I$ + ExS + Bud. See Kalecki, M., 1954, pp. 48-49.

Mathematical Appendix

To study qualitatively the difference equation:

$$I_{t+\tau} = AI_t + B(\Delta I_{t-w}/\Delta t) \quad ,$$

where $0 < A < 1$ and $B > 0$, we could assume that the two time-lags τ and w are equal.

In this case, choosing appropriately the time unit so as to have $\tau = w = 1$, we would obtain a second order linear difference equation,

$$I_{t+1} = AI_t + B\Delta I_{t-1} = (A+B)I_t - BI_{t-1} \quad ,$$

which could easily be analysed with regard to both the stability and cyclical properties:

cycle condition $(A+B)^2 - 4B < 0$

stability condition $1 - B > 0$

However, these are not reasons for assuming that the two time-lags are equal. In fact, it seems more plausible to assume that the time-lag τ, the "gestation period", is greater or equal to the time-lag w, and to analyse, as the most interesting case, the case in which $\tau > w$.

To show how this can be done, let us assume that τ is an integer multiple of w and chose the time unit so as to have w=1.

In this case, we have:

$w = 1$

$\tau = hw = h$,

$\Delta t = t-(t-1) = 1$,

where h = 1, 2, ..., so that the difference equation becomes:

$$I_{t+h} = AI_t + B\Delta I_{t-1} ,$$

with characteristic equation:

$$\varrho^h = (A+B) - B\varrho^{-1} ,$$

where $\varrho = \alpha + i\beta$ ($\beta \geq 0$).

The case of real roots of the characteristic equation can be analysed by means of *STURM's method.*

To this end, let us write the characteristic equation, with $\beta=0$, in the following way:

$$f(\alpha) = \alpha^{h+1} - (A+B)\alpha + B = 0 .$$

The use of Sturm's method (cfr. Baumol, W.J., 1970, pp. 230-246) requires, firstly, the calculation of the polynomials $f_1(\alpha)$, $f_2(\alpha)$, ..., $f_{h+1}(\alpha)$, where:

$$f_1(\alpha) = df/d\alpha ,$$

and $f_2(\alpha)$, ..., $f_{h+1}(\alpha)$, which are derived from $f(\alpha)$ and $f_1(\alpha)$ by a process of long division, are such that:

$$f(\alpha) = f_1(\alpha)g_1(\alpha) - f_2(\alpha) ,$$

....

....

$$f_{h-1}(\alpha) = f_h(\alpha)g_h(\alpha) - f_{h+1}(\alpha) .$$

Secondly, the number of real roots which lie between $\alpha = \alpha_1$ and $\alpha = \alpha_2$ is given by the difference:

$$V(\alpha_1) - V(\alpha_2) ,$$

where $\alpha_1 < \alpha_2$ and where $V(\alpha_1)$ is the number of variations in sign of the Sturm polynomials for $\alpha = \alpha_i$.

As an example, we consider the case in which $h = 2$, i.e., the case in which the gestation period is twice the time-lag w.

In this case, we have:

$$f(\alpha) = \alpha^3 - (A+B)\alpha + B = 0$$

$$f_1(\alpha) = 3\alpha^2 - (A+B),$$

from which:

$$\begin{array}{lcl|l}
\alpha^3 & - (A+B)\alpha & + B & 3\alpha^2 - (A+B) \\ \hline
 & & & (1/3)\alpha \\
-\alpha^3 & + (1/3)(A+B)\alpha & & \\ \hline
 & - (2/3)(A+B)\alpha & + B &
\end{array}$$

so that:

$$f_2(\alpha) = (2/3)(A+B)\alpha - B .$$

In the same way, dividing $f_1(\alpha)$ by $f_2(\alpha)$, we find f_3 which, being the equation of the third order, is simply a constant:

$$f_3 = (A+B) - 27B^2/4(A+B)^2 \gtrless 0.$$

Then, it is possible to determine the condition for real roots of the characteristic equation.

We have:

	$\alpha = -\infty$		$\alpha = +\infty$	
$f(\alpha)$	<0		>0	
$f_1(\alpha)$	>0		>0	
$f_2(\alpha)$	<0		>0	
f_3	<0	>0	<0	>0
$V(\alpha)$	2	3	1	0

so that all roots are real if and only if:

$$f_3 > 0 .$$

Indeed, in this case we have:

$$V(-\infty) - V(+\infty) = 3 .$$

Thus:

$(A + B)^3 - (27/4)\, B^2 > 0$ *condition for (three) real roots.*

Assuming that $\beta > 0$, we can study the case of complex roots of the characteristic equation.

In the general case ($h > 1$), from:

$$\varrho^h = (A + B) - B\varrho^{-1} ,$$

and:

$$\varrho = \alpha + i\beta = r(\cos\omega + i\sin\omega) ,$$

we obtain:

$$r^h (\cos\omega h + i\sin\omega h) = (A + B) - Br^{-1} (\cos\omega - i\sin\omega) .$$

For this equality to hold, we must have:

(a) $r^h \cos\omega h = (A + B) - Br^{-1} \cos\omega ,$

(b) $r^h \sin\omega h = Br^{-1} \sin\omega.$

From (b) we obtain:

$$r = (B \sin\omega / \sin\omega h)^{1/(1+h)} ,$$

and, inserting this expression for r in (a):

$$(B \sin\omega / \sin\omega h)^{h/(1+h)} \cos\omega h = (A + B) +$$

$$- B(B \sin\omega/\sin\omega h)^{-1/(1+h)} \cos\omega ,$$

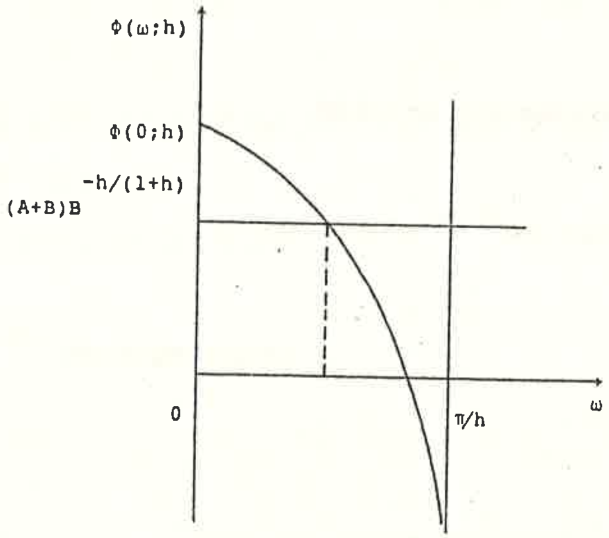
from which:

$$\Phi(\omega;h) = [\cos\omega h + (\sin\omega h/\sin\omega)\cos\omega]\, (\sin\omega/\sin\omega h)^{h/(1+h)} =$$

$$= (A + B)\, B^{-h/(1+h)} .$$

Following Frisch-Holme's procedure (1935), it is now possible to find the condition for cyclical fluctuations.

To this end, consider the graph of the function $\Phi(\omega;h)$ for $h > 1$ and $0 < \omega < \pi/h$:



Assuming that the characteristic roots are complex, we have found that the condition

$\Phi(\omega;h) = (A + B)B^{-h/(1+h)}$ must hold. With the help of the graph we have drawn, we now see that this is possible, in the range $(0, \pi/h)$, if and only if:

$$\Phi(0;h) > (A + B)B^{-h/(1+h)} \quad ;$$

thus:

$$\Phi(0;h) = (1+h)(1/h)^{h/(1+h)} > (A + B)B^{-h/(1+h)}$$ *(major) cycle condition.*

For example, when h=2, i.e. in the case we have analysed with regard to the problem of the existence of real roots, the cycle condition becomes:

$$(A + B)^3 - (27/4)B^2 < 0 \quad ,$$

i.e.:

$$f_3 < 0 \quad .$$

Thus, both conditions we have obtained imply that when the condition $f_3 < 0$ is satisfied we have one real and two complex roots of the characteristic equation.

In the case in which the cycle condition holds, the cycle is damped, of constant amplitude, or explosive, according to whether:

$$r \lesseqgtr 1 \quad ,$$

thus the *stability condition* is:

$$B < \sin \omega h \; / \; \sin \omega$$

The stability condition we have obtained makes possible to understand Kalecki's statement according to which the condition of importance for having cyclical fluctuations is $A < 1$ (see Kalecki, M., 1954, pp. 121-131).

At first sight, this condition appears to be surely not sufficient in order to have cyclical fluctuations.

However, there is a very special case in which the condition $A < 1$ is both necessary and sufficient for having cyclical fluctuations of the investment level, namely the case in which these fluctuations are neither explosive nor damped.

In this case, we have:

$$B = (\sin \omega h / \sin \omega) \ ,$$

and:

$$\Phi(\omega;h) = (\cos \omega h + B\cos \omega)\ B^{-h/(1+h)} \ ,$$

so that the cycle condition becomes:

$$\Phi(0;h) = 1 + B > A + B \ ,$$

which is satisfied if and only if $A < 1$.

Thus, although Kalecki, after the criticism by Frisch and Holme (cfr. Velupillai, K., 1984), never considered explicity again the case of fluctuations of constant amplitude, this is the only case in which, on the basis of the

assumptions made about the values of the coefficients, the second version of Kalecki's "pure business cycle equation" gives rise surely to cyclical fluctuations of the investment level.

Statistical appendix

The data used for the estimation of the equation are given by Kalecki in the Statistical Appendix where the derivation of the S and $\Delta P/\Delta t$ series in also briefly discussed.

The S series, in nominal terms, is obtained from the accounting identities as the sum of gross private investment, export surplus, and budget deficit, and, then, the values so obtained are deflated by the price index of investment goods.

The $\Delta P/\Delta t$ series, in nominal terms, is obtained using data running from mid-year to mid-year, i.e., for example the term $\Delta P_{t-1}/\Delta t$, is calculated as:

$$P_{t-1/2} - P_{t-3/2} \; .$$

To obtain these data running from mid-year to mid-year, the average of profits in two successive years could be used as a first approximation. However, to take this approximation is not satisfactory in this case because the profits series is to serve for the calculation of the change in profits per unit of time.

In this case, on the basis of the approximation, we would obtain:

$$P_{t-1/2} - P_{t-3/2} = [(P_t + P_{t-1})/2] - [(P_{t-1} + P_{t-2})/2] =$$

$$= (P_t - P_{t-2}) / 2 \; ,$$

that is, the change in profits at time t-1 would not have any relation with the level of profits at time t-1. For this reason, it is necessary, in Kalecki's opinion, to correct somehow the average of profits in two successive years and this is done by him postulating that the following relation between profits and private wages plus salaries (WS) holds:

$$(P_{t-1/2})/WS_{t-1/2} = [(1/2)(P_t + P_{t-1})] \; / \; [(1/2)(WS_t + WS_{t-1})] \; .$$

Thus, making use of monthly data on wages and salaries, it is possible to calculate a sort of "correction factor" (CF) which, applied to the average of profits of two successive years, should give a better approximation to profits running from mid-year to mid-year:

$$P_{t-1/2} = (1/2)(P_t + P_{t-1}) \, CF_{t-1/2} \; ,$$

where:

$$CF_{t-1/2} = (WS_{t-1/2})/[(1/2)(WS_t + WS_{t-1})] \; .$$

Finally, the series so obtained is deflated by the price index of investment goods.

It is now possible to estimate Kalecki's investment equation.

The result of the application of the equation to the I, S_{-1}, and $\Delta P_{-1}/\Delta t$ series for the period 1929-40 is the regression equation we have above given, where all estimates are significant, and where R2 = 0.830.

The equation in its original form, making use of the same data, can be estimated for the period 1930-40.

The result of this estimation is the following regression equation:

$$I_t = \underset{(0.87)}{0.332\, S_{t-1}} + \underset{(2.41)}{0.344\, \Delta P_{t-1}} + \underset{(0.71)}{0.341\, I_{t-1}} + \underset{(1.80)}{2.006} \quad , \quad (R2 = 0.830)$$

where, contrary to what we expected, total saving and lagged investment prove to have no influence on investment at all.

To conclude, it is important to notice the arbitrariness of the selection of the time-lag θ between investment decisions and investment realization.

Kalecki, for the selection of this time-lag, does not apply the "goodnes of fit" criterion but, firstly, assumes that θ cannot be shorter than half a year and longer than one year; secondly, renounces choosing the "right" θ within these limits; and, thirdly, produces two variants of the investment equation based on the two limit values of the interval of plausible values of θ.

We have, all through our statistical analysis of the equation, limited the attention to the case of a unitary time-lag.

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