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Pensions and Intergenerational Risk-Sharing When Relative Consumption Matters

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Abstract - Concern for relative consumption introduces an additional source of risk for future pensioners. We study its implications, in terms of optimal risk diversification, for the choice of the mix between a pay-as-you-go and a funded pension systems. We identify a necessary and sufficient condition for the optimal share of pay-as-you-go to be larger when relative consumption matters. We argue that when model parameters assume reasonable values such condition is satisfied.

Keywords: pay-as-you-go pensions, fully funded pensions, relative consumption, inter-generational risk-sharing.

JEL classification: H55.

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1. Introduction

It is well known that the benefits from a fully funded (FF) system depend on the return on financial markets, while in a pay-as-you-go (PAYG) system the relevant variable is the growth of the contribution base, which depends on productivity and labor supply growth. Therefore, in a situation of dynamic efficiency the steady state return from a FF is higher than the steady state return from a PAYG system. However, when risk enters the picture this is not enough to conclude that the optimal pension system should not include a PAYG component: when returns are stochastic and they are not perfectly correlated, it is possible to diversify risks by optimally choosing a mix of PAYG and FF. Indeed, in recent years a number of papers have emphasized the role of social security in providing intergenerational risk sharing with respect to several sources of risk, including return on financial markets, demographic and productivity shocks.[1] According to this perspective, a role for a PAYG pension system may be recognized—at least in principle—as a way to hedge risks to future pensioners' benefits when financial markets are incomplete.

Conclusions on the optimal mix of PAYG and FF have been obtained so far under the standard assumption that individuals care only for their absolute level of consumption. However, there exists now substantial evidence suggesting that people also care about their consumption relative to others' (see Frey and Stutzer, 2000, 2002; Luttmer, 2005; Clark, Frijters and Shields, 2008, and references therein). Although the economic implications of people's concerns for relative standing have been widely investigated,[2] to the best of our knowledge no one has so far inquired the consequences of such an assumption for the insurance properties of pension schemes.

In this paper we show that when model parameters assume reasonable values the relative consumption hypothesis reinforces the case for the PAYG solution. The basic intuition of our result is the following: when people care for their future relative position in terms of consumption, productivity growth affects the value of a given amount of pension because it affects the consumption of the future generation of young. This generates an additional risk that cannot be insured effectively by a FF system, while it can be coped with by an increase of the PAYG component, whose return is itself related

[1] Among others: Gordon and Varian (1988), Shiller (1999), Dutta, Kapur and Orszag (2000) Bohn (2001, 2004), Wagener (2003b), Matsen and Thøgersen (2004), Krueger and Kubler (2006), Ball and Mankiw (2007).

[2] See, for instance, Bagwell and Bernheim (1996), Clark and Oswald (1998), Cole, Mailath and Postlewaite (1992), Cooper, Garcia-Penalosa and Funck (2001), Corneo and Jeanne (1997), Frank (1985), Hopkins and Kornienko (2004), Ireland (2001).

to productivity growth.

The paper is organized as follows. In section 2 we present a simple OLG model suited for our purposes, and we identify the conditions for the optimal mix of FF and PAYG systems when individuals care only about their absolute consumption. In section 3 we modify the model to allow individuals to care also for their relative consumption, and discuss how the optimal mix between PAYG and FF is modified. We find a necessary and sufficient condition identifying the cases in which concern for relative consumption enhances the role of the PAYG, and we argue that this condition will be satisfied within the relevant parameter range. In section 4 we conclude and provide some final remarks about the scope and relevance of our findings.

2. The optimal mix between FF and PAYG when only absolute consumption matters

2.1. The model

We consider a succession of generations $t, t+1, \ldots$, each one represented by a risk averse individual living two periods (we will use the terms "individual" and "generation" interchangeably). By considering a single representative for each generation we will disregard intra-generational distribution and risk sharing. Each representative individual works when young, and lives on savings and social security when old. We indicate consumption of generation t when young (in period t) as $c_{1,t}$, and consumption when old (in period $t+1$) as $c_{2,t}$.

Let the income of individual t be w_t. Income is taxed at a rate τ_t to finance a PAYG pension system; $\tau_t w_t$ is transferred to period t pensioner (i.e. to individual $t-1$). When young, each individual saves a share s_t of his/her income. We make the assumption that first period consumption $c_{1,t}$ always comes in a fixed proportion of w_t, namely $c_{1,t} = \gamma w_t$. In other terms, s_t is chosen by each generation to satisfy

$$\gamma w_t = w_t(1 - \tau - s_t) \tag{1}$$

or $s_t = 1 - \tau_t - \gamma$. Under this assumption, the young will react to a change in τ_t by increasing or decreasing savings, and the choice of τ_t by the government will not affect γ. This assumption is justified by the fact that we are interested in how resources devoted to old age are best divided between s_t and τ_t, i.e. in the mix of savings (in a FF system) and contributions (to a PAYG system), while we will not discuss here the overall pension/saving level.[3]

[3] This is similar to, and no more restrictive than, the assumption usually made in this

We don't explicitly consider capital accumulation. Instead, we take the growth rate g_t from period t to period $t+1$ and the interest rate r_t as exogenously given; this hypothesis corresponds to the case of a small open economy, and allows us to concentrate only on the intergenerational risk-sharing effect of pensions. We assume that g_t and r_t are stochastic, that they are not perfectly correlated, and that their joint distribution is elliptically symmetric.[4] We assume that these variables are independently and identically distributed over time.

In our analysis, we focus on the effect of shocks affecting labour productivity and capital market returns; to make things simpler, we disregard demographic risk by assuming that population does not change from one period to another, and growth is entirely explained by changes in productivity.

It is possible to consider two different notions of intergenerational risk sharing. We might be interested in an ex ante perspective, i.e. we can assume the point of view of each generation before his income in the first period is known, and focus on what perhaps is better described as a problem of optimal redistribution among generations rather than of design of an optimal insurance scheme.[5] However, we are more interested in the attitude of the active population toward social security as a true insurance device. Therefore, we consider the point of view of the individual worker contributing to his/her pension plan when young: we evaluate risk when individuals already know the realization of variables in their first period of life. Namely, utility is evaluated assuming that the individual knows the realization of w_t but considers r_t and g_t – and, hence, $w_{t+1} = (1+g_t)w_t$ – as random variables. In doing so, we adopt the so called interim perspective.

The properties of a PAYG system as an insurance device depends on the nature of the contract among generation implied by the way it adjusts to respond to different shocks.[6] Hence, in one case the pension system might grant each generation of pensioners a given level of benefits, defined as a function of their earnings in the previous period, and τ_t is adjusted in each period to secure that the budget is balanced. Alternatively, the contribution rate τ can be fixed for all generation, and it is the level of benefits which

class of models that consumption by the young is zero. In our model assuming the young do not consume at all would have not allowed us to consider relative consumption level of the old and the young.

[4] The latter property will allow us to pass to a two-moment representation of preferences (Chamberlain, 1983; Eichner and Wagener, 2003). It requires that the level curves of the joint density function are all ellipses obtainable by means of an affine transformation of circumferences. Joint normality is a special case of elliptical symmetry.

[5] Matsen and Thøgersen (2004) refer to the ex ante perspective as "Rawlsian" risk sharing.

[6] For an extensive discussion of possible contracts and their effects, see e.g. Musgrave (1981).

is adjusted to satisfy the budget constraint; under such a provision, each generation transfers a share τ of its income to the previous generation, in exchange for a similar commitment by the next generation. It is clear that under the latter system (but not under the former) the effects of shocks affecting next period wage base can be shared between the next generation of workers and pensioners. Indeed, Hassler and Lindbeck (1998) show that, under the interim perspective on risk sharing (which they refer to as "true risk sharing"), it is only when τ is fixed across generations that intergenerational risk sharing is possible and the pension system can provide hedging against the risky returns of private savings. For this reason we will focus on the case of $\tau_t = \tau$, disregarding other possible intergenerational contracts.[7]

In this framework, $\tau \in [0, 1-\gamma]$ can be interpreted directly as the chosen mix of FF and PAYG.

2.2. Utility maximization and the optimal mix of FF and PAYG

For comparative purposes, we will first study the case where individuals only care about their absolute level of consumption. Later on, we will modify the model assuming that individuals care about their consumption relative to a standard of living which depends on others' consumption. This will allow us to show how the optimal FF-PAYG mix changes when relative consumption matters.

Note that, since γ is exogenously given, consumption in the first period depends only on the realization of w_t. Therefore, the optimal choice of τ can be calculated considering only the expected value of utility in the second period, which we denote as $\mathrm{E}\left[u(c_{2,t})\right]$. Second period consumption depends on the realization of both r_t and g_t and on the level of τ:

$$c_{2,t} = s_t(1+r_t)w_t + \tau(1+g_t)w_t = [(1-\gamma)(1+r_t) + \tau(g_t - r_t)]w_t. \qquad (2)$$

Under the assumption that the joint distribution of r_t and g_t is elliptically symmetric, whatever the function u, we can switch from $\mathrm{E}\left[u(c_{2,t})\right]$ to a two-moment representation of preferences (Eichner and Wagener, 2003), and express the conditions identifying the optimal choice of τ for each generation as a function of the mean, variance and covariance of g_t and r_t. More precisely, our representation will be based on the mean and the standard

[7] A possible interesting alternative from our point of view is a pension contract specifying that the ratio between the representative pension and the representative wage must be kept constant across periods. Note that this corresponds to the case of fixed τ under our hypothesis that population is stationary, while the two cases are not equivalent in general.

deviation of $c_{2,t}$:

$$\mathrm{E}\left[c_{2,t}\right] = w_t[(1-\gamma-\tau)(1+\mu_r)+\tau(1+\mu_g)] \tag{3}$$

$$\mathrm{S}\left[c_{2,t}\right] = \sqrt{\mathrm{Var}\left[c_{2,t}\right]} = w_t[(1-\gamma-\tau)^2\sigma_r^2+\tau^2\sigma_g^2+2\tau(1-\gamma-\tau)\sigma_{rg}]^{1/2} \tag{4}$$

where $\sigma_r^2 = \mathrm{Var}\left[r_t\right]$, $\sigma_g^2 = \mathrm{Var}\left[g_t\right]$ and $\sigma_{rg} = \mathrm{Cov}\left[r_t, g_t\right]$.

Preferences of individual t are now described by the differentiable function

$$V(\mathrm{E}\left[c_{2,t}\right], \mathrm{S}\left[c_{2,t}\right]) \qquad V_E > 0, V_S < 0 \tag{5}$$

where risk-aversion implies that V is concave in its arguments (Meyer, 1987). By differentiating (5) with respect to τ we have that an increase in τ increases utility whenever

$$V_E \frac{\partial \mathrm{E}\left[c_{2,t}\right]}{\partial \tau} + V_S \frac{\partial \mathrm{S}\left[c_{2,t}\right]}{\partial \tau} > 0. \tag{6}$$

The first order condition for an internal optimum, i.e. $0 < \tau^* < 1-\gamma$, can be written as

$$-\frac{V_S}{V_E} = \frac{(\mu_r-\mu_g)\left[(1-\gamma-\tau)^2\sigma_r^2+\tau^2\sigma_g^2+2\tau(1-\gamma-\tau)\sigma_{rg}\right]^{1/2}}{(1-\gamma-\tau)(\sigma_r^2-\sigma_{gr})-\tau(\sigma_g^2-\sigma_{gr})} \tag{7}$$

where the ratio $-V_S/V_E$ is a measure of the local degree of risk aversion (Ormiston and Schlee, 2001; Lajeri and Nielsen, 2000; Wagener, 2003a).

Note that, in general, the optimal τ depends on V_S/V_E, which in turn may depend on the level of w_t. Hence, the optimal τ will be different for generations facing different realizations of w_t. This problem of time inconsistency is well known in the literature (Lindbeck and Persson, 2003; Matsen and Thøgersen, 2004). In order to avoid it, we assume that preferences exhibit constant relative risk aversion.[8] Under this assumption, the ratio V_S/V_E is homogeneous of degree zero in its argument (see Meyer, 1987, Property 6), so that it does not depend on the realization of w_t. This implies that it is possible to identify a single τ^* which is optimal for all generations.

A pure FF system is optimal from the point of view of the individual when young, i.e. $\tau^* = 0$, only if

$$-\frac{\partial \mathrm{E}\left[c_{2,t}\right]}{\partial \tau} = (\mu_r-\mu_g) > -\frac{V_S}{V_E}(\sigma_r-\rho_{gr}\sigma_g) = \frac{V_S}{V_E}\frac{\partial \mathrm{S}\left[c_{2,t}\right]}{\partial \tau} \tag{8}$$

where $\rho_{gr} = \sigma_{gr}/(\sigma_g\sigma_r)$ is the correlation coefficient between r_t and g_t.

[8] This assumption, though quite strong, is consistent with the common observation that individuals usually show decreasing absolute risk aversion.

If returns on financial markets are, on average, lower than the growth of the contribution base, i.e. $\mu_g > \mu_r$, then $\tau^* > 0$ independently of other parameter values. Instead, if $\mu_g < \mu_r$, it is possible that $\tau^* = 0$. In particular, it is more likely that $\tau^* = 0$ the less individuals are risk averse, the more g_t and r_t are correlated and, provided that $\rho_{gr} > 0$, the lower is σ_r with respect to σ_g.[9]

In conclusion, when returns on financial markets are greater on average than the growth of the contribution base, the role of a PAYG system is that of providing risk diversification. This is made clear by the fact that if financial investments are riskless, i.e. $\sigma_r = 0$, condition (8) is always satisfied and we have that $\tau^* = 0$.

3. The optimal mix when relative consumption matters

3.1. Results

In this section we modify the model in order to consider the case where individuals care for relative consumption. More precisely, we assume that utility from consumption depends not only on absolute consumption but also on the current *standard of living*.

The new utility function for the second period (old age) is $u(c_{2,t}, \Sigma_{t+1})$ where Σ_{t+1} is the standard of living at time $t+1$. We assume, as it seems reasonable, that the relevant standard of living is a convex combination of the consumption of the young and the old a time $t+1$. We further assume that a positive change in the standard of living can be translated into a negative change in absolute consumption according to a constant factor $\delta > 0$; the latter variable measures the intensity of concern for the current standard of living. This assumption, together with the fact that Σ_{t+1} is a linear function of both $c_{2,t}$ and $c_{1,t+1}$, implies that we can define $\tilde{u}(c_{2,t} - \beta c_{1,t+1}) = u(c_{2,t}, \Sigma_{t+1})$, where $\beta \in (0, \delta]$ represents how much the old generation values—in terms of their own consumption—an increase in the consumption of the young generation at $t+1$. Since we have that

$$c_{2,t} - \beta c_{1,t+1} = [(\tau - \beta\gamma)(1+g) + (1-\tau-\gamma)(1+r)]w_t \tag{9}$$

we can pass to a two-moment representation of preferences over uncertain outcomes,[10] i.e.

[9] Our conclusion is consistent with what is known in the literature. See e.g. Dutta et al. (2000) and Matsen and Thøgersen (2004, sec. 3.2).

[10] Again, this is possible because the argument of the utility function is a linear combination of stochastic variables which have been assumed to be elliptically symmetric.

$$\tilde{V}(\mathrm{E}\left[c_{2,t}-\beta c_{1,t+1}\right],\mathrm{S}\left[c_{2,t}-\beta c_{1,t+1}\right]). \tag{10}$$

where $\tilde{V}$ is assumed to satisfy the same properties of V and

$$\mathrm{E}\left[c_{2,t}-\beta c_{1,t+1}\right]=w_t[(1-\gamma-\tau)(1+\mu_r)+(\tau-\beta\gamma)(1+\mu_g)] \tag{11}$$

$$\mathrm{S}\left[c_{2,t}-\beta c_{1,t+1}\right]=w_t[(1-\tau-\gamma)^2\sigma_r^2+(\tau-\beta\gamma)^2\sigma_g^2+2(\tau-\beta\gamma)(1-\tau-\gamma)\sigma_{rg}]^{1/2}. \tag{12}$$

The counterpart of the first order condition (7) is

$$-\frac{\tilde{V}_S}{\tilde{V}_E}=\frac{(\mu_r-\mu_g)\left[(1-\gamma-\tau)^2\sigma_r^2+(\tau-\beta\gamma)^2\sigma_g^2+2(\tau-\beta\gamma)(1-\gamma-\tau)\sigma_{rg}\right]^{1/2}}{(1-\gamma-\tau)(\sigma_r^2-\sigma_{gr})-(\tau-\beta\gamma)(\sigma_g^2-\sigma_{gr})}. \tag{13}$$

Note that, by letting individuals care also for Σ_t, we have redefined individuals' preferences. This may preclude a meaningful comparison between the optimal FF-PAYG mix obtained from (7) and the mix obtained under the relative consumption hypothesis. Indeed, if by changing preferences we also change the degree of risk aversion, we may not be able to establish to what extent our conclusions are driven by the relative consumption hypothesis or by such changes. In order to overcome this difficulty, it seems reasonable to normalize the new utility so that, in the situation identified by τ^*, V and $\tilde{V}$ give rise to the same attitude towards risk, i.e. $V_S/V_E=\tilde{V}_S/\tilde{V}_E$ at τ^*.[11]

Under this assumption, we can think of our approach as follows: we know that τ^* is the optimal contribution rate when we observe a certain attitude towards risk and under the assumption that the individuals do not care for relative consumption. Assume instead that, though at τ^* they have the same attitude towards risk, individuals do care for relative consumption: is τ^* underestimating or overestimating the optimal τ?

Let $\tilde{\tau}^*$ be the optimal contribution to the PAYG system when individuals do care for relative consumption. The discussion above allows us to state our main result:

Proposition 1. *Under the assumptions that (i) $\mu_r>\mu_g$ and (ii) $V_S/V_E=\tilde{V}_S/\tilde{V}_E$ at τ^*, we have that*

[11] What we are requiring is that at the state of the world defined by τ^*, the way in which the individual trades-off a small change in the mean and standard deviation of her consumption, when everything else is left unchanged, is independent of whether the individual cares about the standard of living or not.

a) $0 < \tau^* < 1-\gamma$ *implies that* $\tilde{\tau}^* > \tau^*$ *if and only if*

$$\frac{1}{\beta} > \frac{1}{2}\left(\frac{\gamma}{1-\gamma}\right)\left[1 - \frac{(1-\gamma)(\sigma_g^2 - \rho_{gr}\sigma_g\sigma_r)}{(1-\gamma-\tau)(\sigma_r^2 - \rho_{gr}\sigma_g\sigma_r) - \tau(\sigma_g^2 - \rho_{gr}\sigma_g\sigma_r)}\right]; \tag{14}$$

b) $\tau^* = 0$ *implies that* $\tilde{\tau}^* > 0$ *only if (14) holds; in particular, for any* $\beta \in (0, \delta]$*, there exists a high enough degree of risk aversion at* $\tau^* = 0$ *such that* $\tilde{\tau}^* > 0$. □

PROOF. *Part a*). From the fact that V is concave in (E, S), E is linear and S is strictly convex[12] at τ, follows that V is strictly concave in τ. The same is true of $\tilde{V}$.

To simplify notation, we write E' for $\partial \mathrm{E}\left[c_{2,t}\right]/\partial\tau$ and S' for $\partial \mathrm{S}\left[c_{2,t}\right]/\partial\tau$; similarly, let $\tilde{E}'$ be $\partial \mathrm{E}\left[c_{2,t} - \beta c_{1,t+1}\right]/\partial\tau$ and $\tilde{S}' = \partial \mathrm{S}\left[c_{2,t} - \beta c_{1,t+1}\right]/\partial\tau$.

Assume that $0 < \tau^* < 1-\gamma$. Strict concavity implies that $\tilde{\tau}^* > \tau^*$ if and only if $\tilde{V}$ is increasing in τ at τ^*. The first order conditions for a maximum imply that at τ^* we have $V_S/V_E = -E'/S'$, where $E' < 0$ because of (i). From (ii) and from the fact that $E' = \tilde{E}'$ follows that $\tilde{V}$ is increasing at τ^* if and only if

$$\tilde{S}' = \frac{-(1-\gamma-\tau)(\sigma_r^2 - \sigma_{gr}) + (\tau - \beta\gamma)(\sigma_g^2 - \sigma_{gr})}{\left[(1-\gamma-\tau)^2\sigma_r^2 + (\tau-\beta\gamma)^2\sigma_g^2 + 2(\tau-\beta\gamma)(1-\gamma-\tau)\sigma_{gr}\right]^{1/2}} < \frac{-(1-\gamma-\tau)(\sigma_r^2 - \sigma_{gr}) + \tau(\sigma_g^2 - \sigma_{gr})}{\left[(1-\gamma-\tau)^2\sigma_r^2 + \tau^2\sigma_g^2 + 2\tau(1-\gamma-\tau)\sigma_{gr}\right]^{1/2}} = S' \tag{15}$$

from which straightforward calculations lead us to

$$\beta\gamma\left[(1-\gamma-\tau)(\sigma_r^2 - \sigma_g^2) - 2\tau(\sigma_g^2 - \sigma_{gr})\right] < 2(1-\gamma)[(1-\gamma-\tau)(\sigma_r^2 - \sigma_{gr}) - \tau(\sigma_g^2 - \sigma_{gr})]. \tag{16}$$

Note that the term in squared brackets on the right hand side must be positive at τ^*, since the numerator of S' must be negative at an interior optimum. By solving for $1/\beta$ and rearranging terms we obtain condition (14).

Part b). Suppose that $\tau^* = 0$. The first order condition for the maximization of V implies that at $\tau = 0$ we have $S' \geq -(V_E E')/V_S$. From the first

[12] It can be easily checked that the second order derivative of S with respect to τ is positive—hence strict convexity is granted—provided that r_t and g_t are not perfectly correlated.

derivative of $\tilde{V}$ with respect to τ we get that, at $\tau = 0$, $\tilde{V}$ increases in τ only if $\tilde{S}' < -(\tilde{V}_E \tilde{E}')/\tilde{V}_S$. Hypotheses (i)-(ii) imply that, at $\tau = 0$, $\tilde{V}$ increases in τ only if $\tilde{S}' < S'$. As shown above, the latter condition is equivalent to (14). Finally, form strict concavity follows that if $\tilde{S}' < -(\tilde{V}_E \tilde{E}')/\tilde{V}_S$ then $\tilde{\tau}^* > 0$. Since $\tilde{E}'$ is a constant, $\tilde{S}'$ is bounded from below, while $\tilde{V}_S/\tilde{V}_E$ is both negative and unbounded from below, we conclude that for every $\beta \in (0, \delta]$ there exists a degree of risk aversion at $\tau = 0$ such that $\tilde{S}' < -(\tilde{V}_E \tilde{E}')/\tilde{V}_S$. □

Figure 1 shows the optimality conditions in the (S, E)-space when $0 < \tau^* < (1 - \gamma)$ and condition (14) is satisfied. Suppose first that only absolute consumption matters. If $\mu_R > \mu_g$, the locus of feasible combinations of standard errors and expected values determined by τ is given by the curve FF (where E decreases in τ). The slope of the curve FF is equal to the ratio between the derivatives of E and S with respect to τ, and the shape of the curve reflects the fact that E is linear in τ while S is convex in τ. Concavity of V implies that indifference curves are convex. The condition that identifies τ^* is the tangency between the FF curve and the highest feasible indifference curve.

Suppose now that relative consumption matters. Since the attitude towards risk is unchanged at τ^*, also the slope of the indifference curve is unchanged at that point. Hence, a necessary and sufficient condition for $\tilde{\tau^*} > \tau^*$ is that the slope of the new curve $F'F'$ is lower at τ^*, and this is what condition (14) establishes.[13]

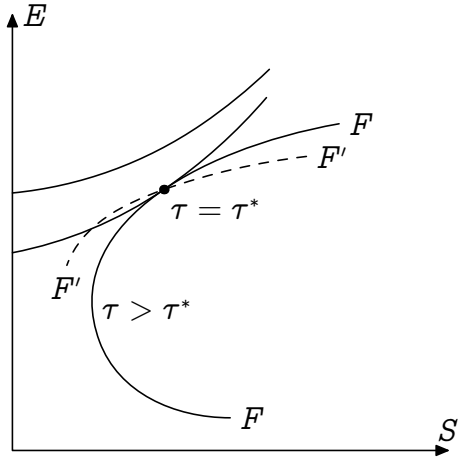

Figure 1

[13] Note that, in order to make the graph of $F'F'$ comparable with that of FF, we have rescaled both horizontal and vertical axis in such a way that $E[c_{2,t}] = E[c_{2,t} - \beta c_{1,t+1}]$ and $S[c_{2,t}] = S[c_{2,t} - \beta c_{1,t+1}]$.

3.2. Discussion

Proposition 1 states that, under a broad range of circumstances, concern for relative consumption enhances the role for a PAYG system. However, it also states that under some circumstances the role for a PAYG system is diminished when individual care for relative consumption.

In order to analyze the point, it is useful to decompose the effect of a marginal increase in τ on $\mathrm{S}\left[c_{2,t} - \beta c_{1,t+1}\right]$, evaluated at τ^*, into two distinct effects: the first due to a reduction in $\mathrm{S}\left[c_{2,t}\right]$; the second due to an increase in the correlation between $c_{2,t}$ and $c_{1,t+1}$. We can rewrite condition (15) as follows:

$$-\frac{\partial \mathrm{S}\left[c_{2,t} - \beta c_{1,t+1}\right]}{\partial \tau} = -\frac{\partial \mathrm{S}\left[c_{2,t}\right]}{\partial \tau} \cdot \frac{\mathrm{S}\left[c_{2,t}\right] - \rho_{12}\beta \mathrm{S}\left[c_{1,t+1}\right]}{\mathrm{S}\left[c_{2,t} - \beta c_{1,t+1}\right]} + \\ + \frac{\partial \rho_{12}}{\partial \tau} \cdot \frac{\beta \mathrm{S}\left[c_{2,t}\right] \mathrm{S}\left[c_{1,t+1}\right]}{\mathrm{S}\left[c_{2,t} - \beta c_{1,t+1}\right]} > -\frac{\partial \mathrm{S}\left[c_{2,t}\right]}{\partial \tau} \quad (17)$$

where ρ_{12} is the correlation coefficient between $c_{2,t}$ and $c_{1,t+1}$.

The intuition behind the claim that $\tilde{\tau}^* > \tau^*$ is that concern for relative consumption makes it more attractive to link variations in $c_{2,t}$ to variations in $c_{1,t+1}$ by increasing τ. This corresponds to the second term in our decomposition, which always makes the marginal effect of τ larger when relative consumption matters.

However, it must be considered that when relative consumption matters a reduction in $\mathrm{S}\left[c_{2,t}\right]$ is important only inasmuch as it results in a reduction in $\mathrm{S}\left[c_{2,t} - \beta c_{1,t+1}\right]$, and the effect of the former on the latter depends on aspects like the overall volatility of consumption and the correlation between the consumption of the old and the young. Since the fraction in the first term of (17) is never higher than one,[14] the benefit from a reduction in $\mathrm{S}\left[c_{2,t}\right]$ is not larger than the effect on $\mathrm{S}\left[c_{2,t}\right]$ itself, which represents the marginal benefit of an increase in τ when individuals are concerned only for their absolute level of consumption.

Thus, the two effects should be balanced one against the other, and it might happen under some circumstances that the diminished benefit from a reduction in $\mathrm{S}\left[c_{2,t}\right]$ offsets the benefits from an increase in ρ_{12}, so that the condition (17) is not verified.

[14] This is easily verified by squaring the fraction and developing the denominator:

$$\frac{\left(\mathrm{S}\left[c_{2,t}\right] - \rho_{12}\beta \mathrm{S}\left[c_{1,t+1}\right]\right)^2}{\mathrm{S}\left[c_{2,t} - \beta c_{1,t+1}\right]^2} = \frac{\mathrm{S}\left[c_{2,t}\right]^2 + \rho_{12}^2 \beta^2 \mathrm{S}\left[c_{1,t+1}\right]^2 - 2\rho_{12}\beta \mathrm{S}\left[c_{1,t+1}\right] \mathrm{S}\left[c_{2,t}\right]}{\mathrm{S}\left[c_{2,t}\right]^2 + \beta^2 \mathrm{S}\left[c_{1,t+1}\right]^2 - 2\rho_{12}\beta \mathrm{S}\left[c_{1,t+1}\right] \mathrm{S}\left[c_{2,t}\right]} \leqslant 1$$

where the equality is only for $|\rho_{12}| = 1$.

As it is clear from (17) itself, the plausibility of such an outcome relies on $\beta \mathrm{S}[c_{1,t+1}]$ being high with respect to $\mathrm{S}[c_{2,t}]$, i.e. on high values of β and γ (the latter parameter affects the relative magnitude of $c_{2,t}$ and $c_{1,t+1}$). When additionally ρ_{12} is so high that $\mathrm{S}[c_{2,t}] < \rho_{12}\beta \mathrm{S}[c_{1,t+1}]$, the numerator of the fraction in the first term of (17) is negative; hence a smaller $\mathrm{S}[c_{2,t}]$ is bad from the point of view of the old who care fore relative consumption, because it increases $\mathrm{S}[c_{2,t} - \beta c_{1,t+1}]$. A high ρ_{12} is the effect of a high ρ_{gr} and τ: in these circumstances, the link between $c_{2,t}$ and $c_{1,t+1}$ cannot be increased much by increasing τ, hence it may happen that the optimal τ is lower when relative consumption matters.

In order to clarify the relevance of the described circumstances, it is useful to identify some sufficient conditions for concern for relative consumption to imply a higher optimal level of τ. If we consider that the term in squared brackets on the right hand side of (14) cannot be larger than 2 for $\tau \leq (1-\gamma)$, and it is certainly not larger than unity when $\sigma_g^2 - \rho_{gr}\sigma_g\sigma_r \geqslant 0$, we have the following:

Corollary 1. *If* $0 < \tau^* < 1-\gamma$, *then* $0 < \beta \leqslant (1-\gamma)/\gamma$ *implies* $\tilde{\tau}^* > \tau^*$. *Moreover, if* $\rho_{gr} \leqslant \sigma_g/\sigma_r$, *then* $0 < \beta \leqslant 2(1-\gamma)/\gamma$ *implies* $\tilde{\tau}^* > \tau^*$. □

There are reasons to believe that these sufficient conditions will be satisfied in practice. First of all, a non-negligible part of a pensioner's reference group is presumably made of other pensioners. Hence, the relevant standard of living will depend only in part on the consumption of the young. This suggests that the parameter β, representing the sensitivity of the old with respect to the young's consumption will be away from its maximum value δ ($\delta \leqslant 1$).

Secondly, it may be argued that only a part of the consumption of the young is used as a reference by the old. Consumption is partly age-specific, and the consumption of the young is usually different from that of the old. Moreover, a fraction of the consumption of the young can actually be an income production cost (e.g. baby sitting, commuting costs, etc.) and, hence, it should not be taken into account in the life standard. This argument too points to a β not so close to δ.

Lastly, there is the issue of how large is γ with respect to $1-\gamma$. In a two period framework, we can think that the desired amount of savings $1-\gamma$ will not be very far from one-half, so that $(1-\gamma)/\gamma$ is close to one. The fact that we usually observe a lower saving rate, closer to one-third, is presumably related to the fact that in the life cycle the average working time is approximately twice as much as the time of retirement. To put it differently: when comparing total consumption during the working age with

total consumption at old age, the different lengths of the two periods over which consumption is spread should be taken into account, and β scaled down accordingly.[15]

The previous arguments suggest that, even in the extreme case where individuals consider a one unit increase in the standard of living equivalent to a one unit decrease in their own consumption, ($\delta = 1$) the coefficient β can be expected to be lower than $(1-\gamma)/\gamma$, so that even the more restrictive sufficient condition reported in Corollary 1 will be satisfied. Moreover, with $\rho_{gr} < \sigma_g/\sigma_r$ the sufficient condition will be violated only for unrealistically high values of γ and β.[16]

It is worth emphasizing that when relative consumption matters the role played by the PAYG pension system is not simply that of risk differentiation in the face of financial markets volatility, as already emphasized in the literature. This should be clear if we consider the hypothetical case in which r_t is deterministic. In this case, when only absolute consumption matters, the condition $\mu_r > \mu_g$ is enough to secure that $\tau^* = 0$. On the contrary, when we consider concern for relative consumption, the optimal value of τ can be positive even if $\mu_r > \mu_g$, and the chance that $\tilde{\tau}^* > 0$ increases with σ_g^2. This is established by the following

Corollary 2. *If r_t is non-stochastic, then $\tilde{\tau}^* > 0$ if and only if*

$$-\frac{V_S}{V_E}\sigma_g > \mu_r - \mu_g \tag{18}$$

□

PROOF. Under the assumption that $\mu_r > \mu_g$, the fact that r_t is non-stochastic implies that $\tau^* = 0$. Therefore, the first order condition for a maximum of $\tilde{V}$ reduces to (18). □

4. Concluding remarks

The introduction of the relative consumption hypothesis in an overlapping generation model produces a further source of risk, namely the risk of a change in one's relative position with respect to the reference standard of living. In this paper we have considered the effect of the assumption that individuals care for their future relative position on the optimal mix between

[15] Note that the inclusion of a positive demographic growth would reinforce this argument, since the same amount of resources would be spread over a number of pensioners which is lower than the number of workers.

[16] Referring to the estimates used by Matsen and Thøgersen (2004, p. 897), the inequality is satisfied for most of the countries considered in that paper.

PAYG and FF pensions, and found that, when model parameters assume reasonable values, the role of PAYG is enhanced.[17] More precisely, for given attitude toward risks and given expected values and variability of the relevant variables, an assessment of the optimal pension mix which overlooks the fact that individual care for relative consumption is likely to underestimate the role of the PAYG component.

A few remarks on our main finding and its scope are worth considering.

In order to focus on the insurance effect, we have abstracted from various important aspects related to pension systems such as the effects on savings and on the labor market (e.g. Lindbeck and Persson, 2003). Of course, the overall desirability of a PAYG system also depends on these issues. In this respect, however, we have to consider the possibility that conventional wisdom may have to be revisited in the light of the relative consumption hypothesis. Indeed, we think that a careful exploration of such a possibility would be a relevant research line to pursue.

Another important restriction is the hypothesis of constant relative risk aversion. This was made to avoid time inconsistency in the optimal size of the PAYG system. Although we cannot dispense with such an assumption if we want to retain time consistency, we emphasize that this is not crucial for our main result. More precisely, the necessary and sufficient condition provided in our proposition 1 would still be true, though it should be checked for each period separately, as the optimal FF-PAYG mix could change over time.

Finally, we have derived our results under the assumption that people care for their relative standing in society in a cardinal way—more precisely, that what matters is the difference between the current consumption and standard of living. As shown by Bilancini and Boncinelli (2008), this may be a non-innocuous assumption. In order to further investigate the robustness of our results, one should repeat the analysis under alternative specifications of concern for relative consumption.

[17] In our proposition we have dealt with the case commonly considered as the relevant one, namely that in which the growth of the contribution base is lower than the return on financial market (in the deterministic case, this corresponds to dynamic efficiency). However, we expect that the same result applies to the case where μ_r is lower than μ_g. Of course, the role of FF and PAYG would be reversed in term of risk diversification, but the main conclusion about the enhanced role of the PAYG remains true.

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