55

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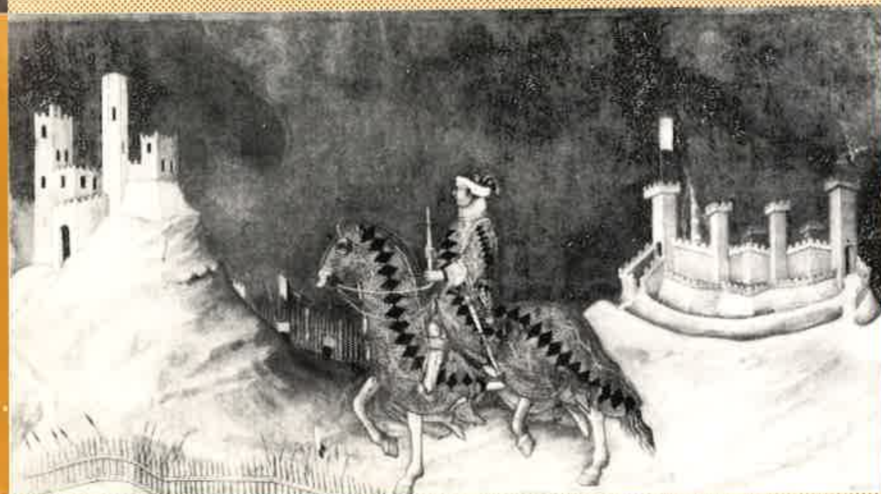
Facoltà di Scienze Economiche e Bancarie

QUADERNI DELL'ISTITUTO DI ECONOMIA

Giulio Cifarelli

EXCHANGE RATES, MARKET EFFICIENCY AND "NEWS". MODEL SPECIFICATION AND ECONOMETRIC IDENTIFICATION



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Giulio Cifarelli

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*1. Introduction**

The aim of this paper is that of investigating the modelling procedures used in the analysis of the relevance of "news" as explanatory factors of exchange rate behaviour.

It can be shown that the commonly used models do not always properly incorporate rational expectations. The restrictions imposed often have no theoretical justification and cannot be set forth a priori. An alternative and more rigorous model is developed in this paper, which allows to introduce rational expectations correctly in the analysis of "news".

The empirical investigation and the associated identification problems bring about additional restrictions, in the simultaneous equations estimation of these models, that often are difficult to accept from an economic point of view. We are left with the choice either of introducing these unrealistic a priori identifying restrictions or of abandoning the simultaneous equations estimation approach and the associated likelihood ratio tests of the validity of the cross equation parameter restrictions associated with the rational expectations hypothesis. A possible solution, in the latter case, is that of using the less efficient but also less restrictive single equation approaches to test the "news".

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2. *The origin of "news"*

In most empirical work on exchange rate markets efficiency, the following equation has been estimated

$$(1) \qquad s^{BA}_{t+1} = a + b\, f^{BA}_{t} + \epsilon_{t+1} \quad ,$$

where s^{BA}_{t+1} is the (logarithm of) the spot exchange rate between the monies of countries B and A, and f^{BA}_{t} is the corresponding (logarithm of) the forward rate for period t+1, set in period t. Under the assumption that the forward rate determined in period t is an unbiased predictor of s^{BA}_{t+1}, a = 0, b = 1.0 and ϵ_{t+1} is a white noise error term.

M. Mussa (1977, 1979), R. Dornbusch (1980), J.A. Frenkel (1981) and others express the spot rate in period t+1 as a function of factors which have been known in advance and are incorporated in the forward rate as well as of "news". They assume that

$$(2) \qquad s^{BA}_{t+1} = a + b\, f^{BA}_{t} + \text{"news"} + \epsilon_{t+1} \; .$$

The main difficulty lies in the quantification of the "news". There have been two main attempts to solve this problem: a single equation (two-step) approach and a simultaneous equations approach.

Alternative versions of the single equation approach have been set forth by R. Dornbusch (1980), J.A. Frenkel (1981), S. Edwards (1982) and

E.J. Bomhoff and P. Korteweg (1983). Various specifications of the "news" are analysed and are used as predetermined variables in the estimation of a relationship analogous to equation (2) above.

The second way to test the "news" is to use nonlinear full information approaches, testing simultaneously for market efficiency and rational expectations. (In the efficient financial markets literature rational expectations are assumed to be implicit in or equivalent to the concept of financial markets "efficiency".) This has been done by P.R. Hartley (1983) in the context of a simple monetary approach and by D.L. Hoffman and D.E. Schlagenhauf (1985) using a more flexible if less rigorous framework(1). They have used the simultaneous equations approach suggested by C.L.F. Attfield, D. Demery and N.W. Duck (1981) and by A. Abel and F.S. Mishkin (1983), which implies the estimation of relationships of the form

$$(3) \qquad s^{BA}_{t+1} - f^{BA}_{t} = \left[X_{t+1} - E(X_{t+1} \mid \Phi_t)\right]\beta + \epsilon_{t+1} \; ,$$

where X_{t+1} is a (1 x k) vector of k variables relevant to the determination of the spot exchange rate, E(.) is the mathematical expectations operator and $E(X_{t+1} \mid \Phi_t)$ is a vector of one period ahead rational forecasts of X_{t+1}, derived in period t, conditional on the relevant information set Φ_t. β is a (k x 1) vector of coefficients and ϵ_{t+1} is a serially uncorrelated scalar disturbance, with the property that $E(\epsilon_{t+1} \mid \Phi_t) = 0$.

Suppose vector X_{t+1} follows the stochastic process

(4) $$X_{t+1} = \sum_{h=1}^{M} Z_{t+1-h}\, \gamma_h + u_{t+1} \quad ,$$

where Z_{t+1} is a (1 x N) vector of N variables contained in Φ_t, useful in predicting X_{t+1}, the γ_h are (N x k) matrices and u_{t+1} is a (1 x k) vector of disturbances.

The rational expectations hypothesis contained in the market efficiency assumption implies that agents use the process (4) in deriving expectations. The following system can then be tested by means of simultaneous equations approaches

$$(I) \begin{cases} (3) \quad s^{BA}_{t+1} - f^{BA}_{t} = \left[X_{t+1} - \sum_{h=1}^{M} Z_{t+1-h}\, \gamma^*_h \right] \beta + \epsilon_{t+1} \\ (4) \quad X_{t+1} = \sum_{h=1}^{M} Z_{t+1-h}\, \gamma_h + u_{t+1} \end{cases}$$

Rational expectations imply that the matrices of coefficients γ_h that determine the stochastic structure of X_{t+1} in (4) coincide with the matrices γ^*_h employed by economic agents in deriving the conditional forecasts of X_{t+1} in (3).

This approach brings about some problems concerning both the theoretical structure and the econometric identification of the model. In section 3 we are going to analyse the main specification problems. They are due to the fact that in asset markets of this kind we are dealing with pairs of countries, X_t, Z_t are vectors of variables which in the two countries involved influence the exchange rate.

In section 4 we are going to analyse the econometric identification problems.

3. The theoretical structure of the asset market monetary model

Consider the following standard asset market model

(5) $$i^B_t - i^A_t = f^{BA}_t - s^{BA}_t$$

(6) $$E(s^{BA}_{t+1} \mid \Phi_t) = f^{BA}_t$$

(7) $$s^{BA}_t - p^B_t + p^A_t = \xi_t \quad ,$$

where (7') $$\xi_t = \delta(L)\, \xi_{t-1} + x_t$$

(8) $$m^B_t - p^B_t = a^B y^B_t - b^B i^B_t$$

(8') $$m^A_t - p^A_t = a^A y^A_t - b^A i^A_t$$

(9) $$i^B_t = r^B_t + E\left[(p^B_{t+1} - p^B_t) \mid \Phi_t \right]$$

(9') $$i^A_t = r^A_t + E\left[(p^A_{t+1} - p^A_t) \mid \Phi_t \right]$$

(10) $\gamma^B_{11}(L)\, m^B_t + \gamma^B_{12}(L)\, y^B_t + \gamma^B_{13}(L)\, r^B_t = u^B_t$

(10') $\gamma^A_{11}(L)\, m^A_t + \gamma^A_{12}(L)\, y^A_t + \gamma^A_{13}(L)\, r^A_t = u^A_t$

(11) $\gamma^B_{21}(L)\, m^B_t + \gamma^B_{22}(L)\, y^B_t + \gamma^B_{23}(L)\, r^B_t = v^B_t$

(11') $\gamma^A_{21}(L)\, m^A_t + \gamma^A_{22}(L)\, y^A_t + \gamma^A_{23}(L)\, r^A_t = v^A_t$

(12) $\gamma^B_{31}(L)\, m^B_t + \gamma^B_{32}(L)\, y^B_t + \gamma^B_{33}(L)\, r^B_t = w^B_t$

(12') $\gamma^A_{31}(L)\, m^A_t + \gamma^A_{32}(L)\, y^A_t + \gamma^A_{33}(L)\, r^A_t = w^A_t$,

where

$$\gamma_{zj}(L) = \sum_{h=0}^{n_{zj}} \gamma_{zj,h}\, L^h$$

and

$$\gamma_{zj,0} = \begin{cases} 1 \text{ if } z=j \\ 0 \text{ if } z\neq j \end{cases}$$

s^{BA}_t = logarithm of the spot rate between the currencies of countries B and A,

f^{BA}_t = logarithm of the corresponding (one period ahead) forward rate,

i^B_t, i^A_t = nominal interest rates on one period bonds in countries B and A respectively,

p^B_t, p^A_t = logarithms of the price levels of countries B and A,

r^B_t, r^A_t = real interest rates in countries B and A,

m^B_t, m^A_t = logarithm of nominal money supply in countries B and A,

y^B_t, y^A_t = logarithm of real output in countries B and A,

$v^B_t, v^A_t, u^B_t, u^A_t, w^B_t, w^A_t$ = independent, serially uncorrelated shocks with zero mean and constant variance.

Equation (5) is the interest arbitrage condition. Equation (6) introduces the assumption that economic agents are risk neutral. Equation (7) defines ξ_t as a temporary deviation from purchasing power parity. It can be autocorrelated (equation (7')). Indeed, as shown by J.A. Frenkel (1981), (1981,a), R. Dornbusch (1980), M.R. Darby (1980) and others, there can be persistent short run deviations from purchasing power parity. Equations (8), (8') are money demand equations in the two countries. Equations (9), (9') are Fisher interest rate relationships. Equations (10), (10'), (11), (11'), (12), (12') depict the evolution over time of the money supply, real output and real interest rate processes in the two countries.

3.1. The Hartley solution: some specification problems

Let us examine a simplified version of the solution proposed by P.R. Hartley (1983).

We want to derive the model solutions for s^{BA}_{t+1} and $f^{BA}_{t}(=E(s^{BA}_{t+1}|\Phi_t))$. We can use them to find an expression for the asset market forecast error that is related to unanticipated changes in the exchange rate determinants or "news". These solutions are obtained in terms of a model consisting of equations (5), (6), (7), (8), (8'), (10), (10'), (11), (11') only.

It is assumed that

$$\gamma^{W}_{13}(L) = \gamma^{W}_{23}(L) = 0 \;, \; W = B, A, \forall L \;,$$

in equations (10), (10'), (11), (11').

Replacing p^B_t and p^A_t by their determinants (obtained from equations (8) and (8')) in equation (7), we obtain the following relationship

$$(13) \quad s^{BA}_t = m^B_t - m^A_t - a^B y^B_t + a^A y^A_t + b^B i^B_t - b^A i^A_t + \xi_t \;.$$

Hartley assumes that interest rate quasi elasticities (of the demand for money) are equal across countries: $b^B = b^A = b$.

Shifting equation (13) forward by one period, we obtain, substituting for the interest rate differential (using equations (5) and (6)),

$$(14) \quad (1+b)\, s^{BA}_{t+1} = m^B_{t+1} - m^A_{t+1} - a^B y^B_{t+1} + a^A y^A_{t+1} + b\, E(s^{BA}_{t+2} \mid \Phi_{t+1}) + \xi_{t+1} \;.^{(2)}$$

Taking expectations and solving forward, we obtain, assuming that $\lambda = \frac{b}{1+b}$,

$$(15) \quad E(s^{BA}_{t+1} \mid \Phi_t) = (1-\lambda)\Big[\sum_{i=0}^{\infty} \lambda^i E(m^B_{t+1+i} \mid \Phi_t) - \sum_{i=0}^{\infty} \lambda^i E(m^A_{t+1+i} \mid \Phi_t) - a^B \sum_{i=0}^{\infty} \lambda^i E(y^B_{t+1+i} \mid \Phi_t) + a^A \sum_{i=0}^{\infty} \lambda^i E(y^A_{t+1+i} \mid \Phi_t) + \sum_{i=0}^{\infty} \lambda^i E(\xi_{t+1+i} \mid \Phi_t)\Big] \;.^{(3)}$$

Substituting back into the exchange rate quasi reduced form, equation (14), we obtain

$$(16) \quad s^{BA}_{t+1} = (1-\lambda)\Big[\sum_{i=0}^{\infty} \lambda^i E(m^B_{t+1+i} \mid \Phi_{t+1}) - \sum_{i=0}^{\infty} \lambda^i E(m^A_{t+1+i} \mid \Phi_{t+1}) - a^B \sum_{i=0}^{\infty} \lambda^i E(y^B_{t+1+i} \mid \Phi_{t+1}) + a^A \sum_{i=0}^{\infty} \lambda^i E(y^A_{t+1+i} \mid \Phi_{t+1}) + \sum_{i=0}^{\infty} \lambda^i E(\xi_{t+1+i} \mid \Phi_{t+1})\Big] \;.$$

Subtracting (15) from (16), we obtain

$$(17)\quad s^{BA}_{t+1} - E(s^{BA}_{t+1} \mid \Phi_t) =$$

$$= (1-\lambda)\ \Big\{ \sum_{i=0}^{\infty} \lambda^i \left[E(m^B_{t+1+i} \mid \Phi_{t+1}) - E(m^B_{t+1+i} \mid \Phi_t) \right] +$$

$$- \sum_{i=0}^{\infty} \lambda^i \left[E(m^A_{t+1+i} \mid \Phi_{t+1}) - E(m^A_{t+1+i} \mid \Phi_t) \right] +$$

$$- a^B \sum_{i=0}^{\infty} \lambda^i \left[E(y^B_{t+1+i} \mid \Phi_{t+1}) - E(y^B_{t+1+i} \mid \Phi_t) \right] +$$

$$+ a^A \sum_{i=0}^{\infty} \lambda^i \left[E(y^A_{t+1+i} \mid \Phi_{t+1}) - E(y^A_{t+1+i} \mid \Phi_t) \right] + \eta_{t+1} \Big\}.$$ [4]

Equations (10), (10'), (11), (11') shifted forward one period can be rewritten in matrix form as follows

$$(18)\quad \begin{bmatrix} \gamma^W_{11}(L) & \gamma^W_{12}(L) \\ \gamma^W_{21}(L) & \gamma^W_{22}(L) \end{bmatrix} \begin{bmatrix} m^W_{t+1} \\ y^W_{t+1} \end{bmatrix} = \begin{bmatrix} u^W_{t+1} \\ v^W_{t+1} \end{bmatrix}, \quad W = A, B\ .$$

Equation (17) can be rewritten as

$$(19)\quad s^{BA}_{t+1} - E(s^{BA}_{t+1} \mid \Phi_t) =$$

$$= (1-\lambda) \Bigg\{ \begin{bmatrix} 1 & 0 \end{bmatrix} \gamma^B(\lambda)^{-1} \begin{bmatrix} m^B_{t+1} - E(m^B_{t+1} \mid \Phi_t) \\ y^B_{t+1} - E(y^B_{t+1} \mid \Phi_t) \end{bmatrix} +$$

$$- \begin{bmatrix} 1 & 0 \end{bmatrix} \gamma^A(\lambda)^{-1} \begin{bmatrix} m^A_{t+1} - E(m^A_{t+1} \mid \Phi_t) \\ y^A_{t+1} - E(y^A_{t+1} \mid \Phi_t) \end{bmatrix} +$$

$$- a^B \begin{bmatrix} 0 & 1 \end{bmatrix} \gamma^B(\lambda)^{-1} \begin{bmatrix} m^B_{t+1} - E(m^B_{t+1} \mid \Phi_t) \\ y^B_{t+1} - E(y^B_{t+1} \mid \Phi_t) \end{bmatrix} +$$

$$+ a^A \begin{bmatrix} 0 & 1 \end{bmatrix} \gamma^A(\lambda)^{-1} \begin{bmatrix} m^A_{t+1} - E(m^A_{t+1} \mid \Phi_t) \\ y^A_{t+1} - E(y^A_{t+1} \mid \Phi_t) \end{bmatrix} + \eta_{t+1} \Bigg\}.$$ [5]

Alternatively, the exchange rate forecast error relationship can be rewritten as

$$(20)\quad s^{BA}_{t+1} - E(s^{BA}_{t+1}|\Phi_t) = (1-\lambda)\left[\frac{\gamma^{B}_{22}(\lambda) + a^{B}\gamma^{B}_{21}(\lambda)}{\Delta^{B}}u^{B}_{t+1} + \right.$$

$$- \frac{\gamma^{A}_{22}(\lambda) + a^{A}\gamma^{A}_{21}(\lambda)}{\Delta^{A}}u^{A}_{t+1} - \frac{\gamma^{B}_{12}(\lambda) + a^{B}\gamma^{B}_{11}(\lambda)}{\Delta^{B}}v^{B}_{t+1} +$$

$$\left. + \frac{\gamma^{A}_{12}(\lambda) + a^{A}\gamma^{A}_{11}(\lambda)}{\Delta^{A}}v^{A}_{t+1} + \eta_{t+1}\right],$$

where

$$\Delta^{W} = \gamma^{W}_{11}(\lambda)\,\gamma^{W}_{22}(\lambda) - \gamma^{W}_{12}(\lambda)\,\gamma^{W}_{21}(\lambda), \quad W = A, B.$$

Equations (20) and (18) constitute a system of equations analogous to system (I) mentioned above, in which the asset market approach economic characteristics are explicitly set forth (in the coefficients of the exchange rate forecast error equation (20)).

It is often assumed (S. Edwards (1982)) that the standard asset market restriction of equality of the structural coefficients of the model across countries (i.e. equality of the corresponding money demand parameters of the two countries) allows to estimate a relationship such as

$$(21)\quad s^{BA}_{t+1} - E(s^{BA}_{t+1}|\Phi_t) = C\left[u^{B}_{t+1} - u^{A}_{t+1}\right] +$$

$$+ D\left[v^{B}_{t+1} - v^{A}_{t+1}\right] + \eta_{t+1}.$$

This specification is correct only if the additional restriction is imposed, that the (corresponding) coefficients of the relationships which depict the behaviour of the stimuli in the two countries are equal, i.e. only if we add the (unrealistic) restrictions that

$$\gamma^{B}_{zj} = \gamma^{A}_{zj} \qquad z,j = 1,2,\ \forall L \text{ i.e. that}$$

$$\gamma^{B}_{zj,h} = \gamma^{A}_{zj,h} \qquad z,j = 1,2,\ \forall h,$$

in the model above.

Theorem 1. It is possible to impose the restriction that (unanticipated) stimuli have effects of equal absolute value (and of opposite sign) on the errors in the forecasts of the one period ahead exchange rate $(s^{BA}_{t+1} - E(s^{BA}_{t+1}|\Phi_t))$ only and only if it is assumed that, besides the corresponding structural parameters of the model, the coefficients of the relationships which depict the behaviour of the stimuli are equal across countries. (The proof is set forth in the Appendix.)

3.2. A more realistic variant of the Hartley solution

Interest rates do not enter explicitly the solution of the asset market model of exchange rate determination set forth above, which explains why Hartley (P.R. Hartley (1983)) finds no reasonable empirical estimates of b, the interest rate quasi elasticity of the demand for money in both countries.

An alternative solution of this model is analysed here in which interest rates are explicitly set forth and are used in the estimation of money demand elasticities.

Consider a model consisting of equations (5), (6), (7), (7'), (8), (8'), (9), (9'), (10), (10'), (11), (11'), (12) and (12'). From equations (5) and (6) we obtain

$$(22) \quad s_t^{BA} = E(s_{t+1}^{BA} \mid \Phi_t) - i_t^B + i_t^A .$$

From equations (7) and (7') we obtain

$$(23) \quad E(s_{t+1}^{BA} \mid \Phi_t) = E(p_{t+1}^B \mid \Phi_t) - E(p_{t+1}^A \mid \Phi_t) + $$

$$+ \delta(L)\,(s_t^{BA} - p_t^B + p_t^A).$$ [6]

Substituting for i_t^B and i_t^A using equations (9) and (9') and for $E(s_{t+1}^{BA} \mid \Phi_t)$ using equation (23), we obtain

$$(24) \quad s_t^{BA} = p_t^B - p_t^A - (1 - \delta(L))^{-1}\, r_t^B + (1 - \delta(L))^{-1}\, r_t^A .$$

We introduce two price relationships. From equations (8) and (8'), substituting for i_t^B and i_t^A, using equations (9) and (9'), rearranging terms, we obtain

$$(25) \quad (1 + b^B)\, p_t^B = m_t^B - a^B y_t^B + b^B r_t^B + b^B E(p_{t+1}^B \mid \Phi_t)$$

$$(26) \quad (1 + b^A)\, p_t^A = m_t^A - a^A y_t^A + b^A r_t^A + b^A E(p_{t+1}^A \mid \Phi_t)$$

Taking expectations of equation (25), solving forward using the methodology set forth in section 3.1. above, and shifting the whole forward by one period, we obtain the following price reduced form

$$(27) \quad p_{t+1}^B = \left[\frac{1}{1+b^B}\right]\left[\sum_{i=0}^{\infty}\left(\frac{b^B}{1+b^B}\right)^i E(m_{t+1+i}^B \mid \Phi_{t+1}) + \right.$$

$$- a^B \sum_{i=0}^{\infty}\left(\frac{b^B}{1+b^B}\right)^i E(y_{t+1+i}^B \mid \Phi_{t+1}) +$$

$$\left. + b^B \sum_{i=0}^{\infty}\left(\frac{b^B}{1+b^B}\right)^i E(r_{t+1+i}^B \mid \Phi_{t+1}) \right] .$$ [7]

In an analogous way we obtain an expression for p_{t+1}^A. In order to obtain a reduced form for s_{t+1}^{BA} we shift the exchange rate relationship (24) forward by one period and substitute p_{t+1}^B and p_{t+1}^A by their determinants.

Assuming that $\lambda = \frac{b^B}{1+b^B}$ and $\theta = \frac{b^A}{1+b^A}$, we obtain

$$(28)\quad s^{BA}_{t+1} = (1-\lambda)\left[\sum_{i=0}^{\infty}\lambda^i E(m^B_{t+1+i} \mid \Phi_{t+1}) - a^B \sum_{i=0}^{\infty}\lambda^i E(y^B_{t+1+i} \mid \Phi_{t+1}) + \right.$$

$$\left. + b^B \sum_{i=0}^{\infty}\lambda^i E(r^B_{t+1+i} \mid \Phi_{t+1})\right] - (1-\theta)$$

$$\times \left[\sum_{i=0}^{\infty}\theta^i E(m^A_{t+1+i} \mid \Phi_{t+1}) - a^A \sum_{i=0}^{\infty}\theta^i E(y^A_{t+1+i} \mid \Phi_{t+1}) + \right.$$

$$\left. + b^A \sum_{i=0}^{\infty}\theta^i E(r^A_{t+1+i} \mid \Phi_{t+1})\right] +$$

$$-\left(\frac{1}{1-\delta(L)}\right) r^B_{t+1} + \left(\frac{1}{1-\delta(L)}\right) r^A_{t+1} \quad . \quad (8)$$

The solution proceeds now as in Hartley's model set forth above. Taking expectations of (28) and subtracting from (28), we obtain an expression for the exchange rate forecast error in terms of "news".

$$(29)\quad s^{BA}_{t+1} - E(s^{BA}_{t+1} \mid \Phi_t) =$$

$$= (1-\lambda)\left\{\sum_{i=0}^{\infty}\lambda^i \left[E(m^B_{t+1+i} \mid \Phi_{t+1}) - E(m^B_{t+1+i} \mid \Phi_t)\right] + \right.$$

$$- a^B \sum_{i=0}^{\infty}\lambda^i \left[E(y^B_{t+1+i} \mid \Phi_{t+1}) - E(y^B_{t+1+i} \mid \Phi_t)\right] +$$

$$\left. + b^B \sum_{i=0}^{\infty}\lambda^i \left[E(r^B_{t+1+i} \mid \Phi_{t+1}) - E(r^B_{t+1+i} \mid \Phi_t)\right]\right\} +$$

$$- (1-\theta)\left\{\sum_{i=0}^{\infty}\theta^i \left[E(m^A_{t+1+i} \mid \Phi_{t+1}) - E(m^A_{t+1+i} \mid \Phi_t)\right] + \right.$$

$$- a^A \sum_{i=0}^{\infty}\theta^i \left[E(y^A_{t+1+i} \mid \Phi_{t+1}) - E(y^A_{t+1+i} \mid \Phi_t)\right] +$$

$$\left. + b^A \sum_{i=0}^{\infty}\theta^i \left[E(r^A_{t+1+i} \mid \Phi_{t+1}) - E(r^A_{t+1+i} \mid \Phi_t)\right]\right\} +$$

$$-\left(1-\delta(L)\right)^{-1}\left[r^B_{t+1} - E(r^B_{t+1} \mid \Phi_t)\right] +$$

$$+\left(1-\delta(L)\right)^{-1}\left[r^A_{t+1} - E(r^A_{t+1} \mid \Phi_t)\right] + \eta_{t+1} \quad .$$

Using, as in section 3.1. above, the Hansen and Sargent approach to express the anticipated stimuli in terms of their determinants and rearranging terms, we obtain the following reduced form

$$(30)\quad s^{BA}_{t+1} - E(s^{BA}_{t+1} \mid \Phi_t) = \frac{1-\lambda}{\Delta^B}\left\{\left[A^B_{11}(\lambda) - a^B A^B_{21}(\lambda) + b^B A^B_{31}(\lambda)\right] u^B_{t+1} + \right.$$

$$+ \left[A^B_{12}(\lambda) - a^B A^B_{22}(\lambda) + b^B A^B_{32}(\lambda)\right] v^B_{t+1} +$$

$$+ \left[A^B_{13}(\lambda) - a^B A^B_{23}(\lambda) + b^B A^B_{33}(\lambda) + \right.$$

$$- \frac{(1 - \delta(L))^{-1} \Delta^B}{1-\lambda} \Bigg] w^B_{t+1} \Bigg\} +$$

$$- \frac{1-\theta}{\Delta^A} \Bigg\{ \Bigg[A^A_{11}(\theta) - a^A A^A_{21}(\theta) + b^A A^A_{31}(\theta) \Bigg] u^A_{t+1} +$$

$$+ \Bigg[A^A_{12}(\theta) - a^A A^A_{22}(\theta) + b^A A^A_{32}(\theta) \Bigg] v^A_{t+1} +$$

$$+ \Bigg[A^A_{13}(\theta) - a^A A^A_{23}(\theta) + b^A A^A_{33}(\theta) +$$

$$- \frac{(1 - \delta(L))^{-1} \Delta^A}{1-\theta} \Bigg] w^A_{t+1} \Bigg\} + \eta_{t+1} \quad . \qquad (9)$$

Δ^B and Δ^A are the determinants of $\gamma^B(\lambda)$ and $\gamma^A(\theta)$ respectively. $A^B_{zj}(\lambda)$ and $A^A_{zj}(\theta)$, (z,j=1,2,3) are elements of the matrices of adjoints of $\gamma^B(\lambda)$ and of $\gamma^A(\theta)$ respectively; where

$$\gamma^B(\lambda)^{-1} = \frac{A^B(\lambda)}{\Delta^B} \quad \text{and} \quad \gamma^A(\theta)^{-1} = \frac{A^A(\theta)}{\Delta^A} . \qquad (10)$$

The reduced form above constitutes, together with the stimuli forecasting relationships, equations (10), (10'), (11), (11'), (12) and (12') a system of equations analogous to system (I) above. In the empirical estimation of relationships of this kind (J.F.O. Bilson (1978), D.L. Hoffman and D.E. Schla-

genhauf (1983), P.R. Hartley (1983) and others) it is usually assumed that money demand interest rate quasi elasticities are equal in the two countries A and B, i.e. that $b^A = b^B = b$.(11)

It should be noticed that the constraints imposed by the rational expectations hypothesis on the coefficients of equations (20) and (30) are highly nonlinear. They can be included in the estimation only if they are relatively simple in structure, i.e. only if the autoregressive structure of the stimuli forecasting relationships is relatively simple.

4. Econometric identification restrictions .

The standard model set forth in section 2 above can be rewritten as:

$$\text{(I)} \quad \begin{cases} (3') \quad s^{BA}_{t+1} - f^{BA}_t = \Big[(X^B_{t+1} - \sum_{h=1}^{M} Z^B_{t+1-h} \gamma^{B*}_h)(X^A_{t+1} - \sum_{h=1}^{M} Z^A_{t+1-h} \gamma^{A*}_h) \Big] \times \begin{bmatrix} \beta^B \\ -\beta^A \end{bmatrix} + \epsilon_{t+1} \\ (4') \quad X^B_{t+1} = \sum_{h=1}^{M} Z^B_{t+1-h} \gamma^B_h + u^B_{t+1} \\ (4'') \quad X^A_{t+1} = \sum_{h=1}^{M} Z^A_{t+1-h} \gamma^A_h + u^A_{t+1} \end{cases}$$

X^A_{t+1}, X^B_{t+1} are $(1 \times k^A)$, $(1 \times k^B)$ vectors of variables relevant to the determination of s^{BA}_{t+1}, Z^{A}_{t+1}, Z^{B}_{t+1} are $(1 \times N^A)$, $(1 \times N^B)$ vectors of variables relevant to the formation of "news", β is a $((k^A + k^B) \times 1)$ vector of coefficients, γ^W_h, γ^{W*}_h are $(N^W \times k^W)$ matrices of parameters, W=A,B. In the models examined above it is assumed that $k^A = k^B$ and $N^A = N^B$, i.e. that the same variables influence exchange rates and "news" in both countries A and B. The results of this section would hold, however, even if this were not the case and $k^A \neq k^B$ and $N^A \neq N^B$.

Identification of the coefficients β, if the equations are estimated by means of a simultaneous equations approach, requires that the exchange rate forecast error equation (equation (3')) be a true reduced form, i.e. that the covariance between the error terms of the stimuli forecasting equations and the error term of the exchange rate forecast error equation be nil(12). As pointed out by M. Obstfeld (1983) this assumption is not always realistic since money supply and real output - elements of X^W_{t+1}, W=A,B - may well respond systematically to contemporaneous exchange rate movements. In that case the simultaneous equations approach is inappropriate since the covariance restrictions are invalid, and the estimated β coefficients will be biased. The model above should then be tested using a less efficient two-step single equation approach which is appropriate even when the covariance between the error terms of the stimuli forecasting equations and of the exchange rate forecast error equation is unknown.

Separate tests of the cross equation constraints, $\gamma = \gamma^*$, imposed by the rational expectations hypothesis bring about additional identification problems. They are performed by comparing the estimates of a version of

system (I) above in which these restrictions are imposed to the estimates of a version of (I) in which these restrictions are relaxed. The number of restrictions - and hence the number of degrees of freedom of the likelihood ratio test - equals the number of identified parameters estimated in the unconstrained system less the number of identified parameters estimated in the constrained system. Alternatively, in model (I) above, the number of restrictions is given by the difference between the number of parameters of the unconstrained and constrained versions of the exchange rate forecast error equation (equation (3')), the number of parameters of the stimuli forecasting equations being the same in the two versions.

These tests can be performed if the parameters of both constrained and unconstrained exchange rate forecast error equations are identified: in both versions the number of parameters to be estimated must not exceed the number of estimable parameters (given by the number of regressors of the unconstrained version of equation (3')), $k^A + k^B + M(N^A + N^B)$, the number of elements of the X^W_{t+1} and Z^W_{t+1-h} vectors respectively, W=A,B, h=1,...,M(13).

Identification of the unconstrained exchange rate forecast error equation will be obtained if $k^A = k^B = 1$ (unanticipated changes in a single variable influence the exchange rate forecast error in each country)(14).

More generally, identification of every coefficient of the unconstrained reduced form (version of equation (3')) will be possible if, in every γ^W_h matrix, W=A,B, h=1,...,M, $k^W - P^{Wh}$ columns are nil and the remaining P^{Wh} columns have ad hoc zero restrictions such that only one nonzero element at most appears in any row of the γ_h^W matrix in question, $1 \leq P^{Wh} \leq k^W$.

The coefficients of the unconstrained version of equation (3') could be identified also with the help of linear constraints. The presence of linear restrictions known a priori between one nonzero coefficient of the γ_h^W matrix and the remaining nonzero coefficients of the same row of the matrix will bring about identification (in terms of the coefficient of reference). It is equivalent to a set of zero restrictions on all the coefficients of this row, with the exception of the coefficient of reference(15). These restrictions are difficult to justify from an economic point of view and have not been used in the estimation of models of this kind.

Identification of the unconstrained version of equation (3') will be obtained also if the stimuli forecasting equations are assumed a priori to be own autoregressions(16).

If the strict (and unrealistic) identifying restrictions on the coefficients of the stimuli forecasting equations mentioned above are not satisfied, we cannot separately test the rational expectations cross equation restrictions $\gamma^W = \gamma^{W*}$ (but, as pointed out by F.S. Mishkin (1983), we can test linear combinations of these restrictions). F.S. Mishkin (1982) and P.R. Hartley (1983) do not impose a priori restrictions on the stimuli forecasting equations and use a partially unconstrained version of equation (3') in likelihood ratio tests of those rational expectations restrictions that are connected with the γ_{jz}^W coefficients that can be identified and estimated.

5. Conclusion

The relevance of "news" as explanatory factors of exchange rate forecast errors has been analysed using the asset market monetary model of exchange rate determination. Rational expectations bring about serious model specification and econometric identification problems because of the highly nonlinear nature of the cross equation parameter restrictions they impose.

The relevance of the cross country restrictions on the absolute values of the stimuli forecasting relationships has been assessed, as well as the possibility of introducing explicitly interest rates in the estimation of the exchange rate models under investigation.

The identification problems that arise in the likelihood ratio tests of the cross equation rational expectations restrictions have also been investigated.

Appendix

Proof of theorem 1

The equality across countries of the γ_{zj} coefficients is both a necessary and a sufficient condition for the equality (of the absolute values) of the unanticipated stimuli coefficients in equation (20).

The proof of the sufficient condition component is straightforward since from equation (20) we see that if $\gamma^B_{zj}(L) = \gamma^A_{zj}(L)$ (and $\gamma^B_{zj}(\lambda) = \gamma^A_{zj}(\lambda)$) z,j=1,2 and $a^A = a^B$, the coefficients of the stimuli u^B_{t+1} and v^B_{t+1} have the same absolute value (and opposite sign) of, respectively, the coefficients of u^A_{t+1} and v^A_{t+1}.

The proof of the necessary condition component is more complicated. We have to show that the hypothesis that

$$\gamma^B_{zj}(L) \neq \gamma^A_{zj}(L) \; (\gamma^B_{zj}(\lambda) \neq \gamma^A_{zj}(\lambda)) \quad z,j=1,2$$

and the hypothesis that the stimuli have coefficients of equal absolute value (and opposite sign) in equation (20) if $a^B = a^A$ are contradictory.

Assume we want to impose the restrictions (in equation (20)):

$$\text{(A.1)} \quad \frac{\gamma^B_{22}(\lambda) + a^B \gamma^B_{21}(\lambda)}{\Delta^B} = \frac{\gamma^A_{22}(\lambda) + a^A \gamma^A_{21}(\lambda)}{\Delta^A}$$

$$\text{(A.2)} \quad \frac{\gamma^B_{12}(\lambda) + a^B \gamma^B_{11}(\lambda)}{\Delta^B} = \frac{\gamma^A_{12}(\lambda) + a^A \gamma^A_{11}(\lambda)}{\Delta^A}$$

We impose the restriction $a^A = a^B$ but assume that

$$\gamma^B_{11}(\lambda) \neq \gamma^A_{11}(\lambda) \; ; \; \gamma^B_{12}(\lambda) \neq \gamma^A_{12}(\lambda)$$

$$\gamma^B_{22}(\lambda) \neq \gamma^A_{22}(\lambda) \; ; \; \gamma^B_{21}(\lambda) \neq \gamma^A_{21}(\lambda)$$

(We assume also that the stimuli coefficients are finite, i.e. that $\Delta^B \neq 0$, $\Delta^A \neq 0$.) From (A.1) we obtain

$$\text{(A.1')} \quad \Delta^A = \frac{\gamma^A_{22}(\lambda) + a^A \gamma^A_{21}(\lambda)}{\gamma^B_{22}(\lambda) + a^A \gamma^B_{21}(\lambda)} \Delta^B$$

From (A.2) we obtain

$$\text{(A.2')} \quad \Delta^A = \frac{\gamma^A_{12}(\lambda) + a^A \gamma^A_{11}(\lambda)}{\gamma^B_{12}(\lambda) + a^A \gamma^B_{11}(\lambda)} \Delta^B$$

From (A.1') and (A.2') we obtain

$$(A.3)\quad \frac{\gamma^{A}_{22}(\lambda) + a^{A}\gamma^{A}_{21}(\lambda)}{\gamma^{B}_{22}(\lambda) + a^{A}\gamma^{B}_{21}(\lambda)} = \frac{\gamma^{A}_{12}(\lambda) + a^{A}\gamma^{A}_{11}(\lambda)}{\gamma^{B}_{12}(\lambda) + a^{A}\gamma^{B}_{11}(\lambda)}$$

This relationship will hold and be compatible with (A.1) and (A.2) if:

1) both sides are nil;

2) both sides are equal to Ψ, $\forall\Psi \neq 0$.

1) Both sides are nil if both nominators are nil in equation (A.3)

$$\gamma^{A}_{22}(\lambda) + a^{A}\gamma^{A}_{21}(\lambda) = 0\ ,$$

$$\gamma^{A}_{12}(\lambda) + a^{A}\gamma^{A}_{11}(\lambda) = 0$$

which, in turn, implies that

$$\text{i)}\quad a^{A} = \frac{-\gamma^{A}_{22}(\lambda)}{\gamma^{A}_{21}(\lambda)} = \frac{-\gamma^{A}_{12}(\lambda)}{\gamma^{A}_{11}(\lambda)}\ ,$$

$$\text{i.e.}\quad \frac{\gamma^{A}_{22}(\lambda)}{\gamma^{A}_{21}(\lambda)} = \frac{\gamma^{A}_{12}(\lambda)}{\gamma^{A}_{11}(\lambda)}$$

or

$$\text{ii)}\quad \gamma^{A}_{22}(\lambda) = \gamma^{A}_{21}(\lambda) = 0\ ,\quad \gamma^{A}_{12}(\lambda) = \gamma^{A}_{11}(\lambda) = 0$$

We have to reject both possibilities since they imply that $\Delta^{A} = \Delta^{B} = 0$, contradicting the assumption made above that $\Delta^{A} \neq 0$, $\Delta^{B} \neq 0$.

2) Both sides of equation (A.3) are equal to Ψ.

$$(A.4)\quad \frac{\gamma^{A}_{22}(\lambda) + a^{A}\gamma^{A}_{21}(\lambda)}{\gamma^{B}_{22}(\lambda) + a^{A}\gamma^{B}_{21}(\lambda)} = \frac{\gamma^{A}_{12}(\lambda) + a^{A}\gamma^{A}_{11}(\lambda)}{\gamma^{B}_{12}(\lambda) + a^{A}\gamma^{B}_{11}(\lambda)} = \Psi$$

$$\frac{\gamma^{A}_{22}(\lambda) + a^{A}\gamma^{A}_{21}(\lambda)}{\gamma^{B}_{22}(\lambda) + a^{A}\gamma^{B}_{21}(\lambda)} = \Psi$$

implies that

$$\gamma^{A}_{22}(\lambda) + a^{A}\gamma^{A}_{21}(\lambda) = \Psi(\gamma^{B}_{22}(\lambda) + a^{A}\gamma^{B}_{21}(\lambda))$$

and

$$(A.5)\quad \gamma^{A}_{22}(\lambda) - \Psi\gamma^{B}_{22}(\lambda) = -a^{A}(\gamma^{A}_{21}(\lambda) - \Psi\gamma^{B}_{21}(\lambda))\ .$$

$$\frac{\gamma^{A}_{12}(\lambda) + a^{A}\gamma^{A}_{11}(\lambda)}{\gamma^{B}_{12}(\lambda) + a^{A}\gamma^{B}_{11}(\lambda)} = \Psi$$

implies that

$$\gamma^{A}_{12}(\lambda) + a^{A}\,\gamma^{A}_{11}(\lambda) = \Psi(\gamma^{B}_{12}(\lambda) + a^{A}\gamma^{B}_{11}(\lambda))$$

and

$$\text{(A.6)}\quad \gamma^{A}_{12}(\lambda) - \Psi\gamma^{B}_{12}(\lambda) = -a^{A}(\gamma^{A}_{11}(\lambda) - \Psi\gamma^{B}_{11}(\lambda)).$$

(A.5) and (A.6) imply that

$$\text{(A.7)}\quad -a^{A} = \frac{\gamma^{A}_{22}(\lambda) - \Psi\gamma^{B}_{22}(\lambda)}{\gamma^{A}_{21}(\lambda) - \Psi\gamma^{B}_{21}(\lambda)} = \frac{\gamma^{A}_{12}(\lambda) - \Psi\gamma^{B}_{12}(\lambda)}{\gamma^{A}_{11}(\lambda) - \Psi\gamma^{B}_{11}(\lambda)}$$

which, in turn, implies that

$$(\gamma^{A}_{22}(\lambda) - \Psi\gamma^{B}_{22}(\lambda))(\gamma^{A}_{11}(\lambda) - \Psi\gamma^{B}_{11}(\lambda)) = (\gamma^{A}_{21}(\lambda) - \Psi\gamma^{B}_{21}(\lambda))(\gamma^{A}_{12}(\lambda) - \Psi\gamma^{B}_{12}(\lambda))$$

Rearranging terms we obtain

$$\text{(A.8)}\quad \Delta^{A} = -\Psi^{2}\Delta^{B} + \Psi(\gamma^{B}_{22}(\lambda)\,\gamma^{A}_{11}(\lambda) - \gamma^{B}_{21}(\lambda)\,\gamma^{A}_{12}(\lambda)) +$$

$$+\,\Psi(\gamma^{A}_{22}(\lambda)\gamma^{B}_{11}(\lambda) - \gamma^{A}_{21}(\lambda)\gamma^{B}_{12}(\lambda)).$$

But from (A.1), (A.2) and (A.4) we have

$$\text{(A.9)}\quad \Delta^{A} = \Psi\Delta^{B}$$

(A.8) will not contradict (A.1) and (A.2), and the necessary condition will not be proved, if it can be shown that, $\forall\ \gamma_{zj}(\lambda)$

$$\text{(A.10)}\quad (\Delta^{A} =)\ \Psi\Delta^{B} = -\Psi^{2}\Delta^{B} + \Psi(\gamma^{B}_{22}(\lambda)\,\gamma^{A}_{11}(\lambda) - \gamma^{B}_{21}(\lambda)\gamma^{A}_{12}(\lambda)) +$$

$$+\,\Psi(\gamma^{A}_{22}(\lambda)\gamma^{B}_{11}(\lambda) - \gamma^{A}_{21}(\lambda)\gamma^{B}_{12}(\lambda))$$

or, if we want, that

$$\text{(A.10')}\quad (1+\Psi)\Delta^{B} - (\gamma^{B}_{22}(\lambda)\gamma^{A}_{11}(\lambda) - \gamma^{B}_{21}(\lambda)\gamma^{A}_{12}(\lambda)) +$$

$$-\,(\gamma^{A}_{22}(\lambda)\gamma^{B}_{11}(\lambda) - \gamma^{A}_{21}(\lambda)\gamma^{B}_{12}(\lambda)) = 0$$

This will be the case if $\gamma^{A}_{zj}(\lambda) = \gamma^{B}_{zj}(\lambda)$, z,j=1,2, a result which we have to reject a priori, since it contradicts the assumption that the stimuli forecasting equations coefficients be different across countries. If $\gamma^{A}_{zj}(\lambda) \neq \gamma^{B}_{zj}(\lambda)$, z,j=1,2, there will be some (values of) $\gamma_{zj}(\lambda)$ for which (A.10') will not hold. Q.E.D.

Notes

(1) D.L. Hoffman and D.E. Schlagenhauf (1985) examine several variants of the asset market model of exchange rate determination. For each model the "news" variant is developed by introducing -as determinants of exchange rate forecast errors- unanticipated shifts in the variables that are suggested by the theory as factors influencing the exchange rate.

(2) From equations (5) and (6),

$$i_t^B - i_t^A = f_t^{BA} - s_t^{BA} = E(s_{t+1}^{BA} \mid \Phi_t) - s_t^{BA} .$$

We can then substitute in equation (13), where it is assumed that $b^A = b^B = b$.

(3) Equation (15) has been derived taking expectations of equation (14), obtaining a left hand side element equal to

$$\left(\frac{1 + b - bL^{-1}}{1 + b}\right) E\,(s_{t+1}^{BA} \mid \Phi_t) = (1 - \frac{b}{1+b} L^{-1}) E(s_{t+1}^{BA} \mid \Phi_t) = (1 - \lambda L^{-1}) E(s_{t+1}^{BA} \mid \Phi_t)$$

and noting that

$$\forall\ \lambda, |\lambda| < 1, (1 - \lambda L^{-1})\ Y_t = X_t$$

implies that

$$Y_t = \sum_{i=0}^{\infty} \lambda^i X_{t+i} .$$

(4) We assume that

$$\sum_{i=0}^{\infty} \lambda^i \left[E(\xi_{t+1+i} \mid \Phi_{t+1}) - E(\xi_{t+1+i} \mid \Phi_t)\right] = \eta_{t+1} .$$

More precisely, as pointed out by Hartley,

$$\eta_{t+1} = A(\xi_{t+1} - E(\xi_{t+1} \mid \Phi_t)) = Ax_{t+1}$$

for some constant A which depends on the δ coefficients of $\delta(L)$. The determination of the autoregressive structure of the ξ_t time series lies outside the scope of this model and is assumed to be known a priori. (It could otherwise be estimated together with the other parameters of the model adding equation (7') to the set of stimuli forecasting equations.)

(5) This relationship has been obtained by applying the well known Hansen and Sargent prediction formulas (L.P. Hansen and T.J. Sargent (1980)) to the stimuli in equation (17) :

$$\sum_{i=0}^{\infty} \lambda^i E(m_{t+1+i}^B \mid \Phi_{t+1}) = \begin{bmatrix} 1 & 0 \end{bmatrix} \gamma^B(\lambda)^{-1} \left[I + \sum_{j=1}^{r-1} \left(\sum_{h=j+1}^{r} \lambda^{h-j}\ \gamma_h^B \right) L^j \right] \begin{bmatrix} m_{t+1}^B \\ y_{t+1}^B \end{bmatrix} ,$$

and

$$\sum_{i=0}^{\infty} \lambda^{i} E(m^{B}_{t+1+i}|\Phi_t) = \begin{bmatrix}1 & 0\end{bmatrix}\gamma^{B}(\lambda)^{-1} \begin{bmatrix} E(m^{B}_{t+1}|\Phi_t) \\ E(y^{B}_{t+1}|\Phi_t) \end{bmatrix} + \begin{bmatrix}1 & 0\end{bmatrix}\gamma^{B}(\lambda)^{-1}$$

$$\sum_{j=1}^{r-1} \left(\sum_{h=j+1}^{r} \lambda^{h-j} \gamma^{B}_{h} \right) L^{j} \begin{bmatrix} m^{B}_{t+1} \\ y^{B}_{t+1} \end{bmatrix},$$

where

$$\gamma^{W}_{h} = \begin{bmatrix} \gamma^{W}_{11,h} & \gamma^{W}_{12,h} \\ \gamma^{W}_{21,h} & \gamma^{W}_{22,h} \end{bmatrix}$$

$$\gamma^{W}_{zj}(L) = \sum_{h=0}^{n_{zj}} \gamma^{W}_{zj,h} L^{h} ,$$

$W = A,B$ and $r = \max_{z,j} (n_{zj})$.

Analogous relationships have been obtained for m^A_t, y^A_t and y^B_t . Replacing in equation (17) and simplifying, we obtain equation (19).

(6) Shifting equation (7) forward by one period, we obtain

$$s^{BA}_{t+1} = p^{B}_{t+1} - p^{A}_{t+1} + \delta(L)\, \xi_{t} + x_{t+1} .$$

Taking expectations, we obtain

$$E(s^{BA}_{t+1} | \Phi_t) = E(p^{B}_{t+1} | \Phi_t) - E(p^{A}_{t+1} | \Phi_t) + \delta(L) E(\xi_t | \Phi_t)$$

or using equation (7),

$$E(s^{BA}_{t+1} | \Phi_t) = E(p^{B}_{t+1} | \Phi_t) - E(p^{A}_{t+1} | \Phi_t) + \delta(L)\,(s^{BA}_{t} - p^{B}_{t} + p^{A}_{t})$$

(7) We shift equation (25) forward by one period, take expectations and rearrange terms in such a way that on the left hand side appears a term like

$$\left(1 - \frac{b^{B}}{1+b^{B}} L^{-1}\right) E(p^{B}_{t+1} | \Phi_t) .$$

We then apply the principle that

$$(1 - \lambda L^{-1})\, Y_t = X_t \Leftrightarrow Y_t = \sum_{i=0}^{\infty} \lambda^{i} X_{t+i}$$

and obtain a forward solution for $E(p^{B}_{t+1} | \Phi_t)$ which we substitute back into equation (25) to obtain, shifting the whole forward by one period, the p^{B}_{t+1} reduced form above.

(8) If we want to estimate this relationship empirically, we add an

error term η_{t+1} to the right hand side of equation (28), which we assume to be serially uncorrelated and with the property that $E(\eta_{t+1} \mid \Phi_t) = 0$.

(9) The stimuli forecasting equations (10), (10'), (11), (11'), (12) and (12') can be written in matrix form

$$\begin{bmatrix} \gamma^W_{11}(L) & \gamma^W_{12}(L) & \gamma^W_{13}(L) \\ \gamma^W_{21}(L) & \gamma^W_{22}(L) & \gamma^W_{23}(L) \\ \gamma^W_{31}(L) & \gamma^W_{32}(L) & \gamma^W_{33}(L) \end{bmatrix} \begin{bmatrix} m^W_{t+1} \\ y^W_{t+1} \\ r^W_{t+1} \end{bmatrix} = \begin{bmatrix} u^W_{t+1} \\ v^W_{t+1} \\ w^W_{t+1} \end{bmatrix},$$

$$W = A,B, \text{ where } \quad \gamma^W_{zj,0} = \begin{cases} 1 \text{ if } z = j \\ 0 \text{ if } z \neq j \end{cases}$$

(10) The matrices of adjoints $A^B(\lambda)$ and $A^A(\theta)$ can be written as

$$A^B(\lambda) = \begin{bmatrix} A^B_{11}(\lambda) & A^B_{12}(\lambda) & A^B_{13}(\lambda) \\ A^B_{21}(\lambda) & A^B_{22}(\lambda) & A^B_{23}(\lambda) \\ A^B_{31}(\lambda) & A^B_{32}(\lambda) & A^B_{33}(\lambda) \end{bmatrix} ;$$

$$A^A(\theta) = \begin{bmatrix} A^A_{11}(\theta) & A^A_{12}(\theta) & A^A_{13}(\theta) \\ A^A_{21}(\theta) & A^A_{22}(\theta) & A^A_{23}(\theta) \\ A^A_{31}(\theta) & A^A_{32}(\theta) & A^A_{33}(\theta) \end{bmatrix}$$

(11) If $b^A = b^B = b$, $\lambda = \dfrac{b}{1+b} = \theta$ and equation (30) becomes:

$$(30') \quad s^{BA}_{t+1} - E(s^{BA}_{t+1} \mid \Phi_t) = \frac{1-\lambda}{\Delta^{B'}} \Bigg\{ \Big[A^B_{11}(\lambda) - a^B A^B_{21}(\lambda) + bA^B_{31}(\lambda) \Big] u^B_{t+1} +$$

$$+ \Big[A^B_{12}(\lambda) - a^B A^B_{22}(\lambda) + bA^B_{32}(\lambda) \Big] v^B_{t+1} +$$

$$+ \Big[A^B_{13}(\lambda) - a^B A^B_{23}(\lambda) + bA^B_{33}(\lambda) - \frac{(1-\delta(L))^{-1}\Delta^{B'}}{1-\lambda} \Big] w^B_{t+1} \Bigg\} +$$

$$- \frac{1-\lambda}{\Delta^{A'}} \Bigg\{ \Big[A^A_{11}(\lambda) - a^A A^A_{21}(\lambda) + bA^A_{31}(\lambda) \Big] u^A_{t+1} +$$

$$+ \Big[A^A_{12}(\lambda) - a^A A^A_{22}(\lambda) + bA^A_{32}(\lambda) \Big] v^A_{t+1} +$$

$$+ \Big[A^A_{13}(\lambda) - a^A A^A_{23}(\lambda) + bA^A_{33}(\lambda) - \frac{(1-\delta(L))^{-1}\Delta^{A'}}{1-\lambda} \Big] w^A_{t+1} \Bigg\} + \eta_{t+1} .$$

$\Delta^{B'}$ and $\Delta^{A'}$ are the determinants of $\gamma^B(\lambda)$ and $\gamma^A(\lambda)$ respectively. $A^B_{zj}(\lambda)$ and $A^A_{zj}(\lambda)$, z,j=1,2,3, are elements of the matrices of adjoints of $\gamma^B(\lambda)$ and $\gamma^A(\lambda)$ respectively.

(12) More precisely, some set of $k(k = k^A + k^B)$ identifying restrictions on the k elements of the covariance between ϵ_{t+1} and u^A_{t+1}, u^B_{t+1} is needed to identify the k β parameters.

(13) The number of parameters of the constrained version of equation

(3') above is $k = k^A + k^B$. (The number of elements of β^A and β^B.) The number of parameters of the unconstrained version of equation (3') is $(k^A + k^B) + M\left[(N^A \times k^A) + (N^B \times k^B)\right]$, the number of elements of the vectors β^A, β^B, and of the matrices $\gamma_h^{A*}, \gamma_h^{B*}$, h=1,...,M, respectively.

Identification is not affected by the cross country restrictions mentioned in section 3 above. If the absolute values of the corresponding parameters of the stimuli forecasting equations and of the structural (money demand) equations are assumed to be equal in the two countries, the number of free parameters is reduced by one half, but so is the number of regressors in equation (3') unconstrained (X_{t+1} and Z_{t+1} are then composed of cross country differentials of the corresponding variables which affect the formation of exchange rates and "news" in both countries). If the structural (money demand) parameters only are restricted across country, the reduction in the number of free parameters will be equal to the reduction in the number of regressors in the unconstrained version of equation (3').

(14) If $k^A = k^B = 1$, the number of parameters of the unconstrained forecast error equation is

$$(k^A + k^B) + M\left[(N^A \times k^A) + (N^B \times k^B)\right] = 2 + M\left[N^A + N^B\right]$$

and coincides with the number of estimable parameters (the number of regressors of the unconstrained forecast error relationship).

(15) The rationale for the identifying restrictions set forth above can be easily seen noting that equation (3') can be rewritten as:

$$(3')\quad s_{t+1}^{BA} - f_t^{BA} = \sum_{j=1}^{k^B} \beta_j^B X_{jt+1}^B - \sum_{j=1}^{k^A} \beta_j^A X_{jt+1}^A + - \sum_{h=1}^{M}\left[\sum_{z=1}^{N^B} Z_{zt+1-h}^B \varphi_z^{hB} - \sum_{z=1}^{N^A} Z_{zt+1-h}^A \varphi_z^{hA}\right] + \epsilon_{t+1},$$

where

$$\varphi_z^{hB} = \sum_{j=1}^{k^B} \gamma_{jz}^{hB*} \beta_j^B,$$

$$\varphi_z^{hA} = \sum_{j=1}^{k^A} \gamma_{jz}^{hA*} \beta_j^A;$$

$$X_{jt+1}^W, \; j=1,\ldots,k^W,$$

are elements of the row vector X_{t+1}^W;

$$Z_{zt+1-h}^W, \quad z=1,\ldots,N^W,$$

are elements of the row vector Z_{t+1-h}^W, W=A,B.

Identification of the coefficients of the unconstrained version of equation (3') relevant for the rational expectations tests requires that

$$\sum_{j=1}^{k^B} \gamma_{jz}^{hB*} \beta_j^B \quad \text{and} \quad \sum_{j=1}^{k^A} \gamma_{jz}^{hA*} \beta_j^A$$

provide separate estimates of γ_{jz}^{hB*} and γ_{jz}^{hA*}.

This will be the case only if the identification restrictions set forth above are satisfied.

(16) An approach of this kind has been used by D.L. Hoffman and D.E. Schlagenhauf (1985). This is but a special case of the identifying restrictions set forth above. The X_{jt+1}^W processes will be own autoregressions when $1 \leq P^{Wh} \leq N^W$, $Z_{t+1}^W = X_{t+1}^W$, W=A,B, and when the off diagonal elements of the γ_h^W matrices are nil $\forall$h, i.e. when only one nonzero element appears in any row of the γ_h^W matrices in question. It should be noticed that if the stimuli forecasting equations are own autoregressions we cannot draw a distinction between anticipated and unanticipated stimuli if the estimation of system (I) is performed by means of a two-step single equation approach (i.e. we can test for efficiency but not for the "news"). As pointed out by B.T. McCallum (1979), observational equivalence in the sense of Sargent (T.J. Sargent (1976)) between the constrained and unconstrained versions of equation (3') will hold unless the cross equation rational expectations restrictions ($\gamma^W = \gamma^{W*}$) are maintained in the estimation of system (I).

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