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The fragility of social capital

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**Abstract** - This paper addresses two hot topics of the contemporary debate, social capital and economic growth. Our theoretical analysis sheds light on decisive but so far neglected issues: how does social capital accumulate over time? Which is the relationship between social capital, technical progress and economic growth in the long run? The analysis shows that the economy may be attracted by alternative steady states, depending on the initial social capital endowments and cultural exogenous parameters representing the relevance of social interaction and trust in well-being and production. When material consumption and relational goods are substitutable, the choice to devote more and more time to private activities may lead the economy to a “social poverty trap”, where the cooling of human relations causes a progressive destruction of the entire stock of social capital. In this case, the relationship of social capital with technical progress is described by an inverted U-shaped curve. However, the possibility exists for the economy to follow a virtuous trajectory where the stock of social capital endogenously and unboundedly grows. Such result may follow from a range of particular conditions, under which the economy behaves as if there was no substitutability between relational activities and material consumption.

**JEL-Classification:** A13, O33, O49, Z13.

**Key-words:** Economic growth, Technical progress, Transitional dynamics, Social capital, Social norms

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# 1 Introduction

The positive role of social capital in growth and development is one of the most popular and controversial theses standing in the contemporary economic debate. Even if theoretical and empirical research have produced a huge amount of papers on the topic, the literature still suffers from some endemic problems. Firstly, economic studies generally focus on the possible effects of social capital on a range of supposed outcomes, with a strong attention to the concepts of innovation and growth (Knack and Keefer, 1997, Rodrik, 1999, Zak and Knack, 2001). However, we lack a micro-founded theory explaining the causal mechanisms, and the reverse effect of growth and technical progress on human interactions is neglected.

Secondly, there is a surprising lack of studies addressing the sources of social capital. The dominant view in sociology is that social capital is a public good incidentally accumulated as a by-product of diverse relational activities (Coleman, 1988). This form of capital exists only when it is shared, i.e. it needs some kind of interaction to be exploited to the pursuit of common interests (Bourdieu, 1980, Coleman, 1990). By contrast, theoretical studies in economics are basically tied down to Becker's (1974) theory of social interaction, where social capital is an individual resource created through rational self-interested investment decisions.

This paper aims to contribute both to the growth and the social capital literature by shedding light on the following questions: which are the dynamics of social capital's interaction with economic growth? How is social capital created and accumulated over time? Does technical progress influence the accumulation dynamics? Do we have to trust the folk wisdom according to which technology is one of the major responsible for the widespread social isolation of our time?

To reach this goal, we develop a dynamic model where social capital enters as an argument in the agents' utility functions and as an input in private and relational goods' production functions. Social capital is here defined as the sum of networks of trust-intensive relations that the agents incidentally develop through the simultaneous production and consumption of relational goods. Relational goods are a distinctive type of good that can only be enjoyed if shared with others<sup>1</sup>.

We integrate hints from sociology and political science into an economic framework by assuming that the stock of social capital accumulated as a by-product of social participation functions as a public good, which benefits all individuals equally. In this way, we explicitly model also one of the most popular claims raised in the social capital debate, i.e. its role in private production. Specifically, the assumption we draw from the empirical literature is that the diffusion of trust boosted by shared values and social participation reduces the average cost of transactions, thereby fostering the economic activity and growth

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<sup>1</sup>Relational goods are different from private goods, which are enjoyed alone, and standard public goods, which can be enjoyed by any number (Uhlener, 1989). A peculiarity of relational goods is that it is virtually impossible to separate their production from consumption, since they easily coincide (e.g. dining out or playing football with friends).

(Knack and Keefer, 1997, Rodrik, 1999, Zak and Knack, 2001, Guiso, Sapienza and Zingales, 2004, Beugelsdijk and van Shaik, 2005, Antoci, Sacco and Vanin, 2007)<sup>2</sup>.

Our analysis starts from the observation that, even if private and relational goods satisfy different needs, the private good can be consumed as a substitute for the relational one. For example, when the social environment is poor, people may be constrained to replace human interactions, e.g. joining the meetings of a cultural circle or playing football with friends, with private consumption, e.g. staying at home and watching a tv show or playing a virtual match against the computer.

The analysis of agents' time allocation choices accounts for the alternative cases of zero or positive substitutability between the two types of good. If the relational good cannot be replaced by the private one, then social participation is constant at a given level independently of the stock of social capital, i.e. it cannot be compressed under a certain threshold. However, it seems to be more realistic to assume a positive degree of substitutability between the two types of goods. In such a case, an increase in the stock of social capital can enhance the productivity of time spent in social interaction, thereby encouraging the consumption of relational goods. This process results in a positive level of participation. As argued by Antoci, Sacco and Vanin (2007), it is in fact more rewarding to interact with people in a context that offers many options for socially enjoyed leisure. Social participation in turn creates and strengthens durable ties increasing the stock of social capital. The model allows to determine the value of participation maximizing the rate of growth of social capital, through a relation that can be interpreted as a "golden rule" for the accumulation of social capital.

The analysis of dynamics produces less optimistic results. The economy may be attracted by alternative steady states, depending on the initial social capital endowments and cultural exogenous parameters representing the importance of social interaction and trust in well-being and production. The possibility exists for the economy to fall down in a "social poverty trap", where agents devote all their time to private activities. In this case, the cooling of relations may cause a progressive destruction of the entire stock of social capital.

According to common wisdom, the link leading from economic growth to social isolation is technology. In an interview to the New York Times<sup>3</sup>, Robert Putnam stated that "The distinctive effect of technology has been to enable us to get entertainment and information while remaining entirely alone. It's fundamentally bad because the lack of social contact, the social isolation means

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<sup>2</sup>The thesis implicitly shared by cited studies is that participation in networks raises the likelihood to repeat interactions, thereby improving both the diffusion of information and the opportunity of retaliation against free-riders. As a consequence, the importance of reputation and the opportunity cost of free-riding are higher. In a social fabric rich of networks and participation opportunities, actors have strong incentives to avoid opportunistic behaviours in prisoners' dilemma kind of situations. The higher likelihood that obligations will be carried out makes actors' behaviour more easily foreseeable, even if they are unknown.

<sup>3</sup>"Who Americans Are and What They Do, in Census Data" by Sam Roberts, published: December 15, 2006

that we don't share information and values and outlook that we should".

Introducing exogenous technical progress in the production function of private goods indeed leads to interesting modifications in social capital's accumulation dynamics. If there is not substitutability between private and relational goods, the stock of social capital can grow indefinitely along any trajectories or alternatively tend to zero according to the value of the model's parameters. Its trend relative to technical progress can be monotonic (always increasing or decreasing) or can experience an initial decline followed by a growth, but not vice versa (a growth followed by a decline is impossible).

Under the more realistic assumption of positive substitutability, two cases are possible: a) if, as a consequence of the substitution process, social capital's productivity in the private sector grows too much, then the stock of social capital may always tend to zero whichever are its initial endowments. Private activities infinitely expand at the expenses of social interaction, thereby trapping the economy in a situation of social poverty. In this case, social capital experiences a growth followed by a decline, so that its relationship with technical progress is described by an inverted-u shaped curve. Since technical progress is in turn positively correlated with GNP, our result sounds as a proof of Putnam's (2000) intuition of the inverted-u shaped relationship between social capital and income. b) Otherwise, if the growth process is not too fast, and if social capital's initial endowments are high enough, its stock can grow indefinitely along certain trajectories. However, other paths exist along which social capital still tends to zero.

The outline of the paper is as follows. Section two briefly reviews the literature on social capital and growth. Sections four and five present the model and analyse its dynamics. Section six studies the effect of exogenous technical progress on social capital's dynamics. The paper is closed by a discussion of results and policy implications.

## 2 Social capital and well-being

Human relations matter for people's happiness and well-being. Such a statement sounds so obvious that most of the people would be surprised to know that the analysis of social interactions is quite a novelty in the contemporary economic debate. What economists have long questioned is whether the social sphere of individuals can influence their *economic* action. The first answers we can find in the economic literature are definitely positive. In the *Lectures on Jurisprudence*, Smith states: "A dealer is afraid of losing his character, and is scrupulous in performing every engagement. When a person makes perhaps 20 contracts in a day, he cannot gain so much by endeavouring to impose on his neighbors, as the very appearance of a cheat would make him lose. Where people seldom deal with one another, we find that they are somewhat disposed to cheat, because they can gain more by a smart trick than they can lose by the injury which it does their character." (1763/1978, 539).

Smith's argument basically refers to trading relationships, but it can be

easily generalized to every kind of interaction. A social environment rich of participation opportunities, which allow people to meet frequently, creates a fertile ground for nurturing trust and shared values. The higher likelihood of repeated interactions increases the opportunity cost of free-riding in prisoners dilemma kind of situations, thereby making the agents behaviour more foreseeable and causing an overall reduction of uncertainty. In other words, social interactions are a vehicle for the diffusion of information and trust which inevitably affect the economic activity, so that the two spheres of individual action continuously fade one another. Such claims more or less explicitly ground most of the contemporary social capital research in economics. Smith's view is similar to modern theories of social capital developed in sociology by Granovetter (1973), who argues that social ties work as *bridges* through which information and trust spread across diverse communities and socioeconomic backgrounds. On the other side, the value of reputation and social approval is considered by Smith one of the main engines of human action. The importance of social approval is further stressed by Bentham (1789), who makes a step forward by mentioning 15 "basic wants" grounding the economic action. Among them, the author lists the pleasures of being on good terms with others, the pleasures of a good name, the pleasures resulting from the view of any pleasure supposed to be possessed by the beings who may be the objects of benevolence, and the pleasures resulting from the view of any pain supposed to be suffered by the beings who may become the objects of malevolence. The agents described by classical economists are thus deeply rooted in the social context, and their economic activities strictly depend on the complex of norms and relationships surrounding them. Personal relations and structures (or networks) of such relations seem to be crucial in generating trust and discouraging malfeasance. This is the same claim advanced by Granovetter (1985) in his essay on the embeddedness of the economic action. Paradoxically, this work stems from a critique to both classical and neoclassical economics, which, according to the author, have in common an undersocialized view of actors. While it is commonly acknowledged that, in the work of Marx and Ricardo, economic actors are deeply socialized, a relevant number of authors find traces of the typical codewords pervading the social capital literature (e.g. trust, norms, values, altruism, sympathy, and so on) in the work of Smith as well (Becker, 1981, Bruni, 2000, Fontaine, 2000).

According to Manski (2000), the narrowing of economics ended by the 1970s: "Since then a new phase has been underway, in which the discipline seeks to broaden its scope while maintaining the rigor that has become emblematic of economic analysis" (2000, 115). The work of Becker (1974) is probably the most notable contribution to the economic analysis of social interaction. In his essay, the author adopts the neoclassical framework "To analyze interactions between the behaviour of some persons and different characteristics of other persons" (1974, 1063). Becker recalls Bentham's concept of "basic wants" to state that the agents' well-being - and their behaviour as well - depend on the satisfaction of a range of fundamental needs, labelled as *commodities*:

$$U_i = U_i(Z_1, \dots, Z_m)$$

Each commodity  $Z_j$  ( $J = 1, \dots, m$ ) is produced through a technology based on market goods and services  $x_j$ , time  $t_j$ , education, experience and environmental variables  $E^i$ , and the characteristics of other persons which may affect the production of the specific  $j$  commodity,  $R_j^1, \dots, R_j^r$ . (Becker, 1974). In other words,  $R_j$  represent the influence of social interaction on the satisfaction of the basic needs given by  $Z$  commodities. The author gives the example of the  $Z_1$  need of  $i$  to stand out in his occupation. In this case,  $R_1^1, \dots, R_1^r$  could be the opinions of  $i$  held by other persons in the same occupation. In the same way, if  $Z_2$  is the need to have a good job,  $R_2^1, \dots, R_2^r$  could be the social contacts helping the individual in his job search. It is straightforward that, if  $Z_3$  is  $i$ 's want to make an impression on his neighbors, their opinions  $R_3^1, \dots, R_3^r$  are complementary to market goods like an expensive car or elegant clothes in the production of  $Z_3$ .

In this way, social interactions are crucial for fulfilling the material needs of life as well as market goods influence the social sphere of agents. Generalizing the definition of relational goods provided in section 1 to include every interaction not directly leading to a market exchange, we could state that, in Becker's model, relational goods enter as arguments in the individuals' production function of certain material goods (which are finally provided through transactions), as well as market goods enter in the production function of relational ones (provided through social interaction). The implicit novelty (in respect to previous neoclassical theory) is that  $i$  can change  $R_j^1, \dots, R_j^r$  by his own effort (e.g. working hard to earn his colleagues' respect, expanding his social network to raise the likelihood to get a good job, and so on). In such a framework, assuming to the seek of simplicity only one single commodity produced with a single good, the agent's problem is as follows:

$$\begin{cases} \text{Max} U_i(x, R) \\ R = D_i + h \\ p_x x + p_r h = I_i \end{cases}$$

where  $h$  measures the effect of  $i$ 's effort,  $D_i$  is the level of  $R$  when  $i$  makes not effort,  $I_i$  is his money income, and  $p_r h$  is the money amount which must be spent to influence  $R$ .

Influences and relations  $R_j^1, \dots, R_j^r$  shaping the social sphere of the individual are a means for the pursuit of his personal interest: they can be considered as Becker's social capital. This theory has been decisively influential in the following literature in the field, but also received wide criticism. We do not want to get the hearth of the debate on the neoclassical approach to social interaction, which has led some authors, and Becker himself, to talk about "economics imperialism". In this context, we just would like to point out that the social capital described by the author is a private good, i.e. an individual resource to be exploited by single agents to the purposes of utility maximization. This view is in part consistent with the early thesis of Banfield (1958). In the *Moral Basis of a Backward Society*, the author explains the backwardness of Southern Italy through the "amoral familism" of its inhabitants, who would not cooperate with

one another outside the boundaries of their immediate families. According to the author, in the Italian Mezzogiorno any family activity was oriented towards the protection and consolidation of the isolated family unit. “Moral” activity (i.e. any action informed by moral norms of trust and reciprocity) was seen as limited to family insiders, with outsiders only being significant as a potential resource to exploit for the family.

Both Becker and Banfield do not explicitly mention the concept of social capital; still, their view of social interaction is commonly considered as an anticipation of the contemporary literature on the topic. The social capital of the authors is not a shared resource, and neither requires the participation of others to be “activated”. Its accumulation is a consequence of rational investment decisions which take place solely at the individual level, since agents’ ties do not add up to create a “public” stock possibly benefiting the community as a whole.

Such position is in sharp contrast with the approaches proposed by the sociological and political science literature, where social capital is treated as a collective resource or, in other terms, as a public good (Bourdieu, 1980, 1986, Coleman, 1988, 1990, Putnam, Leonardi and Nanetti, 1993).

The model presented in section 4 aims to implement the hints coming from modern sociology into a traditional framework of analysis. The creation of social capital does not depend on agents’ rational investment decisions. Rather, it is a by-product of social participation activities related to the production and consumption of relational goods. The stock resulting from its accumulation is a collective resource, which affects the well-being of each individual by contributing to the production of both private and relational goods. In section 5 we will show how the impossibility to internalize the positive externalities of participation may lead to a systematic underinvestment in social capital progressively eroding its stock, and plunging the economy into a situation of social poverty.

### 3 Social capital and growth

In the recent debate, the public good features of social capital are now commonly acknowledged. It is possible to argue that the functioning of the economy itself relies on those institutions (whether formal or informal) that the literature often groups together under the common label of social capital (e.g. norms of trust and reciprocity, moral sanctions, networks of relationships, and organizations). If that is the case, then the economy’s possibility of “reproducing” itself, thereby experiencing sustainable growth, depends also on its ability to foster - or, at least, to preserve - its endowments of social capital. On this basis, the idea is spreading that a better understanding of the role of norms and relationships would be a crucial step for the advancement of modern political economy. In the last two decades, such idea has informed the development of a huge number of empirical studies, exploring the effect of social capital on an immense range of phenomena, from political participation to the institutional performance, from health to corruption, from the efficiency of public services to



the economic success of countries. The seminal study in the field is the so-called “Italian work” by Putnam, Leonardi and Nanetti (1993), which explains the different institutional and economic performance of the Italian regions as the result of the influence exerted by some aspects of the social structure, summarized into the multidimensional concept of “social capital”. Just like Becker’s work, this study received wide criticism in the social science debate of the 1990s. However, it posed a milestone for social capital theory, which registered an explosive development in the following decade, rapidly involving the attention of economists. Empirical research in economics has been prompted by a couple of notable tests of Putnam’s hypotheses: Knack and Keefer (1997) find that trust and civic norms are unrelated to horizontal networks and have a strong impact on economic performance in a sample of 29 market economies. La Porta et al (1997) show that firms’ scale is strongly related to trust in strangers (positively), and trust in family (negatively).

We do not want to discuss the strengths and weaknesses of the empirical research here (see Durlauf and Fafchamps, 2005, and Sabatini, 2007, for exhaustive reviews). To our purposes, it is important to point out that this strand of the literature unanimously converges on the claim that one of the mechanisms through which social capital impacts economic efficiency is by enhancing the level of trust and reducing uncertainty. As anticipated in the previous section, the basic idea is that a social environment rich of participation opportunities is a fertile ground for nurturing trust and shared values, where repeated interactions foster the diffusion of information and raise reputations’ relevance. The higher opportunity cost of free-riding in prisoners’ dilemma kind of situations makes the agents’ behaviour more foreseeable causing an overall reduction of uncertainty. Therefore, an increase in trust-based relations reduce monitoring costs and, more in general, the average cost of transactions. Our model accounts for these claims through the assumption that social capital enters as an input in the production of private goods.

The *direction* of the nexus connecting social capital to development is another subject of contention in the empirical literature. Even if most studies agree on the positive role of networks and trust, we have few evidence on the reverse effect possibly played by growth and development on the accumulation of social capital.

On this regard, two different interpretations can be suggested.

On the one side, higher levels of wealth and development may reinforce social participation and the development of shared norms of trust and reciprocity. For example, the mutual assistance mechanisms developed within the family unit, which Banfield referred to as a result of the “amoral familism”, could be looked on also as a defence reaction against situations of underdevelopment and social poverty, where both the state’s and market’s institutions are weak. In a pioneer study, Bilson (1982) shows that civil liberties are strongly associated with per capita income (and positively but not significantly related to recent income growth). The author’s interpretation is that economic performance determines freedoms, rather than the other way around. More recently, Sabatini (2008) finds that in the Italian regions strong family ties are reinforced by backward-

ness and lower levels of income, while the strength of bridging ties related to voluntary organizations is significantly and positively associated to higher levels of human development.

On the other side, it is possible to argue that the pressure exerted on time by economic growth may act as a factor hampering the consolidation of social ties, thereby leading to an erosion of the social capital's stock. As everyday life experience suggests, time constraints are ever more pressing in modern societies. A historical symptom of the higher opportunity cost of time is the rise of female participation to the labour market, which Putnam (2000) indicates as one of the main causes of the decline of social capital in the United States. In his best-seller *Bowling Alone*, the author shows that several indexes of civic engagement and social capital exhibit an inverted U-shaped trend over time in the United States. Costa and Kahn (2003) confirm that being in the labour force is a statistically significant, negative predictor of membership in organizations, as well as of the probability of visiting friends and entertaining at home. Once again, causality is a major problem: drawing on GSS data, Bartolini and Bilancini (2007) point out the relevance of the reverse effect, finding evidence that social, "non-instrumental", interactions reduce labour supply, with a greater impact on women.

At the theoretical level, the negative externalities of growth have been quite neglected by the literature. Routledge and von Amsberg (2003) show that the technological change and innovation generally associated to growth influence social capital by rising labour mobility: higher levels of turnover may hamper the consolidation of social ties, both inside and outside the workplace. Moreover, we can argue that the uncertainty of future incomes related to increased mobility affects any form of long-term planning of life activities such as marriage and procreation. Antoci, Sacco and Vanin (2007) show that the expansion of market activities implies a growing pressure on time, which compresses the social sphere of individuals. As a consequence, the process of economic growth may be accompanied by a progressive "social impoverishment" of the economy. As the authors state, an early account of this process is given by Hirsch (1976): "As the subjective cost of time rises, pressure for specific balancing of personal advantage in social relationships will increase. ... Perception of the time spent in social relationships as a cost is itself a product of privatized affluence. The effect is to whittle down the amount of friendship and social contact ... . The huge increase in personal mobility in modern economies adds to the problem by making sociability more of a public and less of a private good. The more people move, the lower are the chances of social contacts being reciprocated directly on a bilateral basis" (p.80).

The model in section four addresses all the hypotheses described above: agents chose how to allocate their time between labour, aimed at the production of private goods, and social participation activities, that generate social capital as a by-product. In section five we analyze the long-run dynamics of the interaction between growth and the accumulation of social capital, showing that the economy may fall in a social poverty trap depending on the initial social capital endowments and cultural exogenous parameters representing the

relevance of social interaction and trust in well-being and production. The effects of exogenous technical progress on social capital's dynamics are analyzed in section six.

## 4 The model

In the light of the arguments discussed above, the notion of social capital taken into account within our framework is defined as the sum of networks of trust-intensive relations that the agents develop through the simultaneous production and consumption of relational goods. The accumulation of social capital is highly path-dependent: on the one side, it improves the technology of production of relational goods; on the other side greater social participation taking the form of higher levels of relational goods' production and consumption fosters the consolidation of ties and trust among people, thereby increasing the stock of social capital as a by-product. Of course we cannot exclude the possibility that agents engage in social activities for instrumental purposes (for example, to achieve a better job). However, following hints from rational choice sociology (Coleman, 1990), we assume that most of the times the creation of interpersonal ties does not depend on rational investment decisions. Rather, it is an incidental, not necessary, by-product of social participation. The resulting stock is a public resource, which enters as an argument in every agent's utility function due of its ability to contribute to the production of both private and relational goods.

We consider a population of size 1 constituted by a continuum of individuals. We assume that, in each instant of time  $t$ , the well-being of the individual  $i \in [0, 1]$  depends on the consumption of two goods: a private good,  $C_i(t)$ , and a socially provided good,  $B_i(t)$ . We assume that  $B_i(t)$  is produced by the joint action of the time devoted by agent  $i$  to social activities,  $s_i(t)$ , by the average social participation  $\bar{s}(t) = \int_0^1 s_i(t) di$  and by the stock of social capital  $K_s(t)$ :

$$B_i(t) = F(s_i(t), \bar{s}(t), K_s(t)) \quad (1)$$

The time agent  $i$  does not spend for social participation,  $1 - s_i(t)$ , is used as input in the production of the output  $Y_i(t)$  of the private good. As suggested by Antoci, Sacco and Vanin (2005, 2008), we assume that social capital plays also a role in the production process of the private good. In addition, for simplicity, we assume that  $C_i(t) = Y_i(t)$ , that is  $Y_i(t)$  cannot be accumulated, and that the production process of  $Y_i(t)$  requires only the input  $1 - s_i(t)$  and  $K_s(t)$ :

$$C_i(t) = Y_i(t) = G(1 - s_i(t), K_s(t)) \quad (2)$$

The functions  $F$  and  $G$  in (1) and (2) are assumed to be strictly increasing in each argument. Note that, in such context,  $1 - s_i(t)$  can be interpreted as the time spent both *to produce* and *to consume*  $C_i(t)$ .

As in Antoci, Sacco and Vanin (2005, 2007, 2008), social capital is not accumulated through specific investment decisions but as a by-product of social participation. In addition, since job interactions and the consumption of some kinds

of private goods may stimulate durable social interactions, we assume also a positive spillover on social capital accumulation due to the production/consumption of the private good:

$$\dot{K}_s(t) = H[\bar{B}(t), \bar{Y}(t)] - \eta K_s(t) \quad (3)$$

where  $\dot{K}_s(t)$  indicates the time derivative of  $K_s(t)$ , the parameter  $\eta > 0$  represents the depreciation rate of  $K_s(t)$ ,  $\bar{B}(t) = \int_0^1 B_i(t) di$  and  $\bar{Y}(t) = \bar{C}(t) = \int_0^1 Y_i(t) di$  are the average production/consumption of the socially provided good and the average production/consumption of the private good, respectively.

For simplicity, we consider the following specifications for (1),(2),(3):

$$Y_i(t) = [1 - s_i(t)] \cdot K_s^\alpha(t)$$

$$B_i(t) = s^\varepsilon(t) \cdot \bar{s}_i^{1-\varepsilon}(t) \cdot K_s^\gamma(t)$$

$$\dot{K}_s(t) = [\bar{Y}(t)]^\beta \cdot [\bar{B}(t)]^\delta - \eta K_s(t) \quad (4)$$

with  $\varepsilon \in (0, 1)$ , and  $\alpha, \beta, \gamma, \delta > 0$ .

Note that a positive average social participation  $\bar{s}(t) > 0$  is essential for the production/consumption of  $B_i(t)$ , that is  $B_i(t) = 0$  if  $\bar{s}(t) = 0$  whatever the values of  $s_i(t)$  and  $K_s(t)$  are. If  $\gamma > \alpha$ , then the role of social capital is more relevant in the production/consumption of relational goods than in the production/consumption of private goods.

According to (4),  $\bar{B}(t)$  and  $\bar{Y}(t)$  are both essential factors for social capital accumulation, that is the stock of social capital  $K_s(t)$  decreases (that is  $\dot{K}_s(t) < 0$ ) if  $\bar{B}(t) = 0$  or  $\bar{Y}(t) = 0$ .

Finally, we assume that the instantaneous utility function of individual  $i$  is:

$$U_i[C_i(t), B_i(t)] = \ln C_i(t) + b \ln [B_i(t) + d \cdot C_i(t)] \quad (5)$$

where  $b > 0$  and  $d \geq 0$  are parameters. Notice that, according to this function, the private good can satisfy needs different from those satisfied by  $B_i(t)$  (the part  $\ln C_i(t)$  in the utility function); however, it can also be consumed as a substitute for  $B_i(t)$  (the part  $\ln [B_i(t) + d \cdot C_i(t)]$  in the utility function). The parameter  $d$  measures the degree of substitutability between  $B_i(t)$  and  $C_i(t)$ ; if  $d = 0$ , then there is no substitutability between the two goods. Note that, if  $d > 0$ , the mixed partial derivative of  $U_i$  with respect to  $C_i(t)$  and  $B_i(t)$  is strictly negative:

$$\frac{\partial^2 U_i}{\partial C_i \partial B_i} = -\frac{bd}{(dC_i + B_i)^2} < 0$$

This means that the lower the value of  $B_i(t)$  is, the greater the marginal utility of private consumption  $C_i(t)$  will be. Finally, the parameter  $b$  in (5) measures the relative importance of the consumption of the socially provided good with respect to that of the private good.

Letting  $r$  the discounting rate of future utility, the  $i$ -agent's maximization problem is:

$$\max_{s_i(t)} \int_0^{+\infty} \{\ln C_i(t) + b \ln [B_i(t) + d \cdot C_i(t)]\} e^{-rt} dt \quad (6)$$

subject to the dynamic constraint (4). The agent  $i$  solves problem (6) taking as exogenously given the value of  $K_s(t)$  and average values  $\bar{s}(t)$ ,  $\bar{B}(t)$  and  $\bar{Y}(t)$ ; this is due to the fact that the choice of  $s_i(t)$  by agent  $i$  doesn't modify average values, being economic agents a continuum. As a consequence, by applying the *Maximum Principle* to problem (6) we obtain that the choices of individual  $i$  don't depend on the co-state variable associated to  $K_s(t)$  (that is the "price" of  $K_s(t)$ ) in the maximization problem (6). Consequently, to solve problem (6), agent  $i$ , in each instant  $t$ , chooses the value of  $s_i(t)$  maximizing the value of the instantaneous utility function (5). This implies that the dynamics of  $K_s(t)$  we study do not represent the social optimum. However, since agent  $i$  plays the best response  $s_i(t)$ , given the others' choices, the trajectories followed by  $K_s(t)$  represent Nash equilibria. In fact, along these trajectories, no agent has incentive to modify his choices if the other agents do not revise their ones as well.

To simplify our analysis, in this paper we focus on *symmetric Nash equilibria*. In particular, we assume that individuals are identical and make the same choices. This assumption allows us to study the choices of a *representative agent*; so we can omit the subscript  $i$  in the variables  $s_i(t)$ ,  $B_i(t)$ ,  $Y_i(t)$  and  $C_i(t)$  writing simply  $s(t)$ ,  $B(t)$ ,  $Y(t)$  and  $C(t)$ . In this symmetric Nash equilibrium context, we have that ex ante average values  $\bar{s}(t)$ ,  $\bar{B}(t)$  and  $\bar{Y}(t)$  are considered as exogenously given by the representative agent; however, once chosen  $s(t)$ , ex post it holds:

$$\bar{s}(t) = s(t)$$

$$\bar{B}(t) = \bar{s}^\varepsilon(t) \cdot \bar{s}^{1-\varepsilon}(t) \cdot K_s^\gamma(t) = \bar{s}(t) \cdot K_s^\gamma(t) = s(t) \cdot K_s^\gamma(t)$$

$$\bar{Y}(t) = [1 - \bar{s}(t)] \cdot K_s^\alpha(t) = [1 - s(t)] \cdot K_s^\alpha(t)$$

In this context, the representative agent, in each instant of time  $t$ , chooses  $s(t)$  solving the following static optimization problem:

$$\max_s \left\{ \ln [(1 - s) \cdot K_s^\alpha] + b \ln [s^\varepsilon \bar{s}^{1-\varepsilon} K_s^\gamma + d \cdot (1 - s) \cdot K_s^\alpha] \right\} \quad (7)$$

taking as exogenously given the values of  $\bar{s}$  and  $K_s$ . The solution  $s(t)$  of the problem (7) has to be substituted to  $\bar{s}(t)$  in the equation (4) which, under our symmetric Nash equilibria assumption, can be written as follows:

$$\begin{aligned}
\dot{K}_s(t) &= [\overline{Y}(t)]^\beta \cdot [\overline{B}(t)]^\delta - \eta K_s(t) = \\
&= [[1 - \overline{s}(t)] \cdot K_s^\alpha(t)]^\beta \cdot [\overline{s}^\varepsilon(t) \cdot \overline{s}^{1-\varepsilon}(t) \cdot K_s^\gamma(t)]^\delta - \eta K_s(t) = \\
&= [1 - \overline{s}(t)]^\beta [\overline{s}(t)]^\delta \cdot K_s^{\alpha\beta+\gamma\delta}(t) - \eta K_s(t)
\end{aligned} \tag{8}$$

Where  $\beta$  and  $\delta$  are strictly positive parameters. Note that under dynamics (8), social capital accumulation is negative if  $\overline{s}(t) = 0$  (no social participation) or if  $\overline{s}(t) = 1$  (the production/consumption of the private good is equal to zero).

**Remark:** According to (8), the value of  $\overline{s}(t)$  that, given  $K_s(t)$ , maximizes the rate of growth of  $K_s(t)$  is:

$$\overline{s}(t)=s^g:=\frac{\delta}{\beta+\delta}$$

The latest expression can be interpreted as the “golden rule” for the accumulation of social capital. Note that  $s^g \rightarrow 0$  if  $\delta \rightarrow 0$ , and  $s^g \rightarrow 1$  when the value of  $\beta$  is negligible relative to that of  $\delta$ .

## 5 Analysis of the model

### 5.1 The time allocation choice

For simplicity, we limit our analysis to “robust” cases only, that is those not corresponding to equality conditions on parameters’ values. The following proposition concerns the choice of  $s(t)$  by the representative agent (due to space constraints, propositions’ proofs are omitted if straightforward).

**Proposition 1** *Problem (7) admits solution and the time allocation choice  $s^*(t)$  of the representative agent is:*

1. if  $\gamma - \alpha > 0$

$$s^*(t) = \begin{cases} 0, & \text{if } K_s(t) \leq \left(\frac{d(b+1)}{b\varepsilon}\right)^{\frac{1}{\gamma-\alpha}} \\ \frac{b\varepsilon K_s^{\gamma-\alpha}(t) - d(b+1)}{(1+b\varepsilon)K_s^{\gamma-\alpha}(t) - d(b+1)}, & \text{if } K_s(t) > \left(\frac{d(b+1)}{b\varepsilon}\right)^{\frac{1}{\gamma-\alpha}} \end{cases} \tag{9}$$

2. if  $\gamma - \alpha < 0$

$$s^*(t) = \begin{cases} \frac{b\varepsilon K_s^{\gamma-\alpha}(t) - d(b+1)}{(1+b\varepsilon)K_s^{\gamma-\alpha}(t) - d(b+1)}, & \text{if } K_s(t) \leq \left(\frac{d(b+1)}{b\varepsilon+1}\right)^{\frac{1}{\gamma-\alpha}} \\ 0, & \text{if } K_s(t) > \left(\frac{d(b+1)}{b\varepsilon+1}\right)^{\frac{1}{\gamma-\alpha}} \end{cases} \tag{10}$$

The intuition behind the results of this proposition is simple. In our model, social capital produces contrasting pressures on the representative agent’s time

allocation choices. If  $\gamma - \alpha > 0$  (remember that  $\gamma$  and  $\alpha$  are the exponents of  $K_s$  in the production functions of  $B(t)$  and  $Y(t)$ , respectively), then (ceteris paribus) an increase of the stock of social capital  $K_s$  has the effect to increase the productivity of time spent for social participation  $s(t)$  relative to that of time spent in the production and consumption of the private good. Vice versa if  $\gamma - \alpha < 0$ . Therefore, when  $\gamma - \alpha > 0$ , we have  $s(t) = 0$  (respectively,  $s(t) > 0$ ) for “low” (respectively, “high”) values of the stock of social capital  $K_s$ . Viceversa if  $\gamma - \alpha < 0$ . An implication of such result is that, according to equation (4), the stock of social capital cannot grow indefinitely when  $\gamma - \alpha < 0$  while this may be the case when  $\gamma - \alpha > 0$ .

Notice that if there is no substitutability between private consumption  $C$  and the socially provided good  $B$ , that is  $d = 0$  in (5), it holds  $d(b+1) = 0$ ; therefore  $\left(\frac{d(b+1)}{b\varepsilon}\right)^{\frac{1}{\gamma-\alpha}} = 0$  if  $\gamma - \alpha > 0$ . In such context that, by (9)-(10), the following proposition holds.

**Proposition 2** *Under the assumption  $d = 0$ , problem (7) gives the following time allocation choice  $s^*(t)$  of the representative agent:*

$$s_{d=0}^*(t) = \frac{b\varepsilon}{1 + b\varepsilon}, \text{ whatever the value of } K_s(t) \text{ is;}$$

Therefore, if  $d = 0$ , social participation is constant and such that  $1 > s^*(t) > 0$ , whatever the value of  $K_s(t)$  is. Notice that social participation  $s^*(t)$  in (9)-(10) is (ceteris paribus) a strictly decreasing function of the parameter  $d$ , which measures the degree of substitutability between  $B$  and  $C$ . Therefore, it always holds  $s^*(t) < s_{d=0}^*(t)$ .

## 5.2 Dynamics of social capital accumulation and well-being analysis

Even if we have considered very simple specifications of functions  $F$ ,  $G$  and  $H$ , there may exist multiple steady states and poverty traps, as we will see.

The following result concerns the evolution of representative agent’s well-being along the trajectories under dynamics (8).

**Proposition 3** *Along the trajectories of (8), the values of the utility function  $U$  and of  $K_s$  are positively correlated. This implies that if there exist two steady states  $K_s^1$  and  $K_s^2$  such that  $K_s^2 > K_s^1$ , then  $K_s^2$  Pareto-dominates  $K_s^1$ ; that is  $K_s^1$  is a poverty trap.*

The following proposition defines social capital dynamics resulting from the time allocation choices of the representative agent described in Proposition (1).

**Proposition 4** *If  $\gamma - \alpha > 0$ , social capital dynamics are given by:*

$$\dot{K}_s = \left\{ \left[ \frac{K_s^{\gamma-\alpha}}{(1+b\varepsilon)K_s^{\gamma-\alpha}-d(b+1)} \right]^\beta \cdot \left[ \frac{-\eta K_s}{\frac{b\varepsilon K_s^{\gamma-\alpha}-d(b+1)}{(1+b\varepsilon)K_s^{\gamma-\alpha}-d(b+1)}} \right]^\delta \cdot K_s^{\alpha\beta+\gamma\delta} - \eta K_s \right\} \quad (11)$$

for, respectively,  $K_s \leq \left(\frac{d(b+1)}{b\varepsilon}\right)^{\frac{1}{\gamma-\alpha}}$  and  $K_s > \left(\frac{d(b+1)}{b\varepsilon}\right)^{\frac{1}{\gamma-\alpha}}$ .

If  $\gamma - \alpha < 0$ , they are given by:

$$\dot{K}_s = \left\{ \left[ \frac{K_s^{\gamma-\alpha}}{(1+b\varepsilon)K_s^{\gamma-\alpha}-d(b+1)} \right]^\beta \cdot \left[ \frac{b\varepsilon K_s^{\gamma-\alpha}-d(b+1)}{(1+b\varepsilon)K_s^{\gamma-\alpha}-d(b+1)} \right]^\delta \cdot K_s^{\alpha\beta+\gamma\delta} - \eta K_s \right. \quad (12)$$

for, respectively,  $K_s \leq \left(\frac{d(b+1)}{b\varepsilon+1}\right)^{\frac{1}{\gamma-\alpha}}$  and  $K_s > \left(\frac{d(b+1)}{b\varepsilon+1}\right)^{\frac{1}{\gamma-\alpha}}$ .

Notice that, if  $d = 0$  (that is  $C$  and  $B$  are not substitutes), the dynamics of social capital accumulation become:

$$\dot{K}_s = \frac{(b\varepsilon)^\delta}{(1+b\varepsilon)^{\beta+\delta}} \cdot K_s^{\alpha\beta+\gamma\delta} - \eta K_s \quad (13)$$

and the corresponding dynamic regimes are described by the following proposition.

**Proposition 5** *Under the assumption of no substitutability  $d = 0$ , the basic features of dynamics are the following (whatever the sign of the expression  $\gamma - \alpha$  is):*

*There always exist two steady states:*

$$\overline{K}_s = \left[ \frac{\eta(1+b\varepsilon)^{\beta+\delta}}{(b\varepsilon)^\delta} \right]^{\frac{1}{\alpha\beta+\gamma\delta-1}} \quad \text{and} \quad K_s = 0$$

*If  $\alpha\beta + \gamma\delta < 1$ , then the economy approaches  $\overline{K}_s$  (the steady state  $K_s = 0$  is repulsive) whatever the initial value of  $K_s > 0$  is.*

*If  $\alpha\beta + \gamma\delta > 1$ , then the steady state  $\overline{K}_s$  is always repulsive while the steady state  $K_s = 0$  is locally attractive. In particular, if the initial value of  $K_s$  is lower than the threshold value  $\overline{K}_s$ , the economy reaches the steady state  $K_s = 0$ ; if it is higher than  $\overline{K}_s$ , the economy follows a trajectory along which the value of  $K_s$  grows indefinitely (that is  $K_s \rightarrow +\infty$ ). Thus the steady state  $K_s = 0$  is a poverty trap.*

**Remark:** *Note that equation (13) can be written as follows:*

$$\frac{\dot{K}_s}{K_s} = \frac{(b\varepsilon)^\delta}{(1+b\varepsilon)^{\beta+\delta}} \cdot K_s^{\alpha\beta+\gamma\delta-1} - \eta$$

*Therefore, if the economy follows a trajectory along which  $K_s \rightarrow +\infty$  and if  $\alpha\beta + \gamma\delta > 1$ , then along such trajectory the growth rate  $\dot{K}_s/K_s$  is an increasing function of  $K_s$ . The growth rate  $\dot{K}_s/K_s$  is constant only in the limit case  $\alpha\beta + \gamma\delta = 1$ ; in such case it holds:*

$$\frac{\dot{K}_s}{K_s} = \frac{(b\varepsilon)^\delta}{(1+b\varepsilon)^{\beta+\delta}} - \eta$$



When there is some degree of substitutability between  $C$  and  $B$ , that is  $d > 0$ , dynamics become more complicated and are characterized by the following Propositions.

**Proposition 6** *Under the assumption of substitutability  $d > 0$  and if  $\gamma - \alpha > 0$ , the dynamics are characterized by the following properties:*

1. *The steady state  $K_s = 0$  is always locally attractive (whatever the value of the expression  $\alpha\beta + \gamma\delta$  is).*
2. *If  $\alpha\beta + \gamma\delta < 1$  then the number of steady states with  $K_s > 0$  is (generically) zero (see Figure 1) or two (see Figure 2); if two steady states  $K_s^1$  and  $K_s^2$  ( $K_s^1 < K_s^2$ ) exist, then  $K_s^2$  is attractive while  $K_s^1$  is repulsive.*
3. *If  $\alpha\beta + \gamma\delta > 1$  then there exists at least a steady state with  $K_s > 0$ ; furthermore, the number of steady states with  $K_s > 0$  is one (see Figure 3) or three; if an unique steady state exists, then it is repulsive; if three steady states  $K_s^1$ ,  $K_s^2$  and  $K_s^3$  ( $K_s^1 < K_s^2 < K_s^3$ ) exist, then  $K_s^2$  is attractive while  $K_s^1$  and  $K_s^3$  are repulsive. Finally, if the initial value of  $K_s$  is greater than  $K_s^*$ , where  $K_s^*$  is the steady state with the highest value of  $K_s$ , then there exists a unbounded endogenous growth path with increasing well-being along which<sup>4</sup>  $s \rightarrow \frac{b\varepsilon}{1+b\varepsilon}$ .*

**Proposition 7** *Under the assumption of substitutability ( $d > 0$ ) and if  $\gamma - \alpha < 0$ , the dynamics are characterized by the following properties:*

1. *The value of  $K_s$  always approaches a steady state value lower than the upper bound  $\left(\frac{d(b+1)}{b\varepsilon+1}\right)^{\frac{1}{\gamma-\alpha}}$  (see proposition 4), whatever the value of the expression  $\alpha\beta + \gamma\delta$  is. Consequently, the stock of social capital  $K_s$  cannot grow indefinitely.*
2. *If  $\alpha\beta + \gamma\delta < 1$ , then the steady state  $K_s = 0$  is always repulsive; there exists at least a steady state with  $K_s > 0$ ; the number of steady states with  $K_s > 0$  is (generically) one (see Figure 4) or three (see Figure 5); steady states with an odd index are attractive and those with an even index are repulsive. Whatever the initial value of  $K_s$  is, the economy approaches a steady state with  $K_s > 0$ .*
3. *If  $\alpha\beta + \gamma\delta > 1$ , then the steady state  $K_s = 0$  is always locally attractive; the number of steady states with  $K_s > 0$  is zero (see Figure 6) or*

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<sup>4</sup>Values of simulations in **Figure 1**:  $\alpha=0.01$ ,  $\beta=0.4$ ,  $\gamma=0.82$ ,  $\delta = 1$ ,  $\varepsilon=0.12$ ,  $\eta=0.15$ ,  $b = 4$ ,  $d = 0.03$ ; **Figure 2**:  $\alpha=0.01$ ,  $\beta=0.4$ ,  $\gamma=0.82$ ,  $\delta = 1$ ,  $\varepsilon=0.12$ ,  $\eta=0.04$ ,  $b = 4$ ,  $d = 0.03$ ; **Figure 3**:  $\alpha=0.4$ ,  $\beta=0.9$ ,  $\gamma=0.95$ ,  $\delta = 1$ ,  $\varepsilon=0.7$ ,  $\eta=0.3$ ,  $b = 4$ ,  $d = 0.3$ . For  $\alpha=0.9$ ,  $\beta=0.5$ ,  $\delta = 0.85$ ,  $\gamma = 0.95$ ,  $\delta = 0.85$ ,  $\varepsilon=0.5$ ,  $\eta=0.11$ ,  $b = 4$ ,  $d = 0.05$  we obtain three positive fixed point of coordinates  $K_s^* = 0.005$  (repulsive),  $K_s^{**} = 0.01$  (attractive),  $K_s^{***} = 103$  (repulsive).

two; the steady states with an odd index are repulsive and those with an even index are attractive<sup>5</sup>.

Note that necessary and sufficient conditions allowing for endogenous unbounded growth of the stock of social capital are:

$$\begin{aligned}\gamma - \alpha &> 0 \\ \alpha\beta + \gamma\delta &> 1\end{aligned}$$

When these conditions hold, the poverty trap  $K_s = 0$  is always locally attracting; however, when the initial value of  $K_s$  is high enough, the economy follows a trajectory along which  $K_s \rightarrow +\infty$ . As stated above, along such trajectory we have that:

$$s \rightarrow \frac{b\varepsilon}{1 + b\varepsilon}$$

and the equation (11) “tends” (for  $K_s \rightarrow +\infty$ ) to the equation:

$$\dot{K}_s = \frac{(b\varepsilon)^\delta}{(1 + b\varepsilon)^{\beta+\delta}} \cdot K_s^{\alpha\beta+\gamma\delta} - \eta K_s$$

which coincides with the equation (13) describing social dynamics under the assumption  $d = 0$  (no substitutability between  $C$  and  $B$ ). This allows to say that when the stock of social capital becomes high enough, then the dynamics under the assumption  $d = 0$  and those under the assumption  $d > 0$  become “very similar”<sup>6</sup>.

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<sup>5</sup>Values of simulazions in **Figure 4**:  $\alpha=0.9$ ,  $\beta=0.5$ ,  $\gamma=0.4$ ,  $\delta = 1$ ,  $\varepsilon=0.7$ ,  $\eta=0.04$ ,  $b = 4$ ,  $d = 0.05$ ; **Figure 5**:  $\alpha=0.9$ ,  $\beta=0.8$ ,  $\gamma=0.4$ ,  $\delta = 1$ ,  $\varepsilon=0.7$ ,  $\eta=0.24$ ,  $b = 4$ ,  $d = 0.05$ ; **Figure 6**:  $\alpha=0.9$ ,  $\beta=0.8$ ,  $\gamma=0.7$ ,  $\delta = 1$ ,  $\varepsilon=0.5$ ,  $\eta=0.12$ ,  $b = 4$ ,  $d = 0.4$ . For  $\alpha=0.9$ ,  $\beta=0.5$ ,  $\delta = 0.85$ ,  $\gamma = 0.95$ ,  $\delta = 0.85$ ,  $\varepsilon=0.5$ ,  $\eta=0.11$ ,  $b = 4$ ,  $d = 0.05$  we obtain three positive fixed point of coordinates  $K_s^* = 1.35$  (attractive),  $K_s^{**} = 1.48$  (repulsive),  $K_s^{***} = 1.58$  (attractive).

<sup>6</sup>Remember that from equation (13) it follows that if  $\alpha\beta + \gamma\delta > 1$ , then the growth rate  $\dot{K}_s/K_s$  is positive and increasing along the trajectories where  $K_s \rightarrow +\infty$ . Remember also that unbounded growth of  $K_s$ , with a constant rate of growth  $\dot{K}_s/K_s$  of social capital, can occur if and only if  $\alpha\beta + \gamma\delta = 1$ ; in such case, along the trajectories where  $K_s \rightarrow +\infty$  we have that  $\dot{K}_s/K_s$  approaches the value  $\frac{(b\varepsilon)^\delta}{(1+b\varepsilon)^{\beta+\delta}} - \eta$ .

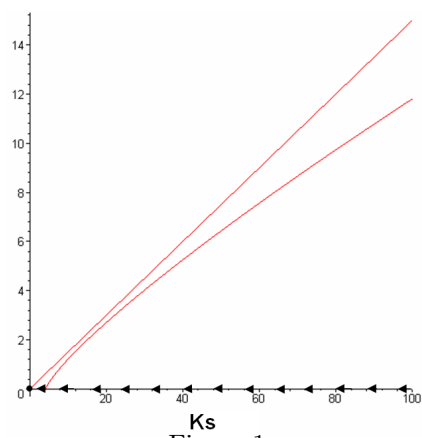


Figure 1

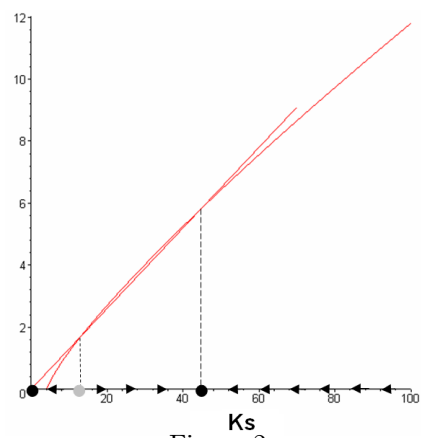


Figure 2

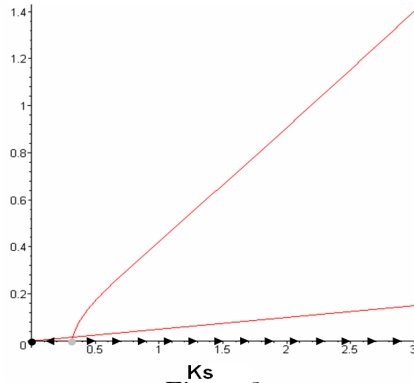


Figure 3

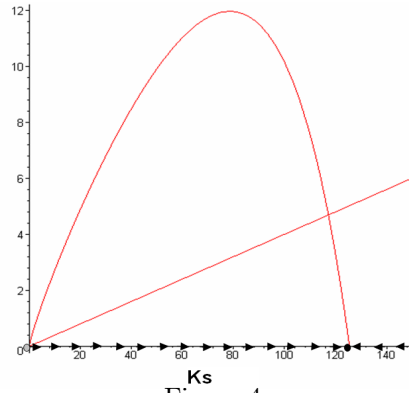


Figure 4

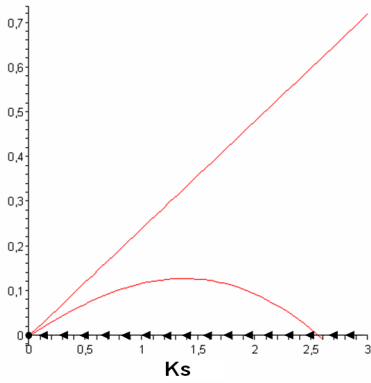


Figure 5

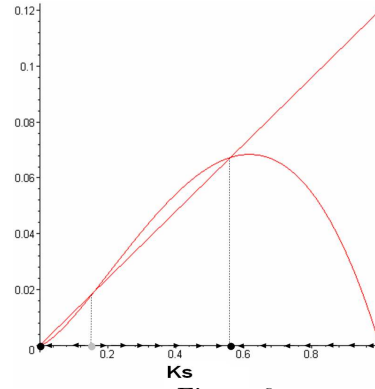


Figure 6

## 6 The effects of (exogenous) technical progress

In this section we study the effects on dynamics generated by the introduction of exogenous technological progress  $T(t)$  in the production function of the private good:

$$Y(t) = T(t) \cdot [1 - s(t)] \cdot K_s^\alpha(t) \quad (14)$$

where the growth rate of  $T$  is assumed to be given by the equation:

$$\dot{T}(t) = \sigma T(t) \quad (15)$$

where  $\sigma$  is a strictly positive parameter representing the growth rate of  $T$ . In such context, the time allocation choice  $s^*(t)$  by the representative agent is described by the following proposition.

**Proposition 8** *If the production function of the private good is given by (14), then problem (7) admits solution and the time allocation choice  $s^*(t)$  of the representative agent is:*

*If  $\gamma - \alpha > 0$*

$$s^*(t) = \begin{cases} 0, & \text{if } K_s(t) \leq \left( \frac{d \cdot (b+1) \cdot T(t)}{b\varepsilon} \right)^{\frac{1}{\gamma-\alpha}} \\ \frac{b\varepsilon K_s^{\gamma-\alpha}(t) - d \cdot (b+1) \cdot T(t)}{(1+b\varepsilon)K_s^{\gamma-\alpha}(t) - d \cdot (b+1) \cdot T(t)}, & \text{if } K_s(t) > \left( \frac{d \cdot (b+1) \cdot T(t)}{b\varepsilon} \right)^{\frac{1}{\gamma-\alpha}} \end{cases} \quad (16)$$

*If  $\gamma - \alpha < 0$*

$$s^*(t) = \begin{cases} \frac{b\varepsilon K_s^{\gamma-\alpha}(t) - d \cdot (b+1) \cdot T(t)}{(1+b\varepsilon)K_s^{\gamma-\alpha}(t) - d \cdot (b+1) \cdot T(t)}, & \text{if } K_s(t) \leq \left( \frac{d \cdot (b+1) \cdot T(t)}{b\varepsilon+1} \right)^{\frac{1}{\gamma-\alpha}} \\ 0, & \text{if } K_s(t) > \left( \frac{d \cdot (b+1) \cdot T(t)}{b\varepsilon+1} \right)^{\frac{1}{\gamma-\alpha}} \end{cases} \quad (17)$$

## 6.1 The case without substitutability between $C$ and $B$

If there is not substitutability between private consumption  $C$  and relational goods  $B$  (i.e. if  $d = 0$ ), social participation is constant and strictly positive  $s^*(t) = b\varepsilon/(1+b\varepsilon)$ ; in such context, the dynamics of social capital are given by the equation:

$$\dot{K}_s = \frac{(b\varepsilon)^\delta}{(1+b\varepsilon)^{\beta+\delta}} \cdot T^\beta K_s^{\alpha\beta+\gamma\delta} - \eta K_s \quad (18)$$

where the evolution of  $T$  is described by the differential equation (15). Note that  $\dot{K}_s = 0$  for  $K_s = 0$  and along the graph of the function:

$$K_s = \left[ \frac{(b\varepsilon)^\delta}{\eta(1+b\varepsilon)^{\beta+\delta}} \right]^{\frac{1}{1-\alpha\beta-\gamma\delta}} \cdot T^{\frac{\beta}{1-\alpha\beta-\gamma\delta}} \quad (19)$$

which is increasing (decreasing) in  $T$  if  $\alpha\beta+\gamma\delta < 1$  (respectively, if  $\alpha\beta+\gamma\delta > 1$ ). Note that, if  $\alpha\beta+\gamma\delta < 1$ , then it holds  $\dot{K}_s < 0$  above the curve (19) and  $\dot{K}_s > 0$  below it; viceversa if  $\alpha\beta+\gamma\delta > 1$ .

The basic features of dynamics under the assumption of no substitutability are described by the following Proposition.

**Proposition 9** *Dynamics (18) have the following properties:*

**1)** *If  $\alpha\beta+\gamma\delta < 1$ , then both  $T$  and  $K_s$  grow without bound (i.e.  $\lim_{t \rightarrow +\infty} T(t) = +\infty$  and  $\lim_{t \rightarrow +\infty} K_s(t) = +\infty$ ) along any trajectory starting with a strictly positive initial value of  $K_s$  (see Figure 7). Among these trajectories, there exists a trajectory represented by the equation:*

$$K_s = \left[ \frac{(1-\alpha\beta-\gamma\delta)(b\varepsilon)^\delta}{[\beta\sigma + \eta(1-\alpha\beta-\gamma\delta)](1+b\varepsilon)^{\beta+\delta}} \right]^{\frac{1}{1-\alpha\beta-\gamma\delta}} \cdot T^{\frac{\beta}{1-\alpha\beta-\gamma\delta}} \quad (20)$$

along which the growth rates of  $T$  and  $K_s$  are given by:

$$\frac{\dot{K}_s}{K_s} = \frac{\beta}{(1 - \alpha\beta - \gamma\delta)} \frac{\dot{T}}{T}. \quad (21)$$

where  $\dot{T}/T = \sigma$  by assumption and  $\frac{\dot{K}_s}{K_s} > \frac{\dot{T}}{T}$  if and only if  $\frac{\beta}{1 - \alpha\beta - \gamma\delta} > 1$ . Along the remaining trajectories, the growth rate of  $K_s$  approaches the value given in (21) as  $t \rightarrow +\infty$ .

2) If  $\alpha\beta + \gamma\delta > 1$ , then two dynamic regimes are possible:

2.1) Case:  $\eta(\alpha\beta + \gamma\delta - 1) - \beta\sigma < 0$ . In such case, both  $T$  and  $K_s$  grow without bound along any trajectory starting with a strictly positive initial value of  $K_s$  (see Figure 8). Along these trajectories, the growth rate of  $K_s$  is always increasing (without upper bound).

2.2) Case:  $\eta(\alpha\beta + \gamma\delta - 1) - \beta\sigma > 0$ . In such case, in the plane  $(T, K_s)$  there exists a trajectory  $\Gamma$  along which the value of the product  $T^\beta \cdot K_s^{\alpha\beta + \gamma\delta - 1}$  is constant (this implies that  $\lim_{t \rightarrow +\infty} K_s(t) = 0$  along such trajectory). The value of  $K_s$  along the trajectories above  $\Gamma$  behaves as in case 2.1) while  $\lim_{t \rightarrow +\infty} K_s(t) = 0$  along the trajectories below  $\Gamma$  (see Figure 9)<sup>7</sup>.

**Proof.** This Proposition can be proved by defining a new variable:

$$x := \frac{T^\beta}{K_s^{1 - (\alpha\beta + \gamma\delta)}} \quad (22)$$

and by calculating its time derivative:

$$\dot{x} = \frac{T^\beta}{K_s^{1 - (\alpha\beta + \gamma\delta)}} \left( \beta \frac{\dot{T}}{T} - (1 - (\alpha\beta + \gamma\delta)) \frac{\dot{K}_s}{K_s} \right) = \quad (23)$$

$$= x \left[ [\beta\sigma + \eta(1 - \alpha\beta - \gamma\delta)] - \frac{(b\varepsilon)^\delta}{(1 + b\varepsilon)^{\beta + \delta}} (1 - \alpha\beta - \gamma\delta)x \right] \quad (24)$$

Equation (24) has two stationary states:

$$x^* = 0 \text{ and } x^{**} = \frac{[\beta\sigma + \eta(1 - \alpha\beta - \gamma\delta)] (1 + b\varepsilon)^{\beta + \delta}}{(1 - \alpha\beta - \gamma\delta)(b\varepsilon)^\delta}$$

Notice that, if  $\alpha\beta + \gamma\delta < 1$ , then  $x^{**} > 0$  is globally attracting in the positive  $x$ -axis; since, by (23),  $\dot{x} = 0$  if and only if the condition (21) holds, point 1) is proved.

To prove point 2), note that, if  $\eta(\alpha\beta + \gamma\delta - 1) - \beta\sigma > 0$ , then  $x^{**} > 0$  is repelling while  $x^*$  is locally attracting; if  $\eta(\alpha\beta + \gamma\delta - 1) - \beta\sigma < 0$

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<sup>7</sup>Values of simulazions in **Figure 7**:  $\alpha=0.3$ ,  $\beta=0.21$ ,  $\gamma=0.2$ ,  $\delta = 0.4$ ,  $\varepsilon=0.4$ ,  $\eta=0.02$ ,  $\sigma = 0.4$ ,  $b = 0.1$ ; **Figure 8**:  $\alpha=0.82$ ,  $\beta=0.71$ ,  $\gamma=0.7$ ,  $\delta = 0.92$ ,  $\varepsilon=0.6$ ,  $\eta=0.02$ ,  $\sigma = 0.4$ ,  $b = 10.1$ ; **Figure 9 and Figure 10**:  $\alpha=0.9$ ,  $\beta=0.91$ ,  $\gamma=0.9$ ,  $\delta = 0.92$ ,  $\varepsilon=0.6$ ,  $\eta=0.02$ ,  $\sigma = 0.01$ ,  $b = 10.1$ .

and  $x^*$  is repelling. Therefore, in case 2.1), if the initial value of  $x$ ,  $x_0$ , coincides with the stationary value  $x^{**}$ , then the economy follows the trajectory  $\Gamma$ . If  $x_0 < x^{**}$ , along the trajectory it holds  $x = T^\beta K_s^{\alpha\beta+\gamma\delta-1} \rightarrow 0$  and consequently  $K_s \rightarrow 0$ , being  $T \rightarrow +\infty$ . Note that the trajectories starting with  $x_0 < x^{**}$  correspond to the trajectories below  $\Gamma$ , in the plane  $(T, K_s)$ . If  $x_0 > x^{**}$ , along this trajectory (which corresponds to a trajectory above  $\Gamma$ , in the plane  $(T, K_s)$ ) it holds  $x = T^\beta K_s^{\alpha\beta+\gamma\delta-1} \rightarrow +\infty$ . This implies that  $K_s \rightarrow +\infty$ ; this is the case since along all the trajectories crossing the curve (19) it holds  $K_s \rightarrow +\infty$ ; so, if there would exist trajectories above  $\Gamma$  along which  $K_s \rightarrow 0$ , then a separatrix should exist between the two sets of trajectories which, by continuity properties, should give rise to a further fixed point of dynamics (24), different from  $x^*$  and  $x^{**}$ . Point 2.2) can be proved by similar arguments. ■

**Remark:** *It is interesting to note that:*

1. *If there is no substitutability between  $C$  and  $B$ , then  $K_s$  can grow without bound, whatever the sign of the expression  $\gamma - \alpha$  (we will show that this result no more holds when there is substitutability).*
2. *Along the trajectories under dynamics (18), the evolution of  $K_s$  is monotonic (always increasing or decreasing) or follows a U-shaped path, according to which  $K_s$  is initially decreasing and then becomes definitively increasing. It is worth to stress such result in that, as we will see, if there is substitutability between  $C$  and  $B$ , then the evolution of  $K_s$  can take the shape of an inverted U curve.*

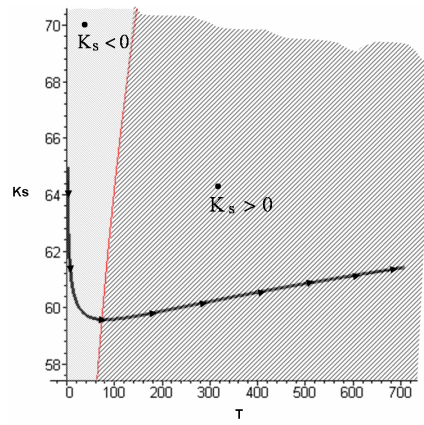


Figure 7

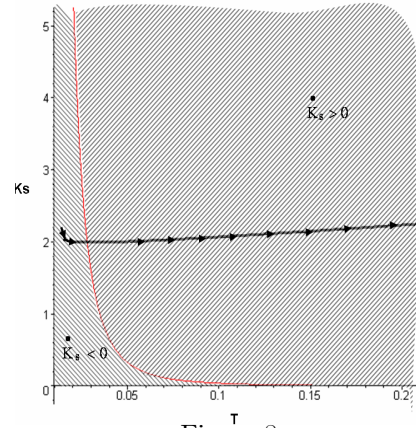


Figure 8

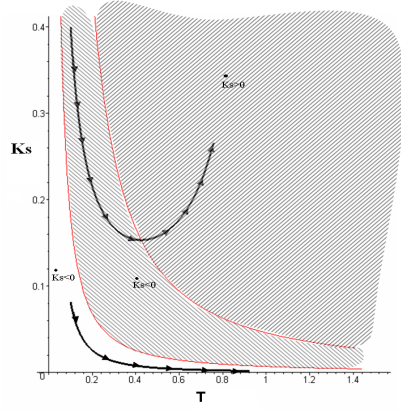


Figure 9

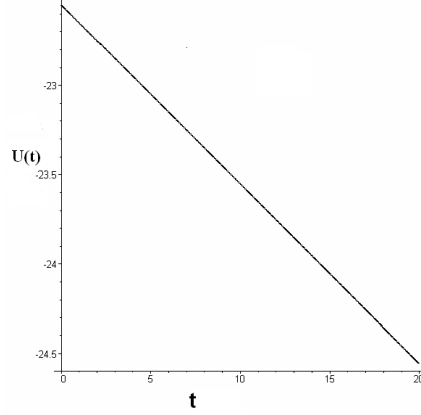


Figure 10

It is easy to check that, along the trajectories below the separatrix  $\Gamma$ , well-being may be decreasing; that is, the increase in  $T$  may be not able to compensate the reduction in  $K_s$ .

## 6.2 The case with substitutability between $C$ and $B$

If  $d > 0$ , then social participation  $s^*$  depends on the values of  $T$  and  $K_s$  and can assume the value 0. In particular, the graph of the function:

$$K_s = \left( \frac{d(b+1)T}{b\varepsilon} \right)^{\frac{1}{\gamma-\alpha}} \quad (25)$$

separates, in the plane  $(T, K_s)$ , the region where  $s^* = 0$  (below the curve) from that where  $s^* > 0$  (above it). Note that the function (25) is increasing (decreasing) in  $T$  if  $\gamma - \alpha > 0$  (respectively, if  $\gamma - \alpha < 0$ ).

In the region where  $s^* = 0$ , social capital dynamics is given by  $\dot{K}_s = -\eta K_s < 0$  and the slope of trajectories  $\frac{dK_s}{dT} = -\frac{\eta K_s}{\sigma T}$  is negative; this implies that along the trajectories below the curve (25),  $T$  increases and  $K_s$  decreases.

Above the curve (25), social capital dynamics are given by:

$$\dot{K}_s = \left[ \frac{K_s^{\gamma-\alpha}}{(1+b\varepsilon)K_s^{\gamma-\alpha} - d(b+1)T} \right]^\beta \cdot \left[ \frac{b\varepsilon K_s^{\gamma-\alpha} - d(b+1)T}{(1+b\varepsilon)K_s^{\gamma-\alpha} - d(b+1)T} \right]^\delta \cdot T^\beta K_s^{\alpha\beta+\gamma\delta} - \eta K_s \quad (26)$$

where  $T$  evolves according to the differential equation (15).

The basic features of dynamics (26) are described in the following Proposition.

**Proposition 10** *If  $\gamma - \alpha > 0$ , then region under the curve (25) is positively invariant under dynamics: every trajectory entering such region cannot leave it. Along the trajectories under the curve (25), the value of  $K_s$  approaches 0 for  $t \rightarrow +\infty$  (see Figure 11.).*



If  $\gamma - \alpha < 0$ , then region above (under) the curve (25) is positively invariant if  $\sigma/\eta(\alpha - \gamma) \geq 1$  (respectively, if  $\sigma/\eta(\alpha - \gamma) < 1$ ); in any case, along every trajectory the value of  $K_s$  approaches 0 for  $t \rightarrow +\infty$  (see Figures 12 and 13<sup>8</sup>..).

**Proof.** To check the results on positive invariance of the sets under or above the curve (25), we have simply to compare the slope of the trajectories  $\frac{dK_s}{dT} = -\frac{\eta K_s}{\sigma T}$ , evaluated along the curve (25), and the slope of (25). To prove the results about the evolution of  $K_s$ , notice that in the set where  $s^* = 0$  it holds  $\frac{dK_s}{dt} = -\eta K_s$  and consequently  $K_s(t) = K_s(0) \cdot e^{-\eta t}$ , where  $K_s(0)$  is the initial value of  $K_s$ . Finally, note that, in case  $\gamma - \alpha < 0$ , every trajectory lies definitively in the set under the curve (25) or in the set above it; in any case, since  $T \rightarrow +\infty$ , the value of  $K_s$  approaches 0. ■

Whatever the sign of the expression  $\gamma - \alpha$  is, along the trajectories crossing the curve (25) we will show that the evolution of  $K_s$  can take an inverted U-shape, differently from the case without substitutability.

According to the above Proposition, a necessary condition to have unbounded growth of  $K_s$  is  $\gamma - \alpha > 0$ ; that is, the importance (measured by  $\alpha$ ) of  $K_s$  as input in the production process of the private good must be lower than its importance (measured by  $\gamma$ ) in the production process of the relational good.

To analyze the behaviour of  $K_s$  in case  $\gamma - \alpha > 0$  we introduce the following definition.

**Definition 11** A Regular Growth Curve (RGC) is a curve in the plane  $(T, K_s)$  along which the rate of growth of  $T$  is equal to the exogenously given value  $\sigma$  while the rate of growth of  $K_s$  is equal to a constant strictly positive value  $g$ , possibly different from  $\sigma$ .

Notice that along a RGC, being  $\frac{\dot{T}}{T} = \sigma$  and  $\frac{\dot{K}_s}{K_s} = g$ , it holds  $\frac{dK_s}{dT} = \frac{\dot{K}_s}{\dot{T}} = \frac{gK_s}{\sigma T}$ ; consequently a RGC is the graph of a function  $K_s(T) = AT^{\frac{g}{\sigma}}$ , where  $A$  is a positive arbitrary constant. RGCs are not trajectories under our dynamics. However, we aim to show that there exist values of  $g$  and  $A$ , that we will denote by  $\bar{g}$  and  $\bar{A}$  respectively, such that along the trajectories starting “near” to  $K_s(T) = \bar{A}T^{\frac{\bar{g}}{\sigma}}$  for  $T$  high enough, it holds  $\frac{\dot{K}_s}{K_s} \rightarrow \bar{g}$  as  $T \rightarrow +\infty$ .

Notice that  $K_s(T) = \bar{A}T^{\frac{\bar{g}}{\sigma}}$  must lie above the curve (25), for  $T$  high enough; this require that  $g$  must satisfy the necessary condition:  $\frac{g}{\sigma} > \frac{1}{\gamma - \alpha}$ , where  $\frac{1}{\gamma - \alpha} > 1$ . That is, it must hold  $g > \frac{\sigma}{\gamma - \alpha}$  (where  $\frac{\sigma}{\gamma - \alpha} > \sigma$ ). So we introduce a further definition.

**Definition 12** A Reachable Regular Growth Curve (RRGC) is a RGC satisfying the condition  $g > \frac{\sigma}{\gamma - \alpha}$ .

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<sup>8</sup> Values of simulazioni in **Figure 11**:  $\alpha=0.7$ ,  $\beta=0.2$ ,  $\gamma=0.2$ ,  $\delta = 0.3$ ,  $\varepsilon=0.4$ ,  $\eta=0.06$ ,  $\sigma = 0.04$ ,  $b = 3$ ,  $d = 0.3$ ; **Figure 12**:  $\alpha=0.3$ ,  $\beta=0.2$ ,  $\gamma=0.71$ ,  $\delta = 0.3$ ,  $\varepsilon=0.4$ ,  $\eta=0.06$ ,  $\sigma = 0.04$ ,  $b = 3$ ,  $d = 0.3$ ; **Figure 13**:  $\alpha=0.82$ ,  $\beta=0.71$ ,  $\gamma=0.7$ ,  $\delta = 0.92$ ,  $\varepsilon=0.6$ ,  $\eta=0.02$ ,  $b = 3$ ,  $d = 0.3$ .

This condition requires a relatively low value of  $\alpha$  compared with that of  $\gamma$ .

Looking at (16), it is easy to check that, along a *RRGC*, the social participation choice  $s^*$  approaches the value  $\frac{b\varepsilon}{(1+b\varepsilon)}$  as  $t \rightarrow +\infty$ . Remember that  $\frac{b\varepsilon}{(1+b\varepsilon)}$  is the value of social participation in the context without substitutability. In other words, if  $T$  and  $K_s$  grow following a *RRGC*, then for  $T$  and  $K_s$  high enough, social participation is “almost” equal to social participation in the context without substitutability. Now the problem is: do values of  $\bar{g}$  and  $\bar{A}$  exist such that along the trajectories starting near the associated *RRGC* it holds  $\frac{\dot{K}_s}{K_s} \rightarrow \bar{g}$  as  $t \rightarrow +\infty$ ? To solve this problem, we analyze the behaviour of the variable:

$$x = \frac{T^\beta}{K_s^{1-\alpha\beta-\gamma\delta}}$$

previously defined (see (22)) and limit our analysis to the case  $1-\alpha\beta-\gamma\delta > 0$ .<sup>9</sup> Remember that in the context in which  $s^* = \frac{b\varepsilon}{(1+b\varepsilon)}$  always (i.e. in the context without substitutability), it holds  $\dot{x} = 0$  (see (23)) along the curve (see (20)):

$$K_s = \bar{A} \cdot T^{\frac{\bar{g}}{\sigma}}$$

where  $\bar{A} := \left[ \frac{(1-\alpha\beta-\gamma\delta)(b\varepsilon)^\delta}{[\beta\sigma+\eta(1-\alpha\beta-\gamma\delta)](1+b\varepsilon)^{\beta+\delta}} \right]^{\frac{1}{1-\alpha\beta-\gamma\delta}}$  and  $\bar{g} := \frac{\beta}{1-\alpha\beta-\gamma\delta}$ . So, if  $\frac{\beta\sigma}{1-\alpha\beta-\gamma\delta} > \frac{\sigma}{\gamma-\alpha}$  (i.e.  $\frac{\beta}{1-\alpha\beta-\gamma\delta}(\gamma-\alpha) > 1$ ), we have that, as  $T \rightarrow +\infty$ , along  $K_s = \bar{A} \cdot T^{\frac{\bar{g}}{\sigma}}$  the value of  $\dot{x}$  approaches 0 while  $\dot{x}$  becomes (see (23)) strictly positive (respectively, strictly negative) along the *RRGCs* corresponding to values of  $g < \bar{g}$  (respectively  $g > \bar{g}$ ), with  $g$  and  $A$  near enough to  $\bar{g}$  and  $\bar{A}$ , respectively. This implies that all trajectories starting (for  $T$  high enough) sufficiently near to  $K_s = \bar{A} \cdot T^{\frac{\bar{g}}{\sigma}}$ , approach  $K_s = \bar{A} \cdot T^{\frac{\bar{g}}{\sigma}}$  as  $T \rightarrow +\infty$ .

Notice that, by Proposition 3, all trajectories in the plane  $(T, K_s)$  can be Pareto-ranked; in particular, we have that given two trajectories  $\hat{K}_s(T)$  and  $\tilde{K}_s(T)$ , with  $\hat{K}_s(T) < \tilde{K}_s(T)$ , then  $\hat{K}_s(T)$  Pareto-dominates  $\tilde{K}_s(T)$ . Furthermore, well-being may be decreasing when the economy follows a trajectory along which  $K_s \rightarrow 0$  (see Figures<sup>10</sup> 14, 15, 16).

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<sup>9</sup>In the opposite case it can be shown that, as in case  $d = 0$ , along trajectories  $K_s$  approaches 0 or  $+\infty$  but, in the latter case,  $\frac{\dot{K}_s}{K_s}$  grows without bound.

<sup>10</sup>Values of simulazions in **Figures 14, 15, 16**:  $\alpha=0.7$ ,  $\beta=0.2$ ,  $\gamma=0.2$ ,  $\delta = 0.3$ ,  $\varepsilon=0.4$ ,  $\eta=0.01$ ,  $\sigma = 0.04$ ,  $b = 3$ ,  $d = 0.3$ .

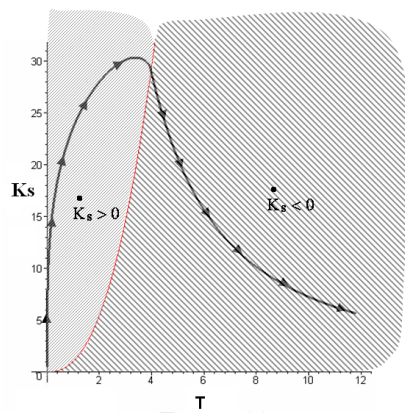


Figure 11

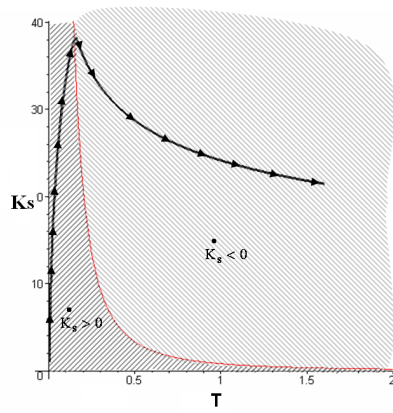


Figure 12

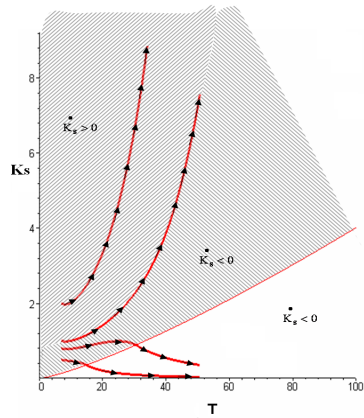


Figure 13

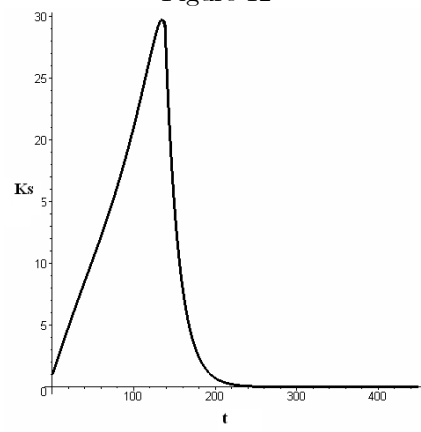


Figure 14

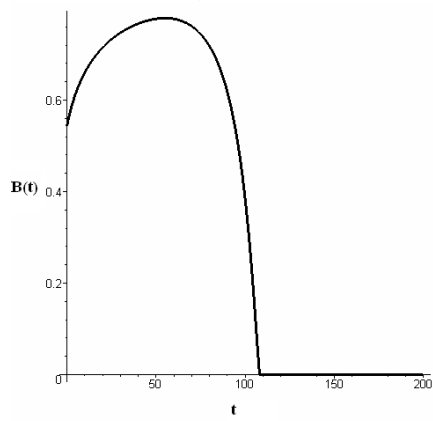


Figure 15

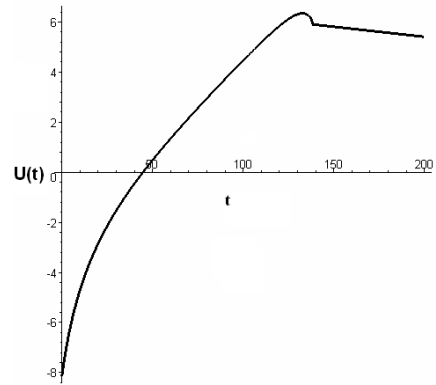


Figure 16

## 7 Concluding remarks

Our framework addresses a range of hypotheses that have never been jointly taken into account within a theoretical model. Agents allocate their time between labour, aimed at the production of private goods, and social participation activities. Private consumption and relational goods are substitutable: if the environment is poor of participation opportunities and social interactions are perceived as costly and frustrating, people can disengage from relational activities and devote more time and resources to private consumption. We consider also the possibility of positive spillovers from private to relational production, due the ability of job interactions to stimulate the creation of durable ties. Following hints from the sociological literature, we assume that most of the times the creation of interpersonal ties does not depend on rational investment decisions. Rather, it is an incidental, not necessary, by-product of social participation. The resulting stock is a public resource, which enters as an argument in agent's utility function and as an input in both "material" and "relational" goods' production functions. Since human relations need a continuous care to be preserved over time, we account for a positive depreciation rate of the stock of social capital. The main results of our study can be summarized as follows.

If social capital is more relevant in the production of relational goods, then the increase in the productivity of time devoted to social participation is higher compared to that of time devoted to the production and consumption of private goods. In such a case, if the stock of social capital is high enough, then the economy may experience positive levels of social participation. As the golden rule pointed out in section four explains, if the impact of private goods in the accumulation of social capital is negligible relative to the role of relational goods, this configuration of the model's parameters allows to preserve (and strengthen) the existing stock of social capital, thereby fostering sustainable growth. The process is highly path-dependent: in a "favourable" situation, the accumulation of social capital improves the technology of production of relational goods, which in turn raises social participation, thereby consolidating social ties and the diffusion of trust. Such a virtuous circle leads to a further strengthening of the stock of social capital.

On the other side, the reverse process may be self-feeding as well: if the economy experiences private growth and a simultaneous decline in social participation and social capital, then the time spent in relational activities becomes more expensive (in terms of opportunity cost) and less productive (in terms of relational goods).

As a long-run result, the economy is attracted by multiple steady states, some of which are social poverty traps, where the stock of social capital is entirely "destroyed" by private activities.

However, under certain conditions, the possibility exists for the economy to follow a virtuous trajectory where the stock of social capital endogenously and unboundedly grows. The achievement of such result relies on a range of particular conditions, under which the economy behaves *as if* there was no substitutability between relational activities and private consumption. In other

words, the possibility to find the path to sustainable growth depends also on the rise of a culture acknowledging the relevance of non market relations and fostering the diffusion of social (or generalized) trust. The building of such conditions does not necessarily need centuries, as Putnam, Leonardi and Nanetti (1993) argued in the pioneer study. Policy holds a decisive role in promoting cooperative values and shaping the moral norms of a society. This statement is less smoky and generic than it may seem, and its discussion requires to address another of the gaps affecting the social capital literature, namely the relationship between policy, inequality and the diffusion of trust. The concept of equality holds at least two dimensions: economic equality and the equality of opportunities. According to Alesina and La Ferrara (2002), the lack of both these dimensions is strongly associated with low trust in the United States<sup>11</sup>. The experimental evidence leads to the same results (Glaeser et al., 2000, Barr, 2004).

Introducing exogenous technical progress in the production function of private goods leads to interesting modifications in social capital's accumulation dynamics. If there is not substitutability between private and relational goods, the stock of social capital can grow without bound along any trajectories when  $\alpha\beta + \gamma\delta < 1$ . If  $\alpha\beta + \gamma\delta > 1$ , there are alternative trajectories where social capital grows indefinitely, or tends to zero when its initial stock is particularly low relative to  $T$ . In both the cases, social capital's trend relative to technical progress can be monotonic (always increasing or decreasing) or experiences an initial decline followed by a growth, but not vice versa (a growth followed by a decline is impossible). Under the assumption of substitutability between private consumption and relational goods, two cases are possible.

If  $\gamma - \alpha < 0$ , i.e. the contribution of social capital is higher in private production than in the production of relational goods, then the stock of social capital always tends to zero whatever are its initial endowments.

In such a case, the stock of social capital experiences a growth followed by a decline, so that its relationship with technical progress is described by an inverted U-shaped curve. Since technical progress is in turn positively correlated with GNP, our result sounds as a proof of Putnam's (2000) intuition of the inverted U-shaped relationship between social capital and development.

When  $\gamma - \alpha > 0$ , the rise of technical progress can be accompanied by a progressive erosion of the entire stock of social capital if its initial endowments are below a critical threshold. However, if the initial endowments are high enough in respect to technical progress, there are trajectories where social capital can grow indefinitely. The possibility of taking trajectories characterized by an unbounded growth of social capital thus depends on its initial endowments: an environment rich of participation opportunities, a culture acknowledging the importance of non market relations, the diffusion of moral norms of reciprocity and cooperation certainly constitute a good, desirable "starting point".

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<sup>11</sup>Drawing on GSS data, Alesina and La Ferrara (2002) find that the strongest factors associated with low trust are (a) belonging to a group that historically felt discriminated against, such as minorities (blacks in particular) and, to a lesser extent, women; (b) being economically unsuccessful in terms of income and education; (c) living in a racially mixed community and/or in one with a high degree of income disparity.

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