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A Model of Imitative Behavior in the Population of Firms and Workers

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Abstract - We study an imitation game of strategic complementarities between the percentage of high-skilled workers and innovative firms, namely, human capital and R&D, respectively. We show that this model has two pure Nash equilibria, one of them with high investment in R&D and skilled workers while the other one, which we interpret as poverty trap, exhibits lack of skills and underinvestment. Furthermore, we show that we can avoid the poverty trap if the number of innovative firms is larger than a threshold value allowing an increment of the number of skilled workers.

JEL classification: C72, C79, D83, O12.

Keywords: Imitative behavior, conformism, poverty traps, strategic complementarities.

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1 Introduction

Nelson and Phelps (1966) offer a first attempt at modeling the idea that the major role of education is to increase the individual's capacity to innovate and to adapt to new technologies, thereby speeding up technological diffusion throughout the economy. This approach provides a couple of interesting insights. First, the level of education attainment is more important than the rate of growth of human capital accumulation (as in Lucas (1988)). Second, the marginal productivity of education attainment is an increasing function of the rate of technological progress. Then, macroeconomic policies which affect rates of innovation and investment will affect the relative demand for workers according to their education levels[1]. In this vein, Azariadis (1996) shows that the initial level of technology is critical for economic growth. In other words, if a country starts above this threshold level of technology, it will follow a sustained path of economic growth. However, if the country's technology level is too low, there will be no R&D and the economy will remain in a poverty trap of zero growth.[2] This approach emphasizes on the role of innovation, in particular that its absence leads to stagnation.

In this paper we deal with imitation factors resulting in performing a wide spectrum of tasks "as others do". In the model we assume that all agents in the population are infinitely lived and interact forever. Each agent sticks to some pure strategy for some time interval and now and then she reviews her strategy, sometimes resulting in a change of strategy (see Weibull, 1995). On imitation theory, Vega-Redondo (1997) and Schlag (1998, 1999) pointed out two approaches based on the idea that individual who face repeated choice problems will imitate others who obtained high payoffs. But despite this basic similarity, the two models differ along two different dimensions, the informational structure ("whom agents imitate") and the behavioral rule ("how agents imitate"). While agents in Vega-Redondo's model observe their immediate competitors, in Schlag model they observe others who are just like them but play in different groups against different opponents. It can be show that the difference between the two models is mainly due to the different informational assumptions rather than the different adjustment rules. So, it is more important whom one imitates than how imitates (see, J. Apesteguia et al., 2007).

In seeking an intentional explanation of imitative behavior, we must search for possible (good) reasons for individuals to imitate others, and only if this endeavor fails should we resort to explanations which assume that actors act instinctually, randomly, or what not. In this sense, a rational imitation can be explained as follows. An actor, A, can be said imitate the behavior of another actor, B, when observation of the behavior of B affects A in such a way that A's subsequent behavior becomes more similar to the observed behavior of B. An

[1] On human capital and growth theory, the main statement, both in Lucas (1988) and Stokey (1991), is that human capital is the "engine" of growth. Similar claims can be found in Mankiw et al., (1992) and Barro (1991).

[2] Azariadis and Starchurski (2005) define *poverty traps* as "any self-reinforcing mechanism which causes poverty to persist".

actor can be said to act rationally when the actor, faced with a choice between different courses of action, chooses the course of action that is best with respect to the actor's interests and her beliefs about possible action opportunities and their effects. Rational imitation hence refers to a situation where an actor acts rationally on the basis of beliefs that have been influenced by observing the past choices of others. To the extent that other actors act reasonably and avoid alternatives that have proven to be inferior, the actor can arrive at better decisions that he or she would make otherwise, by imitating the behavior of others.

The aim of this paper is to study an imitation game where the nature of interactions among individuals creates a potential for multiple equilibria. These equilibria are characterized by different levels of "activity" (*skill workers, innovative firms*) in the economy. To the best of our knowledge, this is the first time that imitation theory is used to explain poverty traps. We show that this model has two Nash equilibria, one of them with high investment in R&D and the presence of high-skilled workers while the other exhibits low investment and skill levels. Hence the possibility of these two kinds of equilibria show that the players, firms and workers, acting under identical settings may perform either well or badly (i.e. grow or stagnate), depending only on the initial conditions. The state ("high" or "low") that a given country may attain depends on the performance and the interactions between two populations, namely those of firms and workers. The investment in R&D is successful only in the presence of high-skilled workers (Aghion and Howitt, 1999). Conversely, the workers increase their skills when a large number of firms invest in high-technology. On the contrary, firms that do not invest in R&D, do not look for high-skilled workers, making the accumulation of skills unprofitable. We show that there exist a threshold number of innovative firms, above which it becomes advantageous to accumulate high skills. This is the mechanism that allows avoiding a poverty trap.

In fact, education (human capital accumulation) is a necessary but insufficient condition for sustained economic growth. In many developing countries, a huge effort in generating a quality educational system did not have an impact on the performance of the economy. For instance, several decades ago Uruguay has invested heavily in public education, in order to generate a highly-skilled workforce. But this effort was not accompanied by a similar investment by the firms in R&D. This happened because Uruguay was a closed country and there was not competitiveness. After this failed attempt the outlays in education decreased, since high wages were not correlated with time spent in schooling (Ros, 2001). This trend deepened after the Uruguayan economy became more open. For another example, the firms using high technology in Mexico are foreign-owned (the so called *maquiladoras*) that import their technology, which is developed abroad, and contract in Mexico only low-skilled workers. Therefore, these firms lack R&D departments in Mexico and do not create incentives to increase the skills of their local workforce.

This paper is organized as follows. Section 2 describes the coordination game which represents a contractual situation among firms and potential workers.

Section 3 presents the dynamic imitation model in which the imitative behavior can lead the economy into a poverty trap. Section 4 concludes the paper.

2 The Game

Consider an economy with two populations: potential workers, W, and firms, F. Each population has two clubs:

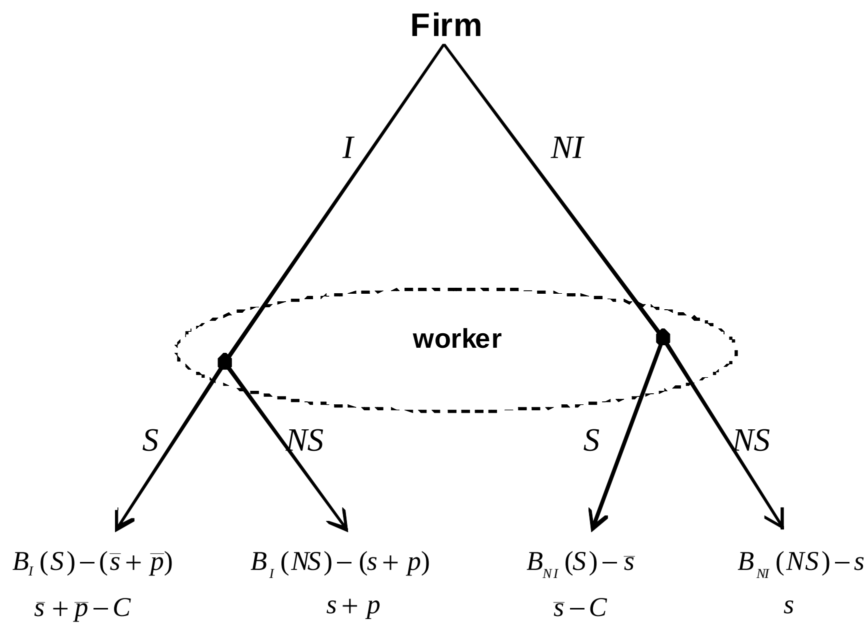
- The W-population has an S-club of strategists that invest in improving their individual skills (high-skill workers), and an NS-club that does not[3].
- The F-population has the I-club of strategists that invest in R&D (innovative firms), and the NI-club of no-innovative firms.

Workers do not know the type of the firm. On the contrary, workers have to certify their skill levels. A S-type strategist worker may invest in education going to a training school, at an associated cost C. This worker gets a salary $\bar{s}$ both in an I or NI firm. Instead, the NS-type workers get a salary $s < \bar{s}$.

Assume that the innovative firm I shares utilities with their workers, each one receiving a premium. NI-firms do not share its benefits. High-skilled workers receive a premium $\bar{p}$ and low-skilled ones p, $(0 < p < \bar{p})$.

[3] Let $I = (1, ..., n)$ be the set of worker positions, ordered by their skill levels, $\{S, NS\}_i$ the pure-strategies of the worker in position i, M_i its mixed-strategy simplex, and $M = \times_{i \in I} M_i$ the polyhedron of mixed-strategy profiles. Let m_{is} denote the mixed-strategy profile corresponding to S .

In summary, we have a two population form game and its extensive-form representation is,



where $B_i(j)$, $\forall \;\; i \in \{I, NI\}$ $j \in \{S, NS\}$, is the gross-benefit of the $i-$firm hiring a $j-$worker.

The expected payoff of a S-type strategist worker, given the chances of being hired either by an I or a NI firm, is:

$$E(S) = \text{prob}(I)\,[\bar{s} + \bar{p}] + \text{prob}(NI)(\bar{s}) - C \tag{1}$$

where prob(I) represents the probability of being contracted by an innovative firm and prob(NI) the probability of being hired by a non-innovative firm. Analogously:

$$E(NS) = \text{prob}(I)\,[s + p] + \text{prob}(NI)(s) \tag{2}$$

A worker prefers to be a S-type strategist when $E(S) > E(NS)$, and reciprocally. This happen if and only if prob(I) is large enough i.e.:

$$\text{prob}(I) > \frac{C - (\bar{s} - s)}{(\bar{p} - p)}. \tag{3}$$

Consider the following remarks:

- Gross-benefits obtained by an innovative firm contracting a high-skilled worker are higher than those obtained by a non-innovative firm contracting the same worker, i.e.: $B_I(S) - \bar{p} > B_{NI}(S)$.
- The gross-benefits of a innovative firm contracting high-skilled workers are higher than those of a non-innovative firm contracting a non-skilled worker, i.e.: $B_I(S) - \bar{p} > B_{NI}(NS)$
- If the firm is innovative, the payoff of a high-skilled worker is larger than the payoff of a non-skilled worker, i.e.: $\bar{s} + \bar{p} - C > s + p$.
- If the firm is non-innovative, the payoff of non-skilled worker is at least as good as the payoff of a high-skilled worker, i.e.: $\bar{s} - C \leq s$.
- If the worker is non-skilled, the benefits of non-innovative firm are higher than those of innovative one, i.e.: $B_I(NS) - (s + p) < B_{NI}(NS) - s$.

This game has two pure strategy Nash equilibria: $A = \{S, I\}$ *and* $B = \{NS, NI\}$*, and a mixed strategy equilibrium. While equilibrium A Pareto-dominates B, in particular, the risk dominant equilibrium is* $\{NS, NI\}$ *while the payoff dominant equilibrium is clearly* $\{S, I\}$*.*

In the sequel, $\text{prob}(I) = PI = QI/Q$ where QI is the number of innovative firms, and Q is total number of firms. Then, $\text{prob}(NI) = PNI = 1 - PI$.

3 Dynamic Imitation

We consider the "behavioral rule with inertia" (see Schlag, 1999) that allows an individual to reconsider his action only with probability $R \in (0, 1)$ in each round. Each round is finished at the end of the contractual situation, and then workers may ask themselves whether to change or not their behavior according to what the others are doing. Consider the i-strategist worker, $i \in \{S, NS\}$. With probability R_i, she will ask herself whether to change or not her behavior. Then, R_i denotes the average time-rate at which an individual worker, that currently uses strategy $i \in \{S, NS\}$, reviews her strategy choice.

Likewise, let P_{ij} be the probability that such a reviewing worker will change to strategy $j \neq i$. Then,

$$P(i \to j) = R_i P_{ij},$$

is the probability that a worker in the $i - th$ club changes to the $j - th$ club.[4] Let $e_S = (1, 0)$ and $e_{NS} = (0, 1)$ indicate the pure strategies, S or NS.With this

[4] In a finite population one may imagine that review times of the S-strategists in population W are modeled as the arrival times of a Poisson process with average (across such individuals) arrival rate R_S, and that at each such arrival time the individual selects a pure strategy according to the conditional probability distribution P_{SNS}. Assuming that all individuals' Poisson processes are statistically independent, the probability that any two individuals happen to review simultaneously is zero, and the aggregate of reviewing time in the W player population among S-strategists is a Poisson process. If strategy choices are statistically independent random variables, the aggregate arrival rate of the Poisson process of individuals who switch from one pure strategy S to another NS is $R_S P_{SNS}$.

specification we can model the flow of high-skilled workers, $\dot{X}_S$, being equal to the number of changing non-skilled workers minus the number of high-skilled workers changing to the non-skilled worker's club. Since we consider large populations, we invoke the law of large numbers and model these aggregate stochastic processes as deterministic flows, each such flow being set equal to the expected rate of the corresponding Poisson arrival process. Rearranging terms, one obtains the system of differential equations that characterizes the dynamic flow of workers:

$$\begin{aligned} \dot{X}_S &= R_{NS}P_{NSS}X_{NS} - R_S P_{SNS} X_S \\ \dot{X}_{NS} &= -\dot{X}_S \end{aligned} \tag{4}$$

being X_S the fraction of high-skilled workers and X_{NS} the fraction of non-skilled workers.

Let us give the following definition on the notion of imitative behavior.

Definition. *Dynamics (4) represent an imitative behavior if two pure strategies $\{S, NS\}$ that currently have the same expected payoff, but NS is currently more popular than S in the sense that more individuals use NS, then the choice probability for NS, P_{SNS} should exceed that of S, P_{NSS}.*

An evaluation rule that seems particularly natural in a context of simple imitation is the "average rule" where each strategy is evaluated according to the average payoff observed in the reference group (see J. Apesteguia et al., 2007). Although a worker does not know all the true values of the payoff of the other workers, she can take a sample of true values in order to estimate the average. Let $\bar{E}(i)$ and $\bar{E}(j)$ be the estimators for the true values $E(i)$ and $E(j)$. Hence, each $i-$worker changes her strategy if and only if $\bar{E}(i) < \bar{E}(j)$.

Therefore, the probability that an $i-$strategist becomes a $j-$strategist is given by $P[\bar{E}(j) - \bar{E}(i) > 0]$, and (4) can be written as:

$$\begin{aligned} \dot{X}_S &= R_{NS}P[\bar{E}(NS) - \bar{E}(S) < 0]X_{NS} - R_S P[\bar{E}(NS) - \bar{E}(S) > 0]X_S, \\ \dot{X}_{NS} &= -\dot{X}_S. \end{aligned} \tag{5}$$

Suppose that the $P[\bar{E}(j) - \bar{E}(i) > 0]$ increases proportionally to the true value $E(j)$, i.e.:

$$P[\bar{E}(j) > \bar{E}(i)] = \begin{cases} \lambda E(j) & if \;\; E(j) > 0 \\ 0 & if \;\; E(j) \leq 0 \end{cases} \tag{6}$$

where $\lambda = \frac{1}{|E(NS)+E(S)|}$. Recall, $E(NS) = (PI)(p) + s \geq 0$, but $E(S) = (PI)(\bar{p}) + \bar{s} - C$ can be positive or negative, depending on the value of PI and C. Then, system (4) takes the form:

$$\begin{aligned} \dot{X}_S &= -\left[R_{NS}\lambda E(S) + R_S(\tfrac{1}{\lambda} - E(S))\right] X_S + R_{NS}\lambda E(S). && (i) \\ \dot{X}_{NS} &= -\dot{X}_S. && (ii) \end{aligned} \tag{7}$$

Let us assume a constant number of innovative firms, and that salaries, premiums and education costs are fixed. Then, $E(S)$ and $E(NS)$ are constant. The following cases apply:

Case 1: If $E(S) > 0$ then, the equation $(7(i))$ takes the form:

$$\dot{X}_S = AX_S + R_{NS}[E(S) + E(NS)]E(S) \tag{8}$$

where:

$$\begin{aligned} A &= E^3(S)R_{NS} + E^2(S)[2E(NS)R_{NS} - R_S] + \\ &\quad +E(S)[E^2(NS)R_{NS} - E(NS)R_S] + R_S \end{aligned}$$

and the solution of the differential equation (8) has the form:

$$X_S(t) = X_{S0}e^{-At} + [R_{NS}\lambda E(S)]t$$

where X_{S0} is the fraction of the high-skill workers at time $t = 0$. It follows that its solution depends on the number of innovative firms. Since $E(S) = (PI)(\bar{p}) + \bar{s} - C$ and $E(NS) = (PI)(p) + s$, the respective values of $E(S)$ and $E(NS)$ are increasing functions of the percentage of innovative firms PI. However if $PI > P_u$ then $E(S)$ increases faster than $E(NS)$ as PI increases, see equations (2) and (1).

For instance, consider two countries, 1 and 2, where the respective percentage of innovative firms are: $PI_1 > PI_2$, so, looking at the solution it follows that the population of high-skilled workers in country 1 is for each $t > t_u$ larger than the population of high-skilled workers in country 2, i.e., $X_{1S}(t) > X_{2S}(t)$, where the value of t_u depends on the parameters of the model, salaries, premiums and probabilities of reviewing workers[5].

For another example assume that $E(S) = (\bar{p})P(I) + \bar{s} - C < 0$. Then, equation $(7\ (i))$ takes the form:

$$\dot{X}_S = -R_S\lambda E(NS)X_S.$$

Hence, the population of high-skilled workers decreases.

Note that there exists a threshold value for the innovative firms

$$p_u = \frac{C - \bar{s}}{\bar{p}}$$

such that, if $PI > p_u$ the population of high-skilled workers is increasing.

The next proposition summarizes our main result.

Proposition 1 *There exists a threshold value, $p_u = \frac{C-\bar{s}}{\bar{p}}$, such that if the initial number of innovative firms PI is larger than this value, $PI > p_u$, then the relative population of high-skilled workers increases. Otherwise, this percentage decreases. As PI increases, $E(S)$ increases faster than $E(NS)$.*

[5]Note that if $A > 0$ then $\dot{X}_s(t) > 0$ for all $t > -\frac{R_{NS}\lambda E(S)}{A}$.

The higher the number of innovative firms, the greater the expected payoff a high-skilled worker will get (it increases the number of high-skilled workers by imitation) in the economy. As the profits of innovative firms increase with the percentage of high-skilled workers, then if at some time $t = t_0$ the number of innovative firms satisfies $PI > p_u$ it follows that the percentage of skilled workers increases. It would then be rational for the firms to be innovative, obtaining higher benefits. The percentage of innovative firms would increase with the percentage of high-skilled workers. Then, the economy would go to the high level equilibrium. Instead, if $PI < p_u$, we obtain a poverty trap, where firms will be no-innovate and workers will remain with low skills.

4 Conclusion

As Accinelli et al. (2007) we obtained that to overcome a poverty trap it is necessary to surpass a threshold number of innovative firms. Then workers will have incentives to improve their skills. Firms, in turn, can obtain more benefits being $I-$type strategists. If this threshold value is not reached by the economy, a policy maker should implement an incentive to reach the high-level equilibrium of innovative firms and high-skilled workers in the economy, for instance, a policy intended to lower the cost of attaining skills. More generally, the goal is to reduce the threshold value, p_u.

Future research indicates that the number of innovative firms should be an increasing function of the number of high-skilled workers $PI = f(X_S)$, $f'(\cdot) > 0$.

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