UNIVERSITÀ DEGLI STUDI DI SIENA

QUADERNI DEL DIPARTIMENTO
DI ECONOMIA POLITICA

Edgar J. Sanchez Carrera

The Evolutionary Game of Poverty Traps

n. 555 – Febbraio 2009

Abstract - We study an evolutionary game in which the individual behavior of the economic agents can lead the economy either into a low-level or a high-level equilibrium. The model represents two asymmetric populations, “leaders and followers”, where in each round an economic agent of population 1 is paired with a member of population 2. Our evolutionary game is a signaling game in which only the leader has private information. The leader moves first; the follower observes the leader's action, but not the leader's type, before choosing her own action. We found the equilibria both as self-confirming and evolutionarily stable strategies. Furthermore, considering an imitative behavior of the followers, we show that to overcome the poverty trap there exists a threshold value equals to the ratio "education costs-efficiency wages" of the number of high-profile economic agents.

Keywords: Evolutionary games, imitation rule, poverty traps, replicator dynamics, signaling games, strategic complementarities.

JEL classification: C70, C72, C73, I30, O10, O40.

Acknowledgments: Many thanks to Elvio Accinelli, Costas Azariadis, Sladjana Pavlinovic and Lionello Punzo for their helpful comments. The usual disclaimer applies. Suggestions and comments are welcome.

Edgar J. Sanchez Carrera, PhD Candidate at Department of Economics, University of Siena, Italy
E-mail: sanchezcarre@unisi.it

.

1 Introduction

A poverty trap or low-level equilibrium is defined as a self-perpetuating condition where the economy caught in a vicious cycle and the economic agents suffer from persistent underdevelopment (see Azariadis and Stachurski, 2005 and Bowles et al., 2006).

In this paper, we model an evolutionary coordination signaling game where economic agents represent institutions such that their rational behavior may raise poverty traps.[1] We argued that poverty traps exist and account due to strategic complementarities between profiles of economic agents (for example: low-skilled workers, no innovative firms, scarcity of human capital and R&D) and that institutions and norms are key causes of their formation and persistence. Institutions and norms are complex cognitive devices, which simultaneously result from the types of economic agents and include a series of intrinsic properties[2]. Hence, it is the combination of multiple elements that may create thresholds effects and entrap groups into low equilibria – economic elements (such as an environment of widespread poverty, commodity dependence), political (predatory regimes) and local social norms.

The aim of this paper is to give a feasible answer to: how do we explain that some countries have actually escaped from long stretches of poverty? Was it just good luck or imitation of successful strategies followed elsewhere? We argue that it is the imitative behavior of successful strategies one of the reasons to escape from low-level equilibria. Because if we imitate by dissatisfaction it is almost sure that we get a low-level equilibrium.

In this vein, the notion of poverty trap has been enriched by Steven Durlauf with a spatial dimension on the idea that an individual's socioeconomic outcomes depend upon the composition of the various groups of which she is a member over the course of her life. That is, the decision for an individual to acquire an education strongly depends on the prior existence of other educated members in a group. This interdependence of behavior induces "neighborhood effects", which generate different types of groups that have different steady states (with/without educated members). This interdependence may be intertemporal, i.e. it affects future social interactions. The dynamics of these combinations explain persistent income inequality: in Durlauf's (1996) model they create incentives for wealthier families to segregate themselves into economically homogeneous neighborhoods. Economic stratification combines with

[1] Game-theoretic models show that solutions and equilibria are multiple, with institutional change being the selection of one equilibrium from many possible ones and which may be sub-optimal. For Aoki (2001), the question of enforcement leads to analyzing the design of institutions that can implement given social goals in a manner that is compatible with the incentives of the players - according to a self-enforceable or an enforcement mechanism (Aoki 2001, p. 6). In this vein, Samuel Bowles has built the seminal concept of 'institutional poverty traps', which emphasizes that coordination failures and poverty traps are induced by the presence of specific institutions. Bowles defines these as institutions that generate "highly unequal divisions of the social product" (Bowles 2006).

[2] Azariadis and Drazen (1990) examined the implications of threshold effects on the supply side, while Acemoglu (1996) and Redding (1996) introduced poverty traps due to coordination problem between firms and workers.

neighborhood effects: their reciprocal feedback transmit different types of economic status across generations. These processes also explain the persistence of poverty in particular areas (such as American inner cities) (Durlauf 2003). It is this concept of neighborhood that for Durlauf allows for the understanding of why poverty traps exist and persist. Hence, poverty traps are defined as a community that if it is composed initially by poor members or low-profile economic agents, then, it will remain in the low-level equilibrium over generations.

Hence, the types of economic agents and their neighborhood effects can explain whether an economy is situated in a low-level or high-level equilibrium. For instance, Nelson and Phelps (1966) offer a first attempt to modeling the idea that the major role of education is to increase the individual's capacity to innovate, first, and to adapt to new technologies, second, thereby speeding up technological diffusion through the economy (Aghion and Howit, 1999). Therefore, high-profile economic agents lead the economy in a high-level equilibrium. In this vein, Azariadis (1996) shows that the initial level of technology is critical for economic growth, that is, if a country to begin with satisfies this threshold level of technology, it will grow. However, if the country's technology level is too low, there will be no R&D and the economy will remain in a poverty trap of zero growth. In this vein, in a seminal paper Lucas (1988) demonstrated that human capital is the "engine" of economic growth.

In this paper, we present a coordination game between "leaders" and "followers" with different profiles, that is, an evolutionary game of the complementarity between the types of profiles of the economic agents. We show that the economy can be located in a low-level equilibrium and that there exists a threshold level to overcome it. We found the self-confirming equilibria and the evolutionarily stable strategies. We conclude that the possibility of either high-level or low-level equilibria implies that players, economic agents, acting under identical settings may experience either adequate living standard or deprivation (growth or crisis), respectively, and it depends only on their histories or initial conditions.

Furthermore, we consider economic agents that stick to some pure strategy for some time interval, and now and then reviews her strategy, sometimes resulting in a change of strategy (early contributions start with Björnerstedt and Weibull, 1995). There are two basic elements common to these models. The first is a specification of the time rate at which agents in the population review their strategy choice. This rate may depend on the current performance of the agent's pure strategy and of other aspects of the current population state. The second element is a specification of the choice probabilities of a reviewing agent. The probability i-strategist will switch to some pure strategy j may depend on the current performance of these strategies and other aspects of the current population state. Hence, imitation can be driven by both dissatisfaction and successful. In seeking an intentional explanation of imitative behavior, we must search for possible (good) reasons for individuals to imitate others, and only if this endeavor fails should we resort to explanations which assume that actors act instinctually, randomly, or what not. In this sense, a rational imitation can be explained as follows. An actor, A, can be said imitate the behavior of another actor, B, when observation of the behavior of B affects A in such a way that A's subsequent behavior becomes more similar to the observed behavior of B. An actor can be said to act rationally when the actor, faced with a choice between different courses of action, chooses the course of action that is best with respect to the actor's interests and her beliefs about possible action opportunities and their effects.

In this vein, theoretical advances to understand imitation have been explored by Vega-Redondo (1997) and Schlag (1998, 1999). Both approaches are based on the idea that individual who face repeated choice problems will imitate others who obtained high payoffs. But despite this basic similarity, the two theories imply markedly different predictions when applied to specific games. For example, for games with a Cournot structure, Schlag's model predicts the Walrasian outcome and Vega-Redondo studies the evolution of Walrasian behavior. Basically, the models differ along two different dimensions, the informational structure ("whom agents imitate") and the behavioral rule ("how agents imitate"). While agents in Vega-Redondo's model observe their immediate competitors, in Schlag model they observe others who are just like them but play in different groups against different opponents. Additionally, agents in Vega-Redondo's model copy the most successful action of the previous period whenever they can. In contrast, Schlag's agents only imitate in a probabilistic fashion and the probability with which they imitate is proportional to the observed difference in payoffs between own and most successful action. It can be show that the difference between the two models is mainly due to the different informational assumptions rather that the different adjustment rules. So, it is more important whom one imitates that how imitates (see, J. Apesteguia et al., 2007).

The remainder of the paper is organized as follows. Section 2 describes the basic game while section 3 starts with the evolutionary game by picking up the replicator dynamic equations and the first two important results on low-level and high-level equilibria. Replicator by imitation is also considered. Section 4 analyzes the imitative behavior and the existence of a threshold level to overcome the poverty trap. Section 5 concludes the paper.

2 The Game

Consider an economy composed by agents with different profiles, that is, the economic agents split in two types: the high- and the low-profiles. We label the vectors $(H, L)\,;(h, l)$ as the strategy space, s_i, denoting high- and low-profiles, respectively, of sub-population $i = 1, 2$. Consider a signaling game where the economic agent of sub-population 2 is a "follower" and the one of sub-population 1 is a "leader" and choosing between high- and low-profiles does not have any cost for the leader[3].

The one-shot game starts with a representative agent of sub-population 1

[3]Signaling games are of incomplete information leader-follower games in which only the leader has private information. The leader moves first; the follower observes the leader's action, but not the leader's type, before choosing her own action (for more details Fundenberg and Tirole, 1991).

who may marriage with a representative agent of sub-population 2. To get profitable outcomes, $\pi_i\left((H,L);(h,l)\right)$, in this economy the player 1 must employ 2 under strategic complementarity, in the sense that, a H-type agent matching with a h-type is more profitable than matching with a l-type, analogously, a L-type agent matching with a l-type is more profitable than matching with a h-type. The follower does not know the type of the leader, but she assumes with probability σ to be hired by a $H-$type and $(1-\sigma)$ by a L-type leader strategist. The follower decidesto beocme a $h-$type facing a training cost or cost of education C while deciding to be a $l-$type has zero cost. An extensive-form representation is,

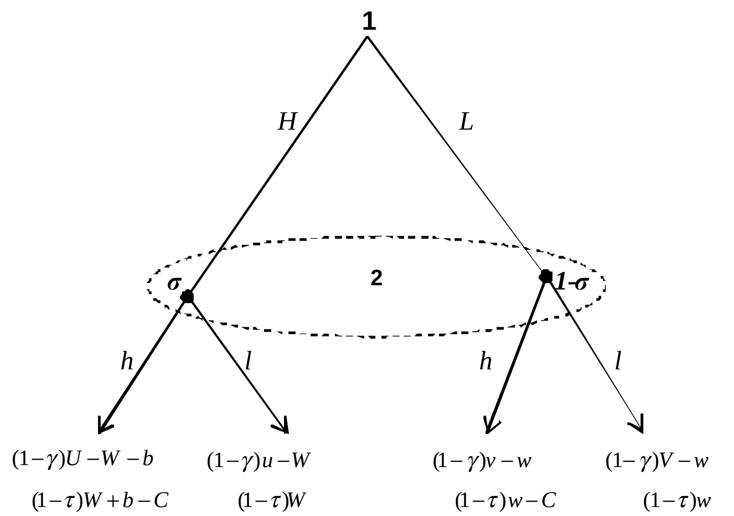

Figure 1. Decisions' tree.

where both agents 1 and 2 face an income tax represented by γ and τ, respectively. Gross-utility of 1 being $H-$type is U or u and being $L-$type is V or v which depends on matching high- or low-profile followers, respectively. The h-type follower gets an income W when is hired by H and w when is hired by L. By complementarity $U>u$; $V>v$ and $W>w$.

The $H-$type leader is looking for $h-$type followers sending a signal, e, of extra-profits or efficiency wages[4], bounded on,

$$0<e<(1-\gamma)(U-u)$$

We are interested in finding the self-confirming equilibria (SCE), since it is based on the idea that player should have correct beliefs about probability

[4]The concept posits a relationship between wages and productivity that over some range is positive. Thus up to some point, raising wages may lower per-unit wage costs (Bellante, 1994).

distributions that they observe sufficiently often. The original definition of SCE assumes that players observe the terminal node that is reached, but in some settings it is natural to assume that they observe less than this. There are several versions of SCE. The most straightforward to define is that of unitary SCE. This requires that each player have beliefs σ over opponents play (ordinarily the space of their behavior strategies) that satisfies two basic criteria. First, players should optimize relative to their beliefs. Second, beliefs should be correct at those information sets on the game tree that are reached with positive probability. Put differently, the beliefs must assign probability one to the set of opponent behavior strategies that are consistent with actual play at those information sets.[5] Nash equilibrium requires that players have correct beliefs about the strategies their opponents use to map their types to their actions, and in order for repeated observations to lead players to learn the distribution of opponents' strategies, the signals observed at the end of each round of play must be sufficiently informative. Such information will tend to lead players to also have correct and hence identical beliefs about the distribution of Nature's moves. Hence, SCE assumes that the players' inferences are consistent with their observations which are consistent with the Perfect Bayesian Equilibria (PBE).[6]

Definition 1 (Fundenberg & Levine, 2007). *σ is a unitary self-confirming equilibrium if for each player $i = 1, 2$ there are beliefs on their payoff functions, π_i, and for each strategy $s_i = \{H, L; h, l\}$ with $\sigma_i(s_i) > 0$ such that*

- *s_i is a best response to π_i and*
- *π_i is correct at every node reached with positive probability under σ.*

To find the SCE let us adopt the principle of backward induction. Firstly, if 2 chooses the strategy of being $h-$type, then, her expected payoff, E^h, is given by:

$$E^h = \sigma\left((1-\tau)W + e\right) + (1-\sigma)(1-\tau)w - C. \tag{1}$$

Alternatively, when 2 is choosing l-type,

$$E^l = (1-\tau)\left(\sigma W + (1-\sigma)w\right). \tag{2}$$

Thus, 2 prefers to be a $h-$type strategist if $E(h) > E(l)$, and it happens if and only if σ is large enough, i.e.

$$\sigma \geq \frac{C}{e}, \tag{3}$$

[5] This version of SCE allows outcomes that are not Nash equilibria, as shown by Fudenberg & Kreps (1988), but it is outcome-equivalent to Nash equilibrium in 2 player games (Battigalli, 1987; Fudenberg & Kreps, 1995).

[6] Recall that a PBE involves optimal actions given beliefs and consistent beliefs in equilibrium (Fudenberg and Tirole, 1991). In cases where more than one PBE is possible, it is also appropriate to examine whether some can be ruled out. In some cases, PBEs rely on unreasonable beliefs that are technically sustainable (because they are off the equilibrium path of behavior) but unlikely to persist if people slightly deviate from equilibrium predictions.

where $\frac{C}{e} \in (0,1)$ is the ratio "education costs-efficiency wages", then, to decrease such value, it should be reduced the costs of education, C, or to increment the signal of efficiency wages, e.

This game has three SCE Nash Equilibria, the first one in mixed strategies and the other two correspond to,

$$\left\{H, h, \sigma \geq \tfrac{C}{e}\right\} \quad \left\{L, l, \sigma < \tfrac{C}{e}\right\} \tag{4}$$

Recall that the term evolutionary process means only that more successful types tend to proliferate while less successful types tend to disappear, an assumption that applies equally well to learning and cultural evolution as well as literal population replacement via natural selection. The model applies as long as people tend to gravitate toward a type that does better than its alternatives.

3 Replicator dynamics

The simplest setting in which to study learning is one in which agents' strategies are completely observed at the end of each round, and agents are randomly matched with a series of anonymous opponents, so that the agents have no impact on what they observe. Hereafter, sub-populations of leaders (1) and followers (2) are denoted by X_1 and X_2, respectively, and they are composed by a large number of individuals which are facing the clue of selecting an adequate level of profiles $\{H, L\}$; $\{h, l\}$, respectively. Let us denote a fraction of individuals of each sub-population as,

$$x_k^i = \frac{X_k^i}{X_K}, \tag{5}$$

for all pair $i \in \{(H, L)\,(h, l)\}$ of sub-population $k \in \{1, 2\}$, respectively. That is, the share of $h-$type strategists is $x_2^h = \frac{X_2^h}{X_2^h + X_2^l}$ and $X_2^h + X_2^l = X_2$ is assumed to be a constant. Assume that both sub-populations are of the same finite size and normalized to 1, $x_1^H + x_1^L = 1$ and $x_2^h + x_2^l = 1$. Note that the probability $\sigma = x_1^H$.

Now, the economic agents of each sub-population have expected payoffs,

$$E_2^h = x_1^H\left((1-\tau)W + e\right) + (1 - x_1^H)(1-\tau)w - C \tag{6}$$

$$E_2^l = x_1^H\,(1-\tau)\,W + (1 - x_1^H)\,(1-\tau)\,w \tag{7}$$

$$E_1^H = x_2^h\left((1-\gamma)\,U - W - e\right) + (1 - x_2^h)\left((1-\gamma)\,u - W\right) \tag{8}$$

$$E_1^L = x_2^h\left((1-\gamma)\,v - w\right) + (1 - x_2^h)\left((1-\gamma)\,V - w\right) \tag{9}$$

We consider the n-population replicator dynamics (Weibull,1995:172) suggested by Taylor, 1979 of the form[7],

$$\dot{x}_k^i = \left[E_k^i - \bar{E}_k\right] x_k^i,$$

where $x_k^i \in [0,1]$ and $\dot{x}_k^i + \dot{x}_k^j = 0$ for all pair $i \neq j \in \{(H,L)\,(h,l)\}$ of subpopulation $k \in \{1,2\}$, respectively[8]. In other words, the growth rate $\frac{\dot{x}_k^i}{x_k^i}$ of the associated subpopulation share equals its excess payoff, $E_k^i - \bar{E}_k$, over the average payoff in its player subpopulation,

$$\bar{E}_k = \frac{X_k^i}{X_K} \cdot E_k^i + \frac{X_k^j}{X_K} \cdot E_k^j,$$

Hence, a replicator dynamic for the $h-$type strategists of sub-population 2, $\dot{x}_2^h = \left[E_2^h - \bar{E}_2\right] x_2^h$, can be written as[9],

$$\begin{aligned}\frac{\dot{x}_2^h}{x_2^h} &= x_1^H\left((1-\tau)W + e\right) + (1-x_1^H)(1-\tau)w - C \\ &\quad -\frac{X_2^h}{X_2^h + X_2^l} \cdot \left(x_1^H\left((1-\tau)W + e\right) + (1-x_1^H)(1-\tau)w - C\right) \\ &\quad -\frac{X_2^l}{X_2^h + X_2^l} \cdot \left(x_1^H\,(1-\tau)\,W + (1-x_1^H)\,(1-\tau)\,w\right).\end{aligned}$$

After some algebraic manipulation this expression yields,

$$\begin{aligned}\dot{x}_2^h &= x_2^h x_2^l [\left(x_1^H\left((1-\tau)W + b\right) + (1-x_1^H)(1-\tau)w - C\right) \\ &\quad - \left(x_1^H\,(1-\tau)\,W + (1-x_1^H)\,(1-\tau)\,w\right)].\end{aligned}$$

Hence, we get the following system of replicator dynamics,

[7] A replicator dynamic equation is a process of change over time in the frequency distribution of the replicators (and in the nature of the environment and the structure of interaction), in which strategies with higher payoffs reproduce faster in some appropriate sense. For instance, it may generate novelty if random errors ("mutations" or "perturbations") occur in the replication process, allowing new replicators to emerge and diffuse into the population if they are relatively well adapted to the replicator system.

[8] This condition assures that the trajectory $x_k(t) = \left\{(x_k^i(t), x_k^j(t)),\ t_0 \leq t\right\}$ is bounded in the unit square $\mathbb{C} = [0,1] \times [0,1]$.

[9] Analogous procedure applies for obtaining the replicator dynamic equation of subpopulation 1.

$$\begin{aligned}\dot{x}_2^h &= x_2^h x_2^l \left[x_1^H e - C\right] \\ \dot{x}_1^H &= x_1^H x_1^L \left[\begin{array}{c} x_2^h \left((1-\gamma)(U-u+V-v)\right) \\ -(1-\gamma)(V+u)-(W-w) \end{array}\right]\end{aligned} \tag{10}$$

In the above nonlinear system, in the steady state $\dot{x}_2^h = 0$, $\dot{x}_1^H = 0$, we obtain five equilibria: $(0,0),(0,1),(1,0),(1,1)$ and the interior $\left(x_2^{h^*}, x_1^{H^*}\right)$, as the interesting case lying in the interior of the square $\mathbb{C} = [0,1] \times [0,1]$, this happening when,

$$\begin{aligned}x_1^{H^*} &= \frac{C}{e}, \\ x_2^{h^*} &= \frac{(1-\gamma)(V-u)+(W-w)}{(1-\gamma)(V-v+U-u)-e}.\end{aligned} \tag{11}$$

while of course the other four equilibria are the corners of the square itself. Figure 2 gives a graphic representation of the vector field and the basins of attraction.

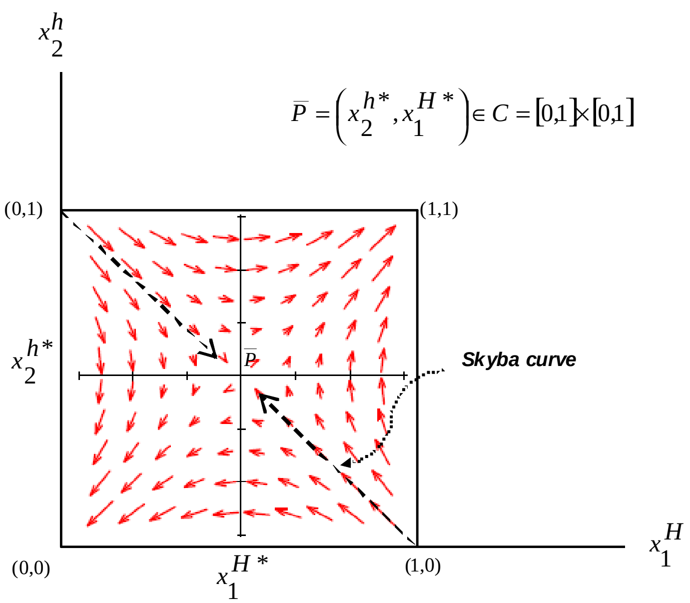

Figure 2. Solution orbits to the replicator dynamic system (10).

3.1 Dynamic stability and equilibria analysis

A strategy is an ESS if a whole population using that strategy cannot be invaded by a small group with a mutant genotype. Similarly, a cultural form is an ESS if, upon being adopted by all members of a society (firm, family, etc.), no small group of individuals using an alternative cultural form can invade. We thus move from explaining the actions of individuals to modeling the diffusion of forms of behavior ("strategies") in society (Gintis, 2000). Recall that, an equilibrium in the replicator dynamics is an evolutionary equilibrium (and is equal to the locally asymptotically stable point in dynamic systems), it is an evolutionarily stable strategy and thus they are ESS of the game (Ross Cressman, 1992 and Shone, 2003). Hence, we can assess whether the five equilibria are ESSs via analyzing Jacobean Matrix of system (10), such that, equilibria fitting $\det(J) > 0$ and $tr(J) < 0$ are asymptotically stable, thus they are ESS of the game. The Jacobian associated to the system is given by,

$$J\left(\dot{x}_2^h, \dot{x}_1^H\right) = \begin{pmatrix} \left(1-2x_2^h\right)\left(x_1^H e - C\right) & x_2^h(1-x_2^h)e \\ x_1^H(1-x_1^H)(1-\gamma)\left(U-u+V-v\right) & \left(1-2x_1^H\right)\mathbb{k} \end{pmatrix}, \qquad (12)$$

where $\mathbb{k} = \left(x_2^h\left((1-\gamma)\left(U-u+V-v\right)\right) - (1-\gamma)\left(V+u\right) - \left(W-w\right)\right)$. Since, the $\mathbb{C}$-square is partitioned into four regions (Figure 2), the point $\left(x_2^{h^*}, x_1^{H^*}\right)$ is a saddle and all other four being local attractors ((0,0) and (1,1)) or repulsors ((0,1) and (1,0)).

That is, (0,0) is the strategy profile in which the economy will converge to (low-profile leaders, low-profile followers) on the contrary (1,1) is the strategy profile that converges to (high-profile leaders, high-profile followers). Notice that history dependence is obtained if and only if, depending on the initial conditions, the optimal solutions of (7) converge towards two or more distinct attractors, like (0,0) and (1,1). These are steady-states.

The thresholds separating different longrun behaviors have been called occasionally Skiba sets in the economic literature (see Brock and Malliaris, 1989). Since the initial values of the adjoint variables differ generically for different trajectories, the control exhibits a jump in a Skiba point. Hence, we found a Skiba point $\bar{P} = \left(x_2^{h^*}, x_1^{H^*}\right)$ separating paths leading to stable high and low level steady-states. Thus, this model can explain the coexistent of countries with low and of countries with high growth as a function of their respective initial conditions alone.

Definition 2 (Skiba, 1978). *The Skiba curve is a set of critical values that converges to $\bar{P}$ into the state of the C-square with the following property: the optimal strategy is different depending on which side of the threshold the current state lies.*

Consequently, the point $\bar{P}$ is a threshold where the dynamics leading to two different long term solutions separate.[10] Next propositions summarize the results.

Proposition 1 *The economy can be located either in a low-level equilibrium* $(0,0)$ *or high-level equilibrium* $(1,1)$ *which are evolutionarily stable strategies.*

Proposition 2 *Equilibria point* $(0,0), (1,1)$ *and* $\left(x_2^{h^*}, x_1^{H^*}\right)$ *are "SCE" to the long-run outcome (steady states or asymptotic steady states) of this economy. Moreover,* $\left(x_2^{h^*}, x_1^{H^*}\right)$ *is a threshold level separating the basin of attractions from the poverty trap to the high-level equilibrium through the Skiba curve.*

Equilibrium (0,0), which is the low level equilibrium, is the most probable outcome in less developed countries in early stages of development. It may often be interpreted as a poverty trap, as it is characterized by low levels of skills and technological profile. On the other hand, the high level equilibrium (1,1) is generally found in developed countries in which the existence of low-profile economic agents may be negligible. However, the mixed equilibrium can be found also in some countries of Latin America (see Ros, 2001).

3.2 Replicator by imitation

Now, economic agents review their strategies and they can observe the performance of their neighbors, so they wonder whether stick to a strategy/club or change over, a function of the type of individuals in their own population they encounter. This is a model of pure imitation driven by dissatisfaction where all reviewing agents adopt the strategy of *the first person that they meet in the street*, picking at random this person from the population (see Alos-Ferrer and Weidenholzer, 2006; Bjornerstedt and Weibull, 1995 and Schlag, 1998; 1999).

Each period an i-type economic agent, $i \in \{h, l; H, L\}$, from sub-population $k \in \{1, 2\}$, reviews her strategy with probability $r_k^i(x)$ wondering whether she may or not change her current strategy, where $x = (x_1^i, x_2^i)$, $\forall i \in \{h, l; H, L\}$.

Let $p_k^{ij}(x)$ be the probability that a reviewing i-strategist really changes to some pure strategy $j \neq i$, $\forall j \in \{h, l; H, L\}$. In the sequel, $s_i = \{(h, l)\,; (H, L)\}$ will indicate vectors of pure strategies independently from population k.

Thus, the *outflow* from club i in population k is $x_k^i r_k^i(x) p_k^{ij}(x)$ and the *inflow* is $x_k^j r_k^j(x) p_k^{ji}(x)$, as defined in the above section x_k^i is the fraction of $i-$type strategists. By the law of large numbers we model these processes as deterministic flows and rearranging terms, we get,

$$\dot{x}_k^i = x_k^j \left[r_k^j(x) p_k^{ji}(x) \right] - x_k^i \left[r_k^i(x) p_k^{ij}(x) \right], \ \forall\, j \neq i \in \{h, l; H, L\}\,,\ k \in \{1, 2\}. \tag{13}$$

System (13) represents the interaction between two groups of economic agents: leaders and followers changing their behaviors under imitations' pressure

[10] Recall that, following the pioneering article of Skiba (1978), such thresholds have been called Skiba points in the economic literature.

driven by dissatisfaction The aim of this model is to capture an evolutionary stable situation in which all members of the two different sub-populations adopt a behavior that is the better possible given the behavior of the individuals of his own sub-population and the characteristic of the agents of the other sub-population.

Let us assume that the decision of an economic agent depends upon the utility associate with her own behavior, given composition of the other population, labeled by the notation $E_k^i(s_i, x_{-k})$, $\forall i \in \{h, l; H, L\}$, of sub-population $k, -k \in \{1, 2\}, k \neq -k$. Let $r_k^i(x)$ be the average time-rate at which an individual that currently uses strategy i, reviews her strategy choice. Then,

$$r_k^i(x) = \ f_k^i\left(E_k^i(s_i, x_{-k}), x\right)$$

The function $f_k^i\left(E_k^i(s_i, x_{-k}), x\right) \in [0, 1]$ is reasonably interpreted as the propensity of a member from the i-th club that considers to switch membership as a function of the expected utility gains from such a choice. Agents with less successful strategies on average review their strategy at a higher rate than agents with more successful strategies.

Once opted for a change, she will adopt the strategy followed by the first population fellow to be encountered (her neighbor), i. e., for any $k \in \{1, 2\}$,

$$p(i \to j/\text{she considers to change her strategy}) = p_k^{ij} = x_k^j, \ \ i, j \in \{h, l; H, L\}, \ i \neq j.$$

Then, equation (13) can be written as:

$$\dot{x}_k^i = x_k^j \ \left[f_k^j\left(E_k^j(s_j, x_{-k})\right) x_k^i\right] - x_k^i\left[f_k^i\left(E_k^i(s_i, x_{-k})\right) x_k^j\right] \tag{14}$$

or

$$\dot{x}_k^i = (1 - x_k^i) x_k^i \left[f_k^j\left(E_k^j(s_j, x_{-k})\right) - f_k^i\left(E_k^i(s_i, x_{-k})\right)\right]. \tag{15}$$

This is a system of four simultaneous equations with four state variables ($x_1^H, x_1^L; x_2^h, x_2^l$ are the state variables) . However, by the normalization rule, $x_1^H + x_1^L = 1$ and $x_2^h + x_2^l = 1$, equation (15) can be reduced to two equations with two independent state variables. Taking the advantage of this property, from now onwards we choose variables x_k^h and x_k^l with their respective equations.

For a first grasp of the problem, let us assume f_k^j to be linear in the utility levels (Weibull, 1995). Thus, this rate is linearly decreasing in payoffs, that is, the average review rate is linearly decreasing in the average payoff, that is,

$$f_k^i\left(E_k^i(s_j, x_{-k})\right) = \alpha_k - \boldsymbol{\beta}_k E_k^i(s_j, x_{-k}) \quad \forall\ i \in \{H, L; h, l\}.$$

with $\alpha_k, \boldsymbol{\beta}_k \geq 0$ and $\frac{\alpha_k}{\boldsymbol{\beta}_k} \geq E_k^i(s_j, x^{-k})$. To get a full linear form, we assume that,

$$E_k^i(s_j, x_{-k}) = s_i A_k x_{-k}, \ \forall\ i \in \{H, L; h, l\};$$

in other words, utility is a linear function of both variables, through a population-specific matrix of weights or constant coefficients, $A_k \in \mathcal{M}_{2\times 2}$, $(k \in \{1, 2\})$.

This latter assumption implies that utility levels reflect population specific (and therefore in principle different) properties, i.e., broadly speaking preference structures over their outcomes. This reduces the previous model to a much simplified version,

$$\dot{x}_k^i = \boldsymbol{\beta}_k x_k^i (1 - x_k^i) \left[(1, -1) A_k x_{-k}^i \right], \quad \forall \; i \in \{H, L; h, l\}, \; \tau \in \{1, 2\}. \tag{16}$$

Now we can study the evolution of the high-profile economic agents by means of their replicator dynamic, that is,

$$\begin{aligned} \dot{x}_1^H &= \beta_1 x_1^H (1 - x_1^H)(a_1 x_2^h + b_1) \\ \dot{x}_2^h &= \beta_2 x_2^h (1 - x_2^h)(a_2 x_1^H + b_2) \end{aligned} \tag{17}$$

where coefficients a and b depend of course upon the entries of the two population-specific matrices, A_k.

3.2.1 Dynamic stability and Nash properties

System (17) admits five stationary states or dynamic equilibria, i.e.

$$(0,0), \; (0,1), \; (1,0), \; (1,1) \text{ and a positive interior equilibrium } \left(\bar{x}_1^H, \bar{x}_2^h\right)$$

where

$$\bar{x}_1^H = -\frac{b_1}{a_1}, \quad \bar{x}_2^h = -\frac{b_2}{a_2}.$$

In fact, the interesting case is when $\bar{P} = \left(\bar{x}_1^H, \bar{x}_2^h\right)$ is an equilibrium lying in the interior of the square $\mathcal{C} = [0,1] \times [0,1]$, this happening when,

$$0 < -\frac{b_1}{a_1} < 1 \text{ and } 0 < -\frac{b_2}{a_2} < 1$$

where vectors (a_1, a_2) and (b_1, b_2) have opposite signs. Next proposition gives a main result.

Proposition 3 *The consistent coefficients range* $\left\{-\frac{1}{2} < (b_1, b_2) < 0 < (a_1, a_2)\right\}$, *which depends on the composition of populations, ensures that the steady states* $(1,1)$ *and* $(0,0)$ *are asymptotically stable equilibria and then ESS, while* $(1,0)$ *and* $(0,1)$ *are non-stable nodes and* $\left(\bar{x}_1^H, \bar{x}_2^h\right)$ *is a saddle point.*

Figure 3 gives a graphic representation to the solution orbits of the standard two-population replicator dynamics (17) driven by imitation in the coordination game of leaders and followers strategists.

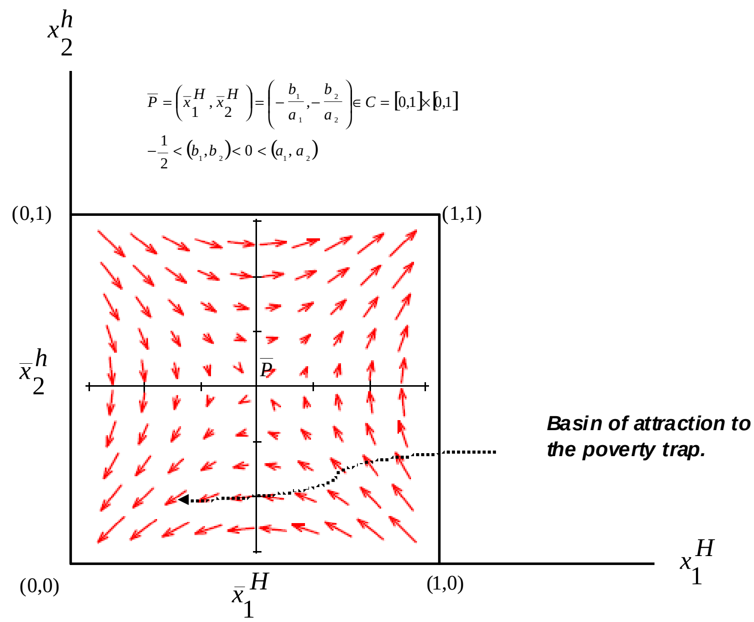

Figure 3. Solution orbits to the system (17) in the Leaders and Followers Coordination Game by imitative behavior.

These equilibria can be interpreted as follows:

- A trivial equilibrium is one where none of the leaders is inclined to be $H-$type and to marry with $h-$type followers and all of them are low-profile economic agents.

- On the other hand there is another equilibrium at the opposite corner (where the sharing clubs involve all of their respective population): this is the case where reciprocal integration, $H-$type married with $h-$type, of the two sub-populations is complete. The two remaining border equilibria show a different club dominating the two sub-populations and in a sense a mismatch between strategies.

- Finally, of course we have the interior equilibrium, this the case of complete segregation. Here the economy is composed by marriages among low- or high-profile economic agents.

4 The imitative behavior of the followers

In this section, we are interested in the behavior of the followers, since they face a training cost to be high-profile economic agents when marriage with $H-$type leaders or they prefer a low-profile when marriage with $L-$type leaders.

Consider that such economic agents review their strategies following a "behavioral rule with inertia" (Schlag, 1999) that allows to reconsider an action with a probability $R \in (0,1)$, that is, an $i-$type strategist, $i \in \{h, l\}$ with probability R_i will ask herself whether to change or not her behavior[11]. Likewise, let P_{ij} be the probability that such an $i-$type reviewing strategist will change to strategy $j \in \{h, l\}$, $j \neq i$. If strategy choices are statistically independent random variables, the aggregate arrival rate of the Poisson process of individuals who switch from one pure strategy i to another j is $R_i P_{ij}$.

Hence, the flow of high-profiles individuals by $\dot{x}_2^h$, is equal to the number of changing low-profiles minus the number of high-profiles individuals changing to the low-profile's club.[12] Rearranging terms, one obtains the system of differential equations that characterizes a dynamic flow of profiles,

$$\begin{aligned} \dot{x}_2^h &= R_l P_{lh} x_2^l - R_h P_{hl} x_2^h \\ \dot{x}_2^l &= -\dot{x}_2^h \end{aligned} \tag{18}$$

as before x_2^h and x_2^l are the fraction of high- and low-profiles economic agents of the follower's sub-population 2. We call an imitative behavior when an economic agent decides to pass from the $i-$ to the $j-$strategy if i is currently more popular than j, $j \preceq i$, in the sense that more individuals of population k currently use i, then the choice probability of i should exceeds that of j.

Definition (Imitative behavior). *A profile's population dynamics* (18) *represents an imitative behavior if* $j \preceq i \Rightarrow P_{ij} > P_{ji}, \;\; \forall i, j \in \{h, l\}, i \neq j$.

Although an individual does not know all the true values of the payoff of the other ones, it is possible that she can take a sample of true values in order to estimate an average. Let $\hat{E}^i$ and $\hat{E}^j$ be the estimators for such true values E^i and E^j. In the particular case that $\hat{E}^l < \hat{E}^h$ or equation (18) holds, then, each $l-$type strategist prefers to be a $h-$type if and only if the probability that an $l-$strategist becomes $h-$strategist is given by $P[\hat{E}^h - \hat{E}^l > 0]$. Hence, (18) can be written as,

$$\begin{aligned} \dot{x}_2^h &= R_l P[\hat{E}^h - \hat{E}^l > 0] x_2^l - R_h P[\hat{E}^l - \hat{E}^h > 0] x_2^h, \\ \dot{x}_2^l &= -\dot{x}_2^h. \end{aligned} \tag{18}$$

[11] Then, R_i denotes the average time-rate at which an individual that currently uses strategy $i \in \{h, l\}$, reviews her strategy choice.

[12] Since we consider large populations, we invoke the law of large numbers and model these aggregate stochastic processes as deterministic flows, each such flow being set equal to the expected rate of the corresponding Poisson arrival process.

Hence, an evaluation rule that seems particularly natural in a context of simple imitation is the "average rule" where each strategy is evaluated according to the average payoff observed in the reference group (see J. Apesteguia et al., 2007). There are idiosyncratic preference differences between economic agents in subpopulations and by the average rule we can get the next result.

Proposition 4 *Consider that* $P[\hat{E}^j - \hat{E}^i > 0]$ *increases proportionally to the true value* E^j*, that is,*

$$P[\hat{E}^j > \hat{E}^i] = \begin{cases} \lambda E^j & if \;\; E^j > 0 \\ \\ 0 & if \;\; E^j \leq 0 \end{cases} \qquad \forall i, j \in \{h, l\}, i \neq j \tag{19}$$

where $\lambda = \frac{1}{|E^l + E^h|}$*. Then, an imitative behavior of the followers,* x_2^h*, leads the economy into a poverty trap if the initial number of* $H-$*type "leaders",* x_1^H*, is lower than a certain threshold value* $\tilde{x}_1^H = \frac{C}{e}$*. Moreover, if* $x_1^H > \tilde{x}_1^H$*, then the relative population of* x_2^h *increases and* E^h *increases faster than* E^l *as* x_1^H *increases.*

Besides, it is important the final position of the economic agents and/or institutions on their types or profiles, that is, the fraction of leaders and followers that are skilled. The initial number of high-profile types of leaders determines the evolution of high-profile followers. For instance, if the initial number of good institutions is high then the economic agents by imitative behavior would decide the strategy of being high-profile types. Hence, it would then be rational for the leaders to be of high-profile because they get larger benefits with the marriage with high-profile followers. Then, the economy would go to the high level equilibrium.

5 Conclusion

We studied an evolutionary game of the complementarity between the types of profiles of the economic agents. We show that the economy can be located in a low-level equilibrium and that there exist a threshold level equals the ratio "education costs-efficiency wages" to overcome the poverty trap. We found the evolutionarily stable strategies of this game and we conclude that the possibility of either high-level and low-level equilibria implies that players, economic agents, acting under identical settings may experience either adequate living standard or deprivation (growth or crisis), respectively, and it depends only on their histories or initial conditions.

Hence, the types of economic agents or institutions can lead the economy in a low-level or high-level equilibrium which are evolutionary outcomes. If the leaders of the economy decide or have the incentives of being $H-$type strategists, then, by an imitative behavior of the followers the economy converges to the high-level equilibrium (1,1).

Countries where the number of economic agents surpasses the threshold levels of physical and human capital can overcome the poverty trap, which is a latent threat for any developing country. Notice that, it is not only necessary the development of human capital to avoid the poverty trap, also it is necessary the accumulation of physical capital or high technology complementing the development of human capital. In fact, in many developing countries, a huge effort in generating a quality educational system did not have an impact on the performance of the economy. For example, several decades ago Uruguay and Argentina have invested heavily in public education, in order to generate a highly-skilled workforce. But this effort was not accompanied by a similar investment by the firms in R&D. This happened because those countries were protectionist and there were not competitiveness incentives to develop R&D departments. After this failed attempt the outlays in education decreased, since high wages were not correlated with time spent in schooling (Ros, 2001). This trend deepened after the Uruguayan economy became more open. On the contrary, the firms using high technology in Mexico are foreign-owned (the so called maquiladoras) that import their technology, which is developed abroad, and contract in Mexico only low-skilled workers. Therefore, these firms lack R&D departments in Mexico and do not create incentives to increase the skills of their local workforce.

Future research indicates that the assumption on a constant number of $H-$type leaders should be modified, as for example, defining the number of $H-$type leaders as an increasing function of the number of high-profile followers, that is, $x_1^H = f(x_2^h)$, $f'(\cdot) > 0$.

We conclude that the market alone is incapable of overcoming this poverty trap, policy makers should intervene, for instance, by providing some kind of financial incentive for R&D investment or by imposing a minimum period of schooling.

Appendix

Proof of Proposition 1. We evaluate $J\left(\dot{x}_2^h, \dot{x}_1^H\right)$,

1. $x_2^h = x_1^H = 0$. The evaluated Jacobean in this case is given by,
   $$J = \begin{bmatrix} -C & 0 \\ 0 & -\left((1-\gamma)(V+u)+W-w\right) \end{bmatrix}$$
   It yields that, $detJ = (W - w + (1-\gamma)(V+u))(C) > 0$ and $trJ < 0$. Hence this equilibrium point (0,0) is an attractor and then ESS.

2. $x_2^h = x_1^H = 1$. The evaluated Jacobean is given by,
   $$J = \begin{bmatrix} -(e-C) & 0 \\ 0 & -\left(W-w+(1-\gamma)(U-2u-v)\right) \end{bmatrix}$$
   Then, $detJ > 0$ and $trJ < 0$. Hence this equilibrium point (1,1) is an attractor and ESS.

3. $x_2^h = 1$, $x_1^H = 0$. The evaluated Jacobean is,
   $$J = \begin{bmatrix} C & 0 \\ 0 & (1-\gamma)(U-2u-v)-W+w \end{bmatrix}$$
   Then, $detJ > 0$ and $trJ > 0$. In this case, the equilibrium point (1,0) is repulsor.

4. $x_2^h = 0$, $x_1^H = 1$. The evaluated Jacobean in this case is,
   $$J = \begin{bmatrix} e-C & 0 \\ 0 & (1-\gamma)(V+u)+W-w \end{bmatrix}$$
   Then, $detJ > 0$ and $trJ > 0$. In this case, the equilibrium point (0,1) is repulsor.

5. The interior equilibrium $x_2^{h^*} = \frac{(1-\gamma)(V-u)+(W-w)}{(1-\gamma)(V-v+U-u)-e}$ and $x_1^{H^*} = \frac{C}{e}$. Evaluating this point in the Jacobean yields,
   $$J = \begin{bmatrix} 0 & x_2^{h^*}(1-x_2^{h^*})(\tau w+e) \\ x_1^{H^*}(1-x_1^{H^*})(1-\gamma)(U-u+V-v) & \left(1-2x_1^{H^*}\right)\mathbb{k} \end{bmatrix}$$
   where $\mathbb{k} = x_2^{h^*}\left((1-\gamma)(U-2u-v)-W+w\right)$. Then, $detJ < 0$ and then, the equilibrium point (n_f^*, n_h^*) is a saddle point.

It characterizes the equilibria which are ESS. ■

Proof of Proposition 2. By the proof of proposition 1 the equilibria point $(0,0),(1,1)$ and $\left(x_2^{h^*}, x_1^{H^*}\right)$ are asymptotic steady states. In addition, for all of the self-confirming equilibria to be possible long-run outcomes, it is necessary that there not be too so much experimentation at any point in the process, as otherwise players might learn the true distribution of off-path play. By equations (1) and (2), player 1 decides being $H-$type if $(1-\gamma)U-W-e \geq (1-\gamma)v-w$ and $\sigma = 1$, which is consistent with the fact that $\sigma > \frac{C}{e}$, hence it is a SCE. But if $(1-\gamma)U-W-e < (1-\gamma)v-w$ and $\sigma = 0$ then 1 decides being $L-$type which is not consistent with $\sigma > \frac{C}{e}$ and then it is not a SCE. Analogously, 2 prefers being $l-$type strategist if $\sigma < \frac{C}{e}$ and 1 chooses $H-$type only if $(1-\gamma)U - W - e \geq (1-\gamma)v - w$ and $\sigma = 1$ which is not consistent, hence it is not a SCE. But 1 chooses $L-$type only if $(1-\gamma)U - W - e < (1-\gamma)v - w$ and $\sigma = 0$, which is consistent with $\sigma < \frac{C}{e}$, and then it is a SCE. In both Nash equilibria $\{H, h\}$ and $\{L, l\}$ the economic agents earn the largest fitness payoffs and would continue to do so with slight variations in the proportion of other mismatching types, or only one type remains in the population and the other type cannot re-enter without earning less than the other type. Since we have a game of incomplete information where it is possible switching among multiple player types, i.e. types of skills $\{H, h; L, l\}$. The SCE $\left\{H, h, \sigma \geq \frac{C}{e}\right\}$ and $\left\{L, l, \sigma < \frac{C}{e}\right\}$ correspond to (1,1) and (0,0) steady states of the evolutionary game, respectively. The mixed strategy corresponds to the interior equilibrium. Both equilibria (0,0) and (1,1) are ESS and they cannot be invaded by any possible mutation or mutant strategy. Moreover, by Definition 2 $\bar{P} = \left(x_2^{h^*}, x_1^{H^*}\right)$ is a threshold in the form of a Skiba point, since, by Definition 2 it separates the basins of attraction of the low-level and high-level equilibria. ■

Proof of Proposition 3. The Jacobean associated to the system (17) is,

$$J = \begin{bmatrix} \beta_1(1-2_1x_1^H)(a_1x_2^h+b_1) & \beta_1 a_1 x_1^H(1-x_1^H) \\ \beta_2 a_2 x_2^h(1-x_2^h) & \beta_2(1-2x_2^h)(a_2x_1^H+b_2) \end{bmatrix}$$

such values, of course, depend on the population specific matrices. Recall that vectors (a_1, a_2) and (b_1, b_2) have opposite signs. Hence,

1. $x_1^H = x_2^h = 1$, the evaluated Jacobean in this case is,

$$J = \begin{bmatrix} -\beta_1(a_1+b_1) & 0 \\ 0 & -\beta_2(a_2+b_2) \end{bmatrix}.$$

   when (b_1, b_2) are negative and (a_1, a_2) positive numbers,[13] then,

$$\begin{aligned} \det(J) &= (-\beta_1(a_1+b_1)) \cdot (-\beta_2(a_2+b_2)) > 0 \\ tr(J) &= -\beta_1(a_1+b_1) - \beta_2(a_2+b_2) < 0, \end{aligned}$$

   and then, this equilibrium point **(1,1) is stable node, and then, it is an ESS.**

2. $x_1^H = x_2^h = 0$, the evaluated Jacobean is,

$$J = \begin{bmatrix} \beta_1 b_1 & 0 \\ 0 & \beta_2 b_2 \end{bmatrix}.$$

   Note that, being (b_1, b_2) negative and (a_1, a_2) positive numbers, then, $\det(J) > 0$ and $tr(J) < 0$, and then, this equilibrium point **(0,0) is asymptotically stable and it is an ESS.**

3. $x_1^H = 1$, $x_2^h = 0$, the evaluated Jacobian is,

$$J = \begin{bmatrix} -\beta_1 b_1 & 0 \\ 0 & \beta_2(a_2+b_2) \end{bmatrix}.$$

   Being (b_1, b_2) negative and (a_1, a_2) positive numbers, then, $\det(J) > 0$ and $tr(J) > 0$, and then, this equilibrium point **(1,0) is a non-stable** node.

[13] Otherwise, if (b_1, b_2) are positive and (a_1, a_2) negative numbers, then, $\det(J) > 0$ and $tr(J) < 0$, and then, this equilibrium point **(1,1) is asymptotically stable and then, it is an ESS.** Therefore, (1,1) is always a stable node.

4. $x_1^H = 0$, $x_2^h = 1$, the evaluated Jacobian is,

$$J = \begin{bmatrix} \beta_1(a_1 + b_1) & 0 \\ 0 & -\beta_2 b_2 \end{bmatrix}.$$

   Being (b_1, b_2) negative and (a_1, a_2) positive numbers, then, $\det(J) > 0$ and $tr(J) > 0$, and then, this equilibrium point **(0,1) is a non-stable node**.

5. The interior equilibrium $\bar{x}_1^H = -\frac{b_1}{a_1}$ and $\bar{x}_2^h = -\frac{b_2}{a2}$, on the Jacobean yields,

$$J = \begin{bmatrix} \beta_1(1 + 2\frac{b_1}{a_1})(-a_1\frac{b_2}{a_2} + b_1) & -\beta_1 b_1(1 + \frac{b_1}{a_1}) \\ -\beta_2 b_2(1 + \frac{b_2}{a_2}) & \beta_2(1 + 2\frac{b_2}{a_2})(-a_2\frac{b_1}{a_1} + b_2) \end{bmatrix}.$$

If $-\frac{1}{2} < (b_1, b_2) < 0$ and (a_1, a_2) are positive numbers, then, $\det(J) < 0$ and then, this equilibrium point $\left(\bar{x}_1^H, \bar{x}_2^h\right)$ is a saddle point. ■

Proof of Proposition 4. If individuals do not observe exact payoffs of neighbors, but they only observe average payoffs in the neighborhood and they imitate the strategy that yields the highest average payoff. The process of copying successful behaviors exhibits *payoff monotonic updating*, since strategies with above-average payoffs are adopted by others and thus increase their share in the population, that is, $P_{ij} > P_{ji},\ \ \forall i, j \in \{h, l\}, i \neq j$. In this way, system (19) takes the form,

$$\begin{aligned} \dot{x}_2^h &= R_l \lambda E^l x_2^l - R_h \lambda E^h x_2^h, \quad (i) \\ \dot{x}_2^l &= -\dot{x}_2^h. \qquad\qquad\qquad\quad (ii) \end{aligned} \tag{20}$$

We know that E^l (equation (2)) is always positive while E^h (equation (1)) can be either positive or negative depending on the training cost, C, of being a $h-$type "follower" which depends on the probability of being hired by a $H-$type strategist "leader", $\sigma = x_1^H \geq \frac{C}{e}$. Then, $E^h > 0$ if $C < \frac{(1-\tau)wb}{(1-\tau)(W-w)+2e}$. Let us assume a constant number of $H-$type strategist of sub-population 1, and that payoff functions E^h and E^l are constant. If $E^h > 0$ then, the equation $(21(i))$ takes the form[14],

$$\begin{aligned} \dot{x}_2^h &= \left(\frac{1}{E^h + E^l}\right)\left(R_l E^l x_2^l - R_h E^h x_2^h\right), \\ \dot{x}_2^h &= \frac{R_l E^{ll} x_2^l - R_h E^h x_2^h}{E^l + E^h}, \\ \dot{x}_2^h &= A x_2^h + R_l[E^h + E^l]E^h. \end{aligned}$$

where,

$$\begin{aligned} A &= \left(E^h\right)^3 R_l + \left(E^h\right)^2 [2E^l R_l - R_h] + \\ &\quad + E^h[\left(E^l\right)^2 R_l - E^h R_h] + R_h \end{aligned}$$

and the solution of this differential equation has the form,

$$x_2^h(t) = x_2^h(0) \exp^{-At} + [R_l \lambda E^h] t$$

where $x_2^h(0)$ is the fraction of the high-profile agents at time $t = 0$. It follows that its solution depends on the number of high-profile agents of sub-population 1, since E^h and E^l are increasing functions of the percentage of x_1^H. However if $x_1^H > \tilde{x}_1^H$ then E^h increases faster than E^l as x_1^H increases[15]. Note that there exists a threshold value for the high-profile agents $\tilde{x}_1^H = \frac{C}{e}$ such that whether $x_1^H > \tilde{x}_1^H$ the population of high-profile agents increases[16]. ■

[14] This is the Brown-von Neumann-Nash (BNN) dynamic which defines an "innovative better reply" dynamics. Indeed, strategies with payoff below average decrease in frequency, while strategies with payoff above average increase, as long as they are rare enough (and even if their frequency is 0).

[15] Note that if $A > 0$ then $\dot{x}_2^h(t) > 0$ for all $t > -\frac{R_l \lambda E^h}{A}$.

[16] If the payoff of high-profile agents is negative, $E^h < 0$, then, equation $(21(i))$ takes the

References

[1] Acemoglu, D. (1996), "A Microfoundation for Social Increasing Returns in Human Capital Accumulation", *Quarterly Journal of Economics*, pp. 779-804.

[2] Alos-Ferrer and S. Weidenholzer (2006), "Imitation, Local Interactions and Efficiency," *Economics Letters* 93(2), pp. 163 - 168.

[3] J. Apesteguia, S. Huck and J. Oechssler (2007), "Imitation-theory and experimental evidence," *Journal of Economic Theory* 136, pp. 217-235.

[4] Aoki, Masahiko (2001), *Toward a Comparative Institutional Analysis*, Cambridge MA, MIT Press.

[5] Aghion P. and Howitt P. (1999), "On the Macroeconomic Consequences of Major TechnologicalChange," in *General Purpose Technologies and Economic Growth*, ed. E. Helpman, Cambridge: MIT Press.

[6] Arthur, W. Brian (1989), "Competing Technologies, Increasing Returns and Lock-In by Historical Events," *Economic Journal* 99(394), pp. 116-131, reprinted in Arthur (1994).

[7] Arthur, W. Brian (1994), *Increasing Returns and Path Dependence in the Economy*, Ann Arbor, University of Michigan Press.

[8] Azariadis, C. and A. Drazen. (1990), "Threshold externalities in economic development", *Quarterly Journal of Economics* 105, pp. 501-526.

[9] Azariadis, C. (1996), "The Economics of Poverty Traps; Part One: Complete Markets," *Journal of Economic Growth* 1(4), pp. 449-486.

[10] Azariadis, C. and Starchuski, H. (2005), "Poverty Traps" in Aghion, P. and Durlauf, S. (eds.) *Handbook of Economic Growth*, Elsevier.

[11] Battigalli P. (1987), *Comportomento razioanle ed eqiulibrio nei giochi e nelle situazioni sociali*, unpublished undergraduate dissertation, Bocconi University, reprinted in translation in *Decisions, Games and Markets*, ed. Battigalli, Montesano, and Panunzi, Springer, 1997.

[12] Bellante, D. (1994), "Sticky Wages, Efficiency Wages, and Market Processes" *The Review ofAustrian Economics* 8(1), pp. 21-33.

[13] Brock, W. A., and Malliaris, A. G. (1989), *Differential Equations, Stability, and Chaos in Dynamical Economics*, North Holland, Amsterdam, Netherlands.

[14] Bjornerstedt, J. and Weibull, J. (1995), "Nash Equilibrium and Evolution by Imitation," in The Rational Foundations of Economic Behavior, (K. Arrow et al, ed.), Macmillan.

form, $\dot{x}_2^h = -R_h \lambda E^l x_2^h$.Hence, the population of high-skilled workers decreases.

[15] Bowles, Samuel (2006), "Institutional Poverty Traps", in Samuel Bowles, Steven N. Durlauf and Karla Hoff eds., *Poverty Traps*, Princeton, Princeton University Press.

[16] Cressman, Ross (1992), *The Stability Concept of Evolutionary Game Theory*. Berlin Heidelberg: Springer-Verlag.

[17] Durlauf, Steven N. (1996), "A Theory of Persistent Income Inequality," *Journal of Economic Growth* 1, pp. 75-93.

[18] Durlauf, Steven N. (2003), "Neighborhood Effects", Madison, University of Wisconsin, department of economics, SSRI working paper 2003-17 (prepared for J. Vernon Henderson and Jacques-François Thisse eds., Handbook of Regional and Urban Economics, vol. 4, Economics).

[19] Fudenberg D, and Kreps D. (1988), *A Theory of Learning, Experimentation, and Equilibrium in Games*. Mimeo.

[20] Fudenberg D, and Kreps D. (1995), Learning in Extensive Games, I: Self-Confirming Equilibrium. *Games and Economic Behavior* 8: 20-55.

[21] Fudenberg D, and Levine DK. (2007), "Self-Confirming Equilibrium and the Lucas Critique," forthcoming in the *Journal of Economic Theory*.

[22] Fundenberg D, and Tirole J. (1991), *Game Theory*, The MIT Press.

[23] Gintis, H (2000), *Game Theory Evolving*, First Edition, Princeton University Press.

[24] J. Hofbauer(1995), *Imitation dynamics for games*, Preprint.

[25] Krugman, Paul (1998), The Legend of Arthur: A Tale of Gullibility at The New Yorker, Slate, 1-14-1998, http://web.mit.edu/krugman/www/legend.html.

[26] Lucas, R. (1976), Econometric Policy Evaluation: A Critique. *Carnegie-Rochester Conference Series on Public Policy* 1: 19.

[27] Lucas, R. (1988), "On the mechanics of economic development," *Journal of Monetary Econimics* 22(1), pp.3-42.

[28] Nelson, R.; Phelps, E. (1966), "Investment in Humans, Technological Diffusion, and Economic Growth," *American Economic Review*, 61:69-75, 1966.

[29] Redding, S.. (1996), "The Low-Skill, Low-Quality Trap: Strategic Complementarities between Human Capital and R&D", *Economic Journal*, 106(435), pp. 458-70.

[30] Ros, J. (2001), *Development Theory and Economics of Growths*, The University of Michigan Press.

[31] Sargent TJ. (2008), "Evolution and Intelligent Design," *AEA Presidential Address*.

[32] Shone, Ronald (2003), *Economic Dynamics*. China Renmin University Press.

[33] Skiba, A. K. (1978), "Optimal Growth with a Convex-Concave Production Function," *Econometrica* 46:527-539.

[34] Schlag, K. H. (1998), "Why Imitate, and if so, How?" A Boundedly Rational Approach to Multi-Armed Bandits", *Journal of Economic Theory* 78(1), 130-156.

[35] Schlag, K. H. (1999), "Which One Should I Imitate", *Journal of Mathematical Economics* 31(4), 493-522.

[36] Taylor, P. (1979), "Evolutionarily Stable Strategies with Two Types of Player", *Journal of Applied Probability* 16: 76-83.

[37] F. Vega-Redondo (1997), "The evolution of Walrasian behavior," *Econometrica* 65, 375–384.

[38] Weibull, W. J. (1995), *Evolutionary Game Theory*, The Mit Press.