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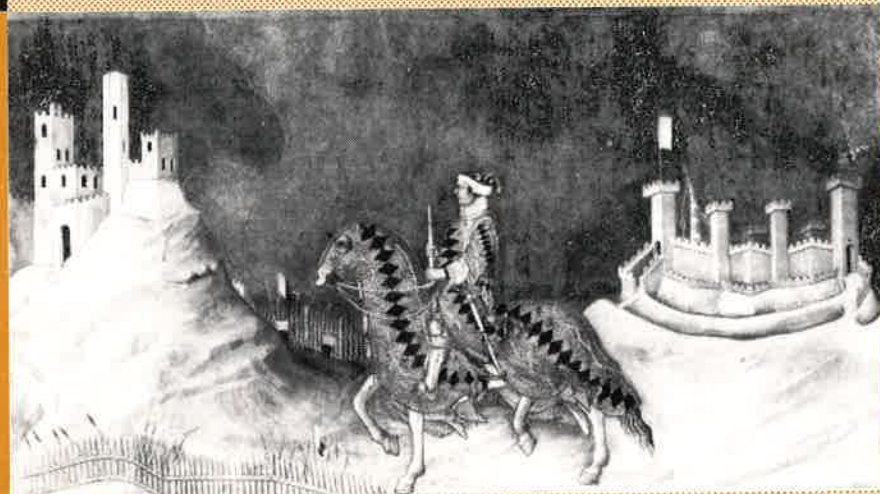
QUADERNI DELL'ISTITUTO DI ECONOMIA

Joan Esteban - Teodoro Millán

COMPETITIVE EQUILIBRIA AND THE CORE OF OVERLAPPING GENERATIONS ECONOMIES



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Joan Esteban - Teodoro Millàn

COMPETITIVE EQUILIBRIA AND THE CORE OF OVERLAPPING GENERATIONS ECONOMIES

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Joan Esteban - Instituto de Anàlisis Econòmico, CSIC and Departament d'Economia i Història Econòmica Universitat Autònoma de Barcelona Bellaterra, Barcelona.

Teodoro Millàn - Departamento de Economia Aplicada Universidad Complutense Madrid

1. Introduction*

Overlapping generations models have been recently analysed from a game theoretical point of view. Specifically, the core of economies with an overlapping generations structure has been studied by Hendricks et al. [11] , Kovenock [13] , Esteban [6] and Chae [3] . Related concepts such as bounded core and short-run core have been dealt with by Hendricks et al. [11] , Chae [3] , Chae and Esteban [4] and Esteban [7] . In this paper we shall examine the relation between competitive equilibria and the core in overlapping generations economies with many agents per generation.

The interest of using a cooperative approach in this type of models might not be immediate. Indeed it is hard to imagine how agents living in unconnected periods can meet and agree on forming a coalition. However, and in spite of the fact that actual bargaining can take place only among agents of coexisting generations, fully informed, rational consumers will behave as if such coalitions could be formed. The sequential structure of the overlapping generations model imposes that only those allocations acceptable for all unborn generations can be acceptable for agents belonging to two coexisting generations. It is in this sense that we can talk of coalitions.

The main purpose of this paper is to analyse whether the well known result that for Arrow-Debreu economies all competitive equilibria belong to the core holds true in overlapping generations economies. In an overlapping generations economy, and in an unrestricted sense, competitive equilibria are not a subset of the core. We define the walrasian set Ω as the set of all those consumption allocations that can be implemented as walrasian equilibria only, i.e. there is no allocation of individual endowments for which the consumption allocation can be obtained as a competitive equilibrium with either outside or inside money (monetary or IOU equilibria). Then, we demonstrate that when we restrict to the walrasian set every competitive equilibrium in this set belongs to the core of the economy.

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For finite horizon Arrow-Debreu economies the relations between efficiency, competitive equilibria and core have been established in a series of well-know theorems, as in Arrow and Hahn [1] for instance. On the one hand, the Fundamental Welfare Theorems state that competitive equilibria are Pareto optimal and that Pareto optimal allocations are implementable as competitive equilibria. On the other hand, we have the result that every consumption allocation implementable as a competitive equilibrium belongs to the core of the economy for all the endowments allocations for which it can be obtained as a competitive equilibrium.

Samuelson [15] himself made it plain that the Fundamental Welfare Theorems are not valid in overlapping generations economies. However, in an important paper Balasko and Shell [2] have demonstrated that an equivalence between competitive and efficient allocations can be obtained by an appropriate redefinition of efficiency. They introduce the notion of weak Pareto optimatily and demonstrate that is satisfied by every competitive equilibrium. Likewise, Chae's [3] work can be seen as an attempt at saving the relation between competitive equilibria and core by redefining the notion of socially viable allocations. He proposes the use of the concept of the bounded core and shows that the competitive equilibria of the economy belong to the bounded core.

The main contributions of this paper are two results establishing the relation between the notions of Pareto optimality, competitive equilibria and core. At variance with the works reported above, our approach does not consist in redefining the concepts of dynamic efficiency and/or core. Instead, we shall establish the relations between the standard concepts but restricted to the walrasian set, Ω. Specifically our results are the following: i) the walrasian set is a subset of the set of Pareto optimal allocations; and ii) every competitive equilibrium in the walrasian set belongs to the core. The fact that competitive equilibria not in the walrasian set may not be Pareto optimal ot not to belong to the core suggests that the fundamental singularity of overlapping generations models lays in the possibility of the occurrence of competitive equilibria in which there is an infinite sequence of consumers violating their budget in every period.

Let us now briefly discuss the relation between these results and the existing literature. For the case of exchange economies with one agent per generation, Hendricks et al. [11] and Esteban [6], have demonstrated that all Pareto optimal walrasian equilibria belong to the core. We know that in overlapping generations economies walrasian equilibria might not be Pareto optimal and thus cannot belong to the core. It can be tempting to conclude, as suggested by Hendricks et al. [11], that the only reason why walrasian equilibria may not belong to the core is that they may fail being dynamically efficient. That this is not the case for economies with many agents per generation has been made plain by Kovenock [13] who has provided an example of a Pareto optimal walrasian equilibrium which does not belong to the core and the core is empty. On this respect, we give here another example of a Pareto optimal walrasian allocation which does not belong to the core, while the core is not empty. Our example is intructive from another point of view. It shows that consumption allocations corresponding to equilibria with inside money might be blocked by the coalition formed by the sequence of lenders. They can grant for themselves a consumption allocation which could be interpreted as a subeconomy that substitutes fiat money for inside money. Further, we show that these examples are by no means pathological. We demonstrate in Proposition 4 that for all competitive consumption allocations with price sequence such that $\text{Lim inf}_{t \to \infty} \|p(t)\| \geq \epsilon > 0$ one can construct distributions of initial endowments for which that equilibrium consumption allocation is not in the core. Moreover, we prove in Proposition 3 that $\text{Lim inf}_{t \to \infty} \|p(t)\| = 0$ is a sufficient condition for a competitive equilibrium to belong to the core of the economy, irrespective of the distribution of endowments. This result is stronger than Chae's [3] Theorem 4.1, where with a continuum of agents he finds that a sufficient condition for an allocation to belong to the core is that the present value of the total endowment be finite.

Besides the eventual interest of these results for general equilibrium theory, the propositions presented in this paper have a special bearing on monetary theory. Specifically, monetary and IOU equilibria can never satisfy the sufficiency condition on prices for a competitive equilibrium to belong to the core. Moreover, by Proposition 4, we can always find examples of Pareto optimal monetary and/or IOU equilibria not belonging to the core of the economy. We devote one section of this paper to the analysis of the specific problems posed by the relation between monetary equilibria and the core. There we show that every monetary equilibrium becomes excluded from the core upon replication of the economy.

All these results are rather negative concerning the suitability of overlapping generations models for the analysis of fiat money. In our discussion at the end of the paper we suggest that our results can be interpreted as a formal demonstration of Clower's [5] observation that, as quoted in de Vries [17] , "money differs from other commodities in being universally acceptable as an exchange intermediary by virtue not of individual choice but rather by virtue of social contrivance" (pp. 14-14).

The paper is structured as follows. Section 2 contains the definitions and assumptions. In section 3 we provide an example of a walrasian Pareto optimal equilibrium which does not belong to the core, while the core is non empty. Section 4 examines the notion of competitive equilibrium and introduces the distinction between walrasian, IOU and monetary equilibria. Further, we define the walrasian set. The relation between efficiency, competitive equilibria and the core of the economy is the object of section 5. There we state our main propositions but the proofs are relegated to section 8. In section 6 and 7 we characterise the walrasian set and prove some auxiliary results which, in fact, are of relevance for monetary theory. Section 9 focusses on monetary equilibria and provides results which complement those obtained in sections 6 and 7. The paper ends with a discussion of the implications of our propositions for monetary theory. We examine the relation with the work of Douglas Gale [8] on the trustworthiness of intertemporal allocations and the role of money. We argue that our results provide a rationale for the lack of trust on the IOU competitive equilibria.

2. *Notation, Assumptions and Definitions*

We shall assume a pure exchange economy with n perishable commodities available at every date. In every period $t, t=1,2,\ldots$, a number m of agents is born(1) and live for two periods. At the beginning of this economy there exists a generation previously born at time $t=0$. Let $c^{j,t}(i,t+s)$ be the consumption of good i $(i=1,2,\ldots)$ at period $t+s$ $(s=0,1)$ $(t=1,2,\ldots)$ by consumer j $(j=1,2,\ldots,m)$ born at t $(t=0,1,2,\ldots)$, $c^{j,t}(t+s) \in R^n_+$ be the consumption vector at period $t+s$ $(s=0,1)$ by agent j born at t, $c^{j,t} \in R^{2n}_+$ the vector of consumptions corresponding to the two periods that agent will be alive, $c^t(t+s) \in R^n_+$ be the vector of consumption by generation t at period $t+s$, and $c^t \in R^{2n}_+$ the vector of consumptions of generation t. For generation $t=0$ we have that $c^0=c^0(1)$ and, of course, $c^0 \in R^n_+$. We shall use c to denote the sequence $c = \{c^0, c^1, c^2, \ldots\}$. Similarly, we shall denote by $w^{j,t}(i,t+s)$ the endowment of good i at $t+s$ by agent born at period t, $w^{j,t}(t+s) \in R^n_+$ be the endowment vector at period $t+s (s=0,1)$ of agent j born at t, $w^{j,t} \in R^{2n}_+$ the vector of endowments corresponding to the two periods that agent will be alive, $w^t(t+s) \in R^n_+$ be the vector of endowments of generation t at period $t+s$, and $w^t \in R^{2n}_+$ the vector of endowments of generation t. Again, for generation $t=0$ we have that $w^0=w^0(1)$ and, of course, $w^0 \in R^n_+$. Finally, w shall denote by w the sequence $w = \{w^0, w^1, w^2, \ldots\}$. Let W be the set of all sequences w which are uniformly bounded from above.

Let us denote by $\bar{w}(t)$ the aggregate endowments available at period t and by $\bar{w}$ the sequence $\bar{w} = \{\bar{w}(1), \bar{w}(2), \ldots\}$. Since $w \in W$ it is obvious that $\bar{w} \in W$ as well. Given a sequence of aggregate endowments $\bar{w}$, we shall denote by $W(\bar{w})$ the set of sequences of individual initial endowments such that

$$w^{t-1}(t)+w^t(t)=\bar{w}^t, \quad w^{t-s}(t) \in R^n_+, \ s=0,1 \text{ and } t=1,2,\ldots .$$

Preferences of consumer j born at t can be represented by a utility function $u^{j,t}: R^{2n}_+ \rightarrow R$, for $t=1,2,\ldots$ and $u^{j,0} R^n_+ \rightarrow R$, for $t=0$, and $j=1,\ldots,m$.

Assumption 1 $u^{j,t}$ has has strictly positive first order partial derivatives and is strictly quasi-concave.

Definition 1 A consumption allocation sequence c is *feasible* if

$$c^{t-1}(t)+c^{t}(t)=c(t) \le w^{t-1}(t)+w^{t}(t)=w(t) \text{ for } t=1,2,\dots .$$

Definition 2 The feasible consumption allocation sequence c is *Pareto optimal* if there is no $\hat{c}$ such that: i) $\hat{c}$ is feasible; and ii) $u^{j,t}(\hat{c}^{j,t}) \ge u^{j,t}(c^{j,t})$ for t=0,1,2,..., and j=1,...,m with at least one strict inequality.

Definition 3 The feasible consumption allocation sequence c is *weakly Pareto optimal* if there is no $\hat{c}$ such that: i) $\hat{c}$ is feasible; ii) $\hat{c}^{j,t}=c^{j,t}$ except for a finite number of periods; and iii) $u^{j,t}(\hat{c}^{j,t}) \ge u^{j,t}(c^{j,t})$ for t=0,1,2,..., and j=1,...m, with at least one strict inequality.

Full information and perfect foresight is assumed throughout the paper. Let p(i,t) denote the price of commodity i delivered at period t,i=1,...,n and t=1,2,..., p(t) the vector $(p(1,t),\dots p(n,t)) \in R^n_+$, and p the price sequence p= $\{p(1),p(2),\dots\}$. We shall normalize by setting p(1,1)=1 and we shall denote by P the set of such price sequences, i.e. $P=\{p/p(1,1)=1, p(t)\in R^n_+\}$.

Definition 4 Let c be a consumption allocation sequence. We shall say that the *price sequence* $p\in P$ *supports* c if and only if, for some suitably chosen endowment sequence $w\in W$, we have that

$$p(t).c^{j,t}(t) + p(t+1).c^{j,t}(t+1) \le p(t).w^{j,t}(t) + p(t+1).w^{j,t}(t+1)$$

and $u^{j,t}(c^{j,t}) \ge u^{j,t}(\hat{c}^{j,t})$ for all $\hat{c}^{j,t}$ satisfying the budget constraint, for j=1,...,m and t=1,2,..., and for t=0

$$p(1).c^{j,0}(1) \le p(1).w^{j,0}(1)$$

and $u^{j,0}(c^{j,0}) \ge u^{j,0}(\hat{c}^{j,0})$ for all $\hat{c}^{j,0}$ satisfying the budget constraint, j=1,...m.

Throughout the paper we shall make extensive use of Balasko and Shell's [2] characterization of Pareto optimum allocations(2). Therefore, we shall assume that the conditions given in their Theorem 5.3 are satisfied.

Assumption 2 (a) The Gaussian curvature of consumer (j,t)'s indifference surface through $c^{j,t}$, $0 < c^{j,t} < [c(t),c(t+1)]$ is uniformly bounded from above and from below away from zero;

(b) There exists a constant P, independent of t, such that

$$0 < P \le p(i,t)/\| p(t) \|, \text{ for } i=1,\dots,n \text{ and } t=1,2,\dots .$$

(c) The sequence c is uniformly bounded from above and from below away from zero by a strictly positive vector.

We shall borrow from Esteban [6] the definitions of coalition and core of the economy and adapt them to the case of many consumers per generation.

Definition 5 A *coalition* is a non-empty connected subset S of the set of all agents.

We shall denote by S_t the set of all agents born at period t which belong to coalition S, so that, $S_t \subseteq S$. Then, by the connectedness of S we mean that if $S_t \ne \emptyset$ and $S_{t+k} \ne \emptyset$, then $S_{t+r} \ne \emptyset$, r=1,...,k-1.

A coalition will thus be formed by a chain of generations, which might not include all their members. We shall denote by f the first of such generations, i.e. $f=\min\{t/S_t \ne \emptyset\}$.

Definition 6 An allocation $\bar{c}$ is *blocked by coalition S* if there exists another allocation c such that:

i) c is feasible for coalition S, i.e.

$$\sum_{j\in S_{t-1}} c^{j,t-1}(t) + \sum_{j\in S_t} c^{j,t}(t) = \sum_{j\in S_{t-1}} w^{j,t-1}(t) + \sum_{j\in S_t} w^{j,t}(t), \text{ for all } S_t \in S,^{(3)}$$

ii) $u^{j,t}(c^{j,t}) \geq u^{j,t}(\bar{c}^{j,t})$ for all agents $(j,t) \in S$, with at least one strict inequality for some agent $(j,t) \in S_t$.

Definition 7 The *core* of an economy is the set of all allocations that are feasible and not blocked by any coalition.

3. *Competitive Equilibria and the Core: an example*

We shall start by giving an example of an economy in which a Pareto optimal non-monetary competitive equilibrium does not belong to the core of the economy, while the set of core allocations in non-empty(4).

Consider a one-good economy with two agents a and b per period with identical preferences $u = [\, c^t(t) c^t(t+1) \,]^{1/2}$ for $t=1,2,\ldots$ and $u = [\, c^0(1)]^{1/2}$ for agents of generation $t=0$. Agents have endowments $w^{a,t}=(1.5,0.5)$ and $w^{b,t}=(0.5,1.5)$ for $t=1,2,\ldots$, and $w^{a,0}=0.5$ and $w^{b,0}=1.5$ for $t=0$.

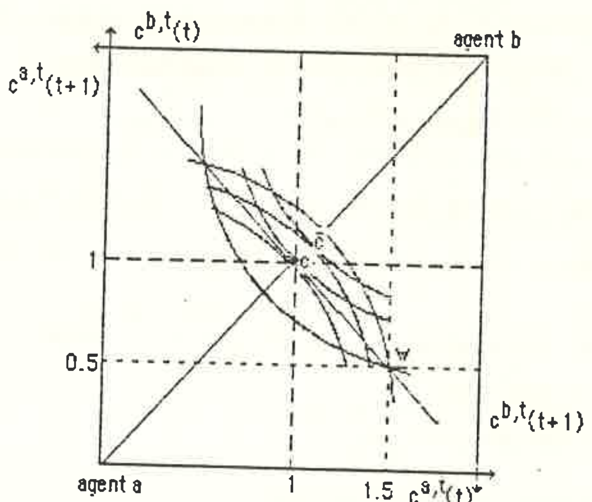

Figure 1

The consumption allocation $c^{a,t}(t)=c^{a,t}(t+1)=c^{b,t}(t)=c^{b,t}(t+1)=1$ is a walrasian equilibrium with equilibrium price sequence $p(t)=1$, for $t=1,2,\ldots$, and $c^{a,0}(1)=0.5$ and $c^{b,0}(1)=1.5$. This walrasian allocation is Pareto optimal because the sum of the inverse of prices diverges, thus satisfying Balasko and Shell's criterion. This allocation however does not belong to the core because it would be blocked by the coalition formed by agents (a,t) $t=1,2,\ldots$ with consumption allocation $\bar{c}^{a,1}(1.5,1)$ and $\bar{c}^{a,t}=(1,1)$ for $t=2,3,\ldots$. Note that the original consumption allocation is a walrasian equilibrium with inside money with agents of type "a" behaving as lenders and those of type "b" as borrowers. The blocking coalition is formed by the sequence of lenders who find preferable substituting outside money for inside money.

The set of core allocations is not empty. Consider for instance the weakly Pareto optimal allocation $\bar{c}^{a,0}(1)=0.5$ and $\bar{c}^{b,0}(1)=1.5$, $\bar{c}^{a,t}=(1.1,1.1)$ and $\bar{c}^{b,t}=(0.9,0.9), t=1,2,\ldots$. This allocation has supporting prices $p(t)=1$ and is thus Pareto optimal. It is a matter of routine to check that this allocation cannot be blocked. Any agent heading a blocking coalition needs a strictly positive compensation in the subsequent period. But no matter the type of agent he tries to get into the coalition, the sequence of compensations needed is strictly increasing and eventually becomes unfeasible.

4. *Competitive Equilibria with Many Agents: Walrasian, IOU and Monetary*

It is obvious that whereas with one agent per generation we could have two types of competitive equilibria, namely walrasian and monetary, with many agents, the possibility of borrowing and lending between agents of the same generation allows for a new type of competitive equilibrium. We shall call IOU the competitive equilibria in which while fiat, outside money is not used agents engage themselves in trades involving borrowing and lending. Let us now introduce the formal definitions.

Definition 8 A *competitive equilibrium* is a sequence of strictly positive commodity prices p, real number M and vector $\mu = (\mu^1, \ldots \mu^m)$, $\mu \in R^m$,

$\Sigma \; |\mu^j| = |\Sigma \mu^j| = |M|$, an endowments allocation sequence $w \in W$, and a consumption allocation sequence c such that:

$$\text{i) } p(t).c^{j,t}(t) + p(t+1).c^{j,t}(t+1) \leq p(t).w^{j,t}(t) + p(t+1).w^{j,t}(t+1)$$

and $u^{j,t}(c^{j,t}) \geq u^{j,t}(\hat{c}^{j,t})$ for all $\hat{c}^t$ satisfying the budget constraint, for $t=1,2,\ldots$ and $j=1,\ldots,m$, and for $t=0$ $p(1).c^{j,0}(1) \leq p(1).w^{j,0}(1)+ \mu^j$ and $u^{j,0}(c^{j,0}) \geq u^{j,0}(\hat{c}^{j,0})$ for all $\hat{c}^{j,0}$ satisfying the budget constraint;

and

$$\text{ii) } c^{t-1}(t) + c^t(t) = w^{t-1}(t) + w^t(t) \text{ for } t=1,2,\ldots .$$

We shall call *monetary equilibria* those competitive equilibria such that $p(t+1).(c^t(t+1)-w^t(t+1))=M$, $t=0,1,2,\ldots$ for some $M \neq O$. The competitive equilibria with M=O can be either IOU or walrasian equilibria. There being many agents per generation, we may have that $p(t+1).(c^{j,t}(t+1)-w^{j,t}(t+1)) \neq 0$ for some j, while M=O for the generation as an aggregate. Indeed agents may not balance their budgets in every period if there is borrowing and lending between agents of the same generation. Further, along the sequence we can have generations in which every agent balances his budget constraint in every period of his life, followed by generations in which there is active borrowing and lending between individuals of the same generation. Therefore, whilst in monetary equilibria fiat money is necessarily purchased in every period along the full sequence, in non-monetary equilibria IOUs may only be used in some periods. Since it is inherent to overlapping generations economies the possibility of analysing the long run behaviour of allocations, we shall call IOU equilibria those competitive equilibria in which IOUs are used in the long run.

In order to make this notion precise, let us define $b^{j,t}(t)$ as,

$$b^{j,t}(t) = \max \{ p(t).[c^{j,t}(t)-w^{j,t}(t)], 0 \}, \; j=1,\ldots,m \text{ and } t=1,2,\ldots ,$$

and $b^t(t)$ and $B^t(t)$ as

$$b^t t = \Sigma \, b^{j,t}(t) \text{ and } B^t(t) = \max \{ b^{1,t}(t),\ldots,b^{j,t}(t),\ldots,b^{m,t}(t) \} .$$

The value $b^{j,t}(t)$ will denote the amount of borrowing engaged by each individual borrower of the economy, $b^t(t)$ the aggregate amount of internal borrowing/lending underwritten by generation t and $B^t(t)$ the maximum amount of individual borrowing in generation t at period t.

Definition 9 Let $w \in W$ and let c be a competitive equilibrium consumption allocation sequence with equilibrium prices $p \in P$ and M=O. We shall say that this competitive equilibrium is an *IOU equilibrium* when $\text{Lim inf}_{t \to \infty} \; b^t(t) \geq \beta > 0$ and a *walrasian equilibrium* otherwise.

Definition 10 Let $w \in W$ be a sequence of aggregate endowment. We shall say that the consumption allocation sequence c is *weakly implementable as a competitive equilibrium* if there exist a price sequence $p \in P$, a real number M and vector $\mu = (\mu^1,\ldots \mu^m)$, $\mu \in R^m$, $\Sigma |\mu^j| = |\Sigma \mu^j| = |M|$, and some suitably chosen endowment sequence $w \in W(\bar{w})$, for which c is a competitive equilibrium accordingly with Definition 8.

Balasko and Shell [2] have demonstrated that every weakly Pareto optimal consumption allocation has supporting prices and can be implemented as a competitive equilibrium. Further, these consumption allocations can always be implemented as walrasian equilibria. For this we need only chosing $w^{j,t}=c^{j,t}$ for every $j=1,\ldots,m$ and $t=0,1,2,\ldots$. From the existence of monetary and IOU equilibria it is obvious that at least some consumption allocations are weakly implementable as monetary and/or IOU equilibria as well as walrasian equilibria. But, as we shall see, there are consumption allocations which are weakly implementable as walrasian equilibria only. We shall call the set of these allocations the walrasian set Ω.

Definition 11 We shall say that the weakly Pareto optimal consumption allocation c belongs to the *walrasian set*, $c \in \Omega$, when it is not weakly implementable as either a monetary or an IOU equilibrium.

Observe that for finite horizon Arrow-Debreu economies all competitive equilibria are walrasian and therefore the walrasian set, as defined here, coincides with the set of Pareto optimal allocations.

5. Efficiency, Core and Competitive Equilibria

In section 3 we have given an example of a Pareto optimal non-monetary competitive equilibrium which does not belong to the core of the economy. Since this consumption allocation can be implemented as an IOU equilibrium it does not belong to the walrasian set. We shall now show that the property of belonging to the walrasian set is a critical condition for a consumption allocation to belong to the core of the economy for all endowments allocations for which it is implementable as a competitive equilibrium.

We have already pointed out in the introduction that for Arrow-Debreu economies the walrasian set coincides with the Pareto set. Thus the standard results relating competitive equilibria, efficiency and core can be restated in terms of the walrasian set. Specifically, we can say that in Arrow-Debreu economies: i) all the allocations belonging to the walrasian set belong to the Pareto set (in fact the two sets coincide) and ii) all the allocations belonging to the walrasian set belong to the core for all the endowment allocations for which they can be implemented as competitive equilibria.

We shall now show that these two propositions hold true for overlapping generations economies. Thus the properties of overlapping generations economies turn out not to be radically different from Arrow-Debreu economies once the analysis is restricted to the walrasian set.

Proposition 1 Let Assumptions 1 and 2 be satisfied. Let c be a consumption allocation sequence belonging to the walrasian set, $c \in \Omega$. Then c is Pareto optimal.

Proposition 2 Let Assumptions 1 and 2 be satisfied. Then the consumption allocation sequence c belongs to the core for every w for which it is a competitive equilibrium if and only if it belongs to the walrasian set, $c \in \Omega$.

The proofs of these Propositions are relegated to section 8. In the next two sections we shall demonstrate a number of auxiliary results which are of interest in their own right.

6. A Characterization of the Walrasian Set

We shall now characterize the consumption allocations sequences in the walrasian set by means of their supporting prices.

Proposition 3 Let Assumptions 1 and 2 be satisfied. The consumption allocation sequence c belongs to the walrasian set if and only if its sequence of supporting prices satisfies that

$$\text{Lim inf}_{t \to \infty} \| p(t) \| = 0.$$

Proof.- It is obvious that any weakly Pareto optimal consumption allocation sequence is weakly implementable as a walrasian equilibrium by means of its supporting prices and chosing w=c. The point that has to be proven is that when the supporting price sequence satisfies the above condition it cannot be implemented either as a monetary or an IOU equilibrium and that when it is not satisfied it can.

In Esteban [6], Proposition 4, it has been shown that the supporting prices to converge to zero is necessary and sufficient for a consumption allocation be weakly implementable as a monetary equilibrium. Hence it only remains to be proven that the same result holds true for IOU equilibria.

Let us start by supposing that the above condition on prices is satisfied and that c is an IOU equilibrium.

By Definition 9 we have that

$$\text{Lim inf}_{t\to\infty} b^t(t) \geq \beta > 0.$$

By the definition of $B^t(t)$ we have the following inequality,

$$m.B^t(t) \geq b^t(t) = \Sigma\, b^{j,t}(t).$$

Let us denote by the superscript k the agent with the highest amount of borrowing in each generation, i.e.

$$B^t(t) = p(t).\,[c^{k,t}(t) - w^{k,t}(t)]\,.$$

It is obvious that

$$\|p(t)\|\,.\,\|c^{k,t}(t)-w^{k,t}(t)\| \geq p(t).\,[c^{k,t}(t)-w^{k,t}(t)] = B^t(t).$$

By Assumption 2(c) the value of $\|c^{k,t}(t)-w^{k,t}(t)\|$ is uniformly bounded above by some $k<+\infty$. Thus we can write the following inequalities

$$m.\,\|p(t)\|\,.K \geq m.\,\|p(t)\|\,.\,\|c^{k,t}(t)-w^{k,t}(t)\| \geq m.p(t).\,[c^{k,t}(t)-w^{k,t}(t)] = m.B^t(t) \geq b^t(t) \geq 0.$$

Therefore when $\text{Lim inf}_{t\to\infty} \|p(t)\| = 0$ it must be that $\text{Lim inf}_{t\to\infty} b^t(t)=0$, thus contradicting the hypothesis that c is an IOU equilibrium. This prove necessity.

For sufficiency one can just follow the same steps as in Proposition 4 in Esteban [6]. At the individual level there is no essential difference between inside and outside money since agents can borrow and lend in both cases, but in IOU equilibria the net borrowing of every generation is nill. When the norm of prices is uniformly bounded from below we can always find an individual allocation of endowments for which the consumption allocation can be implemented as an IOU equilibrium. Consider for instance the endowments allocation sequence w and the sequence of real number $b \in R$ such that $w^{j,t}=c^{j,t}$ $j=3,4,\dots,m$ and $t=0,1,2,\dots$ and $w^{1,t}$ and $w^{2,t}$ satisfying

$$p(t)\ [c^{1,t}-w^{1,t}(t)] = b^{1,t} = -p(t)\ [c^{2,t}(t)-w^{2,t}(t)] = -p(t+1)\ [c^{1,t}(t+1)-w^{1,t}(t+1)] = p(t+1)\,[c^{2,t}(t+1)-w^{2,t}(t+1)\,].$$

By Proposition 4 in Esteban [6] the sequence b exists and is uniformly bounded below by $\beta > 0$. Further, the endowments allocation sequence thus constructed is such that $w \in W$ and hence c is an IOU equilibrium QED.

As we have already pointed out, whereas in finite horizon Arrow-Debreu economies the walrasian set coincides with the set of Pareto optimal allocations, in overlapping generations economies this is not so. We know from the paper by Balasko and Shell [2] that all the consumption allocations with supporting prices p satisfying that $\Sigma\, 1/\,\|p(t)\| = +\infty$ are Pareto optimal. Thus there certainly exist Pareto optimal allocations which do not belong to the walrasian set.

7. *Competitive Equilibria and the Core*

It is quite obvious that any autarkic Pareto optimal competitive equilibrium, i.e. one in which $c^{j,t}=w^{j,t}$ $j=1,\dots m$ and $t=1,2,\dots$, will belong to the core of the economy. We shall study the relationship between non-autarkic competitive equilibria and the core.

Proposition 4 Let Assumption 1 be satisfied. Let c be a consumption allocation sequence and p its supporting price sequence. Then if $\text{Lim inf}_{t\to\infty} \|p(t)\| = 0$ the consumption allocation c belongs to the core for every endowments sequence $w \in W$ for which it is a competitive equilibrium(5).

Proof.- First notice that a price sequence with $\text{Lim inf}_{t\to\infty} \|p(t)\|=0$ cannot correspond to a monetary equilibrium as shown, for instance, in Esteban[6].

Let us assume that c is blocked by coalition S with consumption allocation

$\bar{c}$. Without loss of generality we shall assume that $u^{j,t}(\bar{c}^{j,t}) > u^{j,t}(c^{j,t})$ for some $(j,f) \in S_f$. For any agent $(j,t) \in S_t$, $\bar{c}^{j,t}$, must not be disprefered to $c^{j,t}$, so that,

$$(1) \quad p(t).[\bar{c}^{j,t}(t)-c^{j,t}(t)] + p(t+1).[\bar{c}^{j,t}(t+1)-c^{j,t}(t+1)] \geq 0,$$

for all $(j,t) \in S$.

Since by assumption $\bar{c}$ is strictly prefered to c by some member of generation f, we have that condition (1) holds as a strict inequality for some agent $(j,f) \in S_f$. From the individual budget constraints we have that

$$p(t).c^{j,t}(t)+p(t+1).c^{j,t}(t+1)=p(t).w^{j,t}(t)+p(t+1).w^{j,t}(t+1)$$

for all (j,t).

Substituting in (1) we obtain

$$(2) \quad p(t).[\bar{c}^{j,t}(t)-w^{j,t}(t)] + p(t+1).[\bar{c}^{j,t}(t+1)-w^{j,t}(t+1)] \geq 0,$$

for all $(j,t) \in S$, with strict inequality for some $(j,f) \in S_f$.

Adding over all $(j,t) \in S_t$, we have

$$(3) \quad p(t).\sum_{j \in S_t} [\bar{c}^{j,t}(t)-w^{j,t}(t)] + p(t+1).\sum_{j \in S_t} [\bar{c}^{j,t}(t+1)-w^{j,t}(t+1)] \geq 0$$

for $t \geq f$, with strict inequality for $t=f$.

Consumption allocation $\bar{c}$ must be feasible for the coalition members, that is,

$$(4) \quad \sum_{j \in S_{t-1}} [\bar{c}^{j,t-1}(t)-w^{j,t-1}(t)] + \sum_{j \in S_t} [\bar{c}^{j,t}(t)-w^{j,t}(t)] = 0 \quad \text{for } t \geq f.$$

Combining (3) and (4) we obtain

$$(5) \quad p(t+1).\sum_{j \in S_t} [\bar{c}^{j,t}(t+1)-w^{j,t}(t+1)] \geq p(t).\sum_{j \in S_{t-1}} [\bar{c}^{j,t-1}(t)-w^{j,t-1}(t)] \geq \ldots \geq p(f+1).\sum_{j \in S_f} [\bar{c}^{j,f}(f+1)-w^{j,f}(f+1)],$$

for $t \geq f+1$.

We shall now show that

$$(5) \quad p(f+1).\sum_{j \in S_f} [\bar{c}^{j,f}(f+1)-w^{j,f}(f+1)] > 0.$$

If $f=0$, this follows from the fact that $\bar{c}^{j,0}(1)$ must be strictly prefered to $c^{j,0}(1)$ for some $(j,0) \in S_0$. If $f > 0$ the feasibility condition (4) imposes that

$$\sum_{j \in S_f} [\bar{c}^{j,f}(f)-w^{j,f}(f)] = 0.$$

Thus, from (3) and bearing in mind that $\bar{c}^{j,f}$ must be strictly prefered to $c^{j,f}$, it follows the above inequality.

The terms of the sequence of inequalities in (5) can be bounded above by

$$\| p(t+1) \| . \| \sum_{j \in S_t} [\bar{c}^{j,t}(t+1)-w^{j,t}(t+1)] \| \geq p(t+1). \sum_{j \in S_t} [\bar{c}^{j,t}(t+1)-w^{j,t}(t+1)] .$$

For economies with $w \in W$

$$\| [\sum_{j \in S_t} \bar{c}^{j,t}(t+1)-w^{j,t}(t+1)] \| \text{ is uniformly bounded above.}$$

Hence, if $\text{Lim inf}_{t \to \infty} \| p(t) \| = 0$ must be that

$$\text{Lim inf}_{t \to \infty} p(t+1). \sum_{j \in S_t} [\bar{c}^{j,t}(t+1)-w^{j,t}(t+1)] = 0.$$

Therefore, there does not exist a sequence $\tilde{c}$ satisfying (1) and (4) and c cannot be bloqued. QED.

We shall now study the circumstances under which a competitive equilibrium would not belong to the core of the economy.

Proposition 5 Let Assumptions 1 and 2 be satisfied and let $w \in W$. Let c be a competitive equilibrium consumption allocation and p the equilibrium price sequence with $\text{Lim inf}_{t\to\infty} \|p(t)\| \geq \epsilon > 0$. Then there exists a distribution $w' \in W$ for which c is a competitive equilibrium with the same prices p and does not belong to the core.

Proof.- Let us start by pointing out that for any endowments sequence $w \in W$ such that

(6) $w^t(t)=c^t(t)$ and $w^t(t+1) = c^t(t+1)$, $t=0,1,2,\ldots$, and

(7) $p(t).[c^{j,t}(t) - w^{j,t}(t)] + p(t+1).[c^{j,t}(t+1) - w^{j,t}(t+1)] = 0,$
$j=1,\ldots m$ and $t=0,1,2,\ldots,$

consumption allocation c will be a competitive equilibrium.

In order to prove the proposition we need providing an example of reallocation of initial endowments for which the equilibrium consumption allocation under consideration does not belong to the core. We shall consider in the first place the one-commodity case, i.e. n=1. Moreover, we shall restrict to reallocations of endowments such that $w^{j,t}=c^{j,t}$ for $j=3,4,\ldots m$ and $t=1,2,\ldots$, so that the problem is reduced to finding an appropriate distribution of endowments among two agents in each generation, agents 1 and 2.

A clear case of endowment allocation for which c is not in the core is the one satisfying

(8) $0< c^{1,t}(t+1) - w^{1,t}(t+1) \leqq w^{1,t+1}(t+1)$, $t=1,2,\ldots$

In that case the coalition formed by the sequence of agents (1,t), t=1,2,... would block c.

We need now showing that there exists an endowments allocations sequence satisfying (8) as well as (6) and (7). Using (7), (8) can be rewritten as

(9) $0< c^{1,t}(t+1)-w^{1,t}(t+1) \leqq (p^{t+2}/p^{t+1})(c^{1,t+1}(t+2)-w^{1,t+1}(t+2).$

Let us construct the endowment sequence w satisfying (9) such that

(10) $w^{j,t}(t+1)=c^{j,t}(t+1) - [w^{1,1}(1)-c^{1,1}(1)]/p^{t+1}$, $t=1,2,\ldots$
and with $w^{1,1}(1) - c^{1,1}(1) > 0.$

For every value of $w^{1,1}(1) > C^{1,1}(1)$ and by the use of (10) we can generate a full sequence w (i.e. $w^{1,t}$ and $w^{2,t}$, $t=1,2,\ldots$) satisfying the equilibrium conditions (6) and (7).

It remains only to verify whether at least one of such sequences of endowments satisfies that $w \in W$. By assumption 2(c) the sequence c is uniformly bounded from below by some $\lambda > 0$. It is easy to check that for any sequence of endowments obtained from (10), (6) and (7) such that

(11) $[w^{1,1}(1)-c^{1,1}(1)]/p^t < \lambda$ for $t=1,2,\ldots$

the consumption allocation c is a Walrasian equilibrium, but does not belong to the core by construction. It is now immediate that whenever $\text{Lim inf}_{t\to\infty} \|p(t)\| \geq \epsilon > 0$ there exists some $w^{1,1}(1)$ satisfying (11) such that $w^{1,1}(1) > c^{1,1}(1)$. This completes the proof for n=1.

The extension to the many commodities case is quite straighforward.

From Assumption 2(b) we have that whenever $\text{Lim inf}_{t\to\infty} \|p(t)\| \geq \epsilon > 0$ we have that $\text{Lim inf}_{t\to\infty} p(i,t) \geq P\cdot\epsilon > 0$, $i=1,\ldots,n$. Therefore we can chose $w^{j,t}=c^{j,t}$ for $j=3,4,\ldots,m$ and $t=1,2,\ldots$ and $w^{r,t}(i,t+s)=c^{r,t}(i,t+s)$ for $r=1,2$, $s=1,2$, $i=2,3,\ldots,n$, and $t=1,2,\ldots$. Then our result for n=1 applies. QED.

Proposition 4 gives a sufficient condition for a competitive equilibrium to belong to the core of the economy. The interest of this result lies in the fact proven in Proposition 5 that competitive equilibria do not necessarily belong to the core. Moreover, the sufficiency condition given in Proposition 4 turns out to be critical. As proven by Proposition 5 when this condition is not met one can always find reallocations of initial endowments across individuals of the same generation for which that consumption allocation still is a competitive equilibrium with the same equilibrium prices, but does not belong to the core.

From another point of view, Proposition 5 can be seen as a source of examples of Pareto optimum competitive equilibria that do not belong to the core of the economy. The example provided in Section 3 of a Pareto optimal competitive equilibria not in the core is not exceptional.

8. Proofs of Propositions 1 and 2

Proof of Proposition 1

It follows immediately from our Proposition 3 and taking into account Balasko and Shell's [2] proposition that if the supporting prices of a weakly Pareto optimal consumption allocation satisfy that Lim $\inf_{t\to\infty} \| p(t) \| = 0$ then that consumption allocation is Pareto optimal. QED.

Proof of Proposition 2

Proposition 3 establishes that the supporting prices of a consumption allocation sequence satisfy Lim $\inf_{t\to\infty} \|p(t)\| = 0$ if and only if c belongs to the walrasian set. Thus, Propositions 3 and 4 together imply the sufficiency part of Proposition 2, that is, if a consumption allocation belongs to the walrasian set it belongs to the core for all endowment sequences for which it is a competitive equilibrium. Further, Propositions 3 and 5 together imply the necessity part of Proposition 2. Taken together, they say that if a consumption allocation does not belong to the Pareto set there exists an endowments allocation for which that consumption allocation does not belong to the core while still being

a competitive equilibrium. QED.

9. Some Additional Results on Monetary Equilibria and the Core

We have already pointed out that overlapping generations models have been considered as the most appropriate framework for the analysis of fiat money. It is thus natural paying special attention to the relationship between monetary equilibria and core allocations. Besides the obvious relevance of our previous results to monetary equilibria we shall now introduce to additional result specifically refering to monetary allocations. We start by demonstrating that monetary equilibria with too much money will not belong to the core. But the main result of this section is that as we enlarge to economy by replication every monetary equilibrium becomes eventually excluded from the core.

With many consumers per generation we may have IOU equilibria, monetary equilibria, and a mixture of the two, i.e. simultaneously using IOUs and fiat monetary. The following result refers to economies in which in the long run all intertemporal purchases tend to be made for fiat money. This is an extreme case of a competitive equilibria in which not only fiat money is the only means of transferring purchasing power from present to the future, but money is present in all transactions. We show that these equilibria do not belong to the core.

Proposition 8 Let Assumption 1 be satisfied. Let c be a competitive equilibrium consumption allocation sequence and p the sequence of equilibrium prices. Then if

(12) $\quad \text{Lim inf}_{t\to\infty} p(t+1).\, | c^t(t+1) - w^t(t+1) | / M = 1, \; M > 0,$

the consumption allocation c does not belong to the core.

Proof.- From Definition 4 we know that $p(t+1).\,[c^t(t+1) - w^t(t+1)] = M$, $t=0,1,2,...$

Therefore, if (12) is satisfied we have that

$$\operatorname{Lim\,inf}_{t\to\infty} p(t+1).\,|c^{t}(t+1)-w^{t}(t+1)|\;/p(t+1).\,[c^{t}(t+1)-w^{t}(t+1)] = 1$$

Thus it must be that

$$\operatorname{Lim\,inf}_{t\to\infty}[c^{j,t}(t+1)-w^{j,t}(t+1)] =$$
$$\operatorname{Lim\,inf}_{t\to\infty}|c^{j,t}(t+1)-w^{j,t}(t+1)| = z^{j} \geq z > 0,\ j=1,\dots,m,$$

where $z \in R^{n}_{++}$ and $z(i) = \min\{z^{1}(i),\dots,z^{j}(i),\dots,z^{j}(i),\dots,z^{m}(i)\}$, $i=1,\dots,n$.

Hence there exist T and λ, $0<\lambda\leq z$ such that $[c^{j,t}(t+1)-w^{j,t}(t+1)]\geq\lambda$ for $j=1,\dots,m$ and $t=T,T+1,T+2,\dots$.

Consider now coalition S formed by agents (j,t), $j=1,\dots m$ and $t=T+1,T+2,\dots$ with the consumption allocation $\bar{c}$ such that

$$\bar{c}^{j,t}=c^{j,t} \text{ for all } (j,t)\in S_{t},\ t=T+2,T+3,\dots \text{ and}$$
$$\bar{c}^{j,T+1}=[c^{j,T+1}(T+1)+\lambda\ ,\ c^{j,T+1}(T+2)]\ ,\ j=1,\dots,m.$$

Allocation $\bar{c}$ is feasible for coalition S, is as good as c for all members of S and is strictly prefered by all members of S_{T+1}.
Therefore, consumption allocation c does not belong to the core. QED.

This result seems to substantiate the interpretation given in Esteban [6] as to the effect that it is the acting as a means of exchange rather than a store of value what confers social acceptability to fiat money. We have a case in which in the long run all the exchanges are intertemporal and made by the means of money. Thus, the proposition that no such equilibrium belongs to the core reinforces that view in a many agents economy.

Let us now give a general proposition refered to monetary equilibria. As shows the following Proposition, when the economy is enlarged by replication of the original economy no monetary equilibrium belongs to the core.

Proposition 7 Let ξ be an economy with m agents and n goods in which Assumptions 1 and 2(a) are satisfied. Let c be a monetary equilibrium consumption allocation of the economy ξ and p the equilibrium price sequence. Then, there exists K such that for the K-th replica of the economy $\xi(K)$, the consumption allocation c(K) does not belong to the core.

Proof.- Consider the k+1 replica of the economy and let S be a coalition formed by all agents (j,t), j=1, m, $t=1,2,\dots$, in $\xi(k+1)$ and all agents (j,0), $j=1,\dots,m$, in $\xi(k)$. Since c is a monetary equilibrium we have that $p(1).\,[w^{1}(1)-c^{1}(1)] = M > 0$. Therefore, there exist at least one vector $x^{j,1}(1)\in R^{n}_{+}$ such that $p(1).\,[x^{j,1}(1)-c^{j,1}(1)] > 0$, $j=1,\dots,m$, and $x^{1}(1)-w^{1}(1)\leq 0$. Consider now the consumption allocation $\bar{c}$ for the members of coalition S such that $\bar{c}^{j,t}=c^{j,t}$ for all $(j,t)\in S_{t}$, $t=0,2,3,\dots$ and $\bar{c}^{j,1}=[(x^{j,1}(1)+kc^{j,1}(1))/(k+1),\ c^{j,1}(2)]$ for all $(j,t)\in S_{1}$.

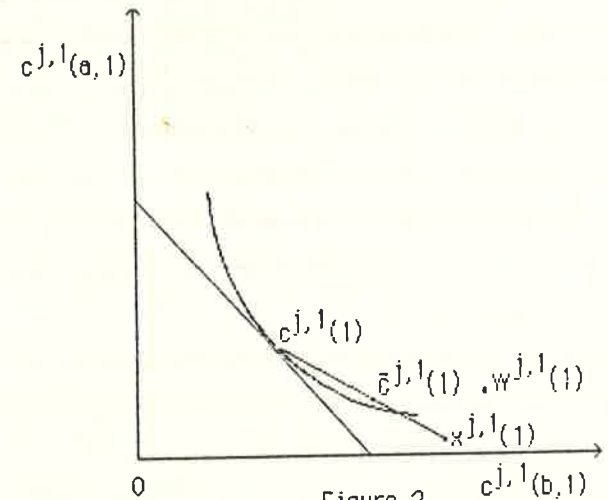

Figure 2

Observe that $\bar{c}$ is obtained as a linear convex combination of vectors $c^{j,1}$ and $\bar{c}^{j,1} = [x^{j,1}(1), c^{j,1}(2)]$ and that as k becomes large $\bar{c}^{j,1} \rightarrow c^{j,1}$. Thus, we have that

$$p(1).\bar{c}^{j,1}(1) + p(2).\bar{c}^{j,1}(2) \geq p(1).c^{j,1}(1) + p(2).c^{j,1}(2), \; j=1,\dots,m.$$

Moreover, it is easy to check that consumption allocation $\bar{c}$ is feasible.

By Assumptions 1 and 2(a) there exists a finite K such that

$$u^{j,1}\,[(x^{j,1}(1)+kc^{j,1}(1))/(k+1),\; c^{j,1}(2)] > u^{j,1}\,[(c^{j,1}(1), c^{j,1}(2)]\;,\; 1,\dots,m.$$

See Figure 2. Since $\bar{c}^{j,t} = c^{j,t}$ for the rest of members of S, S will block consumption allocation c. QED.

The intuition behind this result is the following. In Esteban [6] it has been demonstrated, Proposition 1, that with one agent per generation monetary equilibria can belong to the core provided they do not use too much money. The bounds are given by the value implicitly attributed by individuals to intraperiod exchange. Thus monetary equilibria are not blocked when these intraperiod gains from trading always exceed the intertemporal gain of creating a new currency. It is obvious that, as the number of consumers in each generation increases, the loss associated with not trading with the previous generation in the first period of their lives becomes smaller. Thus, in the limit the intraperiod costs become nill for each generation and no monetary equilibrium can belong to the core of the economy.

This proposition can be interpreted as a formal proof of the observation by Clower that money is not held as the result of individual voluntary choice but by social contrivance, as pointed out in the Introduction.

10. Final Remarks on Some Topics in Monetary Theory

In this section of final remarks we wish to discuss with some detail the implications of our results for some of the current topics in modern general

equilibrium monetary theory. We shall focuss on two areas: monetary theory in overlapping generations models, as developed in Grandmont [10], Kareken and Wallace [12], and Sargent [16], among others, and the role of money in giving trustworthiness to intertemporal allocations, as studied by Douglas Gale [8] and [9].

Monetary equilibria as studied in Esteban [6] pose a specific type of problems. Even with one agent per generation, Pareto optimal monetary equilibria may not belong to the core. Moreover, if we stick at the standard one-good model in which fiat money is analised, no monetary equilibrium belongs to the core. This result can be interpreted as accounting for the fact that, in the absence of other forms of trade, every generation has an incentive to reject the money carried over by the former generation creating its own money instead. With many goods, dynamically efficient monetary equilibria can be in the core if accompanied by a sufficient amount of intergenerational barter, i.e. if the utility gains derived from barter with the former generation fully compensate from the loss incurred in accepting its money. All this is proved in Esteban [6]. But we have just seen that these results are no longer true for economies with many agents per generation. With many agents, refusing the money held by the previous generation does not necessarily entail foregoing any form of intra-period exchange. Agents heading a blocking coalition can always reallocate goods with members of their own generation. The utility loss from not trading with the previous generation is made smaller by enlarging the number of agents in each generation. Hence, as generations become large, the utility losses associated with blocking become small and are ultimately outweighed by the utility gains derived from repudiating the existing money. Indeed we demonstrate that as the economy is enlarged by replication monetary equilibria cease to belong to the core and in the limit all become excluded.

All these results appear to be negative with respect to the important literature on fiat money in overlapping generations economies. However, our results can be interpreted in a more positive spirit as providing a rigorous demonstration of the claim made by Clower [5] that the social acceptance of money is not voluntary and based on its virtue of being a store of value. As a matter of fact, there is nothing terrible or new with this view on money. Douglas Gale [8] and [9] when examining the role of money in the social acceptance of allocations, points out that "it is not the invention of paper money which restores trustworthiness. The Walras allocations are trustworthy in the monetary economy only because there is, in the background, a government which can enforce, evidently at no cost, the payment of money taxes. Thus, we have introduced not just a new commodity (money) but a new social institution" (p. 465). From this point of view it is obvious that fiat money has been introduced in overlapping generations models as a commodity and not as a social institution. On this respect, de Vries [17] has recently examined the case in which the acceptance of fiat money is made compulsory, i.e. money is given the status of "legal tender".

Let us be more specific in comparing our results with Douglas Gale [8] work. As we have already pointed out, he has shown that the introduction of money help in making socially acceptable allocations which would have been blocked without its help. In Gale's model there is a finite number of period and, being an Arrow-Debreu economy in every respect, competitive equilibria belong to the core of the economy. However, he argues that in those equilibria in which there is net borrowing and lending, lenders have good reasons not to trust borrowers. It is in their interest breaking the futures contracts in later periods of their lifes. Gale thus defines the concept of sequential core, i.e. those allocations which belong to the core both in the first and in the subsequent periods, and shows that an allocation is trustworthy if and only if it belongs to the sequential core.

From this point of view, our results can be interpreted in the following way. It seems natural to reconsider his problem in an overlapping generations economy in which one does not need the device of the money tax in order to make money valuable. Then, our findings as to that Pareto optimal monetary competitive equilibria might not belong to the core and that when the economy becomes large by replication no monetary allocation belongs to the core seem

to confirm Gale's assertion that what makes allocations socially stable in his model is not the introduction of fiat money, but making money taxes compulsory.

We can go deeper in comparing our model with Douglas Gale's. We have already pointed out that in order to show that equilibrium allocations involving net borrowing or lending might not be trustworthy he needs assuming that agents can break their futures contracts in later dates. As we shall now argue our results can be seen as providing a rationale for Gale's assumption on agents not honouring their contracts.

Let us start by nothing that an IOU equilibrium in an overlapping generations model can be understood as a sequence of overlapping finite horizon Gale's competitive equilibria. Let us develop this point and focus on IOU competitive equilibria. While there is borrowing and lending, in the IOU equilibrium there is no income transfer across generations. Further, observe that in those equilibria contracts in the futures markets are signed on the two sides by consumers belonging to the same generation. Therefore, as far as the futures markets are concerned, IOU equilibria can be seen as a sequence of isolated generations, i.e. as a sequence of overlapping two-period Gale's equilibria. Alternatively, Gale's model can be considered as isolating one single generation of an IOU equilibrium sequence from an overlapping generations model in order to examine their behaviour in the futures market.

In spite of their similarity, the two models seem to yield different results. While in Gale's model all competitive equilibria belong to the core, in our model their equivalent, i.e. our IOU equilibria, do not. Thus, the fact of placing a collection of selfcontained economies one after the other breaks the relation between competitive equilibria and core. In other words, from a game theoretical point of view it makes a substantial difference considering an isolated finite chain of periods or the full infinite sequence. As we have seen in Proposition 5 only those walrasian equilibria in which there is no borrowing and lending in the long run are always in the core. Hence, the core of an overlapping generations economy formed by changing a sequence of Gale's two period economies would not contain the allocations that do not belong to the sequential core in Gale's model. Therefore, one needs not supposing that agents do not honour their contracts to claim that equilibria which involve borrowing might not be viable. The mere fact that agents live in an endless chain of generations can make IOU equilibria untrustworthy. Our results can thus be considered as a rationale for using the concept of sequential core when one analyses the viability of allocations in finite horizon economies.

Notes

(1) The model can be trivially generalized to a variable number of agents per generation m(t) uniformly bounded above by a finite number m.

(2) As proven by Millan [14], Balasko and Shell's [2] characterization of Pareto optimal allocations can be extended to economies with many agents.

(3) Note that $S_{f-1} = \emptyset$ and hence for the first generation the feasibility condition reads

$$\sum_{j \in S_f} c^{j,f}(f) = \sum_{j \in S_f} w^{j,f}(f).$$

(4) Kovenock 13 has produced another example of a Pareto optimum walrasian equilibrium not in the core and the set of core allocations is empty. In his example the two agents of any generation have preferences defined on different goods. Only the two members of generation t=0 share their preferences for one common good.

(5) Observe that our result is stronger than Chae's [3] Theorem 4.1, where with a continuum of agents he finds that a sufficient condition for an allocation to belong to the core is that the present value of the total endowment be finite.

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