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# The Hold-up Problem Under Common Agency

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**Abstract** - Many real world transactions occur in a common agency environment in which an agent interacts with several principals having competing interests. The hold-up literature, however, has so far neglected to investigate common agency transactions. In this paper, we consider the hold-up problem that arises in a context where there are a monopolistic seller and multiple buyers on the one side and all the parties on the other are required to make specific self-investments. Our contribution is twofold. First, we show that absent initial contracts (i.e., preliminary agreements) between the parties, total efficiency increases when the buyers act competitively using implicit contractual coordination, i.e., contractual menus. Second, we show that introducing initial simple contracts allows parties to reach the first best only under cooperative common agency. Absent this machinery, competition among the principals emerges as a more efficient governance structure for common agency in incomplete transactions.

**Keywords:** incomplete contracts, common agency, mechanism design

**JEL classification:** K12, L22, J41, C70

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# 1 Introduction [1]

Many real world transactions exhibit features of common agency, where several principals contract at once and independently with a common agent. Common agency has been explored in contexts of asymmetric information and complete contracting (Bernheim and Whinston (1986), Chiesa and Denicolò (2009)). Surprisingly, however, no attention has so far been paid to the implications of common agency in incomplete contract contexts. This paper attempts to fill this significant gap.

Common agency in incomplete contracts involve multiple bilateral transactions in which each contractual party makes specific investments, with the peculiarity that the common agent makes the same specific investment in each transaction. In this respect, the common agent can be generally described as the producer of an intermediate good which is suitable for multiple firms, with each firm operating in the same or different market of the final product. Alternatively, the common agent can be generally represented as the unique distributor of several producers. These general representations fit several tangible situations. Private equity funds, for example, typically provide transaction-specific capital to borrowers that operate in a wide variety of industries. Many joint ventures or consortia share similar features. Other cases include franchising arrangements or "dual distribution" contracts, which fall within the category of firms that contract with a common agent while competing in the market for the final product. Still another example of common agency is that resulting from pro-market regulation or antitrust remedies that mandate access to the incumbent's essential facility in liberalized network industries. (On the one hand, regulatory vertical "disintegration" imposes on the incumbent a duty to simultaneously deal with multiple downstream entrants. On the other, both sides of the contract have to make noncontractible specific investments in interconnection and service quality level.) A different general representation of the common agent is that of a buyer contracting with several suppliers that compete in the market for the final product. This is the case, for example, of split-award procurement in which a sponsor procures a good from several suppliers. Another case is given by the grocery retail sector, where many suppliers face the 'buyer power' of a retailer through specific practices such as the use of slotting allowances or loss-leader pricing. Finally, examples of

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non-contractible specific investments on both sides include the expenses a seller may bear to implement a technology designed to lower transportation costs or the resources a buyer invest to build a storage in the nearby of the seller's plant.

Our aim in analyzing common agency in incomplete contracts is twofold. First, we purport to investigate how common agency affects the parties' incentives to make specific investments relative to the standard case of bilateral monopoly (the Williamsonian 'fundamental transformation'). Second, we attempt to understand to what extent different governance arrangements can improve transactional efficiency.

The set of contracts that connect each worker with the firm's manager is a common agency example that has been previously analyzed under the lens of Transaction Cost Economics (TCE) and which is useful to illustrate the intuitions behind our work. In this case, the manager is the common agent representing all the firm's units, whereas each worker's contract governs the relationship between the manager and the worker. For the purpose of this work, assume that both contractual parties (the manager and each worker) are required to make specific investments in human capital to maximize the value of their exchange. Our main research question is which governance structure maximizes joint quasi rents under these circumstances. In particular, we are interested in understanding the efficiency feature of *competitive common agency,* where each worker individually negotiates with the monopolistic manager, versus *cooperative common agency,* where the manager negotiates with a workers union that coordinate the workers' actions. Indeed, the latter governance arrangement can be clearly likened to the Williamsonian bilateral monopoly.

Under TCE analysis, the use of a coordinator (the union) to bargain with the firm's manager serves an efficiency governance function, at least when "both the employer and the employee" perform "tasks that involve the acquisition of significant transaction-specific skills." (Williamson (1985), p. 255). In this respect, cooperative common agency would emerge as a superior governance structure. In other terms, for Williamson transforming a multi-principal transactions into a bilateral monopoly transaction between two agents (the manager and the union) is an effective way to enhance efficiency when transaction-specific investments are required ("the incentive to organize production workers within a collective governance structure increases with the degree of human asset specificity", Williamson, 1985, p. 256).

This conclusion, however, fail to address several issues. First, according to TCE analysis, common agency should be observed only as a temporary governance form, with vertical integration being the ultimate efficient governance structure for transactions involving high degree of asset specificity. This, however, is not the case for many real world transactions, in which competitive, rather than cooperative, common agency is a stable governance structure. Second, "transforming" competitive common agency into bilateral monopoly (i.e., cooperative common agency) may increase the level of specific investment by the principals (i.e., the workers in the example above), since collective bargaining may allow them to reduce quasi-rent appropriation by the common agent. However, we show that this is precisely the reason why the common agent may invest *less* under bilateral monopoly than she would under competitive common agency. Third, and perhaps more fundamentally, bilateral monopoly does not solve the hold-up problem per se, i.e., in the absence of vertical integration. On the contrary, we show that the combination of competitive common agency and contractual menus can solve one side of the hold-up problem. Indeed, because the common agent can extract quasi-rents from uncoordinated principals, she has strong incentive to make first best investments. From our research, bilateral monopoly (i.e., cooperative common agency) emerges as a superior governance arrangement only when the parties are able to enforce an optimal mechanism design which credibly deters ex-post renegotiation. As a consequence, the Williamsonian prediction that competitive common agency is always dominated by bilateral monopoly does not generally hold. In the absence of an optimal mechanism design, we find that competitive common agency dominates bilateral monopoly. In other words, the efficiency of coordination among the principals (such as unionization) depends on the principals' ability to converge towards a precise "contractual machinery". Absent this machinery, competition among the principals emerges as a more efficient governance structure for common agency in incomplete transactions.

## 2 Analytical Framework and Structure of the Paper

In our framework, a monopolistic seller—the common agent—trades with multiple buyers who do not necessarily compete in the downstream market of the final good they produce. All the parties, i.e., the seller and each of the buyers, make non-contractible specific self-investments. Since our main research question is whether, and to what extent, different common agency's governance arrangements influence the parties' incentives to make specific investments, in designing the investment game we consider the alternative cases of cooperative and competitive common agency. Under cooperative common agency, the principals (i.e., the buyers) act collectively by delegating authority over investment decisions to a coordinator. This representation captures several real situations. In addition to the example discussed above of labor unions, consider the following examples: an organization with multiple units where a manager represents all the units vis-à-vis government agencies and industry self-regulatory bodies; a coordination consortium whose goal is to reduce transaction costs for member firms; and a representative of different categories of investors who has authority to deal with the issuer on behalf of all the investors. Conversely, under competitive common agency, each principal (i.e., buyer) engages in individual transactions with the agent (i.e., seller).

When the buyers act cooperatively through a coordinator, the hold-up problem that emerges is equivalent to the one that arises in bilateral monopoly. Accordingly, we use cooperative common agency as the benchmark against which comparing the results obtained under the more interesting case of competitive common agency. In addition, in proceeding backwardly to determine the equilibrium levels of investment, we focus on equilibrium allocation in pure strategy. Indeed, when parties are not restricted to use simple contracts but can exploit the more advanced contractual technology of menus, it is possible to characterize equilibrium allocation in pure strategy (i.e., truthful equilibrium) in common agency environments (Bernheim and Whinston (1986), Chiesa and Denicolò (2009)). This result is significant for the purposes of our research since it allows us to focus on ex-ante efficiency, which is at the core of the hold-up problem.

In the first part of the paper, we investigate the case where the parties do not sign a preliminary contract. Under this circumstance, if the buyers' act competitively, "bargaining externalities" can arise. This occurs because the seller has the power to exclude one or more buyers from trading. The buyers may attempt to reduce these "bargaining externalities" through implicit contractual coordination, which they can implement by offering menus of contracts to the seller. Contractual coordination, however, is costly, since each buyer only receives his marginal surplus in equilibrium, which implies that rents must be left to the seller.[2]Hence, when the buyers act competitively (i.e., engage in individual transactions with the seller), they always receive less than when they delegate allocation authority to a coordinator.[3] In spite of this distributional consequence, we find that in the absence of a preliminary contract both ex-ante and overall efficiency increase when the buyers act competitively through menus. On the one hand, since buyers negotiating individually with the seller expect to receive the surplus "at the margin", the buyers' investment level is unaffected by the governance of common agency (i.e., cooperation or competition). On the other hand, since the seller expects to receive a higher stake of the surplus when the buyers engage in individual transactions, she invests more under competitive common agency.

In the second part of the paper, we investigate whether writing simple contracts can restore the first-best level of investments (Aghion, Dewatripont, Rey (ADR) (1994)). As in ADR, by the term simple contract we refer to an initial contract which the parties (i.e., each buyer and the seller) agree upon and which (i) specifies an initial allocation (i.e., a noncontingent quantity-transfer pair) and (ii) gives one party the exclusive right to make a take-it-or-leave-it offer in the renegotiation stage (i.e., after the realization of uncertainty). Specifically, one party (i.e., the proposer) offers to the other (i.e., the receiver) an initial allocation, which is chosen so that the receiver is always induced to invest optimally and which can be enforced through a contractible default option (e.g., a specific performance remedy). For simplicity, we assume that the proposer also retains exclusive ex-post authority over the renegotiation stage. This makes the proposer the residual claimant of the renegotiation surplus as long as she can guarantee the ex-post level of utility promised to the receiver under the initial contract.

As pointed out by ADR, in a bilateral monopoly context this contractual mechanism always leads to the first best. However, one of the fundamental assumptions for this mechanism to work is that the contracting parties be co-monotonic: that is, there must be gains from renegotiation for whatever realization of the state of nature. This assumption, however, does not always apply when there are multiple buyers, since the buyers may differ ex-post due to different realizations of the state of nature. This implies that even in the presence

[2]This is a classic result in common agency theory, under which the efficient allocation can be supported as an equilibrium only when the principals are able to make the seller's threat of excludability ineffective—in the sense that the seller's payoff is invariant regardless of whether she decides to exclude one buyer.

[3]Indeed, the sum of the (share of the) marginal surplus the buyers receives when their investment decisions are decentralized is always lower than the (share of the) surplus the buyers receive when their investment decisions are centralized.

of a buyer with an high realization, it could be efficient that the seller trades more with other buyers who have relative higher realizations. Thus, in a common agency environment the application of a simple-contract mechanism leads to different results depending on the governance arrangement used in common agency. In the case of cooperative common agency, it is apparent that the ADR result still holds. In the case of competitive common agency, instead, ADR fails to lead to the first best. On the one hand, when the seller is the residual claimant, she may have no incentive to offer the optimal quantity to all the buyers. Consider, for example, the case of poorly correlated buyers. When the seller offers these buyers a fixed quantity, it is like if she provided them with insurance. This may impose costs on the seller that are higher than the expected benefits she receives when the buyers invest optimally. On the other hand, when the buyers are the residual claimants, they anticipate that they may be unable to extract the renegotiation surplus in every state of the world. Indeed, the seller can always enforce the initial contract to maximize her profit, since she is not necessarily co-monotonic with any individual buyer. Hence, both the seller and the buyers underinvest in equilibrium.

Our findings provide important policy implications for the governance of incomplete transactions with asset specificity. On the one hand, when simple contracts are costly to write or unfeasible because of legal constraints (this might well be the case when parties compete in the same industry), it is inefficient to constrain the common agent's bargaining power and/or encourage buyers' coordination. Indeed, this could undermine the common agent's incentives to invest optimally, while leaving unaffected the principals' incentives. On the other hand, when simple contracts are feasible, it is, instead, optimal to encourage principals' coordination *vis-à-vis* the common agent, since this helps to restore first best outcomes.

The paper is organized as follows. In Part 2, we briefly discuss the related literature. In Part 3, we illustrate (i) the setting of the model; (ii) the contractual technology the parties can use to implement the trading; and (iii) the characterization of the first-best that will be the benchmark case for the rest of the paper. In Part 4, we attempt to characterize the investment equilibrium when the parties to each transaction conclude a null initial contract. In Part 5, we apply the ADR mechanism and attempt to analyze the investment equilibrium results. In Part 6, we draw some preliminary conclusions.

## 3 Related Literature

Our research relates to two major—and, yet, so far unrelated—strands of literature: the hold-up literature and the literature on common agency.

As originally characterized in Klein, Crawford, and Alchian (1978) and Williamson ((1979), (1983)), the hold-up problem arises because of two basic reasons. First, contractual parties may be unable to specify the contractual performance prior to making specific investments. Second, parties may be unable to bargain over specific investments once they have been made, because these investments are unverifiable. As a result of the inadequacy of the ex-ante contract, each party is exposed to the risk of ex-post opportunism (i.e., moral hazard) by the other party. This, in turn, generates incentive for bilateral underinvestment. Holmstrom (1982) has provided a formal analysis of this argument, recharacterizing the hold-up problem as a form of moral hazard in teams. A more rigorous formulation of these issues is pioneered by Grossman and Hart (1986) and, later, Hart and Moore (1990) (together, GHM). In GHM, the hold-up problem is linked with the problem of ex-post opportunism by the owner of an asset. In this different framework, it is the lack of property rights (i.e., residual control rights) that leads the non-owner to underinvest. As a result, GHM support a welfare policy under which property rights over assets should be allocated to the party who is required to make (more) specific investments. Within the classic treatment of hold-up, Hart and Moore (1988) on incomplete contracts and renegotiation is probably the more closely related to our research. As in our model, in Hart and Moore (1988) contracting parties conclude an initial contract and are then required to make specific investments that increase the surplus generated by the trading. However, the optimal level of trading can only be determined ex post, after the state of nature reveals. As a result, the initial contract can prove ex-post inadequate, which forces the parties to renegotiate. In the renegotiation stage, opportunism can occur because both the parties' specific investments and the state of nature cannot be verified by a third party adjudicator. Hence, under this framework, it is the difficulty of specifying enforceable contractual contingencies that leads to bilateral underinvestment.

Our research also relates to the literature on hold-up and implementation mechanisms which has variously addresed the underinvestment problem arising from unverifiability. As was mentioned in the introduction, we apply the mechanism introduced by Aghion, Dewatripont, and Rey (ADR) (1994), which is also reproduced in a less formal way in Chung (1991). Noldeke and Schmith (1995) have developed an alternative mechanism to address these problems, proposing an option contract under which one party (i.e., the seller) receives a penalty if trading does not take place. The same party receives, instead, a higher transfer when trading occurs. The solution of Noldeke and Schmith, however, is based on two crucial requirements. First, the buyer must be assigned exclusive bargaining power. Second, the court must be able to verify whether trading occurred. Further our research relates to the literature on hold-up and contractual remedies. In particular, Edlin and Reichelstein (1996) have characterized the equilibrium investments arising in an hold-up context under two different remedies: specific performance and expectation damages. Consistent with ADR, they found that only specific performance leads to optimality when investments are bilateral. Finally, within the hold-up literature, we also touch upon the literature on hybrid governance arrangements: i.e., governance solutions that are located between the classical dichotomy of hierarchy and spot market transactions (Williamson (1979), (1985)). This also links our research ton the literature on to the the stability of plural forms over time (Bradach & Eccles, 1989; Bradach, 1997; Ménard, 2004; Bradach, 1997; Lafontaine & Shaw, 1999, 2005; Baker & Dunt, 2008) and their compared efficiency relative to vertical integration.

The common agency literature is the additional main stream of research to which our work relates. Unlike most studies on common agency—which investigate asymmetric information under complete contracts—we consider the case of symmetric information under incomplete contracts. To this extent, our framework closely relates to the work of Segal (1999) in which an agent trades with several principals. In Segal's model, however, there are direct externalities, which arise because each principals' utility function is affected by the trading of the agent with other principals. Conversely, in our model, there are only contractual (i.e., bargaining) externalities, which arise due to the agent's right to exclude one or more principals from trading. Moreover, in our model, we assume that both the agent and the principal can make a take-it-or-leave-it offer with exogenous probability. Hence, we consider both the offer game *à la* Segal and the bidding game *à la* Bernheim and Whinston (1986). In particular, the trading game of our model is very close to the environment described by Chiesa and Denicolò (2009) in which multiple buyers trade with a common agent (i.e., seller), even though Chiesa and Denicolò consider the minimum rent equilibrium rather than the truthful equilibrium.[4]

# 4 The Model

## 4.1 Environment

We consider a trading game where there are a single seller (female) producing an homogenous good and $N$ identical buyers (males) indexed by $i$ or $j = 1, 2, ...N$, with each buyer entering into a bilateral negotiation with the seller.

Both the buyers and the seller are risk neutral. Each buyer trades a positive amount of the good and pays a fixed transfer to the seller. The basic notation of the trading structure is as follows. The quantity that is traded with buyer $i$ is $q_i \in \mathbb{Q}_i \subseteq \mathbb{R}_+$. The vector of traded quantities is denoted by $\boldsymbol{q}$, and $\boldsymbol{q}_{-i} \equiv (q_1, q_2, ..., q_{i-1}, q_{i+1}, ..., q_N)$ is the vector of quantities that are traded by the seller with all the buyers except buyer $i$. Similarly, the transfer the seller receives from buyer $i$ is denoted by $T_i$.

Uncertainty is represented by a random variable $\theta_i \in \Theta_i$, whose realization affects the future benefits arising from the single trading of buyer $i$ with the seller. Higher realizations of $\theta_i$ implies higher valuation of the good for buyer $i$, but do not necessarily imply an higher level of trading with the seller. Indeed, the level of trading of each buyer also depends on the realization of the random vector $\boldsymbol{\theta}$, whose elements are i.i.d. Uncertainty of the seller is modeled as a linear combination of the individual uncertainty affecting the buyers. This is reasonable if one considers that the seller is producing an intermediate good which is suitable for different industries. However, it is important to note that the seller's technology can also be correlated to a specific industry. As a result, the seller can be completely uncorrelated with one or more buyers. Accordingly, we define the uncertainty of the seller as $\theta_s = \sum_{i=1}^{N} \pi_i \times \theta_i$, where $\pi_i \in [0,\ 1]$ and $\sum_{i=1}^{N} \pi_i = 1$. Note that $\pi_i$ measures the level of correlation of the seller with buyer $i$.

Throughout the paper, we assume that all agents have a common prior on the distribution of the state of nature represented by $F(\theta_i)$. We further assume that everything in the game is common knowledge.

[4] In the minimum rent equilibrium the payoff of each principal exceeds his marginal contribution to social welfare.

As in any incomplete contract model, the benefits from trading are also affected by the level of relationship-specific investment each contracting party makes. To make things interesting, we assume that the parties have to make their specific investments before the realization of uncertainty. This means that the specific investments are made before knowing what the optimal level of trading of each contract is. Specifically, each buyer $i$ chooses his level of investment $\beta_i \in \mathbb{B}_i$. Making the investment $\beta_i$ is costly and this cost is represented by the convex map $\psi_i(\beta_i)$. We denote the vector of the buyers' specific investments as $\boldsymbol{\beta}$, and $\boldsymbol{\beta}_{-i} \equiv (\beta_1, \beta_2, ..., \beta_{i-1}, \beta_{i+1}, ..., \beta_N)$ as the vector of investments of all the buyers except buyer $i$. The seller is also required to make a specific investment before the realization of the state of nature: i.e., she must choose her level of investment $\sigma \in \Sigma$ at the convex cost $\phi_s(\sigma)$.

The gross monetary benefits for each buyer is the map $v_i : \mathbb{Q}_i \times \Theta_i \times \mathbb{B}_i \to \mathbb{R}_+$, which can be represented in functional form as $v_i(q_i, \theta_i, \beta_i)$. Consistent with the risk neutrality assumption, buyers have quasi-linear preferences with utility $U_{B_i} : \mathbb{Q}_i \times T_i \times \Theta_i \times \mathbb{B}_i \to \mathbb{R}$, which can be represented as:

$$U_{B_i}(q_i, \theta_i, \beta_i) = v_i(q_i, \theta_i, \beta_i) - T_i - \psi_i(\beta_i). \tag{1}$$

As standard in the literature, the ex-ante specific investment each buyer makes increases his gross monetary benefits.

Like in the buyers' case, we define the gross monetary cost for the seller as the map $C : \prod_{i=1}^{N} (\mathbb{Q}_i \times \mathbb{B}_i) \times \theta_s \times \Sigma \to \mathbb{R}_+$, which can be represented by the function $C(Q, \theta_s, \sigma)$. Since the seller can trade with all the buyers, it is convenient to define the argument of her cost function in aggregate terms. Therefore, $Q = \sum_{i=1}^{N} q_i$ is the total traded quantity when the seller trades with all the buyers, and $Q_{-i} = \sum_{j \neq i}^{N} q_j$ is the total traded quantity when the seller trades with all the buyers except buyer $i$. The seller also has quasi-linear preferences with utility $U_S : \prod_{i=1}^{N} (\mathbb{Q}_i \times T_i \times \mathbb{B}_i) \times \theta_s \times \Sigma \to \mathbb{R}$, which can be represented as:

$$U_S(T, Q, \theta_s, \sigma) = \sum_{i=1}^{N} T_i - C(Q, \theta_s, \sigma) - \phi_s(\sigma). \tag{2}$$

As also standard in the literature, the seller's ex-ante specific investment reduces her cost of production.

In addition, we assume that the utility functions of the agents are concave in all their arguments.

Finally, the following further assumptions are made throughout the paper:

**Assumption 1.** *$v_{q\beta} > 0$ and $C_{q\sigma} < 0$.*

**Assumption 2.** *(Specificity). $v_i(0, \theta_i, \beta_i) = C(0, \theta_s, \sigma) = 0$.*

**Assumption 3.** *(Inada). $\lim_{q_i \to 0} \frac{\partial v_i}{\partial q_i} = \infty$ and $\lim_{q_i \to \infty} \frac{\partial v_i}{\partial q_i} = 0$ for all $i \in N$, $\lim_{Q \to 0} \frac{\partial C}{\partial Q} = 0$ and $\lim_{Q \to \infty} \frac{\partial C}{\partial Q} = \infty$.*

**Assumption 4.** *(Excludability). The seller can exclude any buyer from trading.*

**Assumption 5.** *(Aggregate Co-monotonicity). For any $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ where $\theta_s = \sum_i \pi_i \times \theta_i$ and for any $\boldsymbol{\theta}' \in \boldsymbol{\Theta}$ where $\theta_s' = \sum_i \pi_i \times \theta_i'$ we have that:*

$$\sum_i v_i(q_i^*, \theta_i, \beta_i) > \sum_i v_i(q_i^*, \theta_i', \beta_i) \Longleftrightarrow C(Q^*, \theta_s, \sigma) < C(Q^*, \theta_s', \sigma)$$

Assumption 1 means that the marginal gross benefits are increasing with the level of specific investments.

Assumption 2 means that the benefit to the buyer of having zero units and the cost to the seller to produce zero units do not depend on the realization of uncertainty and the level of investments.

Assumption 3 means that at the optimum the solution is interior: that is, the seller must trade a positive amount with all the buyers.

Assumption 4 means that each trade must be voluntary. This assumption characterizes the common agency environment.

Assumption 5 means that the weighted realizations of the buyers varies co-monotonically with the seller's uncertainty. However, aggregate co-monotonicity does not imply individual co-monotonicity. Indeed, the latter condition is more stringent than the former and implies that for (i) any $\theta_i \in \Theta_i$ where $\theta_s = \sum_i \pi_i \times \theta_i$, and (ii) for any $\theta_i' \in \Theta_i$ where $\theta_s' = \sum_{j \neq i} \pi_j \times \theta_j + \pi_i \times \theta_i'$, whenever $\theta_i > \theta_i'$, the following condition must be satisfied:

$$v_i(q_i^*, \theta_i, \beta_i) > v_i(q_i^*, \theta_i', \beta_i) \Longleftrightarrow C(Q^*, \theta_s, \sigma) < C(Q^*, \theta_s', \sigma) \quad \forall \theta_j \in \Theta_j \quad \forall i \in N.$$

## 4.2 Contractual Technology and Bargaining

In this part, we describe the contractual technology available to the parties. We assume that at an early stage of the parties' relationship (i.e., before the parties make specific-investment decisions and uncertainty is resolved) either each buyer or the seller can propose an initial contract with some exogenous probability. This contract specifies an initial allocation (i.e., a quantity-transfer pair) and assigns ex-post authority to make a take-it-or-leave-it offer in the renegotiation stage to one contracting party.

We restrict the analysis to simple contracts $m_i = (T_i, q_i)$, which we term allocations. As in ADR, we assume that the parties can use costly collaterals (i.e., hostages) to specify ex-post authority in the renegotiation stage. Also in line with ADR, we focus on extreme allocations of authority, meaning that ex-post authority is exclusively allocated either to the buyers or the seller. However, in contrast to ADR, we assume that the parties can conclude null contracts taking the form $(T_i^0, q_i^0) = (0, 0)$. In this case, determining who has ex-post authority is meaningless. It is also important to emphasize that when the parties sign a null contract, the incomplete contract context is the typical hold-up problem à-la GHM, where parties are required to take some unverifiable actions (i.e., specific investments) before trading occurs.

Contractual parties are protected by specific performance. Under this remedy, the initial contract is enforced by the court unless both parties agree to renegotiate it. Hence, granting parties legal protection in the form of specific performance implies that in the renegotiation stage each contracting party can unilaterally impose the initial allocation on the other party.

Restricting the analysis to simple contracts, however, does not limit the possibility that parties can offer menus composed by multiple simple contracts. This reflects the delegated common agency nature of the game. Indeed, in common agency games principals can implicitly coordinate themselves by proposing latent contracts in addition to the optimal allocation. While a latent contract is never chosen in equilibrium, it serves as a threat against the other principals if they do not choose the optimal allocation proposed in the menu.

The menu for each buyer is defined as $M_i$ and is composed by several quantity-transfer pairs. We denote as $M_i^0$ and $M_i^1$ the menus that are offered, respectively, in the initial stage and the renegotiation stage. The set of contracts in the menu at the initial stage is $\Gamma^0 = \{M^0 \subset \mathbb{R}_+^2 | \text{compact and} (0,0) \in M^0\}$. The set of contracts in the menu in the renegotiation stage is $\Gamma^1 = \{M^1 \subset \mathbb{R}^2_+ | \text{compact and} (T^0, q^0) \in M^1\}$. Note that under Assumption 4 the contract (0, 0) must always be included in $\Gamma^0$, while the initial contract must be included in $\Gamma^1$ by specific performance. In particular:

**Definition 1.** *The cardinality of the set of each menu is denoted by $n(\Gamma) = d$, which represents the number of contracts included in the menu.*

We denote by $\mathbf{M} = (M_1, M_2, ..., M_N) \in \Gamma^N$ the vector of menus offered by all the buyers and by $\mathbf{M}_{-i} = (M_1, M_2, ..., M_{i-1}, M_{i+1}, ..., M_N) \in \Gamma^{N-1}$ the vector of menus offered by all the buyers except buyer $i$.

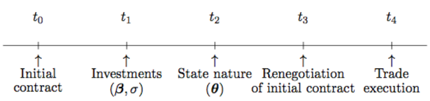
A strategy for the seller is a function $m(\mathbf{M}) : \Gamma^N \to (R^2_+)^N$ and a profile of contracts accepted by the seller is defined by $m = (m_1, m_2, ...m_N)$ such that she maximizes her utility.

Further, contractual possibilities are limited on two grounds:

(i) both the realization of uncertainty and the levels of specific of investments are observable by the contracting parties but not verifiable by a court. For this reason uncertainty and investments are not contractible;

(ii) parties to a single transaction cannot condition the ex-post trading of that transaction on the level of trading of other transactions.

Finally, the timing and actions of the game are represented as follows:




At $t_0$, each buyer $i$ and the seller agree upon an initial simple contract. The initial simple contract can be the null contract.

At $t_1$, parties are required to make their specific investments.

At $t_2$, uncertainty is solved.

At $t_3$, each buyer $i$ and the seller renegotiate the initial contract.

At $t_4$, production and trading take place.

## 4.3 Analysis of the First Best

The level of investments determines ex-ante efficiency, while the trade profile determines ex-post efficiency. However, since the optimal ex-post allocation also depends on the level of ex-ante specific investments, social welfare is maximized only when investments and allocations are both optimal. In this section, we attempt to illustrate first-best results and consider them as a benchmark for the rest of the analysis.

**Definition 2.** *The first-best outcome consists in an ex-ante efficient level of investments* $(\boldsymbol{\beta}^*, \sigma^*)$ *and ex-post contingent allocations* $(\boldsymbol{q}^*)_{\boldsymbol{\theta} \in \boldsymbol{\Theta}}$.

The first-best is determined backwardly. Hence, we first optimize the ex-post contingent allocations. This delivers a parametric solution, which is both state-contingent and investment-contingent. Given the parametrized solution, we solve for the optimal level of investments. Consistently, we will refer to the first optimization problem as the allocation game and to the second optimization problem as the investment game.

The efficient allocation is the vector $\boldsymbol{q}^* \equiv (q_1^*, q_2^*, ..., q_N^*)$ that maximizes the aggregate gross monetary profits of all parties.

For a given $(\boldsymbol{\theta}, \boldsymbol{\beta}, \sigma) \in \mathbb{R}_+^{N+1} \times \Theta_i^N$, we can define the total welfare generated by the exchanges as:

$$\max_{\boldsymbol{q}} W(\boldsymbol{q}, \boldsymbol{\theta}, \boldsymbol{\beta}, \sigma) = \sum_{i=1}^{N} v_i(q_i, \theta_i, \beta_i) - C(Q, \theta_s, \sigma), \tag{3}$$

The solution of this problem delivers, for each contract, the optimal state-contingent and investment-contingent allocation $q_i^*(\boldsymbol{\theta}, \boldsymbol{\beta}, \sigma)$ for $i \in N$[5].

Given the ex-post efficient allocation, the efficient level of investments $(\boldsymbol{\beta}^*, \sigma^*)$ is then obtained by solving the following maximization problem:

$$\max_{\boldsymbol{\beta}, \sigma} \left[ \int_{\Theta} \left[ W(\boldsymbol{q}^*, \boldsymbol{\theta}, \boldsymbol{\beta}, \sigma) \right] dF - \sum_{i=1}^{N} \psi_i(\beta_i) - \phi_s(\sigma) \right], \tag{4}$$

The solution of the problem is characterized by the following system of first-order conditions:

$$\mathbf{E}_{\boldsymbol{\theta}} \left[ \frac{\partial v_i(q_i^*, \theta_i, \beta_i^*)}{\partial \beta_i} \right] = \psi_i^{'}(\beta_i^*) \quad \forall i \in N \tag{5}$$

$$-\mathbf{E}_{\boldsymbol{\theta}} \left[ \frac{\partial C(Q^*, \theta_s, \sigma^*)}{\partial \sigma} \right] = \phi_s^{'}(\sigma^*) \tag{6}$$

Therefore, the investment decision $(\boldsymbol{\beta}^*, \sigma^*)$ is always made in expected terms.

[5] The optimal state-contingent allocation is determined by the following system of equations: $\frac{\partial v_i(q_i^*, \theta_i, \beta_i)}{\partial q_i} = C^{'}(Q^*, \theta_s, \sigma) \quad \forall i \in N$.

# 5 Contract $(T_i, q_i) = (0, 0)$

In this section, we consider the case where the parties (i.e., each buyer and the seller) choose the initial allocation $(T_i, q_i) = (0, 0)$. This case, which we have defined as the null contract, is interesting for three reasons. First, it captures situations in which parties are not permitted to write any contract, because the adjudicator is not able to verify and enforce the initial allocation. This case become more tangible if one considers situations where parties have to make some ex-post actions (rather than exchange quantities) and these actions are not contractible or, similarly, situations where parties have to identify a quality profile rather than a trading profile. To this extent, we can say that the initial contract $(0, 0)$ generalizes the moral hazard problem. Second, the null contract captures situations in which even if the parties can contract on an initial allocation, the law does not permit or protect initial contracts. This could be the case of, for example, antitrust rules that prevent parties from writing any kind of preliminary agreements. Third, the null contract suggests that when parties enter into any initial contract that is likely to be renegotiated, they determine the relevant transfer by anticipating what could be the ex-post bargaining outcome without the initial contract. In other terms, the utility parties expect to obtain under the ex-ante null contract defines the parties' participation constraint when they conclude an initial contract.

As outlined in the introduction, we apply the null contract case to two different scenarios. First, we analyze the environment where the buyers' investment decisions are made by a representative on behalf of all the buyers. Second, we consider the more interesting case where the buyers engage in individual transactions with the seller.

## 5.1 Coordinated Transactions

The representative of the buyers can take binding decisions for all the buyers. As noted above, this makes the hold-up problem in common agency equivalent to the classical hold-up problem where negotiation takes place in bilateral monopoly. In this game, once investment decisions are taken and the state of nature materializes, parties enters into a bilateral bargaining process over the total welfare that can be generated by the exchange of quantities. The feasible bargaining set is, therefore, identified by the total welfare, $W(\boldsymbol{q}, \boldsymbol{\theta}, \boldsymbol{\beta}, \sigma)$.

The bargaining solution concept we apply to this case, as well as in the rest of the paper, is the Nash bargaining solution. Under the Nash bargaining solution, ex-post welfare is divided in accordance with the relevant bargaining power of the parties. We assume that the relative bargaining power of the parties is exogenous. Then, we define $\alpha \in [0,\ 1]$ as the bargaining power of each buyer, and $1-\alpha$ as the bargaining power of the seller when she deals with each buyer. Accordingly, a simple way to represent the Nash bargaining solution is to assume that the buyers can make a take-it-or-leave-it offer with probability $\alpha$. By complementarity, the seller makes a take-it-or-leave-it offer to each buyer with probability $(1-\alpha)$.

Because the representative of the buyer can mandate transfer of wealth from one buyer to the other, it is convenient to consider all the transfers in aggregate, i.e., $T=\sum_{i=1}^{N} T_i$ .

After nature realizes, parties enter into the allocation game. We define an offer as a feasible allocation $\{T, Q\}$. In this context, any menu offered by the parties have active cardinality equal to two, i.e., the null contract and the optimal contract. Whenever one contracting party offers a menu with more than a contract, in fact, there is always a contract that strictly dominates the other. In addition, offering a quantity that is not welfare maximizing is never individually efficient. The following lemma, therefore, states that the allocation problem in this case reduces to a bargaining over the transfers.

**Lemma 1.** *For a given $(\boldsymbol{\theta}, \boldsymbol{\beta}, \sigma) \in \mathbb{R}^{N+1} \times \Theta_i^N$, any bargaining outcome implies that the optimal aggregate quantity $Q^*(\boldsymbol{\theta}, \boldsymbol{\beta}, \sigma)$ is offered.*

*Proof.* We prove that any party's proposal lies on the Pareto frontier. Indeed, by the resource monotonicity property, each party's share of the surplus is increasing with the welfare generated by the transaction. For a given$(\boldsymbol{\theta}, \boldsymbol{\beta}, \sigma) \in \mathbb{R}^{N+1} \times \Theta_i^N$, we then define an offer made by any party as the triplet $\{x_S(Q), x_B(Q), Q\}$, where $x_S(Q) \geq 0$ and $x_B(Q) \geq 0$ is the partition of the surplus that is allocated, respectively, to the seller and the buyer. Let's one party propose $\{x_S(\bar{Q}), x_B(\bar{Q}), \bar{Q}\}$, with $\bar{Q} \neq Q^*$. We know from optimality that by offering quantity $Q^*$, extra welfare equal to $\Delta W(Q^*,\ \bar{Q}) = W(Q^*, \boldsymbol{\theta}, \boldsymbol{\beta}, \sigma) - W(\bar{Q}, \boldsymbol{\theta}, \boldsymbol{\beta}, \sigma) > 0$ can be generated.

We also know that the parties' bargaining set is bounded from above by the total welfare attainable, which includes by optimality $\Delta W(Q^*,\ \bar{Q})$. Therefore, by trading $Q^*$, the new partitions can be represented by, respectively, $x_S(Q^*) = x_S(\bar{Q}) + \alpha \times \Delta W(Q^*,\ \bar{Q})$ and $x_B(Q^*) = x_B(\bar{Q}) + (1-\alpha) \times \Delta W(Q^*,\ \bar{Q})$. Then, by non-satiation, $x_S(Q^*) \underset{S}{\succsim} x_S(\bar{Q})$ and $x_B(Q^*) \underset{B}{\succsim} x_B(\bar{Q})$ hold.

□

From lemma 1, when the buyers' investment decisions are centralized, both the buyers and the seller will reach ex-post efficiency. Specifically, the buyers and the seller will receive, respectively: $\alpha \times W(Q^{*}, \boldsymbol{\theta}, \boldsymbol{\beta}, \sigma)$ and $(1-\alpha) \times W(Q^{*}, \boldsymbol{\theta}, \boldsymbol{\beta}, \sigma)$.

Ex-post efficiency, however, does not imply ex-ante efficiency. Indeed, anticipating that they can only appropriate a portion of the welfare generated by the ex-post trading, the parties underinvest.

The equilibrium levels of investments is determined by the following maximization problems:

$$\max_{\boldsymbol{\beta}} \left[ \alpha \int_{\Theta} \left[ W(Q^{*}, \boldsymbol{\theta}, \boldsymbol{\beta}, \sigma) \right] dF - \sum_{i=1}^{N} \psi_i(\beta_i) \right] \tag{7}$$

$$\max_{\sigma} \left[ (1-\alpha) \int_{\Theta} \left[ W(Q^{*}, \boldsymbol{\theta}, \boldsymbol{\beta}, \sigma) \right] dF - \phi_s(\sigma) \right], \tag{8}$$

whose solutions are characterized by the following system of first-order conditions:

$$\alpha \times \mathbf{E}_{\boldsymbol{\theta}} \left[ \frac{\partial v_i(q_i^{*}, \theta_i, \beta_i^{C})}{\partial \beta_i} \right] = \psi_i^{'}(\beta_i^{C}) \quad \forall i \in N \tag{9}$$

$$-(1-\alpha) \times \mathbf{E}_{\boldsymbol{\theta}} \left[ \frac{\partial C(Q^{*}, \theta_s, \sigma^{C})}{\partial \sigma} \right] = \phi_s^{'}(\sigma^{C}) \tag{10}$$

As a result, $\left(\boldsymbol{\beta}^{\boldsymbol{C}}, \sigma^{C}\right) < (\boldsymbol{\beta}^{*}, \sigma^{*})$ holds. This is the classical hold-up problem under which the existence of incomplete contracts leads to underinvestment.

## 5.2 Individual Transactions

Like in the case of coordinated transactions, we model bargaining in individual transactions so that either the seller or the buyers can make an ex-post take-it-or-leave-it offer according to their relevant bargaining power. In this case, however, the buyers cannot determine their allocation cooperatively. Furthermore, the seller can exclude one or more buyers from trading. If this happens, the seller may increase the level of trading with the non-excluded buyers, who, in turn, may end up receiving a proportionally higher level of welfare. For this simple reason, the existence of multiple buyers creates externalities: positive for the seller and negative for the buyers. Indeed, the level of trading with any given buyer is affected by the total amount the seller trades with the other buyers.

As above, in order to determine the outcome of the investment game, we first consider the allocation game. Parties try to mitigate the effect of the externalities by using menus. As noted earlier, drawing on Bernheim and Whinston (1986) and Chiesa and Denicolò (2009), we focus on pure strategy equilibria in the allocation game in order to determine backwardly the equilibrium of the investment game.

We denote the total surplus that is generated when the seller trades with all the buyers as $W(Q(\beta_i, \boldsymbol{\beta}_{-i}))$, and the total surplus that is generated when the seller trades with all the buyers except buyer $i$ as $W_{-i}(Q_{-i}(\boldsymbol{\beta}_{-i}))$. The difference between the two values is the marginal surplus generated by the transaction with buyer $i$, which can be represented as:

$$\Delta W_i(Q) = W(Q(\beta_i, \boldsymbol{\beta}_{-i})) - W_{-i}(Q_{-i}(\boldsymbol{\beta}_{-i})) \tag{11}$$

In determining the ex-post equilibrium allocation, we first consider the case where each buyer makes a take-it-or-leave-it offer to the seller (i.e., the bidding game) and, then, the opposite case where the seller makes a take-it-or-leave-it offer to the buyers (i.e., the offer game). The two sub-games are played with exogenous probabilities: $\alpha$ and $1-\alpha$, respectively.

In the bidding game, we draw on the results of of Bernheim and Whinston (1986) and Bergermann and Välimäki (2003) on the truthful equilibrium, where each principal (i.e., buyer) in equilibrium cannot earn more than the surplus generated by his individual transaction with the agent. This result is stated in the next proposition.

**Proposition 5.1.** *When buyers make a take-it-or-leave-it offer, the equilibrium level of trading is ex-post efficient and the transfer is such that each buyer gets his marginal contribution to the social welfare. This implies that* $T_{B_i} = v_i(q_i^*) - \Delta W_i(Q^*)$ .

The result is proven in Bernheim and Whinston (1986) and Bergermann and Välimäki (2003) (see pp. 41-42 ). The proof's basic intuition is based on the properties of individual excludability and bilateral efficiency. Individual excludability implies that in any equilibrium the seller can exclude any buyer and still earn her equilibrium payoff. Applied to the case under consideration, this means that, since the buyers are ex-ante identical, each buyer can only appropriate the marginal surplus generated by his transaction with the seller. This is given by (11). Under bilateral efficiency, given the equilibrium menu of contracts offered by the other buyers, for buyer $i$ joint profit maximization with the seller is equivalent to the maximization of his own profit. As a result, each buyer will propose a contract in the menu that specifies the optimal trading. The equilibrium of this game is then truthful in the sense that each buyer trades the optimal quantity and earns his marginal contribution to the surplus.

The efficient equilibrium, however, is supportable only as long as the buyers leave rents to the seller. Since the seller can exclude buyer $i$ and trade more quantity with buyers $-i$ and her cost function is convex, the surplus generated by the transaction between the seller and buyer $i$ is the lowest at the margin. Hence, in equilibrium the seller always enjoys positive externalities (in the form of bargaining rents). This result represents the main difference between the case where the buyers' investment decisions are decentralized or centralized. In the latter case, in fact, no rents are left to the seller, since exclusion of one buyer implies zero level of trading.

As compared to the solution of the bidding game, the solution of the offer game is more straightforward.

**Proposition 5.2.** *When the seller makes a take-it-or-leave-it offer, the equilibrium level of trading is ex-post efficient and the transfer is such that the seller receives the entire surplus. It means that* $T_i^S(q_i^*) = v_i(q_i^*)$.

The proof is immediate since there are no externalities arising from the allocation game. Hence, the menu the seller proposes to each buyer has active cardinality equal to two: the efficient contract and the null contract. Indeed, all the other feasible contracts are strictly dominated by the efficient contract.

As a result in both sub-games, ex-post efficiency is always achieved. Hence, the expected transfer the seller obtains in equilibrium from each buyer can be represented as:

$$T_i^*(q_i^*) = v_i(q_i^*) - \alpha \times \Delta W_i(Q^*). \tag{12}$$

We can now represent the parties' expected payoffs which arise in equilibrium. For each buyer and the seller they are, respectively:

$$\Pi_{B_i} = \alpha \times \Delta W_i(Q^*) \quad \forall i \in N, \tag{13}$$

$$\Pi_s = \left(1 - \sum_{i=1}^{N} \alpha\right) W(Q^*) + \sum_{i=1}^{N} \alpha \times W_{-i}(Q^*_{-i}) \tag{14}$$

Note that, in contrast to the case where the buyers' investment decisions are centralized, in this case the seller always receives positive payoff. In addition the seller's equilibrium payoff when the buyers engage in individual transactions is always higher than her equilibrium payoff when the buyers' investment decisions are centralized. This result is stated in the following lemma.

**Lemma 2.** *1. In any efficient equilibrium, where $\alpha > 0$, the seller always gets bargaining rents equal to:*

$$R_s = \left[N \times W_{-i}(Q^*_{-i}) - (N-1) \times W(Q^*)\right]. \tag{15}$$

*2. The bargaining rents are increasing in the number of buyers.*

*Proof.* The total payoff of the seller is given by equation (14). By subtracting $\sum_i (1-\alpha) \times \Delta W_i(Q^*)$ from equation (14), $R_s$ is obtained. Due to the concavity of the welfare function, we obtain that $\frac{N}{N-1} > \frac{W(Q^*)}{W_{-i}(Q^*_{-i})}$. This proves that bargaining rents are always positive. Proving that bargaining rents increase in the number of buyers requires that the following condition be true:

$W_{-i}(Q^*_{-i}) + N \times \left(W(Q^*) - W_{-i}(Q^*_{-i})\right) - W(Q^*) - (N-1) \times \left(W_{+i}(Q^*_{+i}) - W(Q^*)\right) > 0$

$2(N-1) \times W(Q^*) - (N-1) \times W_{-i}(Q^*_{-i}) - (N-1) \times W_{+i}(Q^*_{+i}) > 0$, where $W_{+i}(Q^*_{+i})$ is the total surplus generated by $(N+1)$ buyers.

The result is proven since the last equation reduces to:

$\left(W(Q^*) - W_{-i}(Q^*_{-i})\right) - \left(W_{+i}(Q^*_{+i}) - W(Q^*)\right) > 0$, which is true by the concavity of the welfare function.

□

Now we can analyze the specific investment decisions. In the game, parties make investment decisions anticipating the ex-post allocation equilibrium. Hence, the problem is to understand whether and to what extent bargaining externalities affect the parties' investment decisions.

The following two propositions state the results of Part 4.

**Proposition 5.3.** *When the buyers engage in individual transactions with the seller, each buyer underinvests. However, the level of underinvestment is the same as in the case where the buyers' investment decisions are centralized.*

*Proof.* Each buyer chooses the level of specific investment to optimize his expected profit. Then:

$$\max_{\beta_i} \left[ \alpha \int_{\Theta} [\Delta W_i(Q^*)] dF - \psi_i(\beta_i)] \right] \quad \forall i \in N. \tag{16}$$

The equilibrium level of specific investments is therefore characterized by the following first order condition:

$$\alpha \times \mathbf{E}_{\boldsymbol{\theta}} \left[ \frac{\partial v_i(q_i^*)}{\partial \beta_i} + \sum_{j=1}^{N} \frac{\partial q_j^*}{\partial \beta_i} \left( \frac{\partial v_j(q_j^*)}{\partial q_j^*} - \frac{\partial C(Q^*)}{\partial q_j^*} \right) \right] = \psi_i'(\beta_i) \quad \forall i \in N. \tag{17}$$

By the envelope theorem the latter equation reduces to:

$$\alpha \times \mathbf{E}_{\boldsymbol{\theta}} \left[ \frac{\partial v_i(q_i^*, \theta_i, \beta_i^e)}{\partial \beta_i} \right] = \psi_i'(\beta_i^e) \quad \forall i \in N. \tag{18}$$

□

A different equilibrium investment is obtained under the seller's program. In the program, the seller makes her decision so to maximize her expected payoff, which depends on her bargaining power and the level of bargaining rents she is able to extract in equilibrium. Hence the existence of bargaining rents makes the seller to invest more than in the case where the buyers' investment decisions are centralized. Additionally, the wedge of the seller's extra-investment is increasing in the bargaining rents she is able to extract.

**Proposition 5.4.** *The seller will underinvest. However,*

1. *her level of underinvestment is less severe than when the buyers' investment decisions are centralized; and*

2. $\forall \alpha > 0$, *her level of investment is increasing in the number of buyers.*

*Proof.* The proof is constructed in three parts.

First, we solve the equilibrium level of investment.

The seller choses the level of specific investment which maximize her expected profits. This means that:

$$\max_{\sigma} \left[ \int_{\Theta} \left[ \left( 1 - \sum_{i=1}^{N} \alpha \right) W(Q^*) + \sum_{i=1}^{N} \alpha \times W_{-i}(Q_{-i}^*) \right] dF - \phi_s(\sigma) \right]. \tag{19}$$

The first order condition gives:

$$\sum_{i=1}^{N}\alpha\times\mathbf{E}_{\boldsymbol{\theta}}\left[\sum_{j\neq i}\frac{\partial q^*_{j-\{i\}}}{\partial\sigma}\times\left(\frac{\partial v_j\left(q^*_{j-\{i\}}\right)}{\partial q^*_{j-\{i\}}}-\frac{\partial C(Q^*_{-i})}{\partial q^*_{j-\{i\}}}\right)-\frac{\partial C(Q^*_{-i})}{\partial\sigma}\right]$$
$$-\left(\sum_{i=1}^{N}\alpha-1\right)\times\mathbf{E}_{\boldsymbol{\theta}}\left(\sum_{i=1}^{N}\frac{\partial q^*_i}{\partial\sigma}\times\left(\frac{\partial v_i(q^*_i)}{\partial q^*_i}-\frac{\partial C(Q^*)}{\partial q^*_i}\right)-\frac{\partial C(Q^*)}{\partial\sigma}\right]=\phi_s^{'}(\sigma),$$

where $\left(q^*_{j-\{i\}}\right)$ is the optimal amount allocated to any buyer $j\neq i$ when buyer $i$ is excluded from trading. By the envelope theorem the latter expression reduces to:

$$-\sum_{i=1}^{N}\alpha\times\mathbf{E}_{\boldsymbol{\theta}}\left[\frac{\partial C(Q^*_{-i},\theta_s,\sigma^e)}{\partial\sigma}\right]+\left(\sum_{i=1}^{N}\alpha-1\right)\times\mathbf{E}_{\boldsymbol{\theta}}\left[\frac{\partial C(Q^*,\theta_s,\sigma^e)}{\partial\sigma}\right]=\phi_s^{'}(\sigma^e). \quad (20)$$

Second, we show that the seller's investment is always higher ijn the case where the buyers' investment decisions are decentralized than where they are centralized.

Note that by convexity of $\phi_s(\cdot)$, the level of specific investment is higher than where the buyers' investment decisions are centralized as long as the LHS of equation (20) is higher than the LHS of equation (10).

This means that:

$$\begin{aligned}-\sum_i\alpha\times\frac{\partial C(Q^*_{-i})}{\partial\sigma}+\left(\sum_i\alpha-1\right)\times\frac{\partial C(Q^*)}{\partial\sigma} &> -(1-\alpha)\times\frac{\partial C(Q^*)}{\partial\sigma}\\ -N\times\frac{\partial C(Q^*_{-i})}{\partial\sigma}+(N-1)\times\frac{\partial C(Q^*)}{\partial\sigma} &> 0.\end{aligned}$$

The last condition can also be rewritten in integral form as:

$$\int_{N\times Q^*_{-i}}^{(N-1)\times Q^*}C_{q\sigma}(x)dx>0.$$

Since $C_{q\sigma}<0$ holds by Assumption 1 and $N\times Q^*_{-i}>(N-1)\times Q^*$ holds by concavity of the welfare function, the result is proven.

Third, we show that the level of specific investments is increasing with the number of buyers.

We have to show that, $\forall\,\alpha$, the LHS of equation (20) is increasing in $N$. This means that:

$$-\frac{\partial C(Q^*_{-i})}{\partial\sigma}-N\times\left(\frac{\partial C(Q^*)}{\partial\sigma}-\frac{\partial C(Q^*_{-i})}{\partial\sigma}\right)+\frac{\partial C(Q^*)}{\partial\sigma}+(N-1)\times\left(\frac{\partial C(Q^*_{+i})}{\partial\sigma}-\frac{\partial C(Q^*)}{\partial\sigma}\right)>0$$
$$-2\times\frac{\partial C(Q^*)}{\partial\sigma}+\frac{\partial C(Q^*_{-i})}{\partial\sigma}+\frac{\partial C(Q^*_{+i})}{\partial\sigma}>0.$$

The last condition can be also written as:

$$\int_{Q^*}^{Q^*_{+i}} C_{q\sigma}(x)dx > \int_{Q^*_{-i}}^{Q^*} C_{q\sigma}(x)dx.$$

Since $C_{q\sigma} < 0$ holds by Assumption 1 and $0 < Q^*_{+i} - Q^* < Q^* - Q^*_{-i}$ holds by concavity of the welfare function, the last condition holds.

□

**Remark 5.1.** *It is apparent that when the buyers' bargaining power is such that $\alpha = 0 \, \forall i \in N$, the seller's level of specific investment is the first best. Additionally, when the buyers engage in individual transactions with the seller, $\forall \alpha > 0$ and for $N$ approaching to infinity the seller's bargaining rents approximate to the whole surplus. This implies that the seller's level of specific investment approximates to the first-best also in this case.*

**Remark 5.2.** *When the buyers have linear utilities, there is no pure strategy equilibrium in the investment game. Since the buyers are ex-ante equal, their expected marginal surplus is zero. This follows from Assumption 4 and linearity. In particular, excludability suggests that buyer $i$ will have positive payoff only when his value function is the highest. The payoff buyer $i$ receives is equal to his valuation and the valuation of the second most efficient buyer. However, if the two most efficient buyers have the same valuation of the good, their marginal surplus is zero. Hence, because payoffs are discontinuous, there is no pure strategy in the investment game.*

From the above analysis, our result is that absent initial contracts, the total equilibrium level of investments is higher when buyers engage in individual transactions with the seller, although it still remains under the first best. What is the value of coordination, then? Our suggestion is that coordination is relevant only from a distributive perspective, but it does not imply an higher level of action from the coordinated parties. This casts a doubt on the alleged efficiency of coordination consortium. While consortium are typically defended as a means to reduce member firms' transaction costs and, therefore, increase allocative efficiency, the preliminary results obtained in our research would suggest that coordination consortium can only improve distributive efficiency. In the next stage of this work, we purport to further investigate this and other practical applications of the theoretical results we have obtained.

In the next section, we will consider the case where parties are able to make contractual commitments before the realization of nature so as to verify wether the first-best level of investments can be restored by contractual mechanisms.

# 6 Contracts $(T_i^0, q_i^0) \neq (0,0)$

In this part, we consider the case where parties conclude bilateral initial contracts à la ADR at $t_0$. We let each party (i.e., the buyers or the seller) propose (i) menu(s) of simple contracts and (ii) an allocation of ex-post authority to the other party. In Part 4, we showed that without initial contracts and under certain conditions, efficient allocations are implementable. However, ex-ante efficiency is not achieved since parties are only partial residual claimants of the surplus. Following the same logic as above, we first construct the equilibrium investment when the buyers' investment decisions are centralized and, then, when they engage in individual transactions with the seller.

## 6.1 Coordinated Transactions

In this case, we assume that at $t_0$ one contracting party (i.e., either the seller or the representative of the buyers) can make a take-it-or-leave-it offer of an initial contract to the other party, with probability that reflects the parties' exogenous bargaining power. Without loss of generality, we assume that the proposer at $t_0$ is also the contracting party who will have ex-post authority in the renegotiation stage. Therefore, the representative of the buyers will be both the proposer of the initial contract and have ex-post authority over the renegotiation stage with probability $\alpha$.

As in Part 4, we first consider the ex-post allocation game and, then, the ex-ante investment game. Since the buyers' decisions are centralized, we deal with aggregate quantities and transfers. We closely follow the ADR implementation mechanism, in which the parties renegotiate the initial contract and trade the optimal quantity. Indeed, for a given level of specific investment $(\boldsymbol{\beta}, \sigma) \in \mathbb{R}^{(N+1)}$, the realization of uncertainty $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ uniquely determines the ex-post efficient level of trading $Q^*(\boldsymbol{\theta}, \boldsymbol{\beta}, \sigma)$. From Assumption 5, renegotiation of the initial allocation always leads to ex-post efficiency. The ex-ante investment problem is then solved by selecting the quantity that induces the receiver (or fixed claimant) to invest optimally.

This result is summarized in the following proposition.

**Proposition 6.1.** *The ex-ante efficient level of investment is achieved when the proposer offers the initial allocation $(\hat{T}^0, \hat{Q}^0)$, where $\hat{Q}^0$ is the aggregate quantity which induces the* *receiver to invest optimally and $\hat{T}^0$ is the expected aggregate transfer that gives the buyers their reservation utility.*

*Proof.* This proof has the same structure as in ADR. We consider the case where the seller is the proposer.

For a given initial allocation $(T^0, Q^0)$, the buyers' level of investment is determined by the following maximization problem:

$$\max_{\boldsymbol{\beta}} \int_{\boldsymbol{\Theta}} \left[V(Q^0, \boldsymbol{\theta}, \boldsymbol{\beta}) - T^0\right] dF(\theta) - \sum_{i=1}^{N} \psi_i(\beta_i), \tag{21}$$

where $V(Q^0, \boldsymbol{\theta}, \boldsymbol{\beta})$ is the aggregate value function of the buyers.

It is possible to define an application $\boldsymbol{\beta}^e(Q^0)$ which maps the initial quantity in the equilibrium level of investments. We define the two arguments of the application that corresponds to the optimal investment when uncertainty realizes, respectively, as the upper bound and the lower bound of the former application. By the intermediate value theorem, there exists an initial allocation $\hat{Q}^0$ such that $\boldsymbol{\beta}^e(\hat{Q}^0) = \boldsymbol{\beta}^*$.

The expected transfer $\hat{T}^0$ is chosen such that the buyers are indifferent between participating or not to the initial contract:

$$\int_{\boldsymbol{\Theta}} \left[V(\hat{Q}^0, \boldsymbol{\theta}, \boldsymbol{\beta}^*) - \hat{T}^0\right] dF(\theta) - \sum_{i=1}^{N} \psi_i(\beta_i^*) = \mathbf{E}_{\boldsymbol{\theta}}\left[\bar{U}^B\right]. \tag{22}$$

where $\mathbf{E}_{\boldsymbol{\theta}}\left[\bar{U}^B\right]$ is the reservation utility of the buyers.

Finally, the seller's problem reduces to the choice of the level of investment that in expectation maximizes her renegotiation surplus. Since she is full residual claimant, she chooses $\sigma^e = \sigma^*$. $\square$

**Remark 6.1.** *The expected initial transfer specified in the initial contract depends on the exogenous bargaining power of the fixed claimant. When the buyers' investment decisions are centralized, their reservation utility is determined as the expected utility they would receive by choosing the null contract. Therefore:*

$$\mathbf{E}_{\boldsymbol{\theta}}\left[\bar{U}^B\right] = \alpha \times \mathbf{E}_{\boldsymbol{\theta}}\left[W(Q^*, \boldsymbol{\theta}, \beta^C, \sigma^C)\right] - \sum_i \psi_i(\beta_i^C). \tag{23}$$

*By substituting (23) into (22), we obtain:*

$$\hat{T}^0(\alpha) = \mathbf{E}_{\boldsymbol{\theta}}\left[V(\hat{Q}^0, \boldsymbol{\theta}, \boldsymbol{\beta}^*) - \alpha \times \left(V(Q^*, \boldsymbol{\theta}, \boldsymbol{\beta}^C) - C\left(Q^*, \theta_s, \sigma^C\right)\right)\right] - \sum_{i=1}^{N} \left[\psi(\beta_i^*) - \psi(\beta_i^C)\right]. \tag{24}$$

*Hence, $\hat{T}^0(\alpha)$ identifies the equilibrium transfer of the initial contract.*[6]

## 6.2 Individual Transactions

As in Part 4, since the seller and the buyers can make a take-it-or-leave-it offer with some exogenous probability, we consider two different games: the offer game (i.e., when the seller is offering) and the bidding game (i.e., when the buyers are offering).

In contrast with the case where the buyers' investment decisions are centralized, here each buyer and the seller individually (i) conclude an initial contract that can be unilaterally enforced; and (ii) renegotiate the initial contract after the realization of nature. As in ADR, we consider the investment decision of the fixed claimant(s) to be a function of the trade profile specified in the initial contract(s). Conversely, the investment decision of the residual claimant depends on the level of ex-post renegotiation surplus she is able to extract.

### 6.2.1 Offer Game

At $t_0$, the seller proposes a menu of initial allocations $M_i^0$ to buyer $i$ and retain the ex-post right to make a take-it-or-leave-it-offer. Buyer $i$ accepts the initial allocation if in expectation he obtains the same level of utility that he would obtain without the initial commitment. Therefore, any initial allocation composed by a transfer and an amount of trade $m^0 = (T^0, q^0)$ needs to satisfy the following participation constraint:

$$\int_{\Theta} \left[ v_i(q_i^0, \theta_i, \beta_i) - T_i^0 \right] dF(\theta) - \psi_i(\beta_i) = \mathbb{E}_{\boldsymbol{\theta}}[\bar{U}_{B_i}] \qquad i \in N. \tag{25}$$

Similarly to the case of the null contract, also at the initial stage the seller will propose two contracts. The following lemma defines the menus of initial contracts.

**Lemma 3.** *The menu of initial allocations offered by the seller has active cardinality $n\left(\Gamma^0\right) = 2$.*

[6] The reservation utility of the buyers depends on the equilibrium outcome that arises when the receiver (i.e., the representative of the buyers) selects the null contract. Also note that, when $\alpha = 0$, the equilibrium transfer is equal to $\hat{T}(0) = \mathbf{E}_{\boldsymbol{\theta}} \left[ V(\hat{Q}^0, \boldsymbol{\theta}, \boldsymbol{\beta}^*) \right] - \sum_{i=1}^{N} \left[ \psi_i(\beta_i^*) - \psi_i(\beta_i^C) \right]$. This means that in expectation the buyers only receive the cost of specific investments.

*Proof.* The menu of initial contracts is composed by the null contract $(T_i^0, q_i^0) = (0, 0)$ and by the contract maximizing the expected utility of the seller. Denoting the latter contract as $\hat{m}_i^0$, we have $\hat{\boldsymbol{m}}^0 = \arg\max \mathbb{E}_{\boldsymbol{\theta}}(U_s)$. Assume that the seller increases the cardinality of the menu offered to buyer $i$, i.e., $n(\Gamma_i^0) = 3$. Under this menu, then, the seller proposes a new allocation $\tilde{m}_i^0$ and keeps the cardinality of all the other menus $n\left(\Gamma_j^0\right) = 2$ for $j \neq i$. However, if buyer $i$ choses $\tilde{m}_i^0$, the seller is worse-off since $(\tilde{m}_i^0, \hat{\boldsymbol{m}}_{-i}^0) \neq \arg\max \mathbb{E}_{\boldsymbol{\theta}}(U_s)$. □

When the buyers' investment decisions are centralized, we have seen that it is optimal for the seller to offer an initial contract where $\forall i \in N$ the amount of initial trade is set to $\hat{q}_i^0$ such that $\beta_i^e(\hat{q}_i^0) = \beta_i^*$. Indeed, under Assumption 5, an increase in the valuation of the good for the buyers implies a lower cost of production for the seller. Therefore, there always exists a Pareto improvement in increasing the level of trading. Conversely, when the buyers' investment decisions are decentralized, the agreement on the terms of renegotiation occurs at the individual level. Therefore, aggregate co-monotonicity does not assure that each buyer's incentive will be aligned with the incentive of the seller in the renegotiation stage. This reduces the renegotiation surplus for the seller. In particular, this problem may arise when the valuation of buyer $i$ increases since $\theta_i \geq \mathbb{E}(\theta_i)$, but ex-post efficiency mandates the seller to trade a quantity lower than $q_i^0$. In this circumstance, to implement the optimal trade profile the seller must leave part of the renegotiation surplus to buyer $i$. Then we can introduce the following proposition.

**Proposition 6.2.** *For the seller is not always individually efficient to offer the initial trade profile that induces all the buyers to invest optimally.*

*Proof.* The proof is by construction and it is developed in three steps. First, we characterize the cardinality of the menus offered in the renegotiation stage; second, we characterize the situation where the seller loses some of the renegotiation surplus in order to implement the optimal trade profile; third, we construct and solve the seller's program.

Following the same argument of lemma 3, the cardinality of the menus offered in the renegotiation stage is $n(\Gamma_i^1) = 2$. Hence, the menus are represented by the initial allocation and the allocation which implements the optimal ex-post trading. Precisely, the ex-post allocation is such that, for a given $(\boldsymbol{\beta}, \sigma) \in \mathbb{R}_+^{(N+1)}$ and $\boldsymbol{\theta} \in \boldsymbol{\Theta}$, $\boldsymbol{q}^* = \arg\max W(\boldsymbol{q}, \boldsymbol{\theta}, \boldsymbol{\beta}, \sigma))$ and $T_i^* = v_i(q_i^*, \theta_i, \beta_i) - v_i(q_i^0, \theta_i, \beta_i) + T_i^0$. The transfer is set so that all the buyers receive the same utility they would obtain under the initial allocation.

We define by $\underline{\boldsymbol{\Theta}} \subset \boldsymbol{\Theta}$ the set of contingencies where buyer $i$ has a high valuation of the good and wants to trade more than the initial allocation, but efficiency requires to trade less:

$$\underline{\boldsymbol{\Theta}} := \{\theta_i \geq \mathbb{E}(\theta_i) : q_i^* < q_i^0\}. \tag{26}$$

In this set of contingencies the renegotiation surplus the seller can extract from buyer $i$ is reduced by the following amount:

$$\zeta_i\left(\beta_i(q_i^0)\right) = v_i\left(q_i^*(\beta_i(q_i^0)), \theta_i, \beta_i(q_i^0)\right) - v_i\left(q_i^0, \theta_i, \beta_i(q_i^0)\right) < 0. \tag{27}$$

The problem of the seller is to choose the vector of initial allocations $\boldsymbol{q^0}$ that maximizes her expected utility subject to the ex-post participation constraint for each buyer. Therefore, the seller program is:

$$\max_{q^0} \mathbb{E}_{\boldsymbol{\theta}}\left[\sum_i T_i^*(q_i^0) - C(\sum_i q_i^*(q_i^0))\right] - \phi(\sigma), \tag{28}$$

subject to

$$T_i^*(q_i^0) = v_i\left(q_i^*(\beta_i(q_i^0)), \theta_i, \beta_i(q_i^0)\right) - v_i\left(q_i^0, \theta_i, \beta_i(q_i^0)\right) + T_i^0 \quad \forall i \in N. \tag{29}$$

Then, in setting the initial trade profile, the seller takes into account the opportunity cost of implementing the optimal trade profile in the renegotiation stage. Indeed, this opportunity cost is incurred by the seller with a non-zero probability. Therefore, the initial trade profile is determined by[7]:

$$\frac{\partial \beta_i}{\partial q_i^0} \times \mathbb{E}_{\boldsymbol{\theta}}\left[\frac{\partial v_i(q_i^*)}{\partial \beta_i} + \frac{\partial \zeta_i\left(\beta_i(q^0)\right)}{\partial \beta_i}\right] = \psi'(\beta_i) \times \frac{\partial \beta_i}{\partial q_i^0} \quad \forall i \in N. \tag{30}$$

Condition (30) describes the trade-off that the seller faces when increasing the initial trade profile. On the one hand, an increase in the initial trade is beneficial as it fosters the buyers' investments, which, in turn, makes the buyers' valuation of the good higher. This effect is represented by $\frac{\partial v_i(q_i^*)}{\partial \beta_i}$ . On the other hand, since individual co-monotonicity fails, increasing the initial allocation may hurt the seller in situations where implementing the optimal allocation requires her to give up some renegotiation surplus. This effect is represented by $\frac{\partial \zeta_i(\beta_i(q_i^0)}{\partial \beta_i}$, which is negative in $\underline{\boldsymbol{\Theta}}$. Therefore, as long as $\underline{\boldsymbol{\Theta}} \neq \emptyset$, the seller may set the initial trade profile such that buyer $i$ does not invest optimally (i.e., $q^0 < \hat{q}^0$ and $\beta_i^e(q_i^0) < \beta_i^*$). [8]

□

**Remark 6.2.** *Note that in this case the seller will invest optimally $\sigma^e = \sigma^*$. Indeed, once the seller sets the initial amount of trade to maximize her expected utility, she is a constrained residual claimant over the renegotiation surplus.*

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[7]When buyer $i$ chooses the optimal allocation and such a choice does not reduce the seller's renegotiation surplus, the optimal initial trade for buyer $i$ is determined by the following condition:

$$\mathbb{E}_{\boldsymbol{\theta}}\left[\frac{\partial \beta_i}{\partial q_i^0} \times \frac{\partial q_i^*}{\partial \beta_i} \times \left(\frac{\partial v_i(q_i^*)}{\partial q_i^*} - \frac{\partial C(Q^*)}{\partial q_i^*}\right) + \frac{\partial \beta_i}{\partial q_i^0} \times \frac{\partial v_i(q_i^*)}{\partial \beta_i} - \frac{\partial v_i(q_i^0)}{\partial \beta_i} \times \frac{\partial \beta_i}{\partial q_i^0} - \frac{\partial v_i(q_i^0)}{\partial q_i^0} + \frac{\partial T_i^0}{\partial q_i^0}\right].$$

By applying the envelope theorem and because $\frac{\partial T_i^0}{\partial q_i^0} = \frac{\partial v_i(q_i^0)}{\partial \beta_i} \times \frac{\partial \beta_i}{\partial q_i^0} + \frac{\partial v_i(q_i^0)}{\partial q_i^0} - \psi'(\beta_i) \times \frac{\partial \beta_i}{\partial q_i^0}$, the previous condition reduces to: $\frac{\partial \beta_i}{\partial q_i^0} \times \mathbb{E}_{\boldsymbol{\theta}}\left[\frac{\partial v_i(q_i^*)}{\partial \beta_i}\right] = \psi'(\beta_i) \times \frac{\partial \beta_i}{\partial q_i^0} \quad \forall i \in N.$

[8]Conversely, when $\underline{\boldsymbol{\Theta}} = \emptyset$, the seller proposes the initial allocation so to induce the optimal level of investment: $q_i^0 = \hat{q}_i^0$ such that $\beta_i^e(\hat{q}_i^0) = \beta_i^*$.

**Remark 6.3.** *The measure of $\underline{\boldsymbol{\Theta}}$ reduces in $\pi_i$. When the level of uncertainty of the seller and buyer i is highly correlated, the probability that buyer i has high valuation of the good and the seller has high cost of production is low. Therefore, high realization of buyer i followed by a reduction of trade is not likely to happen. Indeed, when individual co-monotonicity increases, the probability in $\underline{\boldsymbol{\Theta}}$ has a lower mass. As a result, the initial trade profile the seller offers to buyer i is an increasing function of the level of correlation $q_i^0(\pi_i)$. In the extreme case, where there is only one buyer, the set $\underline{\boldsymbol{\Theta}}$ has zero mass. Indeed, in this case, there always exists mutual agreement on the ex-post level of trading.*[9]

**Remark 6.4.** *Observe that there always is a minimum positive investment undertaken by the buyers. When $(T_i^0, q_i^0) = (0, 0)$ holds, we have seen that the buyers undertake a positive level of investment $\beta_i^e > 0$. However, it might still be beneficial for the seller to offer an initial allocation inducing $\beta_i^e(q_i^0) < \beta_i^e$. The reason for this is that the existence of an initial contract serves as an insurance for the seller, since she obtains ex-post authority to make a take-it-or-leave-it offer.*

### 6.2.2 Bidding Game

Here we consider the case where buyers make a take-it-or-leave-it offer to the seller. In order to determine the equilibrium of investments, we proceed again by backward induction. We first characterize the menus of contracts that the buyers offer in the renegotiation stage, assuming that buyer $i$ proposes the initial allocation $m_i^0 = (T_i^0, q_i^0)$. Given the profile of menus $\mathbf{M}^1 = (M_1^1, M_2^1, ..., M_N^1) \in \Gamma^N$ that the buyers offer in the renegotiation stage, a strategy for the seller is a function $m(\mathbf{M}^1) : \Gamma^N \to (R_+^2)^N$ and a profile of contracts accepted by the seller is defined by $m = (m_1, m_2, ..., m_N)$.

Like in the previous case, since individual co-monotonicity at the individual level is not assured, there are circumstances under which some buyers have different trading preferences than the seller. Hence, it is not possible for the buyers to extract all the renegotiation surplus in the renegotiation stage. This, in turns, leads the buyers to underinvest. The result is given by the following proposition.

**Proposition 6.3.** *Given the profile of menus $\mathbf{M}^1$ offered in the renegotiation stage, the*

[9]This is, indeed, the co-monotonicity condition in ADR where: $\theta \geq \mathbb{E}(\theta) \Longrightarrow q^* \geq q^0$ and $\theta < \mathbb{E}(\theta) \Longrightarrow q^* < q^0$ .

*buyers are not always full residual claimants. Therefore, buyers underinvest.*

*Proof.* The proof is by construction and it is developed in two stages. First, we identify the set of contingencies where buyer $i$ obtains zero renegotiation surplus and the renegotiation surplus of the other buyers is reduced. Second, we set the buyers' problem and determine the equilibrium level of investments.

We define as $\tilde{\mathbf{\Theta}} \subset \mathbf{\Theta}$ the set of contingencies where buyer $i$ is left with zero renegotiation surplus:

$$\tilde{\mathbf{\Theta}} \equiv \{m = (m_i^0, m_j^*)\} \qquad \forall j \neq i. \tag{31}$$

Note that $\tilde{\mathbf{\Theta}} \neq \emptyset$ holds as long as there exists a profile of contracts $m = (m_i^0, m_j^*)$ accepted by the seller. This occurs, for example, when the valuation of buyer $i$ increases since $\theta_i \geq \mathbb{E}(\theta_i)$, but ex-post efficiency mandates the seller to trade a quantity lower than $q_i^0$. In this case, buyer $i$ may be unable to propose an acceptable transfer to the seller, given the transfers that are proposed to the seller by the other buyers. Therefore, in $\tilde{\mathbf{\Theta}}$ there is no optimal contract offered by buyer $i$ that is accepted by the seller, since the seller obtains higher payoff by selecting the initial allocation: $T_i^0 + \sum_{j\neq i} T_j^* - C(q_i^0 + \mathbf{q}_j^*) > \sum_i T_i^* - C(\mathbf{q}^*)$. Anticipating that the seller enforces the initial allocation proposed by buyer $i$, buyer $j$ would propose an allocation which is optimal given the enforcement of allocation $m_i^0$: $m_j(m_i^0) = (T_j^{**}, q_j^{**})$. Under these allocations, the seller gets $T_i^0 + \sum_{j\neq i} T_j^{**} - C(q_i^0 + \mathbf{q}_j^{**}) = T_i^0 + \sum_{j\neq i} T_j^* - C(q_i^0 + \mathbf{q}_j^*)$ and buyer $j$ obtains $v_j(q_j^{**}, \theta_j, \beta_j) - T_j^{**} > v_j(q_j^*, \theta_j, \beta_j) - T_j^*$. Since the seller is fixed claimant, she obtains the same payoffs by choosing $m = (m_i^0, m_j^*)$ or $m = (m_i^0, m_j^{**})$, while buyer $j$ obtains a better payoff. Note, however, that buyer $j$ would obtain a higher payoff if the seller accepted the optimal allocations from all the buyers (i.e $m = (m_i^*, m_j^*)$), since by optimality: $v_j(q_j^{**}, \theta_j, \beta_j) - T_j^{**} < v_j(q_j^*, \theta_j, \beta_j) - T_j^*$.

In what follows, we characterize the optimization problem for buyer $i$ to determine his level of investments.

The expected payoff of buyer $i$ can be represented as:

$$\max_{\beta_i} \mathbb{E}_{\boldsymbol{\theta}} \left[ v_i(q_i^0(\beta_i), \theta_i, \beta_i) - T_i^0 + RS \right] - \psi(\beta_i), \tag{32}$$

where the renegotiation surplus $RS = \max\{0, v_i(q_i^*(\beta_i), \theta_i, \beta_i) - T_i^* - v_i(q_i^0(\beta_i), \theta_i, \beta_i) + T_i^0\}$. Note that $RS$ is equal to zero whenever the seller prefers to enforce the initial allocation. Conversely, whenever the seller accepts the renegotiated contract $m_i^* = (T_i^*, q_i^*) \in M_i^1$, buyer $i$ extracts renegotiation surplus. Therefore, we can define the ex-post loss in the renegotiation surplus as:

$$\tau(\beta_i) = v_i(q_i^0(\beta_i), \theta_i, \beta_i) - T_i^0 - v_i(q_i^*(\beta_i), \theta_i, \beta_i) + T_i^* < 0. \tag{33}$$

The optimal investment level of buyer $i$ is, then, determined by optimizing the buyer's expected payoff. Since buyer $i$ gets zero renegotiation surplus in $\tilde{\mathbf{\Theta}}$, the following condition determines his equilibrium level of investment:

$$\mathbb{E}_{\boldsymbol{\theta}} \left[ \frac{\partial \tau(\beta_i)}{\partial \beta_i} + \frac{\partial v_i(q_i^*)}{\partial \beta_i} \right] = \psi_i^{'}(\beta_i) \forall i \in N. \tag{34}$$

Note that $\left[\frac{\partial \tau(\beta_i)}{\partial \beta_i}\right]$ is negative and occurs only in $\tilde{\mathbf{\Theta}}$. Therefore, buyer $i$ is induced to underinvest, since he is not full residual claimant. Note also that when the seller enforces the initial allocation proposed by buyer $i$, she creates an externality to the other buyers as the renegotiation surplus attainable by each buyer is reduced. Such reduction can be represented by $\tau(\beta_j) = v_j(q_j^{**}(\beta_j), \theta_j, \beta_j) - T_j^{**} - v_j(q_j^*(\beta_j), \theta_j, \beta_j) + T_j^* < 0$. As a result, since also buyer $j$ cannot appropriate all the renegotiation surplus, she is induced to underinvest: $\beta_j^e < \beta^*$. $\square$

Given proposition 5.3. above, we can attempt to analyze the seller's level of investment. To this end, we must characterize the menu of contracts that each buyer offers to the seller at $t_0$.

We have seen that in the renegotiation stage the equilibrium allocations may not be truthful as the efficient level of trade is not implemented under all possible contingencies. Since the buyers are not full residual claimants, they may not have the incentives to set the initial level of trading so as to induce the seller to invest optimally. As a result, it may be the case that, in the menus of initial allocations, the truthful trade profile (i.e the one leading the seller to invest optimally) is not always included.

We will outline the reasoning underpinning the previous statement by characterizing the initial allocation proposed by buyer $i$ when he is full residual claimant. This determines an upper bound of the initial trade profile. In particular, when buyer $i$ is full residual claimant, his problem is $\max_{q_i^0} \mathbb{E}_{\boldsymbol{\theta}} \left[ v_i(q_i^*(\sigma(Q^0), \beta_i), \theta_i, \beta_i) - T_i^* \right] - \psi(\beta_i)$ as long as the seller accepts (i) the allocation proposed in the renegotiation stage $\sum_i T_i^* - C(Q^*) = \sum_i T_i^0 - C(Q^0)$, and (ii) the initial contract $\mathbb{E}_{\boldsymbol{\theta}} \left[ \sum_i T_i^0 - C(Q^0) \right] - \phi(\sigma) = \mathbb{E}_{\boldsymbol{\theta}}[\bar{U}_s]$. Since the investment of the seller is a function of the aggregate initial trade profile $\sigma(Q^0)$, the first order condition gives: $\mathbb{E}_{\boldsymbol{\theta}} \left[ \frac{\partial q_i^*}{\partial \sigma} \times \frac{\partial \sigma}{\partial q_i^0} \times \left( \frac{\partial v_i(q_i^*)}{\partial q_i^*} - \frac{\partial C(Q^*)}{\partial q_i^*} \right) - \frac{\partial C(Q^*)}{\partial \sigma} \times \frac{\partial \sigma}{\partial q_i^0} \right] - \psi'(\sigma) \times \frac{\partial \sigma}{\partial q_i^0} = 0$. By the envelope theorem it reduces to$-\mathbb{E}_{\boldsymbol{\theta}} \left[ \frac{\partial \sigma}{\partial q_i^0} \times \frac{\partial C(Q^*)}{\partial \sigma} \right] = \psi'(\sigma) \times \frac{\partial \sigma}{\partial q_i^0}$. When buyer $i$ is not full residual claimant, his problem, instead, is:$\max_{q_i^0} \mathbb{E}_{\boldsymbol{\theta}} \left[ v_i(q^0(\beta_i), \theta_i, \beta_i) - T_i^0 + RS \right] - \psi(\beta_i)$.

Then, the first order condition of the problem gives: $\frac{\partial v_i(q_i^0)}{\partial q_i^0} - \frac{\partial C(Q^0)}{\partial q_i^0} - \psi'(\sigma) \times \frac{\partial \sigma}{\partial q_i^0} = 0$. Therefore, the initial trade profile is determined by:

$$\mathbb{E}_{\boldsymbol{\theta}} \left[ \left( \frac{\partial v_i(q_i^0)}{\partial q_i^0} - \frac{\partial C(Q^0)}{\partial q_i^0} \right) \mathbf{1}_{\{\mathbf{RS=0}\}} - \left( \frac{\partial \sigma}{\partial q_i^0} \times \frac{\partial C(Q^*)}{\partial \sigma} \right) \mathbf{1}_{\{\mathbf{RS>0}\}} \right] = \psi'(\sigma) \times \frac{\partial \sigma}{\partial q_i^0}$$

.

Since the first term in brackets is always lower than the second, also the seller is induced to underinvest (i.e $\sigma^e < \sigma^*$).

# 7 Preliminary Conclusions

In this work, we have made a first attempt at studying the hold-up problem in a common-agency context with a monopolistic seller and multiple buyers. This framework captures many real-world transactions that do not fall within the traditional bilateral monopoly framework traditionally used to characterize hold-up in incomplete contracts.

We have considered four scenarios: no initial contract with and without buyers' coordination, and simple contracts (*à la* ADR) with and without buyers' coordination. We have shown that when parties are not able to agree upon an initial commitment, underinvestment arises in equilibrium. However, underinvestment is more severe when the buyers act cooperatively through a coordinator. This is so because while cooperation among the buyers does not increase the buyers' investments, it dramatically reduces the common agent's investment.

This result suggests that when the negotiation of ex-ante contractual commitments is costly, the rents that the common agent may extract in competitive common agency (i.e., when the buyers engage in individual negotiations with the common agent) are "beneficial" since they enhance the common agent's incentives to invest, other things being equal. As a consequence, under this circumstance, any countervailing contractual, regulatory or judicial measure aimed at contrasting the common agent bargaining power *vis-à-vis* the principals will worsen total welfare, reducing the total level of specific investments. In other words, when the buyers act competitively , underinvestment is mitigated because the rents the seller can extract induces her to invest more. The optimal level of investment can be approximated only in the ideal case in which the bargaining power is entirely allocated to the buyers and the number of buyers is large enough.

Only when simple contracts à la ADR are feasible and cooperation among the buyers is possible at negligible costs, these contracts can achieve the first best. That is, cooperative common agency is preferable over competitive common agency when the negotiation of ex-ante contractual commitments is feasible. Hence, in common agency contexts the relevance of the hold up problem depends on the principals' ability to coordinate and on the feasibility of simple contracts. When coordination is costly and/or simple contracts are difficult to establish, then it is optimal not to constrain the common agent bargaining power.

# References


[1] Aghion, P., M. Dewatripont, and P. Rey, (1994): "Renegotiation Design with Unverifiable Information", *Econometrica*, 62, 257-282.

[2] Baker, Brent.L.; Dant, Rajiv. P. "Stable plural forms in franchise systems: an examination of the evolution of ownership redirection research". In: Hendrikse, George .; Tuunanen, Mika; Windsperger, Josef.; Cliquet, Gerard. (eds.) Strategy and Governance of Networks. The Netherlands: Springer, 2008.

[3] Bergemann, D. and J. Välimäki, (2003): "Dynamic Common Agency", *Journal of Economic Theory,* 111, 23-48.

[4] Bernheim, B., and M. Whinston (1986): "Menu Auctions, Resource Allocation and Economic Influence", *Quarterly Journal of Economics*, 101, 1-31.

[5] Bradach, Jeffrey. L. "Using the plural form in the management of restaurant chains." Administrative Science Quarterly, vol. 42, n.2 (1997).

[6] Bradach, Jeffrey.L.; Eccles, Robert, G. "Price, authority and trust: from ideal types to plural forms." Annual Review of Sociology, vol. 15 (1989).

[7] Chiesa, G., and V. Denicoló (2009): "Trading with a common agent under complete information: A characterization of Nash equilibria", *Journal of Economic Theory*, 144, 296-311.

[8] Chung, P. (1991): "Incomplete Contracts, Specific Investment and Risk Sharing", *Review of Economics Studies*, 58, 1031-1042.

[9] Edlin, A., and S. Reichelstein, (1996): "Hold-ups, Standard Brach Remedies, and Optimal Investment", *The American Economic Review*, 86, 478-282.

[10] Grossman, S., and O. Hart (1986): "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration", *The Journal of Political Economy*, 94, 691-719.

[11] Hart, O., and J. Moore (1988): "Incomplete Contacts and Renegotiation", *Econometrica*, 56, 755-785.

[12] Hart, O., and J. Moore (1990): “Property Rights and the Nature of the Firm”, *Journal of Political Economy*, 98, 1119-1158.

[13] Holmstrom, B. (1982): “The Moral Hazard in Teams”, *Bell Journal of Economics*, 13, 324-340.

[14] Klein, B., R. Crawford, and A. Alchian (1978): “Vertical Integration, Appropriable Rents and the Competitive Contracting Process”, *Journal of Law and Economics*, 21, 297-326.

[15] Lafontaine, Francine. “Agency theory and franchising: some empirical results.” The Rand Journal of Economics, vol. 23, no. 2 (1992).

[16] Lafontaine, Francine.; Shaw, Kathryn.L. “Targeting managerial control: evidence from franchising”. The Rand Journal of Economics, vol. 36, no. 1 (2005).

[17] Lafontaine, Francine.; Slade, Margaret.E. “Retail contracting: theory and practice.” Journal of Industrial Economics, vol.45, no.1 (1997).

[18] Menard, Claude. “The economics of hybrid organizations.” Journal of Institutional and Theoretical Economics. Vol. 160 (2004).

[19] Noldeke, G., and L. Schmidt (1995): “Option Contracts and Renegotiation: a Solution to the Hold-up Problem”, *RAND Journal of Economics*, 26, 163-179.

[20] Segal, I. (1999): “Contracting with Externalities”, *The Quarterly Journal of Economics*, 114, 337-388.

[21] Williamson, O. (1979): “The Transaction-Cost Economics: the Governance of Contractual relations”, *Journal of Law and Economics*, 22, 233-261.

[22] Williamson, O. (1983): “Credible Commitment: Using Hostages to Support Exchange”, *The American Economic Review*, 73, 519-540.

[23] Williamson, Oliver. E. Markets and hierarchies: analysis and antitrust implications. New York: Free Press, 1975.

[24] Williamson, Oliver. E. The economic institutions of capitalism. New York: The Free Press, 1985.

[25] Williamson, Oliver. E. “Comparative economic organization: the analysis of discrete structural alternatives.” Administrative Science Quaterly, vol. 36, no. 2 (1991).

[26] Williamson, Oliver. E. The mechanisms of governance. Oxford: Oxford University Press, 1996.