64

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A SIMPLE INTRODUCTION TO FLEXIBLE FUNCTIONAL FORMS AND CONSUMER BEHAVIOUR THEORY

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*Abstract**

This paper is an elementary introduction to the flexible functional form approach. The idea is first clearly defined and then its advantages and limitations are described. Furthermore its use to test consumer theory is presented. Far from being an exhaustive survey of the relevant literature, this paper aims to make an increasing number of economists aware of the pros and cons of the present approach to test microeconomic theory.

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1. *Introduction*

Econometric works that impose, or test, the neoclassical microeconomic theory are becoming more and more frequent in the literature (1, 3, 4, 7, 9). A fairly new approach used for this purpose is the flexible functional form approach (4, 7, 9).

The present paper is an elementary introduction to this technique. In section 2 the basic idea is clearly defined and its advantages and limitations are described. The use of flexible functional forms to test consumer theory is presented in Section 3 and in Section 4 the relevant empirical results are surveyed. Sections 5 and 6 are devoted to a more detailed discussion of the limitations of this approach. The very present line of research is briefly mentioned in the conclusions.

2. *The definition of flexible functional form*

Most of the flexible functional forms were devised to generalize two restrictive characteristics of the Arrow, Chenery, Minhas, Solow, better known as CES, production function. Namely to go around the maintained hypothesis, implicit in the use of the CES itself, that the partial elasticities of substitution are constant and, in the case of more than two factors, identical for all the pairs of inputs.

This undesirable property, proved in the impossibility theorem of Uzawa (12), induced many economists working in the area of "production function" to look for more general functional forms. Functional forms able to attain an arbitrary set of elasticities of substitution at a specified set of factors and factor prices. Hence elasticities of substitution no longer constant but depending upon the "specified set of factors and factor prices", and consequently different according to the chosen point of approximation.

Flexible functional forms are specific forms that have enough parameters to be regarded as an approximation, linear in the parameters for obvious estimating reasons, to whatever the true unknown function may be. In general they can be written as (10)

(i) $f^*(\underline{x}) \simeq f(\underline{x}) = \Sigma_i a_i h^i(\underline{x})$

where f^* is the true unknown function, f is the approximating functional form, the a_i's are parameters to be estimated, the h^i's are known functions and $\underline{x}$ is a column vector of independent variables.

The most common method to generate flexible forms is by the use of a Taylor series of second order about a point $\underline{x}^\circ$. In this case the functions h^i in (i) are defined as

(ii) $h^\circ(\underline{x}) = 1$

(iii) $h^1(\underline{x}) = (\underline{x} - \underline{x}^\circ)$

(iv) $h^2(\underline{x}) = (1/2)(\underline{x} - \underline{x}^\circ)(\underline{x} - \underline{x}^\circ)^T$

where T stands for transpose, and the corresponding parameters are

(v) $a_0 = f^*(\underline{x}^\circ)$,

(vi) a_1 is the vector of first order derivatives of $f^*(\underline{x}^\circ)$ with respect to its components,

(vii) a_2 is a square matrix of second order derivatives of f^* with respect to its components.

Consequently, following Diewert's definition (8), it can be stated that a second order approximation of the true function U at point $\bar{\underline{x}}$ is any function $\hat{U}$ that can attain arbitrary elasticities of substitution at any point and furthermore has the following properties:

(a) $\hat{U}(\bar{\underline{x}}) = U(\bar{\underline{x}})$

(b) $\nabla \hat{U}(\bar{\underline{x}}) = \nabla U(\bar{\underline{x}})$

(c) $\nabla^2 \hat{U}(\bar{\underline{x}}) = \nabla^2 U(\underline{x})$

where ∇ is the gradient and ∇^2 is a matrix of second order partial derivatives

of U, or $\hat{U}$, with respect to its components (2).

It has been proved, by the same Diewert (8), that a necessary and sufficient condition for a function to attain arbitrary elasticities is that the function itself can attain arbitrary level, first and second order derivatives. This is achieved using polinomial forms as done in the previous pages.

It is worthed to mention that sometimes tha Laurent series (3), whose second order expansion looks like

(viii) $f(\underline{x}^\circ) = \sum_{i=-2}^{2} a_i h^i(\underline{x}^\circ)^1$,

is preferred to the simpler Taylor series. The main reason is that the former possesses a better behaved remainder term than the latter. By "better behaved" is meant here a remainder term with smoother variations within the convergence region.

3. Flexible functional forms and the consumer behavior theory

The first flexible functional form introduced in the economic literature was the Generalized Leontief, Diewert (8). It was originally used, as it is obvious from the previous discussion, to provide an approximation to an arbitrary twice differentiable cost function. However the ability to produce a good local approximation to any "arbitrary twice differentiable" function was such that its use was extended to the consumer theory. As a result, in the last few years an increasing number of economists have tested the theory of consumer behavior, on actual data, using a flexible functional form to approximate a neoclassical, direct or indirect, utility function.

The most common flexible forms in consumer theory, as well as in production theory, are: the Generalized Leontief (GL), the Generalized Cobb-Douglas

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(1) The meaning of the a_i's and h^i's is the same as before for i=0, 1, 2 whereas for i= -1, -2 the a_i's and h^i's are, respectively, the inverse of a_1, a_2, and h^1, h^2.

(GCD) and the so-called Translog (TLOG). Considering the reciprocal of the indirect utility function, as it is often done because of the fact that it possesses all the properties of a normal utility function (monotonically increasing and strictly quasi concave), they take the following form (4):

A) GCD,

$$h(v) = \pi_i \pi_j [(v_i/2) + (v_j/2)]^{\alpha_{ij}} + \Sigma_n \alpha_{\circ n} \ln v_n + \alpha_\circ$$

where $\alpha_{ij} = \alpha_{ji}$ for all i, j;

(B) GL,

$$h(v) = \Sigma_i \Sigma_j b_{ij} v_i^2 v_j^2 + \Sigma_i b_{\circ i} \ln v_i + b_{\circ\circ}$$

where $b_{ij} = b_{ji}$ for all i, j and $\Sigma_i b_{\circ i} = 0$;

C) TLOG,

$$\ln h(v) = \alpha_\circ + \Sigma_i \alpha_i \ln v_i + (1/2) \Sigma_i \Sigma_j \gamma_{ij} \ln v_i \ln v_j$$

where $\gamma_{ij} = \gamma_{ji}$ for all i,j.

The symmetry restrictions are sometimes tested rather than imposed and the v_i's represent the normalized prices (p_i/m).

The use of normalized prices does not imply any restriction because the functions under consideration are homogeneous of degree zero in the parameters. The homogeneity of degree zero requires to add at least one normalization in order to identify the parameters. One interesting normalization has been used by Berndt et al. (4). They impose the following restrictions on the parameters of the three flexible functional forms above mentioned:

A) GCD:	B) GL:	C) TLOG:
$\Sigma_i \Sigma_j \alpha_{ij} = 1$	$\Sigma_i \Sigma_j b_{ij} = 1$	$\Sigma_i \Sigma_j \gamma_{ij} = 0$
$\Sigma_n \alpha_{\circ n} = 0$	$\Sigma_i b_{\circ i} = 0$	$\Sigma_i \alpha_i = 1$
$\alpha_\circ = 0$	$b_{\circ\circ} = 0$	$\alpha_\circ = 0$

These restrictions make the indirect utility function homogeneous of degree one in income if all prices in the price vector are fixed and equal to each other. In this way they obtain a cardinalization of the utility, or real income.

Finally, another thing that deserves to be pointed out about flexible forms is that, differently from the Rotterdam model and the AIDS model proposed by Deaton and Muellbauer (7), the theoretical restrictions do not apply directly to the parameters.

4. Some empirical results

The flexible functional forms have been used, in general with negative results, to test the microeconomic theory of consumer behavior. For example Berndt et al. (4) have applied the three flexible forms above described to the Canadian data. They have found that the hypothesis that simmetry restrictions are valid is rejected both for the GCD and for the GL while it is accepted for the TLOG. However the hypothesis of homotheticity, conditioned on symmetry, is rejected for all the three forms. An interesting result, reported in this and other studies, is that the three models present own-price elasticities sometimes positive, instead of always negative, and cross-price elasticities erratic when the Slutsky symmetry restrictions are not imposed.

No better results were obtained by Christensen et al. (6) in the paper in which they introduced the translog. Using time series data on U.S. personal consumption expenditure, they were forced to reject symmetry restrictions and all the additional hypothesis about additivity and homotheticity of the utility function. Such results induced them to the "unambiguous rejection" of the theory of demand.

To explain these, and other, negative results it has been argued (7) that there is no reason to suppose that utility function is perfectly translog, Cobb-Douglas or Leontief either for individuals or for the aggregate. Therefore it is impossible to know whether the hypothesis is wrong or the approximation is inaccurate.

To try to understand the reason of this kind of results, it is necessary to go back to the analysis of the theoretical properties of the flexible functional forms.

5. *The limitations of the flexible functional form approach*

As already mentioned, the properties of the flexible functional forms have enhanced considerably the capability for testing structural hypothesis such as homogeneity, symmetry of the Slutsky coefficients and so on. However the approximation properties of these models are known only if integrability of the system of demand functions to a community utility function is imposed a priori at no more than one single infinitesimal point of approximation.

Indeed this class of forms is not able to model as many types of structure as it might appear at first sight. In particular it is not able to provide a second order approximation to any arbitrary weakly separable function in any neighbourhood of a given point (6).

Blackorby et al. (5) prove that the translog and the generalized Leontief, even if can be used to test homogeneity of the approximated function, are separability inflexible. In fact weak separability of the TLOG implies either strong separability or homothetic separability. Much stronger is the "inflexibility" of the GCD. It has been found (5) that it is able to model only strong separability.

Other problems arise when the integrability condition is not required in advance. Wales (13), for instance, analyses the ability of the TLOG and GL functional forms to approximate a CES utility function. That is he assumes that the true utility function is CES depending only upon one composite good and leisure and then studies the approximation properties of the TLOG and GL. He reports that although these functional forms provide a good local approximation they do not always provide a good approximation over the range of observations. And some set of data do not satisfy the regularity conditions, namely monotonicity and quasi-concavity of the utility function, required by microeconomic theory.

The extent to which flexible functional forms satisfy the desired regularity conditions depends on the parameters of the true utility function, the particular choice of flexible form used to approximate the true function and the variation of the determining variables.

A very interesting development, which will have probably far reaching consequences in the area of flexible forms, is the concept of quasi-flexibility recently introduced by Diewert and Wales (9). The main result is that functions globally satisfying regularity conditions can be estimated. This can be achieved by allowing disequalities along the main diagonal of the matrix equation

(ix) $\nabla \hat{U}(\underline{x}) = \nabla U(\underline{x})$

and along the k-th row and column of some specific flexible form.

6. *The strange convention*

Beyond the problems described so far, what seems to represent the most serious reserve toward the use of flexible functional forms in the consumer theory is the fact that they assume the existence of a community utility function. The test of the theory with locally integrable models tests indeed for aggregate integrability and consequently for community utility function. Community utility function, however, existing only under the strong assumption of linear Engel curves which, for any good, are parallel across the consumers.

The condition of parallel Engel curves is necessary and sufficient because of the fact that the aggregation over consumers is obtained using only the first moment of income distribution. In such a case it is assumed that consumers are characterized by the same income elasticities, with respect to the different goods, so that a change in the income distribution does not change the result itself of aggregation. Consequently what is approximated by the locally integrable models exists if and only if Gorman's conditions hold, that is if and only if the representative consumer exists (11).

This has induced some author to write "those who claim superior properties for the class of locally integrable functional forms do so on the ground that such models permit aggregate integrability to a larger class of community utility function over the region of data. It is a strange convention which leads to the selection of a model on the basis of its flexibility under entirely implausible conditions" (1). The "implausible conditions" implicitly maintained, i.e. the existence of a representative consumer, can explain the negative results, previously reported, in testing demand theory with flexible functional forms to a greater extent than the "inaccuracy of approximation".

7. Conclusion

In the previous pages the idea of flexible functional form has been introduced in a very elementary fashion.

After having indicated the theoretical need for this new tool, the precise definition has been given and its properties mentioned. The application of the flexible functional form to test consumer theory has been presented and the relevant empirical results reported. Most of them are negative in the sense of rejecting the consumer theory. Therefore, the last section has been devoted to the possible causes of this phenomenon.

The present line of research is focused toward the removal of all the possible "inaccuracy of determination". The possibility of modelling functional forms globally regular, rather than satisfying the regularity conditions at a single point, is conseguently accurately investigated.

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