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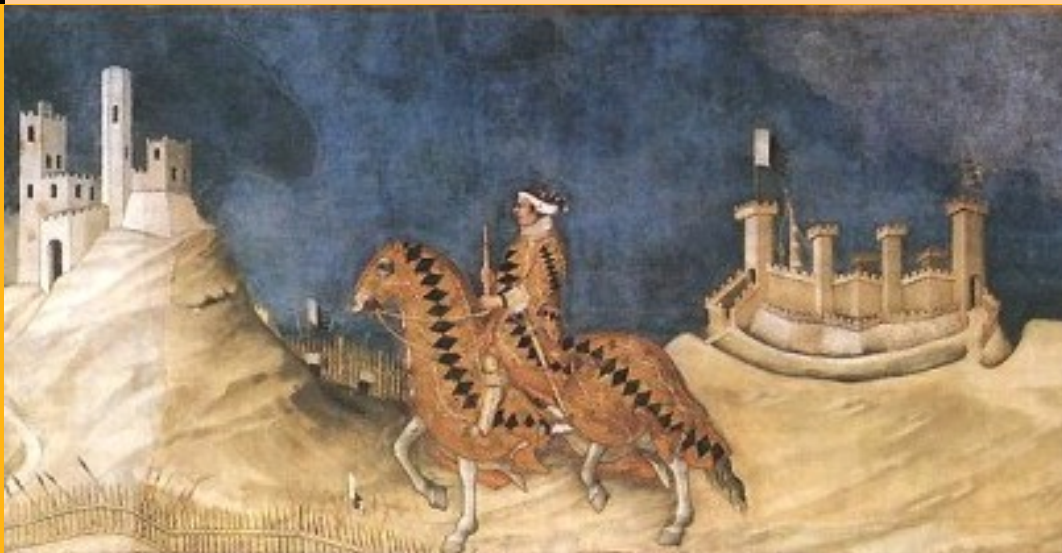
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Re-reading Keynes after the crisis: probability and decision

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Abstract – The recent financial crisis has renewed the interest in Keynes's thought and his analysis of the role played by individual agents in financial markets. George Akerlof and Robert Shiller, in particular, have drawn on the growing interest in behavioural interpretations of financial markets to hold that Keynes's insistence on "the spontaneous urge to action" of individuals is the most relevant message conveyed by the *General Theory*. This paper starts off from a brief summary of Akerlof and Shiller's influential stance and aims to provide an historically motivated assessment of their claim. The paper mostly concentrates on Keynes's *Treatise on Probability* and discusses how Keynes applied his philosophy of probability to decision-making. It is argued that a fresh reading of this part of Keynes's work can contribute to an understanding of how individual agents behave under uncertainty, and that the violations of the Bayesian creed scrutinized in behavioural finance, and in some current proposals to amend mainstream decision theory, were already implicitly discussed by Keynes in his critique of frequency probability.

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1. Introduction

The recent financial crisis has renewed the interest in the works of John Maynard Keynes. In internet blogs, magazines and scientific journals a crowd of commentators have emphasized the need to scrutinize the *General Theory* with the aim to gain a better understanding of the actual dynamics of the world economy, and to devise policy measures apt to foster a recovery from the downturn. Richard Posner (2009), for instance, admitted that he never thought about reading Keynes until recently, mostly on the advice of Milton Friedman. Baffled by an economics profession in disarray over the causes of the crisis, he decided to go through the *General Theory* and found it convincing to such an extent that he published a chronicle on how he became a Keynesian.

Most of the attention has been placed on the mechanics of the economic system. For instance, Gregory Mankiw summarizes Keynes's message by arguing, like in supposedly old-fashioned macro-textbooks, that "the root cause of economic downturns is *insufficient aggregate demand*." When total demand for goods and services declines, businesses throughout the economy see their sales decrease. As a result firms cut back production and lay off workers. As rising unemployment and declining profits further depress demand, the situation reverses only when some event or policy increases aggregate demand. The problem during the current crisis, Mankiw (2008) concludes, is that "it is hard to see where that demand might come from." Paul Krugman, another prominent commentator focusing on Keynesian mechanics, shows even greater confidence in the significance of the *General Theory*: since "depression economics" has returned, he argues, the solution simply is "good, old Keynesian fiscal stimulus" (Krugman 2008, p. 187).

Even authors working in Hyman Minsky's tradition focus on the macro-dynamics of the economy. As it is well-known, Keynes was a close observer of finance and speculation in capitalist economies. On Minsky's (1992, pp. 7-8) elaboration of Keynes's viewpoint it can be shown that "if hedge financing dominates, then the economy may well be an equilibrium-seeking and containing system." However, "the greater the weight of speculative and Ponzi finance, the greater the likelihood that the economy is a deviation-amplifying system." Over a period of prolonged prosperity, the economy "transits from financial relations that make for a stable system to financial relations that make for an unstable system." The current crisis can thus be seen as the inevitable outcome of a fundamental shift of financial economies towards

fragility (Wray 2009) and it cannot be solved without restoring the institutional structure for traditional bank lending (Kregel 2009). Just like in the previous interpretation, Keynes's main point concerns the economy and its financial structure as a macro-system.

But Keynes's thought has been given central prominence also in the analysis of those authors who concentrate on the role played by individual agents. George Akerlof and Robert Shiller (2009), in particular, argue that, animated by "animal spirits," Keynesian agents do not adhere to standard rationality principles and that they reject the maximisation of expected utility. Sophisticated theories of the financial markets that hinge on agents maximizing intertemporal flows of consumption from the holding of assets, are flawed in a Keynesian perspective. Drawing on the growing interest in behavioural interpretations of financial markets, Akerlof and Shiller (2009, p. 3) maintain that Keynes's insistence on "the spontaneous urge to action" which characterizes decision-making in the actual word is the most relevant message conveyed by the *General Theory*. They argue that only an accurate consideration of the effects of individuals' attitude towards phenomena like ambiguity and uncertainty can help understand the causes of the crisis.

On a more general tone, but still emphasizing a decision theoretic perspective, Robert Skidelsky (2009, p. xv) argues that the centrepiece of Keynes's theory is "the existence of inescapable uncertainty about the future." Seemingly endorsing a traditional post Keynesian reading, Skidelsky claims that the relevance of Keynes's thought mostly relies on the acknowledgement that uncertainty cannot be reduced to statistical probabilities. This "irreducible" uncertainty, he argues, has profound implications on the understanding of both human activity in general and the behaviour of individual agents in particular. Uncertainty lies behind panic and bouts of exuberance of economic agents and primarily accounts for the instability of the market economies that we are experiencing nowadays (for a similar viewpoint, see Davidson 2009, and Svetlova and Fiedler 2011).

This paper concentrates on this second main thread of the current Keynesian revival. In particular, starting from a brief summary of Akerlof and Shiller's viewpoint, the paper aims to provide an historically motivated assessment of the claim that Keynesian animal spirits are at the basis of those developments of behavioural finance that explain the failure of conventional models to account for the crisis. Two main points are made in what follows. Firstly, it is argued that a Keynesian explanation of the actual behaviour of individual agents would gain more strength if based on the *Treatise on Probability* rather than on the *General*

Theory alone. Indeed, it is well documented in secondary literature (among others see Lawson 1985, Carabelli 1988, Runde 1994), and recently restated by Skidelsky (2009), that the rationale of Keynes's rejection of "Benthamite calculus" is best provided in the *Treatise.* Secondly, the paper points out that even those authors who focus to the *Treatise* have not provided an account of Keynes's criticism of contemporary probability theory accurate enough to emphasize its positive content and to show the relevance of Keynes even for current decision theory. Drawing on the assessment provided in Basili and Zappia (2009a), this paper concentrates on those elements of Keynes's philosophy of probability that can find "application to conduct" (Keynes 1921, p. 335). This account tries to show that a fresh reading of this part of Keynes's work can contribute to an understanding of how actual individual agents behave under uncertainty, and that the violations of the Bayesian creed scrutinized in certain recent proposals to amend mainstream decision theory (Gilboa 2009), were already implicitly discussed by Keynes in his critique of frequency probability.

It is worth clarifying that this paper does not purport to provide an explanation of the current crisis. This analysis argues nonetheless that modelling individual agents in a stylized, formal analysis of financial markets can be enlightened by a Keynesian viewpoint, and that the understanding of the current financial crisis would gain from an assessment of the principles of decision-making under Keynesian uncertainty as much as from the more conventional one on macroeconomic policy. But it is the *Treatise,* more than the *General Theory*, that provides the rationale for it.

2. Keynesian animal spirits in Akerlof and Shiller

George Akerlof and Robert Shiller (2009) provided an influential analysis of the current crisis. Their understanding of the crisis, and of the inability of mainstream economic theory to predict it, is mostly based on the distinction between the "rational" agents of theoretical models and the "irrational" agents of actual markets. Their emphasis on Keynes is apparent by the title of their recent book, where they are keen to characterize irrationality as "animal spirits." Keynes's *General Theory* is quoted for the discussion of businessmen's calculation. Akerlof and Shiller maintain that Keynes provided the rationale for the crucial question of decision-making in an uncertain environment: decisions are the result of "a spontaneous urge

to action", they are not "the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities" (Keynes 1936, p. 161, reported in Akerlof and Shiller 2009, p. 3).

The connection with current decision theory emphasizing the behavioural roots of individual decisions is made central to Akerlof and Shiller's analysis from the outset: in modern economics, they claim, "*animal spirits* … is an economic term, referring to a restless and inconsistent element in the economy. It refers to our peculiar relationship with ambiguity and uncertainty." The methodological option favoured by Akerlof and Shiller in dealing with this element of "inconsistency" is the one typically followed in recent years by the behavioural approach to economics. Their analysis starts off with an examination of the "deviations [from usual rationality] that actually do occur" as observed in real markets. As a matter of fact, "insofar as animal spirits exist in everyday economy, a description of how the economy really works must consider those animal spirits" (Akerlof and Shiller 2009, p. 5)

Akerlof and Shiller identify five different aspects of animal spirits: confidence, fairness, antisocial behaviour, money illusion, and stories. The connection with Keynes is mostly about confidence as dealt with in Chapter 12 of the *General Theory*. Confidence implies "behaviour that goes beyond a rational approach to decision-making" (Akerlof and Shiller 2009, p. 13). Keynes's main message about animal spirits is that "when people make significant investment decisions, they must depend on confidence." Many of the decisions made are made because people "feel [they are] right." This is of course at odds with standard economic theory that argues that the process of making rational decisions consists of expected utility maximization. Akerlof and Shiller report a number of historical cases in which confidence has played a major role in the actual dynamics of the economy, and stress that even the effect of Keynes's multiplier can be correctly understood only if confidence is taken into account: a policy measure intended to stimulate the economy has greatly different effects under diverse confidence regimes. For instance, during the Great Depression the focus on the necessity to increase the level of nominal wages, with the aim to build up buying power, was misplaced. As Keynes later argued in his *General Theory*, Akerlof and Shiller (2009, p. 69) maintain, "the policy lost sight of the real problem: in the Depression, confidence was so shuttered that banks were holding vast unlent sums, and business did not want to invest in new capital even though interest rates were at abnormal low levels." The general loss in

confidence was the main cause of low demand, and this could not be addressed by a policy focused on nominal wages.

Keynes's understanding of individual behaviour, as a result, is the pivotal element of Akerlof and Shiller's critical analysis of the current mainstream. However, when they indicate how to represent individuals and their animal spirits, Akerlof and Shiller turn their attention to the "irrational" agents of behavioural economics omitting further reference to Keynes's theory. Akerlof and Shiller adhere to a line of thought that in recent years has seen financial modelling to progressively shift from the analysis of rational agents to "noise traders" first (Black 1986), and "biased traders" later (Barberis et al. 2001). Indeed, Shiller is one of the first authors who argued against the efficient market hypothesis in his analysis of the volatility in financial markets (Shiller 1981) thus giving birth to the ensuing empirical literature on financial puzzles. Later, Shiller (2003) endorsed the conclusion that the psychology of decision-making may help explain the behaviour in violation of the Bayesian tenets that is apparent in actual markets, and followed Kahneman and Tversky (1979) in assuming that individuals operating in financial markets tend to overweight recent information and underweight long term tendencies.[1]

Akerlof and Shiller draw on the so-called limited arbitrage critique to the efficient market hypothesis and propose the theoretical strategy of using models that represent investors endowed with a cognitive bias of the Kahneman and Tversky's type, models that have also been used in recent years to explain financial puzzles, over-reaction, herd behaviour and so on (Barberis and Thaler 2003). This is in accordance with the adoption of a broader notion of rationality in the modelling of choice under uncertainty motivated by the huge experimental evidence contradicting rational choice (Camerer 1995). Akerlof and Shiller argue that, in aiming to find a convincing explanation of the crisis, one has to refer to this literature.[2]

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1 As for Akerlof, his main contribution to new Keynesian models, the study of price and wage stickiness as "near-rational" behaviour (Akerlof and Yellen 1985), can be seen as an early attempt to provide those behavioural assumptions able to free traditional Keynesian macroeconomics from ad hockeries, and to give emphasis to the psychological aspect in the *General Theory*, incorporating assumptions deriving from the observation of cognitive biases (Akerlof 2001).

2 On the basis of empirical evidence, the theory of limited arbitrage objects to the efficient market hypothesis that when irrational traders cause deviations from fundamental values, rational traders will often be powerless to do anything about it, since strategies designed to correct mispricings can be both risky and costly, thereby allowing the mispricing to survive. For a survey of these developments see Barberis and Thaler (2003).

Akerlof and Shiller (2009, p. 167) conclude their analysis with a straightforward thesis: "the crisis was not foreseen, and is still not understood … because there have been no principles in conventional economic theories regarding animal spirits." The failure of incorporating animal spirits into economic models has made the economics profession, and key decision-makers, unable to understand the main source of the trouble. This shows, in Akerlof and Shiller's view, in the way actual individuals act. Akerlof and Shiller (2009, p. 174) claim that a "correct" view of how the economy works is instrumental both to "correct individual" and "correct public" decisions. As for Keynes, his critique of standard decision theory is still of major interest since it conveys the main message, but it does not provide valuable analytical tools to move forward. Apparently, thus, Keynes does not know what "correct" individual and public decisions may mean.

3. Keynesian uncertainty

As seen in the previous section, Akerlof and Shiller's discussion of Keynes is mostly based on Chapter 12 of the *General Theory*. Indeed, in his discussion of the "state of long-term expectations" Keynes evidenced that future events conditioning economic activity in general, and investment in particular, "can only be forecasted with more or less confidence." In Keynes's view (1936, p. 148), "the state of confidence, as they term it, is a matter to which practical men always pay the closest and most anxious attention," while economic theory does not. It is this pervading anxiety as to the uncertain environment that makes it important to understand that conventional behaviour may be more relevant than rational behaviour based on "strict mathematical expectation." However, Chapter 12 also contains a crucial reference as to what Keynes meant by "very uncertain," one that Akerlof and Shiller do not discuss, but one that makes Keynes's position more intelligible. In an often-quoted footnote, Keynes (1936, p. 148 n.) argued that "by 'very uncertain' I do not mean the same thing as 'very improbable'" and urged the reader to check his notions of probability and weight of argument in the *Treatise on Probability*. This reference cannot be overlooked as Keynes repeated it in Chapter 17 of the *General Theory*, when discussing the notion of liquidity premium, whose meaning is strictly dependent on the state of confidence in the economy. Here the difference between risk premium and liquidity premium is said to correspond to "the difference between

the best estimate we can make of probabilities and the confidence with which we make them" (Keynes 1936, p. 240).[3]

As it is well-known there exists a long-standing tradition of Keynesian scholars whose assessment provides a thorough examination of the role played by uncertainty in Keynes's thought. Reference to this tradition helps frame the relevance of the question to the current crisis in a more comprehensive historical context than that of Akerlof and Shiller. Akerlof and Shiller's viewpoint that the psychological motivation of individual agents is of crucial, often disregarded importance to the understanding of the crisis is consistent with this tradition. But, in aiming to provide a Keynesian assessment of the crisis, one cannot avoid but deal with the issue of the irreducibility of uncertainty to risk. This crucial aspect of what is usually termed a fundamentalist viewpoint among Keynesian scholars is well summarized in Robert Skidelsky's (2009) recent book on Keynes.

Considering that the current financial crisis was not caused by some external shock, but generated by the financial system itself, an analysis of the endogenous mechanics of the economy is necessary. But this analysis, Skidelsky argues, must take into account those aspects of individual behaviour that are unintelligible in mainstream economics. The study of the influence of the individuals' animal spirits is central to the theory Keynes provided in the 1930s and turns out to be as accurate nowadays as it was in his days. Then as now, Skidelsky contends, economists believed that all uncertainty could be reduced to measurable risk, implying that asset prices always reflected fundamentals, and unregulated markets would in general be very stable.[4]

By contrast, Skidelsky (2009, p. xviii) recalls that "it is 'radical uncertainty' which both makes economies unstable and prevents rapid recovery from 'shocks'." The starting point of Keynesian economics is that not all future events can be reduced to measurable risk.

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3   A further textual piece of evidence supporting this link is provided by Keynes's correspondence with Hugh Townshend (Keynes 1938).

4   Skidelsky's assessment of the crisis starts off from Alan Greenspan's admission that the market regime he oversaw as chairman of the Federal Reserve was deregulated on the basis of an over-esteem of the ability of a free market to self-correct. Greespan's mis-appreciation of the self-destructive power of deregulated mortgage lending appears to Skidelsky (2009, pp. 3-4) the same as the reliance on an "intellectual edifice" that turn out to be flawed, in Greenspan's case the efficient market hypothesis. The simple story that since markets are efficient in pricing assets correctly they would need only the lightest regulation, mostly the received view in mainstream financial modelling, must have induced Greenspan to support extensive financial deregulation and concentrate on keeping interest rates low in the period when the housing bubble was growing. "This was the intellectual edifice, of both theory and policy, which has just been blown sky high," Skidelski (2008) concludes.

As a matter of fact, Skidelsky (2008) argues, any view of the future based on what Keynes called "so flimsy a foundation" is liable to "sudden and violent changes" when the news changes. Investment is sometimes more an act of faith than a scientific calculation of probabilities, and when investors do not know which information is relevant, they do not process new information efficiently, and end up relying on habits. In these instances people may fall back on "conventions," which give them assurance that they are doing the right thing. Individuals make an assumption that the future will be like the past and that current prices correctly sum up "future prospects." In Keynes's (1937, pp. 214-215) words, "to behave in a manner which saves our faces as rational, economic men … we have devised for the purpose a variety of techniques," but "[a]ll these pretty, polite techniques, made for a well-panelled Board Room and a nicely regulated market, are liable to collapse." Hence, conventional behaviour easily turns into herd behaviour, and financial markets are pervaded by alternating currents of euphoria and panic. As regards the understanding of the crisis, Skidelsky (2009, p. 50) disputes that it is quite simple to guess how Keynes would have understood it: "He might not have predicted that the financial collapse would occur when it did … but he would certainly have thought a financial collapse possible, and even likely." Keynes would have pointed out that people in the market were using the wrong model, just like the governments that were monitoring market trends.

Skidelsky can be considered a most prominent figure among the Keynesian scholars who relatively downplay the macro-aspects connected to the mechanics of depression and put major emphasis on the role of uncertainty. Skidelsky places Chapter 12 of the *General Theory* at the centre of his analysis, but correctly adds that the thread that goes from it to Chapter 17 and Keynes's 1937 recapitulation essay constitutes the most enduring aspect of the Keynesian inheritance. Also, Skidelsky (2009, pp. 84-88) makes it clear that this thread had its origins in the *Treatise on Probability*, showing that Keynes focused on the question of how individuals act in the face of uncertainty since his early philosophical studies.

As one would expect from his previous works, Skidelsky provides a masterly assessment of Keynes's thought. As far as we are concerned here, however, Skidelsky's conclusion does not differ much from Akerlof and Shiller's. To be sure, Skidelsky is sceptical about the behavioural economics programme. He (2009, p. 46) claims that, in relying on "human behaviour irrationality," behavioural economics leaves the "epistemological source of this irrationality unexplored." Even if it differs from the New Keynesian explanation, which

typically concentrates on informational problems in an otherwise rational context – an approach repelled by Skidelsky[5] – behavioural economics does not get to the heart of the matter, since it omits considering "the influence of irreducible uncertainty on behaviour."[6] But the outgrowth of Skidelsky's analysis is that no formal modelling of the behaviour of individual agents was suggested by Keynes: indeed, on Skidelsky's viewpoint no formal model can be provided since uncertainty is irreducible on ontological grounds, and some probabilities are not just unknown, but "non-existent" (Skidelsky 2011, p. 3). Apart from the reference to unqualified conventional habits and to the fact that one does not need to invoke "irrationality" to explain conventional behaviour, then, Skidelsky's insistence on the importance of individual behaviour under uncertainty is not substantiated by a specific indication of how Keynesian agents should behave in the market. In this, Skidelsky's assessment is similar to Akerlof and Shiller's. Skidelsky, however, is well aware that the philosophical background of Keynes's understanding of decision-making pertains to his theory of probability. Therefore a closer inspection of the *Treatise* is worth making.

4. Keynes's theory of probability and its application to conduct

The *Treatise on Probability* provides an essential starting point to an assessment of Keynes's understanding of decision-making. In the *Treatise* Keynes specifically questioned what contemporary frequency probability theory could encompass, and put forward an alternative, epistemic notion of probability. But retrospectively assessed his analysis has a much wider significance, since Keynes's critical remarks in the *Treatise* constitute a challenge to any theory of probability which, like frequency probability, is based on a unique additive distribution. Similarly to some current criticism of the Bayesian mainstream, Keynes came to reject the idea that probability functions are always well-defined.[7] This section aims to show

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5 The distinction between "new" and "post" Keynesian models mostly hinges on the rejection by post Keynesians of a methodological approach favouring micro-foundations. The idea of providing microeconomic foundations to Keynesian macroeconomics through the identification of limited, specific deviations from a consolidated mainstream has been disparaged by post Keynesian authors (Davidson 2009). See also Skidelsky (2011, p. 5).

6 Akerlof and Shiller's behaviourally oriented interpretation of Keynes has been the subject of critical scrutinies among post Keynesian scholars. For instance, see Carabelli and Cedrini (2011) and Dow and Dow (2011).

7 As is well-know, these aspects were of concern to Shackle (1949) and the Keynesian scholars who have stressed the role of "fundamental uncertainty" in decision-making (Davidson 1982-83).

that Keynes's critique of frequency probability has a positive content that is usually disregarded among commentators. In particular, as argued in Basili and Zappia (2009a), Keynes's view of the potential incomparability of alternative probability assessments hints at the crucial notion of decision weights that are not usual probabilities, but distorsions of probability measures. Also, Keynes's application of the theory of probability to "human conduct" goes further than a critique of maximisation, and discusses the need for every sensible decision rule under uncertainty to incorporate a measure of the degree of confidence in the probability assessment.

Keynes's theory provides an interpretation of probability different from chance or frequency. Probability is seen as a property of the way individuals think about the world. In Keynes's view (1921, p. 109), probability should not be identified with statistical frequency, as the theory of probability concerns the broader issue of inferring degrees of belief from the available evidence, whereas relative frequencies are a special kind of evidence. Probability, Keynes argued, "is concerned with arguments, that is to say, with the 'bearing' of one set of propositions upon another set." The subject matter of the theory of probability, therefore, is the logical relation of implication between a certain evidence and a conclusion, "a relation, in virtue of which, if we know the first, we can attach to the latter some degree of rational belief" (Keynes 1921, pp. 6-7).[8]

Shackle's non-probabilistic theory of decision under uncertainty contributed much to the claim that a formal alternative to mainstream decision theory could be devised following Keynes. However, it has passed unnoticed for long that Ellsberg (1961) held similar concerns in his critique of Savage's axiomatisation of the Bayesian viewpoint. Since most current efforts to understand individual decisions are motivated by the experimental confirmation of Ellsberg's critique, the link between Keynes, Shackle and Ellsberg is mentioned in what follows (for a more detailed analysis of this link, see Basili and Zappia 2010).

8   Depending on the knowledge on which it is based probability may appear subjective, Keynes (1921, p. 4) maintained, but "the theory of probability is logical … because is concerned with the degree of belief which is *rational* to entertain in given conditions, and not merely with the actual beliefs of particular individuals." The *Treatise*, therefore, shows how to derive knowledge from probability arguments, and the goal of the "logical" approach is to identify the principles of inductive rationality leading different individuals sharing the same evidence to agree on definite probability judgements. This logical perspective met with strong resistance after the emergence of the subjectivist-personalist approach and became a minority viewpoint among philosophical theories of probability. As is well known, while advocating an epistemic approach to probability, neither Ramsey (1931) nor de Finetti (1937) followed Keynes in regarding probability as a purely logical relation. As a matter of fact, Keynes's (1931) reaction to Ramsey's critique of his logical viewpoint is seen by most critics as showing a retreat form the position upheld in the Treatise (for instance, see Bateman 1996). But see Runde (1994) who argues that excerpts from the *General Theory* provide evidence to the contrary.

In the *Treatise* Keynes did not limit himself to a philosophical inquiry, but tried to develop his own formal logic of probability. These formal developments did not get attention among decision theorists after the subjectivist approach shifted emphasis towards consistency as an evaluation criterion for probabilistic axiomatic systems, and remained part of a minority viewpoint (Kyburg and Smokler 1964). But the rationale of Keynes's attempt is of great interest nonetheless for at least three reasons. Firstly, Keynes rejected the idea that probabilities can always be represented through real numbers, and pointed out the limited degree to which probability can be measured. Secondly, Keynes introduced the notion of weight of argument: even when considering a probability measure, the degree of completeness of information is a crucial factor in the subjective assessment of an uncertain environment. Thirdly, he discussed the issue of what kind of decision rule proves to be consistent with his critiques of frequency probability. In what follows, these aspects are briefly examined.

Keynes's rejection of the numerical character of probability was put forward in the first part of the *Treatise*, introducing his "fundamental ideas" on probability. Keynes (1921, p. 21) argued against the generally accepted opinion that "a numerical comparison between the degrees of any pair of probabilities is not only conceivable but it is actually within our power." Being critical of the frequentist viewpoint that the numerical character of probability is necessarily involved in the definition of probability as the ratio between "favourable cases" and the "total number of cases," he analyzed various instances of ordinary life in which "no rational bases have been discovered for numerical comparison" (Keynes 1921, p. 23). Only in "very special case" where the principle of indifference – better known as Laplace's principle of insufficient reason – can be applied, Keynes (1921, p. 32) argued, "a meaning can be given to a *numerical* comparison of magnitude." Keynes conceded that, probability being an intermediate stage between certainty and impossibility, when one argues that one probability is "greater" than another, "this precisely means that the degree of our rational belief in the first case lies *between* certainty and the degree of the rational belief in the second case" (Keynes 1921, p. 37). He also stressed that the probabilities of two quite different arguments can be impossible to compare. Probabilities can be compared if they belong to the same "ordered series," that is, if they "belong to a single set of magnitude measurable in term of a common unit." But there may be more than one "path" from certainty to impossibility and probabilities cannot be compared if they belong to two different paths. Also, when there is

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more than one path probabilities can still be placed into order if they follow the same path, but cannot be numerically measurable.[9]

It is apparent that the issue of "non-numerical" probabilities is reminiscent of the Knightian distinction between risk and uncertainty, with the latter meant to describe situations in which information cannot be summarized by probabilities. That Keynesian uncertainty and Knightian uncertainty overlap on various points has been argued by several authors (in particular see O'Donnell 1989, and Runde 2001). Keynes himself gave support to this viewpoint in the discussion of uncertainty in his 1937 summary of the *General Theory*, an aspect emphasized by those Keynesian scholars who – following Shackle (1967) – claim that "for Keynes uncertainty is an absence of probabilistic reasoning" (Hillard 1992, p. 69). However the Knightian case of unmeasurable uncertainty can be seen as a limit case in Keynes's taxonomy of probabilistic cases: indeed, Keynes's main aim in the *Treatise* was to provide a formal structure for comparisons between probability relations even when the standard approach to probability fails. This was presented in the second part of the book, intended to state the "fundamental theorems" of his theory. Keynes stressed the inductive process of deriving new probability comparisons on the basis of other comparisons constituting direct knowledge, and tried to establish the conditions for the emergence of an ordering of probabilities, well aware that the incompleteness of the probability relation induces only a partial order. Further, he attempted to account for a numerical measure of a relation of probability through the method of "numerical approximation," that he described as "the relating of probabilities, which are not themselves numerical, to probabilities, which are numerical."[10]

The interpretative point here is that Keynes clearly stated that his concern with probabilities that are not numerical could be given theoretical content. Probabilistic weights like the ones represented by non-linear paths from impossibility to certainty can be

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9 Keynes's discussion was summarized in a diagram, featuring different probabilistic paths. A linear path accounts for the usual probabilistic representation, ranging from impossibility to certainty, but other different non-linear paths between the extremes, that do not lie on the straight line, are drawn. These paths represent what Keynes (1921, p. 42) called a "non-numerical probability" or a "numerically undetermined probability." Only probabilities lying on the same path, or on paths that have points in common, can be compared among themselves, but "the legitimacy of such comparison must be a matter for special inquiry in each case" (Keynes 1921, p. 40).

10 Keynes (1921, p. 176) maintained that "many probabilities, which are incapable of numerical measurement, can be placed nevertheless *between* numerical limits. And by taking particular non-numerical probabilities as standards a great number of numerical comparisons or approximate measurements become possible."

approximated through the assigning of intervals of probabilities. Indeed, Keynes pointed to inexact numerical comparison rather than simply to the impossibility of attributing cardinal numbers and deriving probability comparisons: this approach has a precise formal meaning that survived the decline of interest in logical probability thanks to the works of a number of authors adhering to the subjectivist tradition, but critical of its strictly Bayesian version.[11] In the secondary literature on the *Treatise* only a few authors discussed the issue of "non-numerical" probabilities with the aim to analyze which kind of numerical comparisons Keynes had in mind (Brady 1993, Runde 1994, Kyburg 1995), while the standard interpretation of the *Treatise* has treated the philosophical foundations of Keynes's characterization of uncertainty as entailing an epistemic state of individuals in which, using Keynes's (1937, p. 214) words, "we simply do not know." As seen earlier, Skidelsky follows in this tradition.[12]

The traditional reading of the *Treatise* also misses out a related technical aspect. The Keynesian paths describing non-numerical probabilities closely resemble what nowadays would be identified with decision weights, that is, distorsions of an usual probability measure. In modern decision theory this issue has come to the fore after Kahneman and Tversky's (1979) introduction of prospect theory. As it is well-known, in order to justify individuals' unwillingness to use objective probabilities as a basis for decisions under risk, as in the experimental evidence concerning the Allais Paradox, Kahneman and Tversky represented the perception of probabilities through probability weighting functions. It has thus become common to use weighting functions to represent decision-makers who over-weight low probabilities and under-weight high probabilities, a pattern of behaviour regularly observed in actual decision-making under both risk and uncertainty (Wu and Gonzales 1999). Behavioural

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11 Keynes's attempt was taken up by Koopman (1940), Good (1950) and Smith (1961), who built an axiomatic system based on the intuition that probabilities usually provides only a partial ordering, and introduced the notion of upper and lower probabilities. For a restatement of the so-called imprecise probability approach see Walley (1991).

12 Shackle had a somewhat peculiar position on this interpretative issue. On the one hand, he did not refer to Keynes's analysis of non-numerical probabilities, arguably since it originates from the aim to present a logic of probabilistic he rejected. As a result Shackle (1967) focused on Keynes's 1937 insistence on radical, irreducible uncertainty. On the other hand, he never stopped arguing in favour of an axiomatic representation of degrees of belief through a "potential surprise" function, and of the need for a (non-probabilistic) formal model of decision-making under uncertainty. Still in the late *Imagination and the Nature of Choice* (Shackle 1979) his 1949 axiom-system is reproduced in a chapter presenting the formal content of his theory. Basili and Zappia (2009b) claim that Shackle's model anticipates on many grounds the modern non-additive approach to decision-making.

theories of financial markets, like those referred to by Akerlof and Shiller, have argued in recent years that most of what is left unexplained by the theory of efficient markets can be understood by assuming that individual agents deviate from the rationality of subjective expected utility in the specific way proposed by prospect theory and the ensuing developments. On the viewpoint that non-numerical probabilities are a sort of decision weights, it can be maintained that most recent developments have a Keynesian flavour that have been substantially overlooked (Basili and Zappia 2009a).

The second fundamental aspect of Keynes's rationale for criticizing frequency probability was the weight of argument. Keynes emphasized that the uncertainty surrounding the individual cannot be represented only through probability: the confidence in the probability assessment itself is another relevant dimension in the epistemic state of the individual. In Keynes's view the measurement of probabilities should encompass both the magnitude of the probability of an argument and the degree of confidence in it. Probability arguments, he stated (1921, p. 77), depends not only upon the balance between "favourable" and "unfavourable" evidence, but also upon the balance between "the *absolute* amounts of relevant knowledge and of relevant ignorance respectively." Keynes (1921, p. 82) exemplified the way in which the standing of a probability assessment depends on the information on which the assessment is based through the example of coloured balls drawn from urns, the same one later made known in the economics literature by Ellsberg (1961). Keynes claimed that, by virtue of the principle of indifference, the probability of drawing a white ball from an urn known to contain black and white balls in equal proportion is equal to the probability of drawing a white ball from an urn containing an unknown proportion of white and black balls. But he argued nonetheless that, in the first case, a greater weight supports the argument that the probability is ½.[13]

In order to make clear that probability and weight are "independent properties" Keynes (1921, p. 345) added that the weight could be thought as "the degree of completeness of the information upon which a probability is based."[14] The intent was to specify that the

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13 Ellsberg later suggested that decisions contemplating the "unknown" urn can be rationalized as if the probability of drawing a white ball is less than ½. Since the same holds for the drawing of a black ball, either probabilities are non-additive or the decision-maker is using an interval of probability values (Ellsberg 1961, pp. 650)

14 As $p(a/h)$ is the probability of some proposition a, on the basis of the available evidence h, the weight of a certain proposition a given the available evidence h is $w(a/h)$. Following Runde (1990), by using K to denote knowledge and I to denote ignorance, the weight as degree of completeness of information is: $w(a/h)=K/(K+I)$. If $K+I$ is normalized to the unity, w ranges from 0 to 1, as suggested by Keynes in Chapter 26 of the *Treatise*.

weight of argument is not a second order probability. Keynesian scholars reading the *Treatise* in the late 1980s, stressed Keynes's claim that probability and weight are independent properties, and that the weight is intended to provide a deeper explanation of the way individuals assess uncertain phenomena (Lawson 1985; Carabelli 1988; O'Donnell 1989). As recalled above, Keynes himself provided an example of the distinction in his analysis of liquidity preference in the *General Theory* (1936, p. 148 fn., and p. 240). Skidelsky (2009, p. 88) signals its significance for the interpretation of individual behaviour in the markets when claiming: "the greater the amount of evidence supporting an expectation, the more confident we will be in having it."

The distinction between a probability assessment and the degree of confidence in it has no place in a strictly Bayesian set-up, where the decision-maker chooses as if she was guided by precise numerical probabilities of the consequences of her action, and confidence is considered at most a probability distribution over the probability distribution on the set of events, and an axiom of compound probabilities is used (Borch 1968). As a result this distinction has been substantially ignored in mainstream decision theory. However some recent developments endorsing a less strictly Bayesian viewpoint attribute a fundamental role to what Keynes termed weight of argument. This is apparent in the increase of literature on the so-called Ellsberg Paradox. In his study of agents deliberately violating the axioms of Bayesian rationality, Ellsberg (1961, p. 657) remarked that the nature of the individual's information concerning the likelihood of events is a relevant dimension of the decision problem, and proposed to call it the ambiguity of information, "a quality depending on the amount, type, reliability and 'unanimity' of information" expressing the individual's "degree of confidence in an estimate of relative likelihoods." Ambiguity, he argued, can be considered a special case of uncertainty. Following Ellsberg's definition of ambiguity, Levi (1974) and Gärdenfors and Sahlin (1982) introduced the notions of "credal states" and "epistemic reliability" of a probability assessment, and gave birth to a literature on representations of the epistemic state of individuals alternative to subjective expected utility.

The philosophical inspiration of Ellsberg's critique of the Bayesian viewpoint is to be found in Keynes. Ellsberg (2001 [1962], p. 9-13) recognized that the notion of weight of argument is "closely related" to his notion of ambiguity and noted that in situations where information is perceived to be vague the traditional approach to probability is inadequate. Keynes's discussion of probabilities become pivotal since Keynes introduced "formally the

notion of *non-comparability* of beliefs."[15] It is worth noting that all modern current developments of decision theory under uncertainty originated as attempts to account for Ellsberg's ambiguity (Wakker 2008).[16] Ellsberg's analysis and the ensuing formal developments are thus squarely in the Keynesian tradition when they introduce a factor intended to represent the relative ignorance of the individual evaluating an act in ambiguous contexts, that is, his/her confidence in the subjectively held probability measure. Moreover the cognitive unease implicit in both Keynes's advocacy of the weight and Ellsberg's insistence on ambiguity is recognized in current decision theory to such an extent that the literature introducing non-additive probability measures aims to offer a formal solution for the representation of the weight (Kelsey 1994, and Vercelli 1999).[17]

The third, and last aspect of Keynes's rationale for criticizing frequency probability relates to his rejection of the use of mathematical expectation as a criterion for making decisions. As seen above, Keynes's critique of what in his later work in economics he labelled "Benthamite calculus" is widely referred to in the literature on the current crisis, specifically in both Akerlof and Shiller and Skidelsky. It is to this aspect Keynes devoted his investigation of "the application of probability to conduct" in Chapter 26 of the *Treatise*. Here Keynes dealt with the interpretation of "goodness" of choice when "it is not rational for us to believe that the probable is true." Keynes (1921, p. 343) recalled that "normal ethical theory at the present day makes two assumptions: first, that degrees of goodness are numerically measurable and arithmetically additive, and second, that degrees of probability also are numerically measurable." As a result, ethical theory suggested deciding among alternative acts on the basis of their mathematical expectations, which Keynes presented as "a technical expression

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15 Ellsberg did not quote Keynes's *Treatise on Probability* in the 1961 article introducing the paradox of choice, seemingly because he did not read it until working on the philosophical background of his own viewpoint in his Ph. D. thesis, submitted to the Harvard Department of Economics in 1962 (Ellsberg 2011). The thesis remained unpublished and substantially ignored until its publication in 2001.

16 Since Einhorn and Hogart's (1986) assessment of Ellsberg, ambiguity and uncertainty have become almost synonyms in decision theory literature.

17 Following on the experimental evidence on Ellsberg Paradox, the choice of individuals taking decisions in ambiguous contexts is interpreted as showing that they are endowed either with subjectively non-additive probabilities, as in rank dependent expected utility (Quiggin 1982) and Choquet expected utility (Schmeidler 1989), or with a set of additive probabilities, as in maxmin expected utility (Gilboa and Schmeidler 1989). When the decision-maker is not endowed with a unique additive probability measure the degree of ambiguity the decision-maker takes into account can be represented through either the non-additive probabilistic weights or the multiple priors. Gilboa and Schmeidler stated the conditions under which there is an homeomorphism between non-additive probability measures and interval-valued probabilities (Gilboa 2009).

originally derived from the scientific study of gambling and games of chance, … [that] stands for the product of the possible gain with the probability of attaining it."[18] Of course, Keynes (1921, p. 344) disagreed with a generalized application of mathematical expectation, since assuming that "degrees of probability are wholly subject to the law of arithmetic, runs directly counter to the view which has been advocated in part I [of the *Treatise*]." In Keynes's view, "mathematical expectations, of goods or advantage, are not always numerically measurable, and hence even if a meaning can be given to the sum of a series of non-numerical mathematical expectations, not every pair of such sums are numerically comparable in respect of more and less." Also, Keynes contended that mathematical expectation cannot be used in actual conduct since it "ignores what I have termed the weights of arguments, namely the amount of evidence upon which each probability is founded." Therefore Keynes (1921, p. 344-345) concluded that "it is not always possible by a mere process of arithmetic to determine which of the alternative ought be chosen."

Keynes (1921, p. 349) stated that an alternative to the notion of mathematical expectations does not lie, in principle, "in the discovery of some more complicated function of the probability wherewith to compound the proposed good." However, even in this case, he made an effort to analyze constructively. In order to move forward in the search for a decision rule, Keynes argued, probability and weight should be compounded into a coefficient to be used in the shaping of a normative theory of decision making, and he attempted to show how a weight of argument lower than a given maximum would possibly influence decision-making.[19] As noted by Brady (1993), Keynes's coefficient incorporates the properties of a probability weighting function, like Kahneman and Tversky's decision weights, and can thus be used to provide a decision criterion that solves the Ellsberg Paradox (see also Arthmar and

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18 Keynes discussion is based on George E. Moore's analysis of which are the appropriate behavioural rules to be used in ethics. He objected to Moore's contention that mathematical expectation was the appropriate behavioural rule in ethics. On the relationship between Moore and Keynes see Bateman (1989) and Raffaelli (2006).

19 Keynes introduced the following coefficient: $c=2pw/(1+q)(1+w)$, where p is the probability of an event, $q=1-p$ the probability of its complement, and w is the weight, ranging from 0 to 1. Keynes then argued that, in making a decision concerning a possible "amount of good" A which can be expected with probability p, the standard mathematical expectation $E=pA$ should be disposed of in favour of an alternative criterion for choice such as $E'=cA$. Keynes provided also a short discussion of how, for different values of w and p, E' would help order different goods A implying the same E (Keynes 1921, p. 348fn). As Keynes did not use utility values but monetary amounts, in his discussion the coefficient $c=p/1+q$ is intended to take into account the "risk" implicit in taking a decision based on E. Except for the taking of risk into account, when the weight of argument is at its maximum there is no other need to modify E as a decision rule. But when lower than 1, the weight must be taken into account as well.

Brady 2010). It can be concluded then that even with respect to this third aspect of Keynes's probability theory the rationale of Keynes's criticism presents strict similarities with a much favoured theme in current decision theory, that is, Ellsberg's rejection of expected utility maximization.[20]

5. Keynesian decision-making under uncertainty

The previous section pointed out that Keynes's analysis of individual behaviour in the *General Theory* finds a consistent philosophical background in the treatment of probability in the *Treatise.* This background is instrumental in enlightening the issue of how people behave in condition of uncertainty, as Keynes himself stated when he referred to it in the two crucial passages of the *General Theory* mentioned in section 3 above. Under uncertainty, Keynesian decision-makers formulate subjective assessments of the external world that can take the form of "non-numerical" probabilities, that is, probabilities that are not standard additive probability functions. Keynes underlined the need to conceive these non-numerical probabilities in a consistent qualitative, if not quantitative, order, and suggested the method of approximation through intervals of probabilities, when possible. With the benefit of insight, non-numerical probabilities can be interpreted as probabilistic weights, that is, as "subjective" distortions of an "objective" probability function that either cannot be known or is not known due to the ambiguity of the environment perceived by individual agents.[21]

      This framework entails the rejection of the ethical criterion for choice suggested by Moore, and later endorsed in economics through the application of "Benthamite calculus" to

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20 Ellsberg contended that a decision-maker who fails to pick a single distribution out of a set as acceptable, may nevertheless regard one of them as the most reliable, and use it to ponder a stricly conservative criterion like Wald's (1945) maximin. Accordingly the decision rule adopted by Ellsberg (1961, p. 664) was to associate with each act *x* the index $\rho E(x)+(1-\rho)min(x)$, and then choose the act associated with the maximum value of the index. It is apparent that that the parameter ρ can be interpreted as Keynes's confidence in the probability assessment as represented by the weight *w*. When $\rho=1$, confidence is at its maximum and the expected value is a sensible criterion for choice in much the same way as when *w*=1. When $\rho<1$, the lower the confidence value the less the expected value can be sensibly used. The current literature on decision-making has shown that criteria of this kind can be axiomatized by means of a specific class of non-additive probability measures (Eichberger and Kelsey 2007).

21 In certain contexts this ambiguity can be solved by the passing of time, or the replication of an experiment, like in the urn example. However, Keynes seems to suggest that since decision-makers may find difficult to establish how complete their information is, different propensities to act will emerge in general (Feduzi 2010).

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decision under uncertainty. However, this is a prelude to the analysis of alternative criteria Keynes hinted at. Keynesian decision-makers evaluate prospective investments using the maximisation of expected values only when the weight of argument is at its maximum, something Keynes considered the exception rather than the rule. A low weight is associated with situations in which the state of confidence worries the decision-maker to the point of feeling unsure when a probability distribution can be reliable for calculus. In this instance the decision-maker typically shows both risk aversion and uncertainty aversion: while the former can be represented as usual by the shape of the utility function,[22] the latter is caught by a probability measure that is distorted in relation to the weight: the lower its value the more significant the distortion. Or else, when the weight is low, a set of probability functions are deemed to represent the individual's understanding of the uncertain environment. In both cases, the order of acts does not conform to a mathematical expectation, and a more conservative decision rule would be selected.[23] To sum up: it is "Benthamite calculus" that cannot be applied to uncertainty, not calculus iself.[24]

Notably, Keynes suggested a philosophy of decision-making that, after Ellsberg and the related experimental evidence (Camerer 1995), has emerged extensively in an increasingly relevant part of modern decision theory which criticizes the mainstream Bayesian set-up. Decision criteria devised to account for unknown probabilities (such as Wald's maximin), or for probabilities with a low degree of reliability (such as Ellsberg's and Hurwicz's α-maximin), or for multiple probabilities (such as Gilboa and Schmeidler's maximin expected utility), can all be considered different but related ways to account for the behaviour of agents in Keynesian settings. Most of these criteria can be viewed as instances of a generalized procedure consisting in maximizing an expected value where probabilities are non-additive like in Choquet expected utility models (Wakker 2008). The study of this class of what Keynes would term "non-Benthamite" decision rules – which are defined "non-Bayesian" in an increasingly influential current literature (Gilboa, Postlewaite and Schmeidler 2008 and

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22 As already noted in footnote 18, this was represented by Keynes via the "risk" associated with trying to get a profit when taking part in a project that cannot yield it for sure, thus possibly originating a loss.

23 When ambiguous beliefs are represented by a set of probability functions, the expected payoff is measured with respect to more than one probability distribution, and the ambiguity averse decision-maker evaluates an act by the minimum expected value that can be associated with it: the non-Bayesian decision rule requires to compute all possible expected values for each action and then choose the act which has the best minimum expected outcome (Gilboa and Schmeidler 1989).

24 Carabelli (2002) suggests a similar conclusion on the grounds of Keynes's 1910 lectures on speculation.

2012) – has been mostly motivated on axiomatic grounds, since the aim was to account for what is perceived as a normative violation of Savage's axioms.[25] But these decision rules have also been used to rationalize the behavioural theories proposed to account for experimental evidence.[26]

The discussion of the *Treatise* provided in the previous section suggests an assessment of Keynes's (1937, p. 214) famous statement that on some matters – when "there is no scientific basis on which to form any calculable probability whatever" – "we simply do not know," that is alternative to the post Keynesian one Skidelsky hinges on. What individuals do not know can be interpreted as an array of differently characterized situations entailing different degrees of confidence, all of which can be examined without denying the fruitfulness of the Keynesian viewpoint. In uncertain settings, Keynesian agents are not forced to rely on "conventions" or "useful mental habits" as if complete ignorance would represent the entire spectrum of economic activity. Crucially, just after warning the reader that when uncertainty prevails, there is the tendency "to behave exactly as we should if we had behind us a good Benthamite calculation," Keynes (1937, p. 215) concluded: "Tho[ugh] this is how we behave in the market place, the theory we devise in the study of how we behave in the market place should not itself submit to market-place idols." Therefore, in trying to focus on a positive analysis of decision-making under uncertainty, one is simply adhering to Keynes's long-standing commitment to interpret reasonable judgement in a, surely unconventional, probabilistic set-up.

The assessment so far can also help shed light on Keynes's attitute towards two issues that are central in the analysis of the current crisis: financial decision-making and policy-making. The main message of the decision rules devised to deal with an individual's aversion to ambiguity is that a conservative attitude, one that rejects making decision on the basis of subjective expected utility maximization, cannot be deemed "irrational." When applied to financial markets, this theoretical set-up can be used to show that there exists a price interval at which an individual may prefer to take a zero position in risky assets (Dow and Werlang

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25 In his seminal paper introducing Choquet expected utility, Schmeidler (1989) motivates his rejection of Savage's axioms on the grounds that he, as a theoretician, experienced a cognitive unease when examining choice situations of the (Keynes-)Ellsberg type with Savage's lens. No reference to the experimental evidence is made to justify his enquiry. See also Gilboa (2009, p. 136)

26 The literature insisting on behavioural explanations of market choices, though mostly aiming to account for descriptive "anomalies," has been re-interpreted as an application of these decision rules, as in cumulative prospect theory (Tversky and Kahneman 1992).

1992), and that the equilibrium allocation is not Pareto optimal, just like in models with incomplete markets (Mukerji and Tallon 2001). Investors that are averse to ambiguity are shown to be inclined to choose actions whose consequences are more robust to the perceived ambiguity, that is, they hold a portfolio whose value is relatively less affected by the uncertainty about the probability distribution governing future payoffs. Also, if ambiguity emerges as a result of a feeling by the individual investor that her competence in assessing the relevant probabilities is low – and in comparative situations in general (Fox and Tversky 1995) – it can been shown that the structure of the portfolio does not conform to Markowitz's principle of diversification of risks, since it depends on the different degrees of ambiguity across assets, so that the optimal portfolio may contain only the assets in which the investor feels more confident, i.e. those she perceives as less ambiguous (Boyle et al. 2011).[27]

This interpretation of how "irrational" individuals act in financial markets relates well to Keynes's own record as an investor. As it is well-known even in the informed press after the endorsement by an investment giant like Warren Buffet, Keynes's own investment strategy as Bursar of King's College, Cambridge, and a director of the Provincial Insurance Company, did not conform to the practice of diversification of risk. After persuading his college fellows to increase the proportion of funds available for a discretionary strategy, he started investing heavily in equities, and within a few years of activity he came to follow a strategy summarised in a famous letter to F. C. Scott, managing director of the Provincial: "As time goes on I get more and more convinced that the right method in investment is to put fairly large sums into enterprises which one thinks one knows something about and in the management of which one thoroughly believes. It is a mistake to think that one limits one's risk by spreading too much between enterprises about which one knows little and has no reason for special confidence." (Keynes 1934, p. 57). He constructed an highly idiosyncratic portfolio with pronounced size and value tilts that, from the early 1930s on, makes it possible for him to outperform the market on a consistent basis (Chambers and Dimson 2012).

The quest for confidence in the probability assessment also emerges in Keynes's understanding of how government should act in the face of uncertainty. Indeed, the analysis of decision under uncertainty at the individual level can be productively applied to policy decisions. For instance, the policy-maker's decision to intervene in order to regulate the market when a certain information is available can be examined in the following way. Let us

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27 On the relationship between Markowitz and the ensuing capital assets pricing model and subjective expected utility, see Zappia (2012).

assume that a governmental authority, concerned with the potential losses of a systemic crisis, calculates the expected utility of the decision to intervene and check exuberant financial markets, possibly because actual values do not seem to relate to fundamentals. It may well happen that, estimating a "very low," maybe even potentially "unknown," probability of the realisation of the admittedly rare event of the systemic crisis, the expected loss associated with this unfavourable event would be low, even in the face of potentially high losses. The option of inaction, i.e. not to check the bubble, may be considered a preferred choice in view of the expected utility of leaving business to continue as usual, since the expected value of the gains experienced by financial markets associated with it outweighs the expected value of the potential losses calculated on the basis of this "very low" probability. But this deliberate choice, made on the basis of a criterion apt to face familiar events, may well turn out to be difficult to justify when compared to the, seemingly "irrational," precautionary option of avoiding the potentially enormously high losses associated with the catastrophic event. If one looks at the current crisis with the benefit of insight, a choice favouring a prudential attitude like of an intervention intended to check the bubble before it eventually burst, would have been advisable. This is a main element in many Keynesian inspired analyses, including those of Akerlof, Shiller and Skidelsky.

This is a point that conforms to Keynes's philosophy of decision to such an extent that he (1921, p. 344) made it explicitly in the *Treatise* when contending that "the doctrine that the 'mathematical expectations' of alternative courses of action are the proper measures of our degrees of preference is open to doubt … because it ignores the element of 'risk' and assumes that an even chance of heaven or hell is precisely as much to be desired as the certain attainment of a state of mediocrity." Keynes's argument cannot be interpreted simply as an allusion to risk aversion, or to the variance of the expected outcomes, as it preludes to the analysis of the coefficient compounding probability and weight. A more persuading interpretation is that a conservative attitude in making decisions is fundamental when the numbers at stake are huge and the probabilities are "unknown". Here Skidelsky's intellectual assessment of Keynes helps when pointing out that the main underlying idea behind Keynes's political philosophy was "prudence." In his early thoughts on Edmund Burke's political philosophy Keynes argued that the expected probable consequences of achieving one's goal must be pondered and "we should be very chary of sacrificing large numbers of people for the sake of a contingent end, however advantageous that may appear" (quoted in Skidelsky 2009,

p. 156). Even with reference to other thoughts about government's action Keynes made during his political life, Skidelsky (2009, p. 158) summarizes this attitude as follows: "Prudence in the face of unknown is the key to Keynes's philosophy of statesmanship." Following on the interpretation of the *Treatise* just provided, Keynes's quest for prudence is reflected in his application of probability to human conduct.

6. Concluding remarks

The re-reading of Keynes originated by the current financial crisis includes a critical analysis of Keynes's decision theory under uncertainty. Akerlof and Shiller, in particular, claim that, if the aim is an understanding of the crisis, the "spontaneous urge to action" of Keynes's animal spirits must find room in economic and financial modelling. They argue in favour of the application of the tenets of behavioural economics to financial issues, rejecting the idea of rational individual agents and efficient financial markets, and they concentrate on how people "really" behave in the market. Limiting their inspiration to only one chapter of the *General Theory*, though, Akerlof and Shiller cannot find any positive element in Keynes's thought. Most of all, Keynes was right in pointing out that most decisions depend on confidence, and that when confidence is low a straightforward rational calculation of expected outcomes is not available, but, they conclude that it is only through behavioural economics that his message can survive in economic analysis.

Akerlof and Shiller's viewpoint may appear justified in view of the existing critical literature on Keynes and his dealing with uncertainty. As a matter of fact, even those scholars who correctly refer to the *Treatise* for the more persuasive motivation of Keynes's advocacy of uncertainty, like Skidelsky, rely on an assessment that rejects the possibility of discussing uncertainty in a formal context. Eventually, the conventional behaviour to which individuals are supposed to adhere when confidence is low does not find a representation in Skidelsky's understanding of Keynes.

This paper has pointed out that Keynes's discussion of probability in the *Treatise* can help enlighten this issue with specific regard to the kind of decision rule Keynes would have favoured. The proposed reading of this part of Keynes's work, putting emphasis on some technical aspects of his probability theory, can indeed contribute to an understanding of how

actual individual agents behave under uncertainty. In conclusion, this paper has tried to show that those violations of the Bayesian creed scrutinized in some current proposals to amend mainstream decision theory were discussed at length by Keynes.

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