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The interaction between natural resources- and physical capital-intensive sectors in a behavioral model of economic growth

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Abstract - In this paper we examine the role played by environmental externalities in shaping the dynamics of a small open economy with two sectors (a farming sector and an industrial one), free inter-sectoral labor mobility and heterogeneous agents (workers/farmers and industrial entrepreneurs). We find that the stability properties of the equilibria and their features in terms of environmental preservation, welfare outcomes and sectoral allocation of labor are sensitive to the relative level of carrying capacity with respect to the rate of environmental pressure of the economic activities. We show that an endogenous process of industrialization associated to a reduction in farmers/workers' welfare can emerge.

JEL classification: D62; O11; O13; O15; O41; Q20
Keywords: Environmental negative externalities; welfare reducing industrialization; economic growth with heterogeneous agents

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1 Introduction

In his seminal work about the role of structural change in economic growth, Matsuyama [14] suggests that the link between labor productivity in farming and manufacturing growth changes according to the degree of trade openness. In small open economies, if productivity in the agricultural sector is low, the industrial sector could benefit from a large supply of labor at low cost and, in this way, the economy could gain a comparative advantage in manufacturing production. The opposite relation holds for closed economies.

The positive role of industrialization for economic growth is usually combined with the idea that industrialization is a necessary, albeit not a sufficient, condition for poverty reduction. Industrialization is indeed a key feature of the growth processes of those countries which have been successful in combating poverty and in ensuring satisfactory living conditions for vast sections of the population. The fact that several countries have experienced higher labor productivity and industrialization without poverty reduction is often traced back to low absorption of labor in higher productivity sectors and to lack of labor transfer from rural subsistence to modern activities with the consequent expansion of the urban informal sector (Ocampo et al. [15], Easterly [7]).

Within this conceptual framework, less attention has been paid to the role that environmental externalities may have for economic growth and poverty reduction during the industrialization processes.[1] This paper seeks to make a contribution towards filling this gap. We adjust the framework proposed in Matsuyama [14] in order to take into account the effects of industrial and agricultural pollution on capital accumulation and welfare. We consider exogenous prices in order to focus on small open economies since, in recent decades, several developing countries have undertaken processes of trade liberalization.

The interactions between economic development, sectoral output composition and the environment are discussed by modeling an economy where environmental degradation affects workers' incentive to move out from environmental sensitive sectors. This reduces the impact of these activities, as well as workers' dependence on natural resources, but it can also fuel a self-reinforcing process of growth in industrial production and pollution. In doing this, our paper contributes to a growing body of literature which studies how, in multisectoral economies where natural resources are used

[1] An emblematic example of the possibile effects of pollution when natural resources are used as productive inputs is reported in Reddy and Behera [17, pp. 530-534]. By analsying the impact of industrial water pollution in a village of Andhra Pradesh, they find that "Majority of the cattle is becoming sick over the years [...] The amount of land under cultivation has declined substantially (88%) due to the incidence of pollution [...] Most of the people who were depending on agriculture before pollution have shifted to industry, business and other sources. Majority of them have become daily laborers".

as productive inputs, structural change processes and reallocation of labor across sectors can emerge as endogenous adjustments to a reduction in natural capital affecting economic growth and social welfare (see Peretto [16], Bretschger and Smulders [6], López [12] and López et al. [13]). Most of these models are concerned with the role of both the substitution between natural resources and man-made inputs (or labor) and the change in natural resource prices in driving the economy towards a sectoral shift which allows sustainable growth. These models, however, abstract from the distributional implications associated with such processes and, with the exception of López [12], identify resource-using and resource-impacting activities.

In our paper we build on this literature by taking a broader distributional perspective. More precisely, we analyze a two-sector model in a small open economy with free access renewable natural resources as factors of production. In our model, the physical capital is specific to the industrial sector, whereas the natural capital is specific to farming, but both sectors employ labor and produce environmental externalities which, in turn, affect labor costs and labor productivity. There are no constraints to inter-sector labor mobility and, as a consequence, labor productivity gains in the economy are equally shared among workers and there is no risk that possible benefits of industrialization are offset by a low absorption of labor in higher productivity sectors. Moreover, we exclude the impact of domestic food supply and domestic demand on prices of the goods produced by the two sectors. In this way, we concentrate on the role of resource-based activities in setting the basic opportunity cost for labor in the whole economy. As in López et al. [13], we find that a decline in labor share employed in the natural resource-intensive sector can arise in the absence of biased technological progress and can be an endogenous response to low labor productivity in this sector. However, structural change can emerge even in the absence of a change in the relative prices of goods produced in the economy.

We follow the basic features of the set-up proposed by Antoci et al. [1, 2], but we assume that only the farming sector is dependent on natural resources. The main difference as compared to the first of these two articles is that in the present paper we do not take physical capital as exogenously given; while the main difference between the present paper and the second article lies in the way in which we model the accumulation of capital. We indeed assume that agents' decisions to invest in physical capital are based on a behavioral equation, as in Solow's model [18], rather than on an intertemporal optimization problem. While in Antoci et al. [2] the analysis could not go beyond the local stability of the stationary states, the behavioral approach we adopt in this paper makes the formalization much simpler and makes it possible to draw conclusions about the global dynamics of the model. Despite this crucial difference in the modeling of capital accumulation, the results by Antoci et al. [2] are substantially confirmed, indicating that they do not critically depend on the hypotheses about agents' rationality. Our main findings seem, rather, to depend on the inability of natural resource-dependent agents to coordinate and internalize the externalities of economic activities. We find that there is no unique relation between capital accumulation, environmental pressure and agents' welfare. Regimes with multiple attractive equilibria are possible and capital accumulation can lead to either an increase or a decrease in inequality between the two population groups.[2] The analysis of the model explains how welfare, distributive and environmental outcomes of industrialization depends on two main factors: the initial endowment of natural capital and the level of pollution intensity compared to the environmental carrying capacity.

Economies where the pollution intensity of economic activities is very high compared to the existing carrying capacity tends to undertake a process of complete industrialization with negative impacts on the environment and social equity, i.e. an immiserizing and unsustainable complete industrialization. In all other cases, the dynamics is shaped by the initial conditions. In economies where environmental carrying capacity is particularly high, this type of negative industrialization can be avoided for sufficiently high values of initial natural capital. In this case, the transition from a complete agrarian economy to a diversification towards industrial production allows a Pareto improvement. However, we show that, under some conditions, a cyclical disequilibrium dynamics can also emerge implying a persistent conflict between the two groups.

The rest of the paper is organized as follows. The model is presented in Section 2, where the equilibrium points of its dynamic system are also derived. Section 3 contains both an analysis of the global/local stability properties and welfare implications of the equilibrium points and a characterization of the different dynamic regimes that can emerge. Section 4 concludes. All proofs and lengthy computations are contained in the Appendix.

2 The model

We consider a small open economy model in which economic agents belong to two different communities, one consisting of 'farmers', the other of 'industrial entrepreneurs'. The former are endowed with their own working capacity only which they use partly in their fields for the production of farming goods with the use of a natural resource[3] and partly as employees of the

[2] In light of this possible negative correlation between farmers and entrepreneurs' welfare, we do not study the social optimum. In this context, indeed, the benevolent social planner's decisions would be strictly linked to the 'weights' he or she assigns to the two typologies of agents.

[3] In line with the empirical evidence (see, e.g., Barbier [5]), we assume that farmers cannot accumulate physical capital. In doing this, we depart from Antoci et al. [3] where a similar model is analyzed, which, however, also includes the possibility that farmers invest

industrial entrepreneurs. In turn, the latter produce industrial goods with the physical capital they own and the labor force they hire. Accordingly, economic activity is divided into two sectors which we define as the 'F-sector' (Farming) and the 'I-sector' (Industry). Given the small size of the economy, the prices of both goods are exogenously determined regardless of what happens within it.

2.1 The production in the two sectors

Let us start with a description of the production in the two sectors. The productivity of labor in the F-sector depends on the stock of the natural renewable resource E and, as a consequence, production in this sector is intensive in E. On the other hand, production in the I-sector depends on the stock of the capital K owned by industrial entrepreneurs and on the labor force L provided by farmers. For simplicity, we assume that industrial entrepreneurs do not invest in the F-sector and that the latter is only composed of small firms run by farmers. In everything that follows, we focus on the behavior of two representative agents, one for each sector.

The production function of the firm run by the representative farmer ('F-agent') is given by:

$$Y_F = L^{\alpha} E^{\beta} \qquad \alpha, \beta > 0,\ \alpha + \beta \leq 1 \tag{1}$$

where $1 \geq L \geq 0$ is his or her labor input and E the stock of a natural renewable resource.[4]

The production function faced by the representative industrial entrepreneur (*I-agent*), on the other hand, has constant returns to scale and is given by:

$$Y_I = (1 - L)^{\alpha} K^{1-\alpha} \tag{2}$$

For simplicity, both the elasticities of Y_F and Y_I with respect to L and $1-L$, respectively, are assumed to be equal to α. Thus, possible differences in the productivity of labor employed in the two sectors depend on the levels of the stocks E and K only.

2.2 Economic agents' choices

Let us now analyze agents' choices. To simplify, we assume that the unit prices of both the agriculture and industrial goods are equal to unity and that both the I-agent and the F-agent take the wage rate w as exogenously given.

their savings in physical capital (although, unlike entrepreneurs, they cannot borrow).

[4]In the case in which the F-agent also owns a fixed amount T of land or other forms of capital, an equation of the same type as (1) could be obtained by writing the production function as $Y_F = L^{\alpha} T^{1-\alpha} E^{\beta}$ and then normalizing to one the fixed amount T.

In each instant of time t, the I-agent maximizes with respect to $1 - L$ his or her revenue, given by:

$$R_I = (1 - L)^{\alpha} K^{1-\alpha} - w(1 - L) \tag{3}$$

Analogously, the F-agent maximizes with respect to L his or her revenues, measured by the function:

$$R_F = L^{\alpha} E^{\beta} + w(1 - L) \tag{4}$$

Thus, in each instant of time, the I-agent solves the following optimization problem:

$$\underset{1-L}{\text{Max}} \left\{(1 - L)^{\alpha} K^{1-\alpha} - w(1 - L)\right\}$$

which gives rise to the following first order condition:

$$\alpha(1 - L)^{\alpha-1} K^{1-\alpha} = w \tag{5}$$

Analogously, the optimization problem of the F-agent can be expressed as:

$$\underset{L}{\text{Max}} \left\{L^{\alpha} E^{\beta} + w(1 - L)\right\}$$

such that the corresponding first order condition is:

$$\alpha L^{\alpha-1} E^{\beta} = w \tag{6}$$

Finally, by equalizing the left-hand sides of (5) and (6), we obtain the following equilibrium value of the variable L:

$$L = \frac{E^{\frac{\beta}{1-\alpha}}}{E^{\frac{\beta}{1-\alpha}} + K} \tag{7}$$

2.3 The dynamic system

With regard to the dynamics of K and E, we assume first of all that the accumulation process of the former is driven by a behavioral mechanism as in Solow's [18] growth model. Thus, indicating with a dot over a variable the first derivative with respect to time, we have:

$$\dot{K} = sR_I - dK \tag{8}$$

where the parameters s, $d \in (0, 1)$ represent the marginal propensity to save and the depreciation rate of the capital stock, respectively.

The time evolution of the latter, on the other hand, is assumed to be given by:

$$\dot{E} = \begin{cases} E(\bar{E} - E) - \delta\bar{L} - \varepsilon\bar{Y}_I \text{ , for } E > 0 \\ 0, \text{ for } E = 0 \end{cases} \tag{9}$$

where the parameters $\delta > 0$ and $\varepsilon > 0$ measure the environmental impact of the production in the F- and I-sector, respectively, the parameter $\bar{E} > 0$ represents the carrying capacity of the natural resource and $\bar{L}$ and $\bar{Y}_I$ are the average values of L and Y_I, respectively, taken as exogenously given by the two agents. Consequently, both sectors produce environmentally negative externalities that agents are not able to internalize due to coordination problems. As will soon be evident, this assumption plays a crucial role in shaping the results of our model, much more than the behavioral assumption about the accumulation process of physical capital. It is meant to take account of the fact that environmental externalities have a strong impact on economic activities, especially in developing countries, where property rights tend to be ill-defined and ill-protected, environmental institutions and regulations are weak and natural resources are more fragile than in developed countries (López [10, 11]).

Finally, by substituting (7) into equations (8) and (9), and taking account also of equations (5) and (6), we obtain the following nonlinear dynamic system in the two variables E and K:

$$\dot{E} = \begin{cases} E(\bar{E} - E) - \frac{\delta E^{\frac{\beta}{1-\alpha}}}{E^{\frac{\beta}{1-\alpha}} + K} - \frac{\varepsilon K}{\left(E^{\frac{\beta}{1-\alpha}} + K\right)^{\alpha}}, \text{ for } E > 0 \\ 0, \text{ for } E = 0 \end{cases} \tag{10}$$

$$\dot{K} = \left[\frac{s(1-\alpha)}{\left(E^{\frac{\beta}{1-\alpha}} + K\right)^{\alpha}} - d \right] K \tag{11}$$

which, clearly, is not defined for $E = K = 0$.

2.3.1 Isoclines and equilibrium points

We now turn to the problem of the determination and characterization of the equilibrium points of our model. First of all we notice that, from (11), it follows that $\dot{K} = 0$ holds if either $K = 0$ or:

$$K = \theta - E^{\frac{\beta}{1-\alpha}} \tag{12}$$

where $\theta := [s(1-\alpha)/d]^{1/\alpha} > 0$ and $1 > \beta/(1-\alpha) > 0$. Thus, for $K > 0$, (12) in the (E, K)-plane is a strictly decreasing function of E which intersects the E-axis at $(\theta^{(1-\alpha)/\beta}, 0)$ and the K-axis at $(0, \theta)$. Moreover, $\dot{K} < 0$ ($\dot{K} > 0$) holds above (respectively, below) the curve of equation (12).

Analogously, $\dot{E} = 0$ holds if either $E = 0$ or:

$$E(\bar{E} - E) - \frac{\delta E^{\frac{\beta}{1-\alpha}}}{E^{\frac{\beta}{1-\alpha}} + K} - \frac{\varepsilon K}{\left(E^{\frac{\beta}{1-\alpha}} + K\right)^{\alpha}} = 0 \tag{13}$$

Unlike equation (12), equation (13) defines a relation between E and K only implicitly. As a consequence, it cannot be used to draw the graph of this branch of the $\dot{E} = 0$-isocline in the (E, K)-plane. However, it turns out to be very useful in order to highlight some of its basic properties. Let us first notice that we always have $\dot{E} < 0$ if $E \geq \bar{E}$ and therefore that the $\dot{E} = 0$-isocline lies on the left of the vertical line $E = \bar{E}$. Let us also notice that in the interval $[0, \bar{E}]$ the following inequality holds:

$$\dot{E} < \frac{\bar{E}^2}{4} - \frac{\varepsilon K}{\left(\bar{E}^{\frac{\beta}{1-\alpha}} + K\right)^{\alpha}}$$

where $\bar{E}^2/4$ is the maximum of $E(\bar{E} - E)$ and $\bar{E}$ the maximum of E.

Furthermore, given that $\lim_{K \to +\infty} \varepsilon K / \left(\bar{E}^{\beta/(1-\alpha)} + K\right)^{\alpha} = +\infty$, there exists a value $\bar{K}$ such that $\dot{E} < 0$ for every $E > 0$ and $K > \bar{K}$. This implies that the $\dot{E} = 0$-isocline is bounded also from above.

Some additional properties of the $\dot{E} = 0$-isocline can be highlighted by evaluating the partial derivative of $\dot{E}$ with respect to K, given by:

$$\frac{\partial \dot{E}}{\partial K} = \frac{\delta E^{\frac{\beta}{1-\alpha}} - \varepsilon \left(E^{\frac{\beta}{1-\alpha}} + K\right)^{1-\alpha} \left[E^{\frac{\beta}{1-\alpha}} + (1-\alpha)K\right]}{\left(E^{\frac{\beta}{1-\alpha}} + K\right)^{2}}$$

which is positive if and only if its numerator is positive. Thus, given E and K, $\partial \dot{E}/\partial K > 0$ holds if ceteris paribus δ is high enough with respect to ε, i.e., if the environmental impact of the F-sector is high enough with respect to that of the I-sector. When this is the case, an increase in K lessens the negative impact of economic activity on the environmental resource. This happens because such an increase generates a shift of labor forces from the environmentally damaging F-sector towards the environmentally less impacting I-sector. However, $\partial \dot{E}/\partial K$ always becomes strictly negative if ceteris paribus the value of K becomes high enough. In fact, in this case the labor input L in the F-sector, as defined in (7), is near to zero and a further increase in K does not generate a reduction in the activity level of the F-sector able to compensate the growth in the negative impact of the I-sector due to the increase in its activity level. Furthermore, we notice that if K increases, for a given $E > 0$, then the sign of $\partial \dot{E}/\partial K$ may change at most once; in particular, if $\partial \dot{E}/\partial K \leq 0$ holds for $K = 0$ (this is the case if $\delta - \varepsilon E^{\beta} \leq 0$), then $\partial \dot{E}/\partial K < 0$ holds for every $K > 0$, while if $\partial \dot{E}/\partial K > 0$ holds for $K = 0$, then $\partial \dot{E}/\partial K$ becomes definitively negative for K high enough. These properties allow us to say that any given vertical line $E = \tilde{E} > 0$ can intersect the $\dot{E} = 0$-isocline at most twice. If $E = \tilde{E}$ intersects the $E = 0$-isocline at a point $(\tilde{E}, \tilde{K})$ such that $\dot{E} < 0$ for $K < \tilde{K}$, then $E = \tilde{E}$ must intersect the $\dot{E} = 0$-isocline also at another point $(\tilde{E}, \hat{K})$, $\hat{K} > \tilde{K}$, such that $\dot{E} > 0$ ($\dot{E} < 0$) for $\hat{K} > K > \tilde{K}$ (respectively, $K > \hat{K}$).

If, on the contrary, the line $E = \tilde{E}$ intersects the $\dot{E} = 0$-isocline at a point $(\tilde{E}, \tilde{K})$ such that $\dot{E} > 0$ for $K < \tilde{K}$, then $\dot{E} < 0$ holds for every $K > \tilde{K}$. An analogous property can also be observed moving along any given horizontal line $K = \tilde{K}$. In this case, if $E > 0$ is low enough, then $\dot{E} < 0$ always holds; furthermore, if $K = \tilde{K}$ meets the $\dot{E} = 0$-isocline at a point $(\tilde{E}, \tilde{K})$, then it must intersect the isocline also at another point $(\hat{E}, \tilde{K})$, $\hat{E} > \tilde{E}$, such that $\dot{E} > 0$ ($\dot{E} < 0$) for $\hat{E} > E > \tilde{E}$ (respectively, $E > \hat{E}$). It is easy to check that these properties imply that the $\dot{E} = 0$-isocline is given by the intersection between a closed curve and the positive quadrant of the plane (E, K). Moreover, $\dot{E} > 0$ ($\dot{E} < 0$) holds inside (respectively, outside) it.

Equilibrium points with either $E = 0$ or $K = 0$ The intersection point between the branch of the $\dot{K} = 0$-isocline defined by (12), and the K-axis (along which $\dot{E} = 0$), i.e.:

$$(0, K_0) = (0, \theta) \tag{14}$$

is always an equilibrium point of the dynamic system (10)-(11).

Furthermore, equilibrium points with $K = 0$ and $E > 0$ may also exist, at the intersections between the branch of the $\dot{E} = 0$-isocline of equation (13) and the E-axis. To find them, notice that, for $K = 0$, $\dot{E} = E\left(\bar{E} - E\right) - \delta$ holds. Consequently, if $\bar{E} > 2\sqrt{\delta}$, i.e. if the environmental impact of the resource-intensive sector is low enough relatively to the carrying capacity $\bar{E}$ of the environmental resource, system (10)-(11) admits also the following pair of equilibrium points with $K = 0$:

$$(E_1, 0) = \left(\frac{1}{2}\left(\bar{E} - \sqrt{\bar{E}^2 - 4\delta}\right), 0\right) \tag{15}$$

$$(E_2, 0) = \left(\frac{1}{2}\left(\bar{E} + \sqrt{\bar{E}^2 - 4\delta}\right), 0\right) \tag{16}$$

Internal equilibrium points By substituting K as defined in equation (12) into equation (10), the latter becomes:

$$\dot{E} = f_1(E) - f_2(E) \tag{17}$$

where:

$$f_1(E) := E\left(\bar{E} - E\right) - \varepsilon\theta^{1-\alpha} \tag{18}$$

$$f_2(E) := \Omega E^{\frac{\beta}{1-\alpha}} \tag{19}$$

and

$$\Omega := \frac{\delta - \varepsilon\theta^{1-\alpha}}{\theta} \tag{20}$$

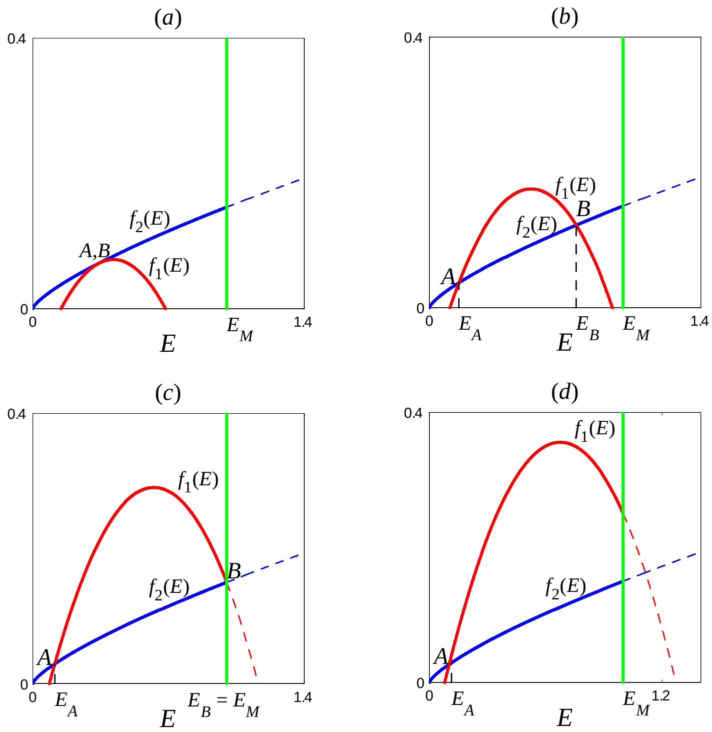

Figure 1: A taxonomy of the number of positive equilibrium points with $\delta = 0.25$, $\varepsilon = 0.1$ and different values of the carrying capacity of environment: (a) $\bar{E} \approx 0.8305$, (b) $\bar{E} = 1.05$, (c) $\bar{E} = 1.25$ and (d) $\bar{E} = 1.35$

Equation (17) may be used to evaluate the value of $\dot{E}$ along the $\dot{K} = 0$-isocline. A value $E = E^* > 0$ identifies an equilibrium point with $E, K > 0$ if $f_1(E^*) = f_2(E^*)$ and if, when substituted in (12), gives a value of K which is strictly positive, i.e. if:

$$E^* < E_M := \theta^{\frac{1-\alpha}{\beta}} \tag{21}$$

where E_M – which is independent of $\bar{E}$ – is the value of E at which the $\dot{K} = 0$-isocline intersects the E-axis.

As it is easy to check, the graphs of $f_1(E)$ and $f_2(E)$ can have at most two intersections and consequently at most two internal equilibrium points can exist. We shall indicate by A (respectively, by B) the intersection point in correspondence of which $f_1'(E) > f_2'(E)$ (respectively, $f_1'(E) < f_2'(E)$).

Fig. 1, in which the parameters are such that $\Omega > 0$, illustrates the complete taxonomy of possible cases in which at least one positive equilibrium point exists.[5] They are obtained by varying only the value of the carrying

[5]In this figure and in all others that follow, when not otherwise stated, we fix the

capacity $\bar{E}$, which is increasing from $\bar{E} \approx 0.8305$ in Fig. 1(a) to $\bar{E} = 1.35$ in Fig. 1(d). The case of tangency between $f_1(E)$ and $f_2(E)$ is shown in Fig. 1(a). For values of $\bar{E}$ less than the value we have used to generate this figure, clearly, the two curves do not intersect in the first quadrant and, therefore, the system does not admit internal equilibrium points. When the value of $\bar{E}$ increases, the curve $f_2(E)$ does not move, whereas the curve $f_1(E)$ 'expands' itself in such a way that two points of intersection between the two curves emerge (one of type A and another of type B, as shown in Fig. 1(b)). However, the intersection points of the two curves correspond to internal equilibrium points only if they are located on the left of the vertical line $E = E_M$. Thus, when $\bar{E}$ is further increased beyond the value used to generate the 'critical' case of Fig. 1(c), there exists only one positive equilibrium of type A, as in Fig. 1(d).

The following proposition summarizes the cases just described.

Proposition 1 *A necessary and sufficient condition for the existence of a unique internal equilibrium point (which is always of type A) is:*

$$\bar{E} > \theta^{\frac{1-\alpha}{\beta}} \left(1 + \delta\theta^{-\frac{2(1-\alpha)}{\beta}}\right) = E_M + \frac{\delta}{E_M} \tag{22}$$

When condition (22) is not satisfied, the dynamic system (10)-(11) generically admits either zero or two internal equilibrium points.[6] *In this case, a sufficient condition for the non existence of internal equilibrium points is:*[7]

$$\begin{cases} \bar{E} \leq 2\sqrt{\varepsilon\theta^{1-\alpha}}, \text{ when } \Omega > 0 \\ \bar{E} \leq 2\sqrt{\delta}, \text{ when } \Omega < 0 \end{cases} \tag{23}$$

while a sufficient condition for the existence of two internal equilibrium points (one of type A and the other of type B) is:[8]

$$\left(\frac{\bar{E}}{2}\right)^2 - \Omega\left(\frac{\bar{E}}{2}\right)^{\frac{\beta}{1-\alpha}} > \varepsilon\theta^{1-\alpha} \text{ and } \frac{\bar{E}}{2} < E_M \tag{24}$$

Proof. See Sect. A.1 in the Mathematical Appendix ■

Remark 1 *When condition (23) is satisfied, the two equilibria on the E-axis do not exist.*

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parameters of the production function, the depreciation rate and the marginal propensity to save at $\beta = 0.3$, $\alpha = 0.6$, $\delta = 0.1$ and $s = 0.25$, respectevely, and allow the environmental parameters $\bar{E}$, δ and ε to vary.

[6] Also the case of a unique internal equilibrium point is again possible, but only in the critical case in which $f_1(E)$ and $f_2(E)$ are tangent.

[7] Given the definition of Ω in (20), it is easy to check that the two conditions in (23) can equivalently be written as $\bar{E} \leq \min(2\sqrt{\varepsilon\theta^{1-\alpha}}), 2\sqrt{\delta})$.

[8] Necessary and sufficient conditions for this case could also be indicated. They are however rather complicated and not easily interpreted.

3 Global/local stability properties of the equilibrium points and fundamental dynamic regimes

To go deeper into the understanding of the dynamics generated by our model, and in the attempt to identify and characterize the main dynamic regimes that can emerge, we now turn to the problem of the stability properties of the equilibrium points and of their welfare and distribution implications.

3.1 Stability properties

The global and local stability properties of the dynamics generated by system (10)-(11) are well illustrated by two propositions, that we are now going to introduce.

Proposition 2 *The set:*

$$Q = \left\{(E, K) : 0 \leq E \leq \bar{E} \text{ and } 0 \leq K \leq \theta\right\}$$

is positively invariant under the dynamic system (10)-(11); furthermore, every trajectory starting outside it either enters it in finite time or approaches the equilibrium point $(0, K_0)$ lying on the boundary of Q. Consequently, by the Poincaré-Bendixson Theorem, each trajectory approaches either an equilibrium point or a limit cycle surrounding an internal equilibrium point of type B.

Proof. See Sect. A.2 in the Mathematical Appendix ■

Thus, first of all, if no internal equilibrium point exists, then all the trajectories of the dynamic system (10)-(11) approach either the equilibrium point with $E = 0$ – i.e., $(0, K_0)$ – or with $K = 0$ – i.e., $(E_1, 0)$ or $(E_2, 0)$. Moreover, the stability properties of the equilibrium point $(0, K_0)$ can be easily determined by simply remembering that $\dot{E} < 0$ holds if $E > 0$ is low enough and that $\dot{K} < 0$ (respectively, $\dot{K} > 0$) holds above (respectively, below) the $\dot{K} = 0$-isocline. As a result, the set $\{(E, K) : 0 \leq E \leq a, \theta - b \leq K \leq \theta + b\}$ containing $(0, K_0)$ is positively invariant for $a > 0$ and $b > 0$ small enough. This implies that $(0, K_0)$ is always locally attractive.

In turn, the stability properties of the remaining equilibrium points are described by the following proposition.

Proposition 3 *The equilibrium point of type A is always a saddle point while that of type B may be either locally attractive or repulsive. The equilibrium $(E_1, 0)$ is repulsive if $E_1 < E_M$ while it is a saddle point (with unstable manifold lying in the E-axis) if $E_1 > E_M$. The equilibrium $(E_2, 0)$ is a saddle point (with stable manifold lying in the E-axis) if $E_2 < E_M$ while it is locally attractive if $E_2 > E_M$.*

Proof. See Sect. A.3 in the Mathematical Appendix ■

In short, when a unique internal equilibrium point exists, then $E_1 < E_M < E_2$ holds and therefore the equilibrium $(E_1, 0)$ is repulsive while $(E_2, 0)$ is attractive, while if no internal equilibrium point exists, then either $E_1 < E_2 \leq E_M$ or $E_M \leq E_1 < E_2$ holds.

3.2 Welfare in the attractive equilibrium points

As a last step, before concentrating on the main results concerning the dynamics of the model, it is useful to compare the various (globally and locally) stable equilibrium points of the dynamic system of our model, in terms of the revenues of the two representative agents.

As we have seen, the F-agent uses a natural resource which is available at zero cost but is exposed to negative externalities. The I-agent, on the other hand, does not use free environmental resources, but has to save in order to accumulate physical capital. His or her advantage is that, unlike the F-agent, he or she can hire wage labor and expand his or her physical capital over time, not harmed by environmental externalities. The F-agent is indirectly affected by physical capital accumulation through two different channels: on one hand, a rise in labor productivity in the I-sector due to an increase in K has a positive effect on the equilibrium wage rate; on the other hand, the resulting net environmental impact, due to the combination of scale and labor sectoral composition effects, influences the productivity of agricultural labor and consequently the opportunity cost of wage labor. To analyze the welfare properties associated with each equilibrium point, we should remember that the revenues of the F-agent are measured by equation (4). By substituting in it the equilibrium values of w and L given by (5) and (7), respectively, equation (4) becomes:

$$R_F = \frac{E^{\frac{\beta}{1-\alpha}} + \alpha K}{\left(E^{\frac{\beta}{1-\alpha}} + K\right)^{\alpha}} \tag{25}$$

such that $\partial R_F / \partial E > 0$ and $\partial R_F / \partial K > 0$. Furthermore, since from (12) it follows that along the $\dot{K} = 0$-isocline the condition $E^{\beta/(1-\alpha)} + K = \theta$ holds, the value of R_F evaluated along it can be expressed as a function of the value of the stock E only, i.e.:

$$R_F\left(E\right) = \frac{1-\alpha}{\theta^{\alpha}} E^{\frac{\beta}{1-\alpha}} + \alpha\theta^{1-\alpha} \tag{26}$$

Let us now consider the I-agent revenue, which, given equation (3), can be expressed as:

$$R_I = \frac{\left(1-\alpha\right) K}{\left(E^{\frac{\beta}{1-\alpha}} + K\right)^{\alpha}} \tag{27}$$

such that $\partial R_I/\partial E < 0$ and $\partial R_I/\partial K > 0$. By substituting in (27) the equilibrium condition (12), it follows that, along the $\dot{K} = 0$-isocline, the value of R_I can be expressed as a function of K only, i.e.:

$$R_I(K) = \frac{(1-\alpha)K}{\theta^\alpha} \tag{28}$$

The following proposition sums up the above results.

Proposition 4 *Along the $\dot{K} = 0$-isocline, the revenue of the I-agent is represented by the strictly increasing function of K shown in (28), whereas those of the F-agent by the strictly increasing function of E shown in (26).*

Equations (26) and (28) tell us something rather interesting: once we have fixed the parameters, the welfare of the F-agent is positively correlated with E and does not depend on K, even in the presence of diversification in income sources. Vice versa, the revenue of the I-agent is positively correlated with K and does not depend on E. Consequently, along the $\dot{K} = 0$-isocline, to which the equilibrium points $(0, K_0)$ and (E_B, K_B) belong, there is a trade-off between the I-agent's revenue and that of the F-agent. This implies that in $(0, K_0)$, the revenue of the I-agent is higher than in (E_B, K_B), and vice versa for the revenues of the F-agent, being $E_B > 0$ and $K_B < K_0$.

Furthermore, notice that, if the equilibrium $(E_2, 0)$ is such that $E_2 > E_M$ (that is, if it is attractive), then $R_F(E_2) > R_F(E_M)$ holds. This implies that F-agent's revenues are higher in $(E_2, 0)$ than in $(0, K_0)$, whereas the opposite holds for I-agent's revenues. Consequently, the following proposition can be stated.

Proposition 5 *In the bistable dynamic regimes in which the attractive equilibrium point $(0, K_0)$ coexists with another attractive equilibrium (either (E_B, K_B) or $(E_2, 0)$), the revenue of the I-agent is higher in $(0, K_0)$ than in (E_B, K_B) or $(E_2, 0)$, whereas the opposite holds for the revenues of the F-agent.*

Notice that $(E_2, 0)$ is the equilibrium point that the economy would reach in the absence of the I-sector (that is, if $K = 0$) starting from an initial value of E greater than E_1. It is easy to check that, if $E_2 < E_B$, then $R_F(E_B, K_B) > R_F(E_2, 0)$. In fact, since $\partial R_F/\partial E > 0$ and $\partial R_F/\partial K > 0$, $R_F(E_B, K_B) > R_F(E_B, 0) > R_F(E_2, 0)$ holds. As a result, the revenues of the F-agent are higher in the equilibrium point where the two sectors coexist than in the equilibrium with $K = 0$. Notice, moreover, that $E_2 < E_B$ holds if and only if $\partial \dot{E}/K > 0$ holds in $(E_2, 0)$, i.e., if the value of δ is high enough with respect to the value of ε. In this case, an increase in K alleviates the negative impact on the environmental resource due to economic activity because it generates a shift of labor forces from the environmentally damaging F-sector towards the environmentally less impacting I-sector. In this context, therefore, a transition from an agrarian to a diversified economy is a win-win solution which produces positive effects on the environment and on both types of agents.

3.3 Dynamic regimes: some numerical simulations

A number of numerical simulations, showing the various types of dynamics that can be generated by our model are illustrated in the figures below.[9] These simulations confirm that the regimes which can be generated by the dynamic system of our model depend crucially on the relative value of $\bar{E}$ with respect to δ and ε. Relative changes in the values of these parameters, implying different proportions between the rate of environmental pressure of economic activities and the carrying capacity of the environment, mark out the transition from a dynamic regime to another one.

Let us first notice that, starting from the parameter values we have used in Fig. 1, and taking account of the conditions given in Proposition 1, the following three *fundamental* dynamic regimes can be defined, by simply letting $\bar{E}$ vary:

1. The dynamic regime for the case in which our model economy meets condition (22) and therefore, given that its carrying capacity is above a certain threshold, can be considered as a 'resource rich' economy

   In this regime, obtained for example for $\bar{E} = 1.35 > E_M + \delta/E_M = 1.25$ as in Fig. 1(d), a unique internal equilibrium point exists, which is a saddle point as shown in Fig. 2. In addition, there are two coexisting locally attractive equilibrium points on the axes, namely, the equilibrium $(0, K_0)$ with full specialization in the I-sector and the equilibrium $(E_2, 0)$ with full specialization in the F-sector, with basins of attraction separated by the stable manifold of (E_A, K_A). Thus, in a case like this of natural resource abundance, the economy cannot approach an equilibrium state with the coexistence of the two sectors and its dynamics is path-dependent. If the economy starts near enough to $(0, K_0)$ (respectively, to $(E_2, 0)$), then the stationary state with full industrial (respectively, farming) specialization is reached. This implies that entrepreneurs are more likely to maximize their revenues if in the initial conditions the economy is characterized by a highly deteriorated environment. In this context, the initial stock of natural capital might be very low and entrepreneurs can benefit from a progressive acceleration of labor force shift from the F-sector. In contrast, for higher levels of initial natural capital, industrialization is hampered and the economy is more likely to converge to a full specialization in the natural-resource based sector. This latter outcome, given the

[9] In all of them, the attractive equilibrium points are marked by full dots (•), the repulsive equilibrium points by open dots (○) and saddle points by squares (□).

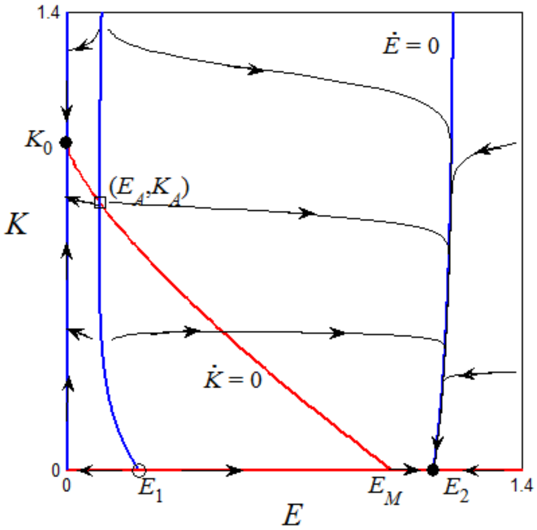

Figure 2: The case of coexistence of two locally stable equilibria on the axes with basins of attraction separated by the stable manifold of the equilibrium point A (with $\bar{E} = 1.35$ such that condition (22) is satisfied)

relative size of the basins of attraction of the two equilibrium points, appears to be the most probable one.

2. The dynamic regime for the case in which our model economy satisfies condition (23) and therefore, given that its carrying capacity is relatively low compared to the pollution intensity of the existing economic activities, can be considered as a 'resource-poor' economy.

   In this regime, obtained for example for $\bar{E} = 0.63$ – a value of the carrying capacity less than the 'critical' value used to generate Fig. 1(a) – the dynamic system (10)-(11) admits no internal equilibrium points as shown in Fig. 3. Therefore, as in the previous regime, the coexistence of the two sectors in a non transient way is excluded and the equilibrium $(0, K_0)$ is globally attractive. The result is a full industrialization accompanied by inequality and depletion of environmental resources. In this regime, indeed, physical capital accumulation feeds industrialization to the detriment of the F-agent's revenues. The economy converges to the equilibrium $(0, K_0)$ which ensures the highest revenue for the I-agent but the lowest stationary value of R_F. In this context, there is a negative link between agricultural productivity and industrialization: environmental externalities, reducing return to farming labor, help industrial growth since they push labor force out of the F-sector exerting down pressure on the wage rate.

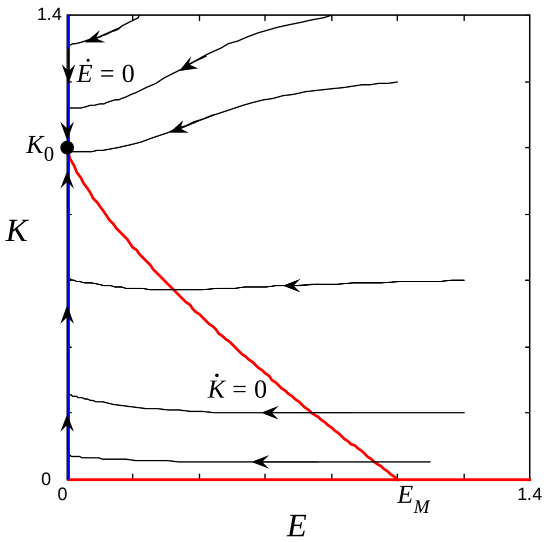

Figure 3: The case of $(0, K_0)$ globally attractive (with $\bar{E} = 0.63$ such that condition (23) holds)

3. The dynamic regime for the intermediate case in which our model economy meets neither condition (22), nor condition (23).

   This regime, obtained for example for $\bar{E} = 1.05$ as in Fig. 1(b), well illustrates the implications of the previous propositions as shown in Fig. 4. Two internal equilibrium points exist, one of type A and the other of type B. The former, as expected, turns out to be a saddle point, whereas the latter is locally stable and thus such that the coexistence of the two sectors can be observed in a non transient way. As a result, a new type of bistable regime is observed where the locally stable equilibrium point on the K-axis coexists with the locally stable internal equilibrium point (E_B, K_B), with the stable manifold of the saddle point (E_A, K_A) separating the basin of attraction of $(0, K_0)$ from that of (E_B, K_B). Finally, two equilibrium points on the E-axis also exist, namely, $(E_1, 0)$, where $E_1 < E_M$, which is repulsive and $(E_2, 0)$, where $E_2 < E_M$, which is a saddle point with stable manifold on the E-axis.[10]

In this intermediate regime, with two internal equilibrium points, when

[10] Other simulations we have performed suggest that, ceteris paribus, as ε increases, the basin of attraction of the locally stable equilibrium (E_B, K_B) with the coexistence of the two sectors narrows whereas that of the fixed point with complete industrialization expands. In short, with a higher environmental impact of the I-sector, the economy is more likely to converge to a stable equilibrium which benefits the I-agent and damages the F-agent.

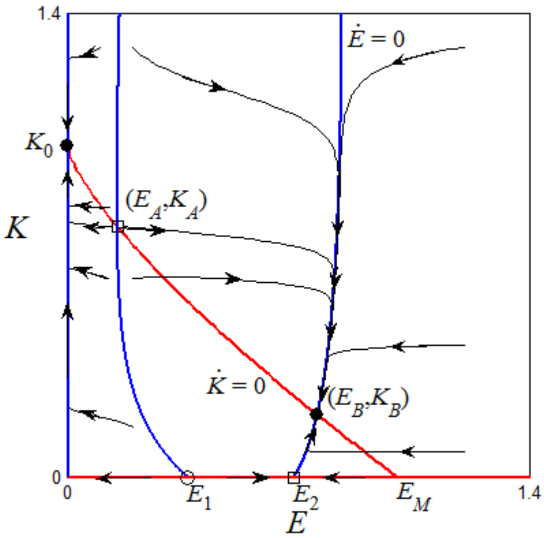

Figure 4: The case of coexistence of one locally stable internal equilibrium point and a locally stable equilibrium point on the K-axis (with $\bar{E} = 1.05$ such that condition (24) holds)

further parameters are allowed to vary, such that also the $\dot{K} = 0$-isocline moves, other interesting typologies of dynamics may also be observed. In particular, it may happen that the internal equilibrium points (E_A, K_A) and (E_B, K_B) exist, but that the latter is not attractive. When this is the case, all trajectories that do not approach $(0, K_0)$ approach either a limit cycle or the equilibrium point $(E_2, 0)$ (with $E_2 > E_M$), as shown respectively by the numerical simulations in Figs 5 and 6.

To summarize, in countries with 'intermediate' levels of $\bar{E}$, if (E_B, K_B) is stable or it is surrounded by a stable limit cycle, two main paths are admissible: (i) if the economy starts from low initial values of E, it will undertake a process of full industrialization and sustained physical capital accumulation, associated with environmental degradation and impoverishment of the F-agent. This scenario can emerge even if K is initially very low; (ii) for higher values of initial environmental resources, a poor economy, characterized by the same low initial endowments of physical capital, through industrialization, albeit incomplete, could converge to the fixed point of type B or to a stable limit circle around it ensuring a higher welfare for the F-agent compared to the alternative equilibrium $(0, K_0)$. In the latter case, industrialization helps the economy to follow a more equitable and sustainable development path.

The case shown in Fig. 5, in which the dynamic system generates a limit cycle around the repulsive equilibrium point (E_B, K_B), is particularly

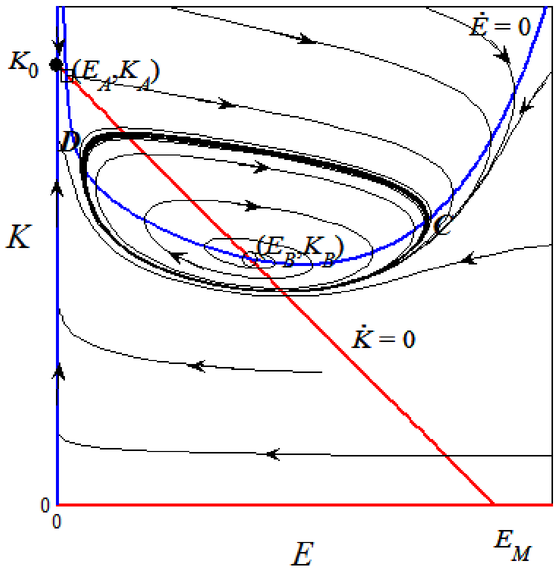

Figure 5: The case of persistent fluctuations of the variables (with $\alpha = 0.7$, $\beta = 0.3$, $\delta = 0.53$ and $\varepsilon = 0.005$)

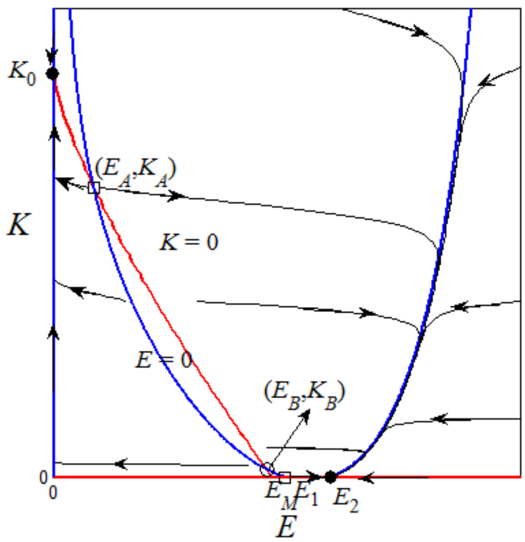

Figure 6: The case of an unstable equilibrium point of type B not surrounded by a limit cycle (with $\bar{E} = 1.42$, $\delta = 0.5$, $\varepsilon = 0.01$, $s = 0.2$).

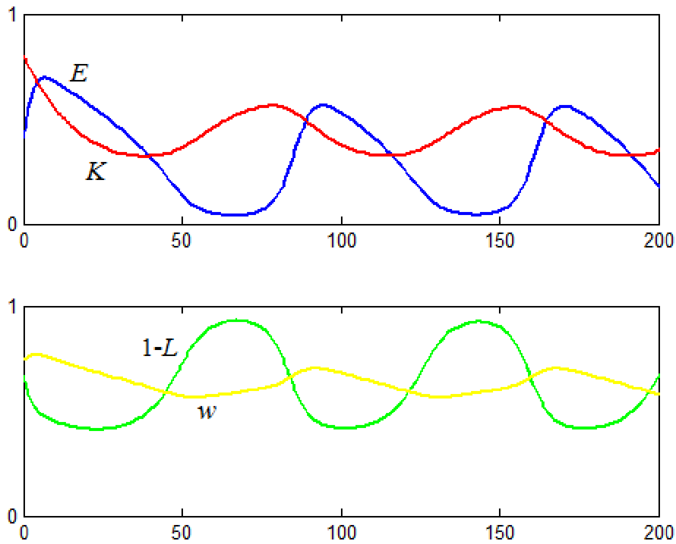

Figure 7: The main variables of the model versus time evaluated along the limit cycle illustrated in Fig. 5

relevant and deserves additional attention. As shown in the figure, the limit cycle coexists together with the locally stable equilibrium on the K-axis and, if the initial condition is such that the system converges to it, the economy never reaches a stationary state, with the consequence that the two variables undergo persistent fluctuations.

To give an intuitive idea of what happens during the cycle and to understand what is the crucial mechanism that generates it, it is useful to make an analogy with Goodwin's [9] growth cycle model. Doing this, we can say that, whereas in the latter model, formalized in terms of Lotka-Volterra's equations (see, for example, Volterra [19] and Gandolfo [8, pp. 448-463]), the cyclical dynamics is the outcome of the conflict between workers and capitalists, in our model it is explained in terms of the conflict between the F-agent and the I-agent. Let us consider for example the case in which the economy starts from the maximum value of the natural renewable resource E over the cycle and an intermediate level of K (as in point C of Fig. 5). In this situation, labor productivity in the F-sector is high (see Fig. 7), the F-agent tends to use more of his or her time to work in this sector (so that $1 - L$ decreases) and the opportunity cost of wage labor starts to increase.

As a consequence, the I-agent needs to offer an higher wage to attract the F-agent and this reduces his or her profits and investment in physical capital. The consequent decline in physical capital leads to a reduction in labor demand in the I-sector producing downward pressure on wages. Moreover, since the environmental impact of the F-sector is larger than that of the I-sector, E begins to decline, until a minimum value over the cycle as in point D of Fig. 5 is reached. This decline, however, in turn, generates further downward pressure on wages and reduces labor productivity in farming activities pushing the F-agent out of the F-sector. Indeed, as wages become sufficiently low, investments recover and physical capital starts to grow. The decrease in wages accelerates physical capital accumulation which in turn is associated with labor productivity growth and increases in the production and labor demand of the I-sector. Initially, the first effect prevails and thus industrial labor share declines while the natural resource starts to increase. As this process continues, wage growth eventually discourages investment in physical capital and the dynamics restarts from the beginning.

4 Conclusions

This work draws on the intuition of Matsuyama's influential model [14] which predicts a negative relationship between agricultural productivity and industrialization (and consequently economic growth) in small open economies. Our model is built on the idea that a large stock of natural capital, by ensuring high productivity in the F-sector, can squeeze out the manufacturing sector in a small open economy with a constant labor supply and free intersectoral labor mobility. However, we depart from this common starting point in that we include environmental externalities, agents' heterogeneity and capital accumulation and we exclude the possibility that *learning-by-doing* is a process specific to industrial production. As a result, we found that there is no unique relation between the abundance of environmental resources, industrialization and agents' welfare. Moreover, some of the welfare implications of Matsuyama's model are reversed. Regimes with multiple attractive equilibria are possible. The initial stocks of physical and natural capital, the carrying capacity of the economy and the pollution intensity produced by the two sectors affect the type of development path that the economy follows. Natural resource rich countries are likely to converge to a stationary state with a complete specialization in the F-sector, but if the economy starts from a very deteriorated environment, it can also converge to full industrialization. The farmers obtain higher revenues in the first scenario, but, in this case, the possibility of exploiting the benefits of physical capital accumulation is ruled out. Therefore, in the long run, also this equilibrium might represent a poverty-trap. The system is likely to converge to complete industrialization, regardless of the initial endowments of physical capital, also when the economy is very vulnerable to environmental degradation since its carrying capacity is particularly low with respect to the pollution intensity of economic activities. This path tends to the highest stationary value of physical capital, but it is unequal and environmentally unsustainable since it leads to exhaustion of natural resources and to the lowest and highest equilibrium values of workers' and entrepreneurs' revenues, respectively. As a consequence, processes of capital accumulation and industrialization are accompanied by a growth of poverty since environmental degradation reduces remuneration of industrial labor and agricultural labor productivity at the same time. Finally, for sets of parameters which determine intermediate levels of environmental pressure and endowments, the economy can undertake a process of industrialization and converge to a stationary state or a stable limit cycle where both sectors coexist. In this case, industrialization is more consistent with equity and sustainable development, but some trade-off between the welfare of the two groups persist. If the economy converges to a stable limit cycle, dynamics of natural and physical capital, the key assets used by the two categories of agents we consider, follow phases of positive and negative correlation.

All these alternative scenarios show that environmental externalities and agents' heterogeneity in terms of dependence on natural resources and ability to accumulate physical capital, are factors which deserve great attention since they significantly affect the distributive and welfare outcomes of industrialization processes and, more generally, of structural changes. The introduction of environmental externalities in a modified Lewis dual-sector model, therefore, shows that, because of environmental dynamics, a new type of industrialization without development can emerge even if there are no barriers to labor employment in the I-sector and prices are exogenously given.

A Mathematical appendix

A.1 Proof of Proposition 1

Proof. Observe that the difference $f(E) := f_1(E) - f_2(E)$ is such that $f(0) < 0$ and $\lim_{E\to+\infty} f(E) = -\infty$ and remember that, by assumption, $\alpha + \beta \leq 1$ and consequently $\beta/(1-\alpha) \leq 1$. Furthermore, notice that $f_1(E_M) > f_2(E_M)$ holds for:

$$\bar{E} > \Omega\theta^{1-\frac{1-\alpha}{\beta}} + \theta^{\frac{1-\alpha}{\beta}} + \varepsilon\theta^{1-\alpha-\frac{1-\alpha}{\beta}} = \theta^{\frac{1-\alpha}{\beta}}\left(1 + \delta\theta^{-2\frac{1-\alpha}{\beta}}\right) = E_M + \frac{\delta}{E_M}$$

Such a condition holds if and only if $f_1(E)$ and $f_2(E)$ have two intersection points, one (which is clearly of type A) on the left of the vertical line $E = E_M$ and the other (of type B) on the right of it. In other words, (22) is a necessary and sufficient condition for the existence of a unique equilibrium point with $E, K > 0$.

Two internal equilibrium points can exist only if condition (22) does not hold. When this is the case, a sufficient condition for the existence of two equilibrium points is $f_1\left(\bar{E}/2\right) > f_2\left(\bar{E}/2\right)$ and $\bar{E}/2 < E_M$, i.e.:

$$\left(\frac{\bar{E}}{2}\right)^2 - \Omega\left(\frac{\bar{E}}{2}\right)^{\frac{\beta}{1-\alpha}} > \varepsilon\theta^{1-\alpha} \text{ and } \frac{\bar{E}}{2} < E_M$$

where $\bar{E}/2$ is the value of E maximizing $f_1(E)$.

Let us now prove the sufficiency of condition (23). If $\Omega > 0$, then the graph of $f_2(E) = \Omega E^{\beta/(1-\alpha)}$ lies above the E-axis, therefore a sufficient condition for the non existence of equilibrium points can be obtained by imposing that:

$$f_1\left(\frac{\bar{E}}{2}\right) = \frac{\bar{E}^2}{4} - \varepsilon\theta^{1-\alpha} \leq 0$$

where $f_1\left(\bar{E}/2\right)$ is the maximum of $f_1(E)$; such a condition is satisfied for $\bar{E} \leq 2\left(\varepsilon\theta^{1-\alpha}\right)^{1/2}$.

If $\Omega < 0$, then the graph of $f_2(E)$ lies below the E-axis and therefore a sufficient condition for the non existence of equilibrium points can be obtained by imposing that:

$$f_1\left(\frac{\bar{E}}{2}\right) \leq f_2(E_M)$$

where $f_2(E_M) = \delta - \varepsilon\theta^{1-\alpha}$ is the minimum of the function $f_2(E)$ in the interval $[0, E_M]$. Such a condition is satisfied for $\bar{E} \leq 2\delta^{1/2}$. ■

A.2 Proof of Proposition 2

Proof. To prove this proposition, remember that the equilibrium point $(0, K_0)$ coincides with the intersection between the $\dot{K} = 0$-isocline and the K-axis. Since $\dot{K} < 0$ holds above this isocline, all trajectories crossing the side with $K \leq \theta$ of the rectangle Q enter Q. Analogously, since $\dot{E} < 0$ for $E = \bar{E}$, all trajectories crossing the side with $E = \bar{E}$ of Q enter Q. Furthermore, notice that every rectangle R containing Q is a positively invariant set. This implies, by the Poincaré-Bendixson Theorem, that every trajectory in R has as ω-limit set either an equilibrium point or a limit cycle surrounding the equilibrium point of type B. Since all equilibrium points belong to the rectangle Q and a limit cycle, if existing, must lie in the interior of Q, the proposition is proven. ■

A.3 Proof of Proposition 3

Proof. Linearizing the dynamic system (10)-(11) around the internal equilibrium points, and taking account of the fact that from (12) it follows that for all equilibrium points of this type it must be true that:

$$E^{\frac{\beta}{1-\alpha}} + K = \theta$$

we obtain the following expressions for the elements of the Jacobian matrix J:

$$\frac{\partial \dot{E}}{\partial E} = \bar{E} - 2E - \frac{\beta}{1-\alpha}\left[\frac{\delta}{\theta^2} - \frac{\alpha\varepsilon}{\theta^{1+\alpha}}\right] E^{\frac{\alpha+\beta-1}{1-\alpha}} K$$
$$\frac{\partial \dot{E}}{\partial K} = \frac{\delta}{\theta^2} E^{\frac{\beta}{1-\alpha}} + \frac{\alpha\varepsilon}{\theta^{1+\alpha}} K - \frac{\varepsilon}{\theta^{\alpha}}$$
$$\frac{\partial \dot{K}}{\partial E} = -\frac{\alpha\beta s}{\theta^{1+\alpha}} E^{\frac{\alpha+\beta-1}{1-\alpha}} K < 0$$
$$\frac{\partial \dot{K}}{\partial K} = -\frac{\alpha s(1-\alpha)}{\theta^{1+\alpha}} K < 0$$

Then, by substituting $K = \theta - E^{\beta/(1-\alpha)}$, the determinant of J can be written as follows:

$$\text{Det}J = -(1-\alpha)\left(\bar{E} - 2E\right) + \frac{\beta}{\theta}\left(\delta - \varepsilon\theta^{1-\alpha}\right) E^{\frac{\alpha+\beta-1}{1-\alpha}}$$

which is such that $\text{Det}J < 0$ if $f_1'(E) > f_2'(E)$. Consequently, the equilibrium point of type A is always a saddle point while that of type B may be locally attractive or repulsive.

Notice that, since $\partial \dot{K}/\partial K < 0$ always holds, the equilibrium point of type B can be repulsive only if $\partial \dot{E}/\partial K > 0$. Let us now analyze the local stability of $(E_1, 0)$ and $(E_2, 0)$. It is easy to check that the Jacobian matrix of the dynamic system (10)-(11), evaluated at $(E_1, 0)$, is a triangular matrix with eigenvalues $\bar{E} - 2E_1 = \sqrt{\bar{E}^2 - 4\delta} > 0$ (in direction of the E-axis) and $[s(1-\alpha)/E_1^{\alpha\beta/(1-\alpha)}] - d$ (> 0 for $E_1 < E_M$) while the Jacobian matrix evaluated at $(E_2, 0)$ is a triangular matrix with eigenvalues $\bar{E} - 2E_2 = -\sqrt{\bar{E}^2 - 4\delta} < 0$ (in direction of the E-axis) and $[s(1-\alpha)\gamma^{1/(1-\alpha)}/E_2^{\alpha\beta/(1-\alpha)}] - d$ (> 0 for $E_2 < E_M$). ■

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