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Ellsberg Rules and Keynes's State of Long-Term Expectation: More Than an Accordance

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Abstract - This paper advances an intuitive way to represent Keynes's notion of long-term expectation. The epsilon-contamination approach is introduced and a rational and coherent decision rule is derived. The result is evidence that Ellsberg and Keynes share the notion of uncertainty and adopt the same class of decision rules.

Keywords: Keynes, Ellsberg, expectation, consensus distribution, uncertainty, epsilon-contamination.

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1. Introduction

Economic literature reports a large debate about the correct interpretation of Keynes's notion of long-term expectation, which determines investment decisions, between mainstream theory (see Lucas 1980; Begg 1982; Sargent 1983) and radical critique of Keynesian fundamentalists (see Lawson 1985; Carabelli 1988; O'Donnell 1989; Gerrard 1994; Dow 1995). This article points out a different interpretation of Keynes's theory of long-term expectation that is intimately related to his notion of uncertainty as different from mathematical risk.

Keynesian fundamentalists focused on Keynes's contributions to differentiate uncertainty from risk, but different interpretations emerge among them. These interpretations can be summarized by Lawson's and Gerrard's viewpoints that earlier appeared in the *Economic Journal*. Lawson (1985, 926) 'focused upon Keynes' conclusion that it is generally impossible, even in probabilistic terms, to evaluate the future outcomes of all possible current actions a viewpoint that is integral to Keynes' interpretation of uncertainty... [he]…attempted to assess the analytical implications of this conclusion allowing for the observation that people are highly knowledgeable of current institutions and social practices'. Gerrard (1994, 335) introduces a functional form to represent the Keynesian Uncertainty Hypothesis (KUH), according to which 'the propensity to act on an expectation depends on the credence of the expectation where credence reflects the agent's assessment of the adequacy of the available evidence' that is, the behavioural function that 'allows for the possibility that changes in the available evidence may affect behaviour by operating on expectations or credence or both'. If Lawson's viewpoint refers to a notion of uncertainty that evokes numerically indeterminate or non-comparable probability relations, on the contrary, Gerrard's KUH seems suggestive of a new line of research focused on the rational degree of beliefs and the weight of an argument. In fact, Gerrard puts in evidence that Keynes distinguishes short-term expectation based on certainty equivalent modelling strategy, and long-term expectation, 'upon which our decisions are based, does not solely depend, therefore, on the most probable forecast we can make. It also depends on the *confidence* with which we make this forecast—on how highly we rate the likelihood of our best forecast turning out quite wrong. If we expect large changes but are very uncertain as to what precise form these changes will take, then our confidence will be weak' (1936, 133). Under the KUH, Gerrard summarizes Keynesian long-term expectation through the behavioural function: $x(t) \equiv [s^e(T), \delta(T)]$, where $s^e(T)$ is the rational expectation of future states of the world (Ω) and $\delta(T)$ is the credence of $s^e(T)$, and emphasizes that 'credence represents the weight of evidence which, in turn, determines the completeness of the information set and the shape and fuzziness of the probability distribution' (Gerrard 1995, 335).

This paper posits a simple and direct approach to representing Keynes's rational behaviour in a scenario characterised by imprecise knowledge. It is suggested that uncertainty in prior distributions about expected value of an asset or long-term expectations can be modelled by using a class of epsilon-contaminated probabilities, where the parameter $\varepsilon \in [0,1]$ indicates the lack of information about the relevant odds. That is, instead of completely committing to the elicited probability distribution, an economic agent assumes that elicited probability distribution may not be not the right prior, and its lack of reliability is revealed by the parameter epsilon. The epsilon-contamination (*ε-contamination*) approach can easily encompass Gerrard's behavioural function derived from Keynes's analysis.

Through the ε-contamination approach, it appears that Ellsberg's optimal decision rules are only a case of the Keynes's rational decision rule and modern decision theory under ambiguity the tight development of Keynes model of rational behaviour. The paper is organized as follows: Section 2 discusses Ellsberg's rules; Section 3 provides a discussion of Keynes's theory of long-term expectation; Section 4 introduces the epsilon-contamination approach; Section 5 includes the re-interpretation of Ellsberg's and Keynes's viewpoints; Section 6 provides concluding remarks.

2. Ellsberg's Rules

The publication of 'Risk, Ambiguity, and the Savage Axioms' in the *Quarterly Journal of Economics* (1961, 646) is a crucial turning point in decision theory. Ellsberg gives a clear definition of his aim: 'I propose to indicate a class of choice-situations in which many otherwise reasonable people neither wish nor tend to conform to the Savage postulates, nor to the other axiom sets that have been devised'. Ellsberg (656) calls this class of situations where the decision maker faces uncertainties that are not risks 'some situations in which the Savage axioms do not seem so plausible'. Ellsberg elucidates the decision making process under uncertainty by using gambling choices as examples; indeed, the legendary hypothetical urn experiments with either two-colour problems (violation of Insufficient Reason Principle) or three-colour problems (violation of Sure-thing Principle or Savage P2 axiom) describe the Ellsberg Paradox. The Ellsberg Paradox describes decision makers who deliberately violate the maximization of the expected utility and show reversal preference when probabilities are indeterminate.[1] Ellsberg's thought experiments resulted in

[1] It is worth noting that Keynes (1921, 75) and also Knight (1921, 111) use the urn with the two-colour problem to represent situations that are uncertain but not risky (Ellsberg 1961, 653). In Chapter 6 of the *Treatise on Probabilities* (the weight of arguments), distinguishing between the weight of arguments and probable error, Keynes makes use of cases of black and white balls drawn from an urn when they are in equal proportions or the proportion of each colour is

evidence of choices that represent the intentional and persistent violation of any normative and descriptive criteria for decision making under uncertainty, when decision makers 'are not "minimaxing", nor are applying a "Hurwicz criterion," maximizing a weighted average of minimum and maximum pay-off for each strategy' (1961, 656). Ellsberg (657) asserts that violators' behaviour does not depend on the relative desirability and likelihood of consequences, but rather derives from 'a third dimension of the problem of choice: the nature of one's information concerning the relative likelihood of events. What is at issue might be called the ambiguity of this information, a quality depending on the amount, type, reliability and "unanimity" of information, and giving rise to one's degree of "confidence" in an estimate of relative likelihoods'. Ambiguity encompasses situations between ignorance and risk when a decision maker has more than one probability distribution in mind, none of which is considered fully reliable or sure, even if a subset of them (Y^0) can be considered more reasonable. Ellsberg (667) considers violator behaviour as conservatism, where each subject 'chooses to act "as though" the worst were somewhat more likely than his best estimates of likelihood would indicate…[the subject] "distorts" his best estimates of likelihood, in the direction of increased emphasis on the less favorable outcomes and to a degree depending on ρ, his confidence in his best estimate'. For each action (act) $x \in X$, payoffs (consequences) are evaluated with the functional $I(x)$ induced by a decision rule derived from the Restricted Bayes Criterion (Hodges and Lehmann 1952), a convex combination with respect to the parameter $\rho \in [0,1]$, which reveals the degree of confidence between the expected payoff corresponding to the best estimated distribution y^0 and its minimum expected payoff, with respect to the set of reasonable probability distributions Y^0 (security level): $I(x) = [\rho y^0 + (1-\rho) y_x^{min}](x)$.

In his doctoral dissertation 'Risk, Ambiguity and Decision', submitted to the Economics Department of Harvard University in April 1962, Ellsberg introduces the Restricted Bayes/Hurwicz Criterion as a guide for decision making under ambiguity, which amends some deficiencies of the Restricted Bayes Criterion recommended in his 1961 article. In the Restricted Bayes/Hurwicz Criterion, Ellsberg conjugates the Hurwicz optimism-pessimism rule, and the Restricted Bayes Criterion, which reflects the degree of confidence about the reasonable set of probability distributions that is, 'after you have eliminated certain possible probability distributions over the states of the world as unacceptable representation of your opinions, leaving a set of reasonably acceptable distributions, you force yourself to make further comparisons, probing your less definite options' (2001, 191). If the parameter $\alpha \in [0,1]$ expresses the decision maker's

unknown. Keynes (1921, 75) remarks that 'it is evident that in either case the probability of drawing a white ball is ½ but that the weight of the argument in favor of this conclusion is greater in the first case'.

optimism/pessimism coefficient (ambiguity attitude), the parameter $\rho \in [0,1]$ reveals the degree to which information is perceived as ambiguous. Explicitly referring to Knight and Keynes,[2] Ellsberg (2001, 193) specifies the reason for his determination to distinguish the 'relative influence of favorable versus unfavorable possibilities…dependent upon the personality (and perhaps, official "role" and overall situation) of the decision-maker', symbolized by the parameter α, and the degree of confidence or definiteness of opinions,[3] reflected in the parameter ρ. As a result, the Restricted Bayes/Hurwicz Criterion emerges as the decision rule that combines Bayesian and α-minimax principles and generates all choice patterns, observed in the urn experiments, that Ellsberg considers reasonable and rational. Applying the Restricted Bayes/Hurwicz Criterion, each act $x \in X$ is evaluated by the functional $I(X){:}\, I(x) = \{\rho y^0 + (1-\rho)\left[\alpha y_x^{max} + (1-\alpha) y_x^{min}\right]\}(x)$.

3. Keynes's Theory of Long-Term Expectation

In Chapter 12 of *The General Theory of Employment, Interest and Money*, Keynes claimed that 'the state of long-term expectation, upon which our decisions are based, does not solely depend, therefore, on the most probable forecast we can make. It also depends on the *confidence* with which we make this forecast on how highly we rate the likelihood of our best forecast turning out quite wrong. If we expect large changes but are very uncertain as to what precise form these changes will take, then our confidence will be weak' (1936, 133). Keynes explicitly affirms 'it would be foolish, in forming our expectations, to attach great weight to matters facts about which we feel somewhat confident, even though they may be less decisively relevant to the issue than other facts about which our knowledge is vague and scanty' (148). However in financial markets, even if the state of

[2] 'Many writers, including Frank Knight and Lord Keynes, have insisted upon the feasibility and relevance of this sort of judgment, without indicating precisely how it might affect decision-making; we shall consider [it] a meaningful role' (2001, 193).

[3] Ellsberg (2001, 192) clarifies what he means by confidence or definitiveness through the example of a 'decision-maker who relies upon a panel of experts to guide his official opinions, and who finds in particular case that each consultant produces a different, definitive probability distribution. It may be convenient to think of some of the members of Y^0 in the concrete form of probability distributions each written down on a separate piece of paper with the name of the forecaster attached. For simplicity, we may imagine a decision-maker who is compelled, in some sense, to base his own opinions, and hence his action, upon this set of conflicting forecasts…In the end, we can imagine this decision-maker evolving a particular distribution y^0 over the relevant events, representing his own "best guess" opinions on all these questions that may influence, directly or remotely, his judgments of the relative probabilities of those events…[H]is occasion (or frequent) failure to *act upon y^0 exclusively* reflects *another* sort of judgment, concerning the reliability, credibility or adequacy of his information, experience, advice, intuition taken as a whole, not about the relative support it may give to one hypothesis as opposed to another, but about its ability to lend support to any hypothesis—any set of definitive opinions—at all'.

confidence is crucial in determining the rate of investment, representing the reliability of knowledge about a vague and distant future, "practical men" need to act.

Crucially, what is and how does the state of confidence change ? Keynes (1936, 150) observes that 'knowledge of the factors which will govern the yield of an investment some years hence is very slight and often negligible', even in a very short period. Different from heroic times, when investment 'was partly a lottery, though with the ultimate result largely governed by whether the abilities and character of the managers were above or below the average', when the separation between ownership and management prevails, then 'certain classes of investment are governed by the average expectation of those who deal on the Stock Exchange as revealed in the price of shares, rather than by the genuine expectations of the professional entrepreneur'. Keynes condenses the process that induces the change of convention in the famous metaphor of financial markets as a newspaper beauty contest in which 'the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view. It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest' (1936, 155). Keynes maintains that an investor does not have to anticipate what will be the fundamental value of a firm in the future, but rather should estimate other investors' value. The individual estimated value is different from 'the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities'; rather, it depends on how *animal spirits* and competence decode-read it. In fact, to make an investment decision, 'we are assuming, in effect, that the existing market valuation, however arrived at, is uniquely *correct* in relation to our existing knowledge of the facts which will influence the yield of the investment, and that it will only change in proportion to changes in this knowledge; though, philosophically speaking it cannot be uniquely correct, since our existing knowledge does not provide a sufficient basis for a calculated mathematical expectation. In point of fact, all sorts of considerations enter into the market valuation which are in no way relevant to the prospective yield' (152).

Keynes considers the stock exchange ruled by professional investors and speculators who are forced to anticipate the mass psychology of the market that is, to inform and foresee 'changes in the conventional basis of valuation a short time ahead of the general public'. As a consequence, the behaviour of professional investors and speculators is the result of two different components: 'the average expectation of those who deal on the Stock Exchange as revealed in the price of shares' and the competence 'to anticipat[e] what average opinion expects the average opinion to be' (1936, 157–158).

In 'The General Theory of Employment,' published in the *Quarterly Journal of Economics* in 1937, Keynes definitively elucidates how to replace the Benthamite calculation (mathematical expectation) with a behaviour which allows rational, economically inclined men to save face. He suggests that such a practical theory should be founded on three main principles: '(1) We assume that the present is a much more serviceable guide to the future than a candid examination of past experience would show it to have been hitherto. In other words we largely ignore the prospect of future changes about the actual character of which we know nothing. (2) We assume that the existing state of opinion as expressed in prices and the character of existing output is based on a correct summing up of future prospects, so that we can accept it as such unless and until something new and relevant comes into the picture. (3) Knowing that our own individual judgment is worthless, we endeavor to fall back on the judgment of the rest of the world which is perhaps better informed. That is, we endeavor to conform with the behavior of the majority or the average. The psychology of a society of individuals each of whom is endeavoring to copy the others' leads to what we may strictly term a conventional judgment' (1937, 214).

An interpretation of how "practical men" need to act can be derived by Keynes's trading activity as manager of the King's College endowment. As an investor, Keynes 'was able to construct idiosyncratic portfolios, concentrating on relatively few sectors and adopting a small cap bias…[he] eschewed extreme diversification in favour of large exposures to securities that reflected his preferences and skills' (Chambers and Dimson 2012, 40). Acting as an investor who beats the market, Keynes rejects conventional approach, putting in evidence an investment strategy that, according to his credit-cycling approach, exhibits overconfidence, home bias effect, and disposition effect. In fact 'as time goes on, I get more and more convinced that the right method of investment is to put fairly large sums into enterprises which one thinks on knows something about and in the management of which one thoroughly believes' (Keynes CWK XII, 57).

4. The Epsilon-Contamination Approach

As noted by Ellsberg (1961), Hodges and Lehmann (1952, 396) introduce an 'approach to the problem of optimal decisions that utilizes the available information but at the same time provides a safeguard in case this information is not correct', such that 'instead of minimizing the maximum risk it is proposed to restrict attention to decision procedures whose maximum risk does not exceed the minimax risk by more than a given amount'. Hodges and Lehmann define which sufficient conditions make their decision procedure a Restricted Bayes Criterion (Bayes solution subject to the constrain of the minimax solution) that is, a convex combination with respect to the parameter $\rho \in (0,1]$, which indicates confidence in the available information. Using Ellsberg's notation, Hodges and Lehmann's decision rule evaluates the feasible action $x \in X$ by the functional $I(x)$: $I(x) = [\rho y^0 + (1-\rho)y_x^{min}](x)$.

Hodges and Lehmann's approach can be considered an ancestor (another is Good [1950]), of the so-called epsilon-contamination (*ε-contamination*) structure in the empirical Bayes analysis of individual beliefs or robust Bayesian analysis. The Robust Bayesian viewpoint affirms that one of the main justifications for using Bayesian analysis is a belief that prior distributions can never be quantified or elicited exactly (i.e., without error), especially in a finite amount of time (Berger 1984). Crucially, if this assumption is enough for Frequentism to reject Bayesian analysis, for a Bayesian statistician it only 'precludes the obvious Bayesian solution of writing down a single prior distribution and doing a Bayesian analysis. Instead, the viewpoint is essentially that one should strive for Bayesian behavior which is satisfactory for all prior distributions which remain plausible after the prior elicitation process has been terminated. I call this the robust Bayesian viewpoint' (Berger 1984, 65).

On this viewpoint, an attractive method of modelling uncertainty in the prior distribution is through use of ε-contamination classes that is, classes of distributions which have the form $\pi = (1-\epsilon)\pi_0 + \epsilon q$, where π_0 is the elicited prior, q is a contamination or perturbation of π_0, and ε reflects the amount of error in π_0 that is considered possible. Then ε-contamination emerges as a robust Bayesian method to quantify, in terms of a class of possible distributions, how partial and incomplete is the subjective information encompassed in a single prior distribution. In fact, 'quantification of prior beliefs can never be done without error, and hence that one is left at the end of the elicitation process with a set Γ of prior distributions which reflect true prior belief; i.e., π_T is an unknown element of Γ' (Berger 1984, 73).[4] From the perspective of this paper, the fact that partial prior knowledge or worry about prior misspecification is modelled with a class Γ of priors makes the Robust Bayes approach the natural candidate for representing Keynes's long-term expectation approach.

[4] The class Γ of prior distributions such that $\Gamma = \{\pi: \pi = (1-\epsilon)\pi_0 + \epsilon q\}$, where $\varepsilon \in [0,1]$ is given, π_0 is a particular prior distribution, and $q \in Q \subseteq P$ is a probability distribution in some subset Q of the probability space P, has some interesting properties: it is surprisingly easy to work with it and is very flexible through the choice of Q (Berger and Berliner 1982).

5. Keynes and Ellsberg from the Epsilon-Contamination Viewpoint

In standard financial theory, under the efficient-market hypothesis, costly arbitrage (transaction and holding costs) and the actions of marginal investors who correct noise make the stock price equal to the discounted expected value of future payoffs. If an asset price is the discounted expected value of future payoffs, it has the martingale property what average opinion expects the average opinion to be are redundant, because of the law of iterated expectations[5] and the Bayes rule.[6]

Keynes considers investors' long-term expectations about an asset as the combination of the asset price, the sum of future prospects, and estimation of other investors' asset value. An asset price is interpreted as the expected value that is, price times probability at each future state of the world. Probability distribution of the asset price emerges from the set of investor probability distributions about future asset value. Because of uncertainty, it can be assumed that each investor does not have a unique prior on states of the world, but rather a finite set of probability distributions (multiple priors), none of which is considered fully reliable.[7] Keynes states that an individual asset evaluation 'also depends on the *confidence* with which we make this forecast' (1936, 148). He believes that in financial markets, two different attitudes, corresponding to different classes of investors, coexist: speculators who are concerned 'not with making superior long-term forecasts of the probable yield of an investment over its whole life, but with foreseeing changes in the conventional basis of valuation a short time ahead of the general public' and skilled individuals who act on the base of 'the best genuine long-term expectations' (153–154).

From this perspective, long-term expectations can be considered as the result of the combination, through the degree of confidence, of the probability distribution that represents expectation of future payoffs (γ_0) and the probability distribution that characterizes investors' most reliable evaluation of the asset (Γ). That is, an investor's long-term expectations can be thought of as the parametric combination, with respect to the error that is deemed possible, of the prior that

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[5] The law of iterated expectations is the foundation of many theorems in applied statistics. The basic statement is as follows: $E(Y) = E(E(Y|X))$.

[6] Allen et al. (2006, 741) show 'that the law of iterated expectations fails to hold for average opinion when there is differential information. It is not the case that the average expectation today of the average expectation tomorrow of future payoffs is equal to the average expectation of future payoffs'. Allen et al. explain that 'the noisy rational expectations model with short-lived traders exhibits the following features: prices reflect average expectations of average expectations of asset returns; prices are overly sensitive to public information; and traders underweight their private information'.

[7] Basili and Zappia (2009).

represents the actual convection and the probability distribution that characterises an alternative possible common evaluation of the asset.[8]

In the class of possible distributions, the investor could extract another distribution that in her opinion better represents investors' asset value, or what she considers the consensus distribution. Keynes's problem of assessing conventional judgment can be represented as the problem of aggregating probability distributions of investors, finding proper pooling methods to not only discover other opinions, in this case asset value, but also to judge how well informed they are. In short, Keynes's issue is how to form and elicit a consensus distribution.[9]

If the parameter $\varepsilon \in [0,1]$ expresses the individual confidence in market evaluation or ε-contamination of confidence, each investor's expectation can formally be denoted by $\gamma = \{\varepsilon\gamma_0 + (1-\varepsilon)\Gamma\}$. The investor's expectation reveals that she is $\varepsilon \times 100\%$ certain that the uncertainty she faces is summarized by the market price probability distribution, but at the same time she is aware that with $(1-\varepsilon) \times 100\%$ chance, uncertainty could be better represented by another probability distribution in the set of all reasonable evaluations of other investors' true asset value. In sum, the ε-contamination interpretation of investors' expectations allows describing imprecision of a priori knowledge and the behavioural effect of its awareness.[10]

Significantly, in legitimating the Robust Bayesian approach, Berger (1984, 73) affirms that 'another situation in which working with a class of priors is clearly unavoidable is when group conclusions or decisions must be made and the prior of all members of the group must be considered…. The issue of scientific communication is related to this, the (often unattainable) ideal

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[8] 'This battle of wits to anticipate the basis of conventional valuation a few months hence, rather than the prospective yield of an investment over a long term of years, requires no gulls amongst the public to feed the maws of the professionals; it can be played by professionals amongst themselves. Nor is it necessary that anyone should keep his simple faith in the conventional basis of valuation having any genuine long-term validity' (Keynes 1936, 155).

[9] The Bayesian axiomatic approach to consensus distribution would not appear satisfying, not even in the sophisticated versions (copula models) and elicitation based on behavioural combination approaches (e.g., DeGroot and Montera 1991). If investors' opinions are not all independent and equally likely, each investor has to cope with ambiguity and stochastically dependent evaluations. As a consequence, each investor could calibrate the aggregation of investors' opinions through her confidence or degree of belief by pooling methods based on Dempster's rule of combination or theory of evidence, combination rules based on possibility distributions and fuzzy measures, or aggregation based on multiple priors or capacity (DeMiguel et al. 2009; Huang 2010; Basili and Chateauneuf 2011, 2013; Basili and Pratelli 2013; Boyle et al. 2012).

[10] Basili and Chateauneuf (2011) suggest a multiple quantile utility model where the notion of reliability and error in the elicitation of the prior distribution is represented through two ϵ-contaminations weighted by the asymmetric ambiguity attitude with respect to gains and losses. Cardin (2012) generalizes that approach by introducing classes of aggregation function that extend Choquet and Sugeno integrals.

being that of presenting a conclusion which would be the conclusion for any reasonable prior that a user of the information might have'.

The previous interpretation of financial assets pricing might appear a courageous vision of Keynes's idea of long-term expectation. Nonetheless, if some recent unconventional interpretations of Keynes's Boolean algebra-logic approach to the construction of probability interval estimates, such as Hailperin (1986) and Brady (1993), are considered, then astonishment vanishes. In a recent paper, Arthmar and Brady (2012) show that 'Keynes provided a complete mathematical structure for his system of probability, which he called approximation, in the *A Treatise on Probability*... he provided a solid mathematical structure for his nonlinear, non-additive decision theory approach'. From this critical stance, the Keynesian conventional coefficient of risk and weight c[11] is the probability weighting function in cumulative prospect theory (Tversky and Kahneman 1992).[12] It is well known that Choquet Expected Utility (CEU) is equivalent to cumulative prospect theory[13] and that CEU can be represented[14] by the parametric structure of ε-contamination of probability distributions. In fact, a representation with ε-contamination complies with CEU[15] and a capacity υ corresponds to the ε-contamination of a probability $\pi \in \Pi$ if given a set Ω of states of the world, the set Σ of all subsets if for all $A \subseteq \Sigma$: $\upsilon(A) = (1-\varepsilon)\pi(A)$ if $A \neq \Sigma$ and $\upsilon(A) = 1$ if $A = \Sigma$.

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[11] The conventional coefficient of risk and weight $c = 2pw/[(1+q)(1+w)]$, where p is the probability of success, q is the probability of failure, such that $p + q = 1$, and w is the weight of the evidence.

[12] Arthmar and Brady (2010) states that 'it is a simple case of arithmetic to obtain the same solutions provided by the majority of the K-T experimental subjects in the following categories of decision problems: (a) certainty effects, (b) reflection effects, (c) translation effects, (d) Allais paradox effects, and (e) preference reversal effects. The crossover points, relative to the p axis and the weighting function axis, $\pi = f(p)$, where the K-T weighting function π is a function of p, are obtained easily by taking linear combinations of p, $p[(1/(1+q)]$, and $p[(1+q)]$. So, $p[(1/(1+q)]$ generates a convex curvature while $p[(1+q)]$ generates a concave one. Linear combinations of these two different curvatures (first convex then becoming concave or *vice versa*) result in *S*-shaped curves that cross over the 45 degree line specifying where $\pi(p)=p$. For example, an expression such as $ap[(1/(1+q)]+(1-a)p[(1+q)]$, where a and $(1-a)$ sum to one, generates one of many different, possible such *S*-shapes. Three-dimensional graphics are easily obtained by using the *Mathematica* program'.

[13] Tversky and Wakker (1993); Basili and Chateauneuf (2011).

[14] Nishimura and Ozaki (2006).

[15] Formally, let $\Omega = \{\omega 1, \dots, \omega n\}$ be a non-empty set of states of the world and let $\Sigma = 2^{\Omega}$ be the set of all events. A function $\upsilon: \Sigma \rightarrow \mathbb{R}_{+}$ is a non-necessarily additive probability measure, or a capacity, if (I) $\upsilon(\emptyset) = 0$, (ii) $\upsilon(\Omega) = 1$ and if (iii) for all $A, B \epsilon \Sigma$ such that $B \subseteq A, \upsilon(A) \geq \upsilon(B)$. A capacity is said to be convex (super-additive) if (iv) $\upsilon(A \cup B) \geq \upsilon(A) + \upsilon(B) - \upsilon(A \cap B)$. The Choquet Integral (Choquet 1954) of an act $x \epsilon X$ with respect to a capacity υ is $\int x d\upsilon = \int_0^{\infty} \upsilon(\{\omega \epsilon \Omega \mid x(\omega) \geq t\})dt + \int_{-\infty}^{0} [\upsilon(\{\omega \epsilon \Omega | x(\omega) \geq t\}) - 1]dt$.

It is worth pointing out that the investor's expectation $\gamma = \{\varepsilon\gamma_0 + (1-\varepsilon)\Gamma\}$ can be considered as a functional that generalizes Gerrard's KUH form.[16] In fact, along with the expectations about future outcomes weighted by the credence attached to them, as Gerrard assumes, the ε-contamination representation also includes the expectations elicited through the consensus distribution. Thus the ε-contamination representation of long-term expectation gives a functional form to Keynes's contrast between actual beliefs and reasonable beliefs, which appears in *A Treatise on Probability* and makes evident continuity and coherence of his notion of probability in *The General Theory of Employment, Interest and Money* (e.g., Carabelli and Cedrini 2013).

The representation of the Ellsberg Paradox with relation to ε-contamination is direct and axiomatised by some authors (Nishimura and Ozaki 2006; Chateauneuf et al. 2007 and Kopylov 2009). Crucially, Ellsberg defines what could be considered as the set of reasonable acceptable distributions in the case of an urn that contains 90 balls of three possible colours. Ellsberg introduces a representation that can be easily depicted as the ε-contamination of the probability distribution considered more reasonable in the set Δ of all possible probability distributions defined for the urn. The decision maker sorts gambling choices (prospects or lotteries) by a 'mixture of the worst belief in the set Δ with the probability measure $p \subseteq \Delta$. It is natural to interpreter p as the decision-maker ex ante probabilistic belief' (Kopylov 2009, 201). Nishimura and Ozaki, Chateauneuf et al. and Kopylov give different sets of axioms by which investors' preferences with regard to uncertain acts (prospects or lotteries) can be represented by the CEU with the ε-contamination of confidence.

Until now, decision theorists have simply assumed that Keynes is the noble father of current decision theory, since he confirms and shows the distinction between risk and uncertainty. This paper shows that current decision rules, which formalize the decision-maker attitude with respect to ambiguous events by a capacity or a set of multiple priors, evaluate feasible acts by the Choquet integral, and solve the Ellsberg Paradox (Schmeidler 1989, Gilboa and Schmeidler 1994), are analogous to Keynes's decision rule when it is interpreted in terms of the ε-contamination of confidence.

6. Conclusion

This paper proposes a different interpretation of Keynes's theory of long-term expectation based on the ε-contamination approach of probability distributions. The suggested interpretation sharpens the relationship between Keynes's and Ellsberg's representations of uncertainty as

[16] Gerrard's behavioural function $x(t) \equiv [s^e(T), \delta(T)]$ can be represented by the capacity υ obtained by the ε-contamination of the probability distribution $\gamma \in \Gamma$, such that $\upsilon(A) = (1-\varepsilon)\gamma(A)$, $\forall A \neq \Sigma$ and $\varepsilon \in [0,1]$.

distinguished from risk: they share the same notion. This fact makes Ellsberg's urn example not a simple accordance or chance but, rather, a unique representation of the same intuition. The ε-contamination interpretation of Keynes's long-term expectation theory makes direct and explicit the relationship between his decision rule and contemporary decision theory originated by the Ellsberg Paradox, and illuminates a long diatribe, dated to Ramsey, on Keynes's approach to probability and its use in science. It is possible to affirm that the Bayesian approach to Ramsey, based on additive probability, is only a special case of Keynes's general (non-additive) theory of probability. Remarkably, the novel representation of long-term expectation sheds new light on Keynes's view of stock exchanges like casinos, where speculators make the market (sequential trades in thin markets, algorithmic trading, high frequency trading, over-the-counter trading, etc.), and induce herding behaviour in bounded rational investors. Finally, the new viewpoint appears to be coherent with Keynes's own trading activity as documented in Chambers and Dimson (2012).

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