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Abstract

The linearisation approach to approximating variance of complex non-linear statistics is a well-established procedure. The basis of this approach is to reduce non-linear statistics to a linear form, justified on the basis of asymptotic properties of large populations and samples. For diverse cross-sectional measures of poverty and inequality such linearised forms are available, though the derivations involved can be complex.

Replication methods based on repeated resampling of the parent sample provide an alternative approach to variance estimation of complex statistics from complex samples. These procedures can be computationally demanding but tend to be straightforward technically. Perhaps the simplest and the best established among these is the Jackknife Repeated Replication (JRR) method.

Recently the JRR method has been shown to produce comparable variance for cross-sectional poverty measures; and it has also been extended to estimate the variance of longitudinal poverty measures for which Taylor approximation is not currently available, or at least cannot be easily derived. This paper extends the JRR methodology further to the estimation of variance of differences and averages of poverty measures. It illustrates the application of JRR methodology using data from first four waves of the ECHP for Italy. JRR variance and design effect for a number of cross-sectional poverty indicators are computed and compared with those obtained from linearisation methodology. For cross-sectional measures design effect can be decomposed into the effect of clustering and stratification, and that of weighting under both methodologies. For differences and averages of these poverty measures JRR method is applied to compute variance and three separate components of the design effect – effect of clustering and stratification, effect of weighting, and an additional effect due to correlation of different cross-sections from panel data – combining these the overall design effect can be estimated.

Keywords: variance; linearisation; Jackkinfe; ECHP; poverty; design effect; weighting.

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1. Introduction

Often, linearisation methods are used to estimate standard error of poverty and inequality measures. The basis of this approach is to reduce non-linear statistics to a linear form, justified on the basis of asymptotic properties of large populations and samples. For diverse cross-sectional measures of poverty and inequality such linearised forms are available, though the derivations involved can be complex. However, for a range of complex measures of poverty and inequality based on panels or other repeated cross-sectional samples, the linearised form is not currently available. For these measures the linearisation method cannot be used for variance estimation. Replication methods based on repeated resampling of the parent sample provide a practical approach to variance estimation in such cases. These procedures can be computationally demanding but tend to be straightforward technically. Perhaps the simplest and the best established among these is the Jackknife Repeated Replication (JRR) method.

This paper provides practical details and results of the application of JRR methodology for the estimation of the variance of differences and averages of poverty measures from different waves of a panel. It is important to note that linearised forms for the estimation of variance for differences and averages of poverty and inequality measures are not currently available. It has been reported in Verma and Betti (2005a) that for the cross-sectional measures of poverty and inequality linearisation and JRR methods provide comparable estimates of variance. It has also been shown in Gagliardi *et al.* (2006) that the JRR methodology can be extended to estimate the variance for longitudinal measures of poverty, such as individuals' persistence to remain in the state of poverty over a period of time. This paper extends the JRR methodology further to the estimation of variance of net differences and averages over time of cross-sectional poverty and inequality measures. We use data from the first four waves of the European Community Household Panel (ECHP) survey for Italy, and construct measures of net differences of poverty indicators between waves and of averages of poverty indicators over two or more waves. For differences and averages of poverty measures JRR method is also applied to compute design effects. It cannot be estimated directly, but it is possible to compute three components into which design effect can be broken down – effect of clustering and stratification, effect of weighting, and an additional effect due to correlation of different cross-sections from a panel. Combining these, the overall design effect can be estimated.

1.1 Differences and averages of poverty measures

The following indicators of poverty are used as the basis for the construction of net differences and average measures: the Head Count Ratio, the Mean Equivalised Income, the Gini Index and the Share Ratio S80/S20.[1] Suppose that W1, W2, W3 and W4 stand for the cross-sectional measures based on any of the above mentioned indicators of

[1] A brief definition of the indicators is the following.
Head Count Ratio or at-risk-of-poverty rate: proportion of the population with equivalised income below 60% of the national median.
Mean equivalised income: constructed using the equivalised income, defined as the total disposable household income divided by equivalent household size (constructed using the modified-OECD scale which gives a weight of 1.0 to the first adult in a household, 0.5 to each subsequent member aged 14 and over, and 0.3 to each child aged under 14), is ascribed to each member of the household.
Inequality of income distribution Gini coefficient: it is defined as the relationship of cumulative shares of the population arranged according to the level of equivalised disposable income, to the cumulative share of the equivalised total disposable income received by that population.
Inequality of income distribution S80/S20 income quintile share ratio: ratio of the shares of equivalised income of the top and the bottom 20% of the population.

poverty/inequality for four consecutive years. Then nine measures of difference and average are constructed as follows:

Measures of differences between waves: (W1-W2), (W2-W3), (W3-W4)

Measures of mean of two waves: (W1+W2)/2, (W2+W3)/2, (W3+W4)/2

Measures of mean of three waves: (W1+W2+W3)/3, (W2+W3+W4)/3

Measures of mean of four waves: (W1+W2+W3+W4)/4.

This paper is concerned with the estimation of sampling errors of these measures of net change in the poverty rate from one year (cross-section) to another, and of poverty rates averaged over two or more waves. We are not aware how this kind of situation can be captured in the form a linearised variate *defined at the individual level*.

1.2 Variance estimation

The procedure for variance estimation used is Jackknife Repeated Replication (JRR). The standard JRR approach can be extended to such measures on the following lines.

(1) Using the common sample structure of the cross-sections in a panel, a common set of JRR replications is defined in the usual way. (Constructing a 'common set of replications' essentially requires that when an element is to be excluded in the construction of a particular replication, it must be excluded simultaneously from every cross-sectional sample where it appears.)

(2) For each replication, the required measure is constructed for each cross-section involved at the aggregate level.

(3) These replication-specific cross-sectional measures are differenced/averaged to obtain the required net-change and average measures corresponding to the replication.

(4) Variance is then estimated from the resulting replicated measures in the usual way.

In estimating the variance it is necessary also to consider the non independence between different cross-sections of the panel. This issue is analysed in Section 3.

1.3 Design Effect

Design effect is the ratio of the variance under the given sample design, to the variance under a simple random sample of the same size. (We use $deft^2$ for the ratio of variances, and *deft* for the corresponding ratio of standard errors.)

Using JRR methodology it is not possible to directly estimate deft. From previous literature (Verma and Betti, 2005b), it is known that deft can be decomposed into two separate components: effect of clustering and stratification and effect of weighting. These two components can be estimated with JRR separately. These procedures can be easily extended to longitudinal measures, given the fixed structure of the sample (Gagliardi *et al.*, 2006).

For measures of net differences and averages, design effects can be estimated in a way similar to that for ordinary cross-sectional estimates. However, in application additional factors or complications have to be considered, as explained in detail in Sections 4 and 5 below.

(a) Randomisation (for estimating the design effect due to clustering and stratification) cannot be done independently for individual cross-sections. This is because replications of the randomised sample must also meet the requirement noted in (1) of Section 1.2.

(b) The effect of weighting has to be estimated separately for each cross-section, and then put together over the cross-sections involved.

(c) Apart from the two components (clustering/stratification, and weighting), a third component is also involved in deft: the effect of correlation at the level of individual elements.

1.4 The data Set

We have used the JRR methodology for estimating standard errors and design effects for net changes in poverty rates between pairs of adjacent waves, and for poverty rates averaged over 2, 3 and 4 waves, using first four waves of Italian ECHP. The data mostly come from the User's Data Base (ECHP-UDB)[2]. Unfortunately, sufficient information is not provided in ECHP-UDB for the identification of the sample structure, in particular the identification of the stratum and PSU to which a household or person belongs, nor on how the PSUs have been selected. Such information is available in the Production Data Base (ECHP-PDB), D-File, which is not available to the researcher outside the National agency conducting the ECHP or Eurostat. In our case, thanks to the co-operation of Istat we have been able to use the ECHP-PDB for Italy that provides information about the structure of the sample.

But even here, the information available on the sample structure in the Italian ECHP-PDB is not complete, and cannot be easily connected to the available documentation on the survey. In fact the final sample structure as we have used comes not only from the UDB and its PDB version, but also from descriptions of the sample provided in various other documents by Istat.

The rest of the paper is organised as follows. Section 2 gives our main results – variance and design effect of differences and averages of poverty measures. Section 3 presents a brief description of variance estimation using JRR methodology, in particular its extension to measures defined over multiple cross-sections of a household panel. Section 4 provides details of the estimation of components of the design effect under this methodology. Further technical details of estimation of variance and design effects of difference and average measures are presented in Section 5.

2. Results for measures of net difference and average

This section provides the main results of our analysis. We have used the JRR methodology for estimating standard errors and design effects for net changes in poverty rates between pairs of adjacent waves, and for poverty rates averaged over 2, 3 and 4 waves, using first four waves of Italian ECHP. This has also been done for the following measures: mean equivalised income, income share ratio (S80/S20), and Gini coefficient.

With a panel design, the statistical problem is the following.

A large proportion of the individuals are common in the different cross-sections of the panel. However, a certain proportion of individuals are different from one wave to the other. The cross-sectional samples are not independent, resulting in correlation between measures from different waves. Apart from correlations at the individual level, we have to deal also with additional correlation that arises because of the same structure (stratification and clustering)

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[2] Eurostat (2003a, 2003b, 2003c).

of the waves of a panel. Such correlation would exist, for instance, in samples coming from the same clusters even if there is no overlap in terms of individual households.

The estimation of these measures (differences or averages of poverty measures) is straightforward. However, the correlation because of non-independent cross-sections complicates the estimation of variance of the measures.

In order to estimate the variances of difference or average measures, one has to take into account also the covariance of the constituent cross-sectional measures.

Consideration of this correlation implies that the variance of the difference, say of (W1-W2), can be written as:

$$\mathrm{V(W1-W2)} = \mathrm{V(W1)} + \mathrm{V(W2)} - 2\rho_{12}\sqrt{\mathrm{V(W1)}\cdot\mathrm{V(W2)}} \qquad [1]$$

where ρ_{12} is the coefficient of correlation between W1 and W2.

The variance of the mean, say of (W1+W2)/2, can be written as:

$$\mathrm{V}\left(\frac{\mathrm{W1+W2}}{2}\right) = \frac{1}{4}\cdot\left(\mathrm{V(W1)} + \mathrm{V(W2)} + 2\rho_{12}\sqrt{\mathrm{V(W1)}\cdot\mathrm{V(W2)}}\right) \qquad [2]$$

In the expressions above, we have expressed the covariance term as a combination of variances and correlation. A part of the present study is devoted to analysing these correlations between waves.

Table 1 presents results for nine measures based on each of the four poverty indicators, defined in Section 1. The estimates and their standard errors are obtained using JRR methodology detailed in subsequent sections. Here we provide and exposition of the main results.

In column (1) we present the estimation of the measures, in column (2) their standard errors and in column (4) the corresponding variances.

For each indicator, the estimates of the average measures are very stable regardless of the number of waves incorporated. The estimates for differences between adjacent waves indicate a trend of increasing income levels and a slight reduction in inequality and poverty.

Looking at the standard errors, we see that the Head Count Ratio is the index with greatest differences among the nine measures, in fact they range from 0.47 to 1.01. The standard errors for the differences are higher (from 0.77 to 1.01) than those for averages (from 0.47 to 0.59), the latter decreasing with the number of waves averaged.

For the other three indicators, standard errors are much more similar across the various difference and average measures. As will be seen later, this reflects generally stronger correlations across waves for these indicators. In particular the one of Gini ranges from 0.38 to 0.43 and the S80/S20 from 0.15 to 0.19.

Column (5) is the mean of the wave-by-wave cross-sectional variances considered for that particular measure. For example, for (W1-W2) column (5) is the mean of the variances for W1 and W2. This provides a point of reference for column (4); also, we need this measure for the estimation of a synthetic coefficient of correlation described in Section 5.3.

Column (6) presents this synthetic coefficient of correlation of the measures for adjacent waves. Among the four indices, HCR is the measure with the lowest correlation, (from 0.12 to 0.37). The other three indices have much higher correlations (exceeding 0.50), generally increasing as we consider average measures for more than two waves (more than 0.64).

Column (3) is the overall design effect for each measure. For all the four indicators it increases sharply from measures of difference to those of average over four waves. This reflects the loss in efficiency in cumulating across positively correlated waves. See Section 5.2 for discussion of these results.

Table 1. Estimate, standard error and design effect of difference and average measures

Head Count Ratio						
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Measure	**est**	**stat_se**	**Deft**	**VAR**	*****	**Rho**
W1-W2	-0,1	0,81	1,03	0,65	0,47	0,31
W2-W3	0,3	0,77	1,06	0,60	0,37	0,20
W3-W4	0,6	1,01	1,35	1,02	0,58	0,12
(W1+W2)/2	20,5	0,56	1,42	0,31	0,47	0,32
(W2+W3)/2	20,3	0,50	1,36	0,25	0,37	0,32
(W3+W4)/2	19,9	0,59	1,58	0,35	0,58	0,19
(W1+W2+W3)/3	20,4	0,47	1,53	0,22	0,45	0,30
(W2+W3+W4)/3	20,1	0,51	1,65	0,26	0,51	0,35
(W1+W2+W3+W4)/4	20,2	0,48	1,77	0,23	0,53	0,37
Equivalised Income Mean						
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Measure	**est**	**stat_se**	**Deft**	**VAR**	*****	**Rho**
W1-W2	-1.215	169	0,82	28.655	40.794	0,65
W2-W3	-712	147	0,69	21.643	45.620	0,76
W3-W4	-329	146	0,68	21.202	39.893	0,73
(W1+W2)/2	16.459	183	1,79	33.654	40.794	0,59
(W2+W3)/2	17.422	200	1,87	39.986	45.620	0,56
(W3+W4)/2	17.943	191	1,80	36.450	39.893	0,54
(W1+W2+W3)/3	16.899	181	2,14	32.871	41.845	0,74
(W2+W3+W4)/3	17.651	191	2,18	36.302	42.360	0,83
(W1+W2+W3+W4)/4	17.201	179	2,42	31.898	40.344	0,74
Gini						
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Measure	**est**	**stat_se**	**Deft**	**VAR**	*****	**Rho**
W1-W2	0,0	0,43	0,83	0,19	0,23	0,59
W2-W3	1,1	0,38	0,75	0,15	0,22	0,66
W3-W4	1,3	0,38	0,76	0,15	0,20	0,63
(W1+W2)/2	33,2	0,42	1,62	0,18	0,23	0,55
(W2+W3)/2	32,6	0,43	1,68	0,19	0,22	0,68
(W3+W4)/2	31,5	0,40	1,60	0,16	0,20	0,65
(W1+W2+W3)/3	32,8	0,40	1,92	0,16	0,23	0,64
(W2+W3+W4)/3	32,0	0,41	1,94	0,16	0,21	0,75
(W1+W2+W3+W4)/4	32,3	0,39	2,16	0,15	0,21	0,74
S80/S20						
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Measure	**est**	**stat_se**	**Deft**	**VAR**	*****	**Rho**
W1-W2	0,2	0,19	0,79	0,04	0,04	0,56
W2-W3	0,3	0,19	0,80	0,04	0,04	0,51
W3-W4	0,3	0,16	0,85	0,03	0,03	0,52
(W1+W2)/2	6,0	0,19	1,52	0,03	0,04	0,64
(W2+W3)/2	5,8	0,17	1,41	0,03	0,04	0,52
(W3+W4)/2	5,5	0,15	1,58	0,02	0,03	0,63
(W1+W2+W3)/3	5,9	0,16	1,76	0,03	0,04	0,64
(W2+W3+W4)/3	5,6	0,16	1,73	0,02	0,03	0,70
(W1+W2+W3+W4)/4	5,8	0,16	2,05	0,03	0,04	0,75

* mean of the wave-by-wave cross-sectional variances

3. Methodology: Jackkinfe Repeated Replication

3.1 Jackknife Repeated Replication (JRR): basic cross-sectional application

JRR is one of a class of variance estimation methods based on comparisons among replications generated through repeated re-sampling of the same present sample. For a stratified multi-stage sample, the standard version of JRR involves constructing replications by eliminating one sample PSU at a time and increasing the sample weight of the remaining PSU's in its stratum in compensation.

The attraction of the approach is its versatility and simplicity. Once a fixed structure of the sample is defined (strata and primary sampling units 'PSU'), the variance of any kind of statistic can be estimated.

The same variance estimation algorithm can be applied – once the set of replications has been appropriately defined for any complex design - to a statistic of any complexity. The main computation involved is repeating the estimation of the statistic of interest for each replication. In principle, the methods captures the effect on variance of those design features and processes – such as calibration and imputation – which are re-performed for each replication as was done originally for the parent sample.

Let u be a full-sample estimate of any complexity, and $u_{(hi)}$ be the estimate produced using the same procedure after eliminating primary unit i in stratum h and increasing the weight of the remaining (a_h-1) units in the stratum by an appropriate factor $g_h = w_h / (w_h - w_{hi})$. Let $u_{(h)}$ be the simple average of the $u_{(hi)}$ over the a_h values of i in h. The variance of u is then estimated as:

$$\mathrm{var}(u) = \sum_h \left[(1 - f_h). \frac{a_h - 1}{a_h} . \sum_i \left(u_{(hi)} - u_{(h)} \right)^2 \right]. \qquad [3]$$

A major advantage of a procedure like the JRR is that, under quite general conditions for the application of the procedure, the same and relatively simple variance estimation formula [3] holds for u of any complexity.[3]

3.2 Jackknife Repeated Replication (JRR): extension beyond a single cross-section

The strength of the JRR methodology is the following: it can be easily extended to longitudinal samples and to measures between different cross-sections. Once a common structure (PSUs and strata) is defined for all waves of a data set, the JRR can be used to estimate variance of any measure that incorporate measures from different waves.

Therefore, in order to estimate the variance of longitudinal measures or of difference or average measures of different waves, it is necessary to construct a structure common to all the waves of the data set.

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[3] We have used the above formulation in our applications presented here. Two common variations may be mentioned.

A 'conservative' alternative to [3] is to replace $u_{(h)}$, the simple average of the $u_{(hi)}$ over the a_h replications created from h, by the full-sample estimate u:

$$\mathrm{var}(u) = \sum_h \left[(1 - f_h). \frac{a_h - 1}{a_h} . \sum_i \left(u_{(hi)} - u \right)^2 \right].$$

Secondly, factor g_h is often defined as $a_h/(a_h-1)$; this alternative does not maintain a fixed sum of sample weights from one replication to another.

The data set that we use possesses such a common structure. In the ECHP survey, the initial sample gives a common structure to all the following waves. This is because, once selected, the position of an individual in the sample structure remains unchanged over waves even if the individual moves to a new location during this period. This structure is defined on the basis of geographic area where individuals are *originally selected*. Furthermore, all the individuals that enter in the survey after the first wave are either associated to other individuals (hence households) that have been already interviewed in the first wave, or are already in the original structure even if for some reason are not interviewed in the 1st wave. This implies that the structure of the sample remains fixed over waves, despite entries to or exits from the panel after the first wave.

This common sample structure for Italy is characterised by 253 PSUs and 121 strata. See Annex 1 for details.

4. Estimation of the design effect

The design effect (deft) is estimated by the ratio of actual standard error (se) of a statistics under the given sample design, to standard error (se_srs) under a simple random sample of same size.

In this section we describe the basic procedure for estimating the design effect using the JRR variance estimation method, as originally developed for cross-sectional measures (Verma and Betti, 2005b). The procedure is easily extended to longitudinal measures (Gagliardi *et al.*, 2006), and to measures of difference and average over cross-sections (Section 5).

4.1 Design effect with linearisation method

This method is based on defining a linearised variable (say $\mathrm{u}_{\mathrm{hij}}$ for ultimate unit j in PSU i, stratum h) such that the ordinary expression for variance of the total of this linearised statistics gives an estimate of variance of the complex statistic of interest. This is:

$$se^2 = \mathrm{var}(u) = \sum_h \left[\frac{a_h}{a_h - 1} \sum_i \left(u_{hi} - \frac{u_h}{a_h} \right)^2 \right] \qquad [4]$$

where $\mathrm{u}_{\mathrm{hi}} = \sum_{\mathrm{j}} \mathrm{w}_{\mathrm{hij}} \cdot \mathrm{u}_{\mathrm{hij}}$, $u_h = \sum_{i-1}^{a_h} u_{hi}$, $u = \sum_h u_h$, and the finite population correction is disregarded.

Once u_{hij} has been defined, it can be used to directly estimate variance under a equivalent simple random sample:

$$(se\_srs)^2 \equiv \sum_{j \in S} w_j \cdot u_j^2 / \sum_{j \in S} w_j \qquad [5]$$

where the sum is over all ultimate units j in the sample *S*. [4]

Hence, the ratio of the two quantities defined above,[4]/[5], directly gives the required design effect for the linearised method, at least for cross-sectional measures for which the required linearised variable (u_{hij}) has been developed.

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4 There in no need to identify different h and i values in [5]. Note that [5] defines the denominator of deft^2 in terms of a simple random sample of households.

For the general application of the JRR approach, specially to longitudinal and other more complex situations to which the linearised approach is not easily extended, we have to assume that the required linearised form (u_{hij}) is not available, so that (se_srs), and hence deft, cannot be estimated directly.

4.2 Decomposition of the design effect

In order to solve the above described problem, an indirect approach is required. One such approach involves identifying components making up the design effect, each of which can be separately estimated without requiring the fully linearised variable (u_{hij}). The required components are (1) the effect of sample weights on variance, and (2) the effect of clustering, stratification and other aspects of the design.

In fact, the identification of the effect of weighting is in itself of substantive interest apart from its usefulness for the above purpose. A question of great practical interest is the following. How does the weighting affect variances? There are effects in both directions:

(i) Calibration weights and other weighting correlated with the survey variables <u>can</u> reduce not only bias, but also variances. (Optimal allocation in stratified samples and the corresponding weighting involved is an obvious example.)

(ii) On the other hand, very often weighting is determined on the basis of 'external' factors (e.g., the need to over-sample small regions; compensation for high non-response in particular sample areas, etc.). Such weighting, essentially uncorrelated with survey variables, results in increased variance.

Generally, the second of the above effects is found to predominate in practice. That is, usually the net effect of weighing is to inflate variances[5].

Again, when the full linearised variable (u_{hij}) is available, variance of an equivalent element sample but retaining the effect of weighting can be directly estimated as:

$$(se\_wtd)^2 = n.\sum w_j^2 \cdot u_j^2 / \left(\sum w_j\right)^2 \qquad [6]$$

Hence, deft can be directly decomposed as:

$$deft = \left(\frac{se}{se\_wtd}\right) \cdot \left(\frac{se\_wtd}{se\_srs}\right) = D_u \cdot (Kish\_taylor) \qquad [7]$$

The first factor in the above (D_u) is the effect of sample design features other than weighting. The second factor:

$$(Kish\_taylor)^2 = \left(\frac{se\_wtd}{se\_srs}\right)^2_{Taylor} = \left(\frac{n}{\sum w_j}\right) \cdot \frac{\sum w_j{}^2 u_j{}^2}{\sum w_j u_j{}^2} \qquad [8]$$

is an estimate of the effect of weighting. We have termed it "Kish_taylor" for the following reason.

This formula can be directly compared with the well known "Kish factor", proposed by Kish to estimate the effect of essentially "random" weights on variance:

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5 Proper weighting should of course reduce mean-squared error, by controlling bias even if there is some increase in variance.

$$(Kish\_Factor)^2 = D_w{}^2 = n \cdot \frac{\sum w_j{}^2}{\left(\sum w_j\right)^2} = \left(\frac{n}{\sum w_j}\right) \cdot \frac{\sum w_j{}^2}{\sum w_j} = 1 + cv^2\left(w_j\right), \qquad [9]$$

where w_j is the sample weight of unit j, and the sum is over n units in the sample; *cv* is the coefficient of variation of unit weights. D_w indicates the design effect (deft) due to weighting: standard errors are inflated by D_w.

The above factor is simple and general as it is determined simply from the variability of weights in the sample, independently of any substantive variable.

The factor (Kish_taylor) is an alternative and in principle a more accurate expression of the effect of weighting than the simple (Kish_factor). This applies specifically in situations where the sample weights are not "random" but are systematically correlated with substantive variables of interest – as is the case with calibration and similar types of adjustments applied to the sample weights.

4.3 Decomposition under JRR method

An indirect procedure is required if we assume that the full expression for the linearised variable (u_{hij}) cannot be assumed to be available when using JRR rather than the linearisation methodology.

As we have already noted, it is not possible to estimate deft directly with JRR. This problem can be solved using an alternative approach that consists in estimating separately the two effects of clustering/stratification and weighting. Once these are estimated, they can be combined in order to obtain deft.

An alternative estimate of variance of a weighted element sample, (se_wtd), can be obtained by "randomising" the sample as follows.

A randomised sample is created from the actual sample by completely randomising the position of individual elements (households, persons) within the sample structure. In principle, this creates a random element sample which is not subject to clustering or stratification effects, and differs from a true simple random sample simply because of the presence of unequal weights.

Random groupings of the elements can be formed to serve as clusters and strata in the variance estimation without affecting the expected variances.

We term the standard error estimated from such a randomised sample using JRR as (se_rnd). This, in theory, is the same as (se_wtd) defined earlier, and their empirical closeness has been verified in Verma *et al.* (2006). Consequently we can estimate the effect of clustering, stratification and any factors other than weighting as:

$$\text{Effect of clustering and stratification} = \left(\frac{se}{se\_rnd}\right). \qquad [10]$$

Now concerning the estimation of the effect of weighting under the JRR approach, again, we have to assume that the full expression for the linearised form (u_j) is not available, as for instance in the case of longitudinal measures.

The linearised expression for any (u_j) is developed to have two parts. The first part, say (u_{1j}), is simple and corresponds to treating the complex statistics under consideration as a simple ratio. The second part is the effect of further complexity of the actual statistics. For instance, for poverty rate (p), the first part is simply:

$$u_{1j} = (p_j - p)\,. \qquad [11]$$

The above is exactly the linearised form for variance estimation of an *ordinary ratio*. Hence, this first part for the linearised form is always available for any complex statistics without evoking the full linearisation methodology.

In Verma *et al.* (2006), it has been empirically demonstrated, at least for a wide variety of cross-sectional measures of poverty and inequality, that the expression:

$$(Kish\_Jrr)^2 = \left[\frac{n}{\sum w_j}\right] \cdot \frac{\sum w_j^2 \cdot u_{1j}^2}{\sum w_j \cdot u_{1j}^2} \qquad [12]$$

provides a *very close approximation* to the effect of weighting computed above as:

$$(Kish\_taylor)^2 = \frac{n}{\sum w_j} \cdot \frac{\sum w_j^2 \cdot u_j^2}{\sum w_j \cdot u_j^2}\,. \qquad [13]$$

This relationship remains valid also in case of more complex longitudinal measures (Gagliardi *et al.* 2006).

To summarise, the motivation of introducing (Kish_Jrr) is the following. The full expression u_i is available only in the context of the linearisation method. It may not even be possible (or at least not easy) to develop the required expression, for instance, for certain complex measures of poverty. The simplified form u_{1i} is the well-known one for ratios. It is always available for application of the JRR method, even when the full linearised form u_i is not (or cannot be) developed. It is for this reason that we have subscripted the above quantities as "_Jrr".

Finally, the design effect is estimated as the product of the components defined above:

$$deft = \left(\frac{se}{se\_rnd}\right) \cdot (Kish\_Jrr) \qquad [14]$$

For measures of difference and average between different waves of a samples also a third effect should be considered in dealing with deft: correlation at micro-level. This issue is analysed in Section 5.

5. Further technical details of estimation of variance, design effects and synthetic correlations

In this section we explain technical details of the estimation of standard errors, design effects and correlations for the nine measures of difference or average listed in Section 1.1. Because of the particularity of these measures - they are based on cross-sectional measures from panel data - all the estimations are done not in the same manner as described in Section 3 for pure cross-sectional measures. Here we describe the differences in the estimation techniques.

5.1 Standard errors

The estimation has been done only with the JRR methodology described above, given that we are not able to apply the linearisation for such measures.

It is important to underline that for estimating variance of measures involving more than one wave, all the waves should have the same sample structure – the same number of PSUs and strata. In the 4th wave in our data set, however, we have one PSU and one stratum less than the first three waves due to sample attrition. So, when we combine this wave with the other waves, we have to change the structure of other waves in order to be consistent with the 4th one. This procedure is described in Annex 1.

Once we have obtained the estimates of the statistic for the waves concerned (for example, W1 and W2), we have the required measure of their difference or average (such as W1-W2). The required measure can be constructed for each replication using the same procedure. These after the JRR variance estimation procedure is identical to that for any ordinary cross-sectional measure.

5.2 Design Effect

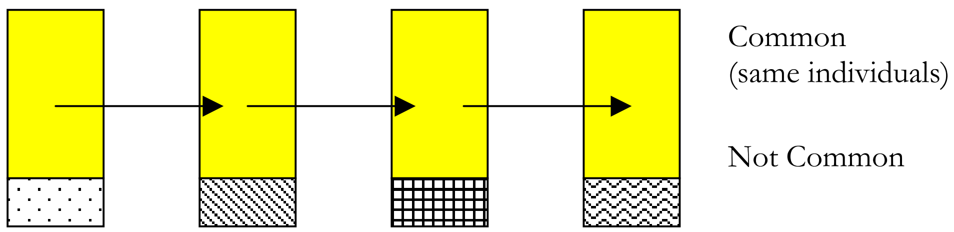
The design effect, as described in Section 1.3, is the ratio between the standard error of the measure considered based on the actual sample and the standard error of the same measure under the assumption of simple random sampling.

So, as example, the squared design effect for the measure of difference between first two waves can be expressed as:

$$\mathrm{Deft}^2 = \frac{\mathrm{V(W1-W2)}}{\mathrm{V_{srs}(W1)+V_{srs}(W2)}} \qquad [15]$$

The design effect, as described in Section 4, can be decomposed into components: the effect of clustering and stratification and the effect of weighting. Here, because of the panel nature of the data, we have also a third effect: the effect of correlation. This correlation arises from two sources. The first is the common structure (stratification and clustering) of the waves of a panel. This correlation exists even if there is no overlap between waves at the level of individual households or persons. Further correlation arises from overlap between waves at the individual level as follows.

We have four waves of a panel survey. Because of the panel nature of the survey, a large proportion of the individuals are common in these four waves. However, a small but non-negligible proportion of individuals are different from one wave to the other.



As shown in the figure above, the four cross-sectional samples largely overlap and are not independent. This causes correlation between measures from different waves.

For the measure of difference between two waves, formula [15] can be expressed as:

$$\frac{V(W1-W2)}{V_{srs}(W1)+V_{srs}(W2)} = \frac{V(W1-W2)}{V_{rnd}(W1-W2)} \cdot \frac{V_{rnd}(W1)+V_{rnd}(W2)}{V_{srs}(W1)+V_{srs}(W2)} \cdot \frac{V_{rnd}(W1-W2)}{V_{rnd}(W1)+V_{rnd}(W2)} \qquad [16]$$

The first term on the right hand side stands for the effect of clustering and stratification, the 2nd term for the effect of weighting and the 3rd for the effect of correlation. In this expression we can estimate all the variance terms except for $V_{srs}(.)$. For the calculation of the second term (the effects of weighting) that involves $V_{srs}(.)$, we proceed as follows. As noted in Section 4.3 empirically we find that $\frac{V_{wtd}(.)}{V_{srs}(.)} = (\text{Kish\_taylor})^2 \cong (\text{Kish\_Jrr})^2$, and theoretically the following holds $V_{wtd} = V_{rnd}$.

Using these two results, the $V_{srs}(.)$ in the second term can be substituted by $\frac{V_{rnd}(.)}{K(.)}$, where $K(.)$ stands for (Kish_Jrr .) Hence we can write the design effect of (W_1-W_2) as:

$$\frac{V(W1-W2)}{V_{srs}(W1)+V_{srs}(W2)} = \frac{V(W1-W2)}{V_{rnd}(W1-W2)} \cdot \frac{V_{rnd}(W1)+V_{rnd}(W2)}{\frac{V_{rnd}(W1)}{K(W1)}+\frac{V_{rnd}(W2)}{K(W2)}} \cdot \frac{V_{rnd}(W1-W2)}{V_{rnd}(W1)+V_{rnd}(W2)} \qquad [17]$$

Similar expressions can be derived for the design effects for (W2-W3) and (W3-W4).

Similarly, the design effect for measures of average over waves can be expressed as follows:

$$\frac{V((W1+W2)/2)}{(V_{srs}(W1)+V_{srs}(W2))/4} = \frac{V((W1+W2)/2)}{V_{rnd}((W1+W2)/2)} \cdot \frac{V_{rnd}(W1)+V_{rnd}(W2)}{\frac{V_{rnd}(W1)}{K(W1)}+\frac{V_{rnd}(W2)}{K(W2)}} \cdot \frac{V_{rnd}((W1+W2)/2)}{(V_{rnd}(W1)+V_{rnd}(W2))/4} \qquad [18]$$

Similar expressions can be derived for average of W2 and W3, and average of W3 and W4. For the average over three waves we have:

$$\frac{V((W1+W2+W3)/3)}{(V(W1)+V_{srs}(W2)+V_{srs}(W3))/9} = \frac{V((W1+W2+W3)/3)}{V_{rnd}((W1+W2+W3)/3)} \cdot \frac{V_{rnd}(W1)+V_{rnd}(W2)+V_{rnd}(W3)}{\frac{V_{rnd}(W1)}{K(W1)}+\frac{V_{rnd}(W2)}{K(W2)}+\frac{V_{rnd}(W3)}{K(W3)}} \cdot \frac{V_{rnd}((W1+W2+W3)/3)}{(V_{rnd}(W1)+V_{rnd}(W2)+V_{rnd}(W3))/9} \qquad [19]$$

Similar expression can be written for the average of W2, W3 and W4. Finally, for the average over four waves we have:

$$\frac{V((W1+W2+W3+W4)/4)}{(V(W1)+V(W2)+V_{srs}(W3)+V_{srs}(W4))/16}$$
$$=\frac{V((W1+W2+W3+W4)/4)}{V_{rnd}((W1+W2+W3+W4)/4)}$$
$$\cdot\frac{V_{rnd}(W1)+V_{rnd}(W2)+V_{rnd}(W3)+V_{rnd}(W4)}{\frac{V_{rnd}(W1)}{K(W1)}+\frac{V_{rnd}(W2)}{K(W2)}+\frac{V_{rnd}(W3)}{K(W3)}+\frac{V_{rnd}(W4)}{K(W4)}} \qquad [20]$$
$$\cdot\frac{V_{rnd}((W1+W2+W3+W4)/4)}{(V_{rnd}(W1)+V_{rnd}(W2)+V_{rnd}(W3)+V_{rnd}(W4))/16}$$

Table 2 presents the three factors, described above, which make up the design effect. Column (3), the ratio between column (1) and (2), represents the effect of clustering and stratification; column (4) is the effect of weighting; and column (5) is the effect of the correlation among waves. Column (6) is the overall design effect.

We can see that for any given indicator the effect of weighting is very stable for any comparison between waves. There is a moderate variation across indicators.

Looking at the effect of the correlation, we can see that it increases as we move down the column for any indicator, from the difference between two adjacent waves to the average over all four waves.

The overall design effect increases in the same manner as of the effect of correlation.

The design effect for the differences between two waves for the Equivalised Income mean, for the Gini and for the S80/S20 is less then 1. This implies that the effect of positive correlation between waves (reducing variance of measures difference between them) is predominant over other effects such as clustering and weighting which normally increase variance.

On the opposite for the measures of average over two, three or four waves, the design effect is bigger than 1. For example, for the mean between four waves for the HCR we have a design effect of 1.77. This means that, given the sample that we have used, the same result could be obtained with a sample size $(1.77)^2$ times smaller than our sample size, if each wave was based on a simple random sample and entirely independent from one wave to another.

5.3 Synthetic correlations

As mentioned above, the correlation between two or more waves in a panel, due to repeated observations on same individuals and clustering, affects the variance of the statistics that incorporate poverty measures from different waves of a panel.

Let W1 and W2 be some poverty measures from two waves of a panel. Then the variance of difference and average measures can be expressed as seen in Section 2 by:

$$\mathrm{V}(\mathrm{W1}-\mathrm{W2})=\mathrm{V}(\mathrm{W1})+\mathrm{V}(\mathrm{W2})-2\rho_{12}\sqrt{\mathrm{V}(\mathrm{W1}).\mathrm{V}(\mathrm{W2})}$$

$$\mathrm{V}((\mathrm{W1}+\mathrm{W2})/2)=\frac{1}{4}(\mathrm{V}(\mathrm{W1})+\mathrm{V}(\mathrm{W2})+2\rho_{12}\sqrt{\mathrm{V}(\mathrm{W1}).\mathrm{V}(\mathrm{W2})})$$

Similarly we derive variance expression for average measure of three waves, (W1+W2+W3)/3 and four waves, (W1+W2+W3+W4)/4.

Table 2. Break-down of design effect of difference and average

HCR

Measure	(1) stat_se	(2) se randomised	(3) Effect of clustering and stratification	(4) Effect of weightings	(5) Effect of Correlation	(6) Deft
W1-W2	0,81	0,65	1,24	1,16	0,72	1,03
W2-W3	0,77	0,66	1,17	1,16	0,78	1,06
W3-W4	1,01	0,71	1,43	1,13	0,84	1,35
(W1+W2)/2	0,56	0,45	1,24	1,16	0,99	1,42
(W2+W3)/2	0,50	0,43	1,17	1,16	1,01	1,36
(W3+W4)/2	0,59	0,45	1,30	1,13	1,07	1,58
(W1+W2+W3)/3	0,47	0,38	1,24	1,15	1,07	1,53
(W2+W3+W4)/3	0,51	0,40	1,29	1,14	1,12	1,65
(W1+W2+W3+W4)/4	0,48	0,35	1,37	1,14	1,12	1,77

Equivalised Income Mean

Measure	(1) stat_se	(2) se randomised	(3) Effect of clustering and stratification	(4) Effect of weightings	(5) Effect of Correlation	(6) Deft
W1-W2	169	143	1,19	1,11	0,63	0,82
W2-W3	147	128	1,15	1,10	0,54	0,69
W3-W4	146	128	1,14	1,09	0,55	0,68
(W1+W2)/2	183	146	1,25	1,11	1,29	1,79
(W2+W3)/2	200	155	1,29	1,10	1,31	1,87
(W3+W4)/2	191	152	1,26	1,09	1,31	1,80
(W1+W2+W3)/3	181	144	1,26	1,10	1,54	2,14
(W2+W3+W4)/3	191	148	1,29	1,10	1,54	2,18
(W1+W2+W3+W4)/4	179	140	1,27	1,10	1,73	2,42

Gini

Measure	(1) stat_se	(2) se randomised	(3) Effect of clustering and stratification	(4) Effect of weightings	(5) Effect of Correlation	(6) Deft
W1-W2	0,43	0,41	1,07	1,09	0,72	0,83
W2-W3	0,38	0,37	1,03	1,08	0,67	0,75
W3-W4	0,38	0,36	1,07	1,07	0,66	0,76
(W1+W2)/2	0,42	0,35	1,19	1,09	1,25	1,62
(W2+W3)/2	0,43	0,35	1,25	1,08	1,24	1,68
(W3+W4)/2	0,40	0,34	1,18	1,07	1,27	1,60
(W1+W2+W3)/3	0,40	0,32	1,24	1,08	1,43	1,92
(W2+W3+W4)/3	0,41	0,33	1,23	1,08	1,46	1,94
(W1+W2+W3+W4)/4	0,39	0,32	1,24	1,08	1,62	2,16

S80/S20

Measure	(1) stat_se	(2) se randomised	(3) Effect of clustering and stratification	(4) Effect of weightings	(5) Effect of Correlation	(6) Deft
W1-W2	0,19	0,22	0,88	1,13	0,80	0,79
W2-W3	0,19	0,21	0,90	1,14	0,78	0,80
W3-W4	0,16	0,15	1,07	1,11	0,72	0,85
(W1+W2)/2	0,19	0,16	1,15	1,13	1,16	1,52
(W2+W3)/2	0,17	0,16	1,01	1,14	1,23	1,41
(W3+W4)/2	0,15	0,13	1,17	1,11	1,21	1,58
(W1+W2+W3)/3	0,16	0,14	1,16	1,13	1,34	1,76
(W2+W3+W4)/3	0,16	0,14	1,14	1,13	1,35	1,73
(W1+W2+W3+W4)/4	0,16	0,13	1,24	1,13	1,47	2,05

The variances are:

$$\mathrm{V}\left(\frac{\mathrm{W1}+\mathrm{W2}+\mathrm{W3}}{3}\right)=\frac{1}{9}\cdot[\mathrm{V(W1)}+\mathrm{V(W2)}+\mathrm{V(W3)}+2\rho_{12}\sqrt{\mathrm{V(W1)}\cdot\mathrm{V(W2)}}+$$
$$+2\rho_{23}\sqrt{\mathrm{V(W2)}\cdot\mathrm{V(W3)}}+2\rho_{13}\sqrt{\mathrm{V(W1)}\cdot\mathrm{V(W3)}}]$$

This formula can be used also for the mean between W2, W3, and W4. In four waves we have:

$$\mathrm{V}\left(\frac{\mathrm{W1}+\mathrm{W2}+\mathrm{W3}+\mathrm{W4}}{4}\right)=\frac{1}{16}\cdot[\mathrm{V(W1)}+\mathrm{V(W2)}+\mathrm{V(W3)}+\mathrm{V(W4)}+$$
$$+2\rho_{12}\sqrt{\mathrm{V(W1)}\cdot\mathrm{V(W2)}}+2\rho_{23}\sqrt{\mathrm{V(W2)}\cdot\mathrm{V(W3)}}+2\rho_{34}\sqrt{\mathrm{V(W3)}\cdot\mathrm{V(W4)}}+$$
$$+2\rho_{13}\sqrt{\mathrm{V(W1)}\cdot\mathrm{V(W3)}}+2\rho_{24}\sqrt{\mathrm{V(W2)}\cdot\mathrm{V(W4)}}+2\rho_{14}\sqrt{\mathrm{V(W1)}\cdot\mathrm{V(W4)}}]$$

In the above expressions ρ_{kl} stands for the correlation coefficient between waves *k* and *l*; *k*=1,2,3,4 and *l*=2,3,4.

Assuming $\mathrm{V(W1)}=\mathrm{V(W2)}=\mathrm{V}$, say the average between the two variances, we have the following simple formulas for an 'average' value, ρ, of the correlation:

$$V(W_i - W_{i+1}) = 2V(1-\rho) \qquad [21]$$

$$V((W_i + W_{i+1})/2) = \frac{V}{2}(1+\rho) \qquad [22]$$

where *i=1, 2 and 3* define the various measures involving two adjacent waves *i* and *(i+1)*.

For three or four waves, we assume that the correlation between waves declines with the interval between them as follows: $\rho_{12}=\rho_{23}=\rho_{34}=\rho$, $\rho_{13}=\rho_{24}=\rho^2$ and $\rho_{14}=\rho^3$. This gives the approximate relations:

$$V((W1+W2+W3)/3) = V((W2+W3+W4)/3) = \frac{V}{3}(1+\frac{4}{3}\rho+\frac{2}{3}\rho^2) \qquad [23]$$

$$\mathrm{V((W1+W2+W3+W4)/4)} = \frac{\mathrm{V}}{4}(1+\frac{3}{2}\rho+\rho^2+\frac{1}{2}\rho^3) \qquad [24]$$

Given the variance of the average and difference measures and the variance of the cross-sectional measures from different waves of the panel, the last set of equations can be used to estimate the value of ρ. This measure of correlation is a synthetic measure of the average correlation between consecutive waves.

A part of this correlation arises from having the same individuals present in different waves, and an additional part from the different sample having the same clusters. The first component can be estimated separately by considering the samples 'randomised' as described above.

Annex 1

Sample structure, computational structure and construction of data files

This section gives a brief description of the sample structure as it has been implemented by Istat for the purpose of ECHP data collection in Italy. For the selection of the sample Istat has used a two stage sampling design with stratification of the primary selection units on the basis of region and population size. Within each stratum a unique municipality (commune) has been selected with the probability of selection proportional to the population size. The municipalities of large size (23 in total) have been considered as self-representative, and each of them constituted a stratum. For every municipality (commune), then a systematic sample of families has been selected from the list in population registers. The initial sample consisted of 7,989 families and 24,063 individuals.

The data set of the ECHP consists of data from three levels: household (H-file), personal (P-file) covering individual persons aged 16+, and register (R-file) covering all persons in enumerated households. For Italy, in the 1st wave (1994) the number of households interviewed was 7,115. The number of household interviewed in the 2nd (1995), 3rd (1996) and 4th (1997) waves were 7,128, 7,132 and 6,713 respectively.

However, it should be noted that the analysis presented in this study uses a somewhat smaller data set than the numbers mentioned above because of missing values of household income. After omitting households with missing income, we have 6,915 households in the 1st wave, 7,004 households in the 2nd, 7,026 households in the 3rd, and 6,627 households in the 4th.

The sample structure, the PSUs and strata, are constructed as follows. The first step of constructing a computational structure involves identifying the PSUs. From the User Data Base (UDB) it is not possible to identify the structure of the sample (strata and PSUs) that has been used by Istat for the collection data. Thanks to the cooperation of Istat, we have been able to use the Production Data Base (PDB) that provides more information about the structure of the sample.

From the information in the D-file, mentioned above, we have identified 21 self-representing provinces (mainly large cities) that formed 33 strata and 66 PSUs (two PSUs for each stratum). Six large cities among them are assigned more than one stratum: Genova, Palermo and Torino two strata each, Napoli and Milano three strata each, and Roma six strata. Within each stratum we randomised the households and then assigned them to one of the two PSUs.

For the non-self-representing units, we assigned the PSU code equal to the value of the variable called “d01smst2” (primary strata) in the D-file. Within a region two PSU were coupled to form a stratum. When an odd number of PSUs was present within a region, the last stratum of that region was formed to contain three PSUs. Using this process we obtained a total of 121 strata and 253 PSUs.

This structure, constructed as above, is attached to the each H-file of the 4 waves. The cross-sectional analysis uses data from the household (H) files of four waves (1994, 1995, 1996 and 1997). The sample structure is the same for the four waves.

We also randomised our sample in order to estimate particular components of the design effect. The randomised sample structure consists of only one stratum and 50 PSUs. The randomisation is achieved by generating random numbers from a uniform distribution and ordering the sample on the basis of these numbers. The PSUs are defined as follows: after randomisation the sum of all the cross-sectional weights is divided by the number of PSUs required (50 in our case) to obtain the target sum of weights within a PSU. Then we sum up the weights of units in sequence till we reach (or just cross) the sum required within each

PSU, and all the households considered in this sum constitute a PSU. In the randomised data set as well, the structure is common to all the waves.

For the cross-sectional analysis we have used the cross-sectional weights of each wave. We trimmed the weights as recorded in the ECHP-UDB to remove very extreme values. For example, in the second wave the maximum weight was 37.16 and the minimum 0.077. This means that the lowest weight is more than 450 times smaller that the highest. We have trimmed them looking at the distribution of weights. The trimmed is done in the following way: weights above 99% of the weight distribution are replaced by the value of weight at the 99^{th} percentile, and weights less than 1% are replaced by the value of weight at the 1^{st} percentile. Such simple procedures of trimming of the weights are quite commonly used in survey analysis.

Data Problem

It is important to note that in the fourth wave we have an empty PSU, because of sample attrition. So there is a stratum with just one PSU in the 4^{th} wave. This amounts to a computational problem since the variance estimation methodology cannot work with less then two PSUs per stratum. In order to circumvent this problem we have merged this stratum with only one PSU with the previous stratum. This leads to a structure with 252 PSUs and 120 strata for the 4^{th} wave.

This point is important in the present study since it is necessary to have a common structure for all the waves for the estimation of variance of mean and difference measures. Since only in the 4^{th} wave we have a different structure because of an empty PSU, in the estimation of variance of a measure that includes an indicator from the 4^{th} wave we have to change the structure of the other waves involved in of the measure to be consistent with the 4^{th} wave. For example, in the estimation of the variance of W3-W4, the structure of the 3^{rd} wave is changed to that of the 4^{th} wave (252 PSUs and 120 strata). However, when a measure does not involve an indicator from the 4^{th} wave, the original computation structure is retained (253 PSUs and 121 strata).

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