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The Demise of Marx's Labour Theory of Value and the ‘New Interpretation’: A Recap Note

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by Ernesto Screpanti

Abstract. Marx’s theory of labour value is flawed. This note summarizes the main reasons why this is so. At the same time, it claims that the theory of exploitation does not depend on a labour embodied valuation and can be expounded by resorting to the theory of production prices. Almost all Marxists have now accepted this truth. Most of them have been convinced by a ‘new interpretation’ which has been able to translate the price of net output into an amount of ‘living labour’ and the rate of exploitation into a ratio between unpaid and paid labour. What produced such a surprising result is the use of labour productivity as a numeraire.

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Introduction

Once upon a time there were orthodox Marxists who believed that Marx is infallible and therefore the labour theory of value cannot be wrong. After 1982 orthodox Marxists still exist, but they have changed their minds. They think that Marx is infallible and therefore does not have a labour theory of value. This nonetheless constitutes progress, as it is now possible to critically reconsider Marx’s theory of value with some serenity. Which is what I intend to do in the present note. In an attempt to clarify the erroneous aspects of Marx’s theory of value and bring to light its valid core as a theory of social relations, I will use some simple analytical tools and play with numeraires to expound an argument that today seems to be almost unanimously accepted.

All the debates on Marx’s theory of value have been sent off track by the way the value problem is set up by Marx himself: namely as a problem of the transformation of labour values into production prices. Marx does not realize that there is a difficulty stemming from deep philosophical reasons, which is independent of the transformation problem. Indeed, although he appreciates Adam Smith’s notion of ‘labour commanded’, he does not grasp his reasons for rejecting embodied labour as a measure of value in a capitalist economy (Marx, 1861-63, 31, 153), the reasons being that value determination based on embodied labour is only valid in a non-capitalist economy.

Labours commanded are production prices, and are determined in a way that makes them a correct expression of the technical and social conditions of production. Labour values, instead, only express technical conditions, and only when just one technique

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exists. To prove these propositions, I first present the naive model of a corn-corn economy, which is sufficient to show the validity of a labour commanded measure and the fallacy of a labour embodied one. Then I show that generalization to an economy producing \( m \) commodities does not modify this result.

After that, I tackle the transformation problem: given a double system approach, with a labour value system and a production price system, is it possible to transform the former into the latter while keeping the profit and exploitation rates invariant? I argue that, even when some aggregate invariance postulates are validated with opportune normalization, the basic problem remains unsolved – the problem of the inability of labour values to correctly express the social relations of production. In particular I argue that no reasonable normalization can achieve the invariance of the rate of exploitation and the rate of profit, which is tantamount to concluding that labour values are unsuitable to measure exploitation.

Finally I show that, by normalizing prices with the average productivity of labour, it is possible to define the rate of exploitation as a ratio between unpaid and paid labour without using the labour theory of value. This is how many contemporary Marxists, resorting to a ‘new interpretation’, have come to accept the need to abandon the notion of ‘embodied labour’ as a substance of value.

Labour embodied and labour commanded

Let us start with the model of an economy producing corn by means of itself and labour. Let \( a < 1 \) be the quantity of corn required to produce one unit of corn, \( l \) the labour coefficient, \( v \) the labour embodied in one unit of corn, \( p \) the production price, and \( r \) the rate of profit. The money wage is posited as equal to 1, so the production price is measured in labour commanded. The labour value and the production price of one unit of corn are determined with the equations:

\[
\begin{align*}
v &= l + va \\
p &= l + (1 + r)pa
\end{align*}
\]

whose solutions are:

\[
\begin{align*}
v &= l(l - a)^{-1} = l(1 + a + \cdots a^n) \\
p &= l(l - (1 + r)a)^{-1} = l(1 + (1 + r)a + \cdots (1 + r)^n a^n)
\end{align*}
\]

where \( l = 1 \), and \( n \) tends to infinity. Notice that the wage rate does not appear in any equation, but for distinctive reasons: in (1) and (3) because it is not required to determine embodied labour, in (2) and (4) because it is the numeraire. Equation (3) makes it clear that the labour value of output is the quantity of labour directly and indirectly embodied in it. Equations (3) and (4) reveal that, with \( 0 \leq r < r_{\text{max}} \), it holds \( p \geq v \).

As shown in figure 1, the labour commanded by one unit of corn is greater than the labour embodied in it, and the greater the higher the profit rate. Labour commanded is
a correct expression of value in an economy in which wage workers are exploited by capitalists, since it rises when exploitation rises. Labour embodied does not change with exploitation, and therefore labour values do not correctly express social relations in a capitalist economy.

Now let us reinterpret (1)-(4) as matrix equations. $v$ and $p$ are vectors of labour values and production prices respectively, $l$ a vector of homogenous labour coefficients, and $I$ the identity matrix. The scalar $a$ becomes an indecomposable matrix of technical coefficients $a = \{a_{ij}\} < 1$. It remains true that, since $0 \leq r < r_{\text{max}}$, it is $p \geq v$. In fact $(I - (1 + r)I)^{-1} \geq (I - a)^{-1}$.

Whatever the numeraire, $p \neq v$ holds generically. Since production prices differ from labour values, the question arises: which give a significant theory of value, i.e. a theory that expresses social relations? The answer is immediate: only prices yield a correct valuation, for they change when social relations change.

Among all the possible price vectors, those normalized with labour commanded have a peculiar property: they are increasing functions of $r$. They are a transparent measure of value – transparent with respect to social relations, as the labours commanded by all commodities rise with exploitation.

Such a property may be seen at work in the definition of the rate of exploitation in labour commanded, $e_c$:

$$e_c = \frac{ps-pas-L}{L} = \frac{p(l-a)s}{L} - 1$$  \hspace{1cm} (5)

where $s$ is the vector of gross outputs (all postulated as equal to 1) and $L = ls$ is the aggregate labour force.

This measure for the rate of exploitation is a ratio between the labour commanded by surplus value and that commanded by the wage. Capitalists have bought command

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2 Apart from when $r = 0$, a special case in which the labour theory of value holds strictly ($p = v$) is when $l$ is an eigenvector of $a$ (Kurz & Salvadori, 1995, 110-3).

3 The first derivatives of prices are $p'(r) = l(l - (1 + r)a)^{-1}a(l - (1 + r)a)^{-1} > 0$. The derivatives of any degree are also positive (Kurz & Salvadori, 1995, 125).
over $L$ workers, then they have exerted command over them in the production process so as to make them produce a surplus value, which may buy command over a further potential amount of labour, $L^* = p(s - a)s - L$.

Rather interesting is the factor of exploitation, $1 + e_c = (L^* + L)/L$, which is equal to the inverse of the wage share in net output. It is a ratio between the labour commanded by the net output and that used to produce it (Screpanti, 2003), in other words, the productivity of labour. In the presence of exploitation, this factor is greater than 1 as ‘the value of the total product can [...] buy more living labour than is contained in it’ (Marx, 1861-63, 31, 153).

The transformation of labour values into production prices

Marx knows that relative prices are different from relative labour values, but he thinks there is no problem in the aggregate. To his soul as an essentialist philosopher, ‘abstract labour’ is the substance of value (Screpanti, 2008), whilst production prices only express the surface appearance of market exchanges in a reproduction equilibrium, and the appearance should not alter the substance.

He seems to believe in a sort of a law of value conservation, and is confident that exchanges at production prices only redistribute value and surplus value among the different industries without altering their overall amount, so that the aggregate ‘surplus value’, ‘value of labour power’ and ‘dead labour’ are not modified by exchanges at production prices. If this were so, he could go on using the rate of exploitation and the rate of profit measured in labour values even when dealing with the price system.

Let $e_v$ and $e_p$ represent the rate of exploitation in labour values and production prices respectively; $r_v$ and $r_p$ the rate of profit in labour values and production prices respectively; $\hat{p}$ the price vector with a new normalization; $w_v$ and $w_p$ the unit ‘value of labour power’ and the money wage. And let us consider the following:

a) Fundamental invariance postulates

1. $e_v = e_p$
2. $r_v = r_p$

b) Secondary invariance postulates

1. $v(l - a)s = \hat{p} (l - a)s$
2. $w_vL = w_pL$
3. $vas = \hat{p} as$

The transformation problem boils down to finding a diagonal matrix $d$ such as $vd = \hat{p}$. In general, prices are determined up to a proportionality factor. Therefore there are

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4 Other invariance postulates can be deduced from these three. The invariance of gross output results from (b.1) and (b.3). The invariance of surplus value, from (b.1) and (b.2).
many $d$, one for each possible numeraire, and the standard can be chosen in order to obtain one of the secondary invariance postulates. So, let us normalize prices by positing

$$\hat{p}(1 - a)s = L$$

Since $v(1 - a)s = L$ by equation (1), postulate (b.1) is validated.

Embodied labour is a natural standard in the labour value system, which is made up of $m$ scalar equations with $m$ unknowns. Once labour values have been determined, it is sufficient to fix a basket of wage goods, $b$, to determine the ‘value of labour power’, $V = w_pL = vbL$, and the surplus value, $S = L - vbL$. There are no degrees of freedom, because labour values are determined without knowing the distributive variables. The case of the price system, in which prices depend on labour costs and the rate of profit, is different. Since social and political forces exogenously determine either $w_p$ or $r_p$, the system is made up of $m$ scalar equations with $m+1$ unknowns. There is one degree of freedom, and the possibility of introducing a normalization equation to validate one invariance postulate – but only one.\(^5\) Therefore, if (b.1) is validated, (b.2) and (b.3) cannot be.

**The fundamental invariance postulates do not hold**

Validation of a secondary invariance postulate does not imply validation of the fundamental ones. The rates of exploitation in the two systems are:

$$e_v = \frac{v(1-a)s - w_pL}{w_pL}$$

$$e_p = \frac{\hat{p}(1-a)s - w_pL}{w_pL}$$

The rates of profits are:

$$r_v = \frac{v(1-a)s - w_pL}{v\hat{a}s}$$

$$r_p = \frac{\hat{p}(1-a)s - w_pL}{\hat{p}\hat{a}s}$$

It is easy to see that $e_v = e_p$ and $r_v = r_p$ if and only if (b.1), (b.2) and (b.3) hold, which is not the case.

\(^{5}\) However, it is possible to force a further invariability assumption. If we postulate that both distributive variables are unknown, i.e. that neither of them is determined exogenously, we end up with $m+2$ variables. Thence we can posit two secondary invariance postulates and obtain, as a result, a fundamental one. For instance, by positing (b.1) and (b.2), (a.1) holds too. Loranger (2004) posits (a.2), which implies the invariance of aggregate capital and surplus value. Not surprisingly, these devices have not met with much success among Marxist economists, as they boil down to an imaginative theory of exploitation according to which the profit or the exploitation rates are determined not by the social and political forces of class struggle, but by the theoretical requirements of an ingenious thinker.
The result causes a serious problem for Marx. In fact, suppose $e_v < e_p$ and renormalize prices (now $\tilde{p}$) in such a way as to yield $w_pL = w_pL$. Then the ‘value of labour power’ is the same in the value and price systems; in other words, the wage in the price system coincides with the quantity of labour embodied in the workers’ consumption. The new normalization transforms the price of labour into its labour value.

Now, since the rate of exploitation is a pure number, it does not change with a change of standard. Therefore, $v(l - a)s - w_pL < \bar{p}(l - a)s - w_pL$, which means that, notwithstanding the value of labour power is identical in the labour value and price systems, the surplus value produced in the latter is greater than that produced in the former. It is as if the surface appearance of market exchanges had produced a surplus value over and above that produced in the labour value system.

Summing up, if value is a social relation, as required by Marx the social scientist, then production prices are meaningful measures of value, for they convey information about both the technical and the distribution conditions of production, and change when exploitation changes. Labour values, instead, are insensitive to changes in exploitation conditions. This is the reason why the rate of exploitation and the profit rate are not invariant in the transformation procedure. Now, $r_p$ is the actual rate of profit and is uniquely associated with $e_p$, not with $e_v$. Therefore the latter is an erroneous measure of exploitation.

One might observe that the labour and capital coefficients of production convey information about the way society allocates necessary labour among the various industries, and therefore the labour values they determine do, in fact, represent social relations. This proposition is correct if only one technique is available, yet it does not endorse the superiority of labour values, for all such ‘social’ information conveyed by them is also conveyed by production prices.

However, if more than one technique exists, then the labour value system may not convey correct information on the technical conditions of production. This is a big problem for Marx, given the great importance he attributes to technical change in the process of capital accumulation and in class struggle, and in fact has disappointing consequences on the law of the falling profit rate. The problem was highlighted by Okisho (1961), who proved that, since the choice of techniques is motivated by profit, the evolution of technical progress in a capitalist economy cannot be understood by using labour values. If there are two techniques, for instance, the price system correctly reveals which one is chosen by the capitalists, whilst the value system could lead to the wrong technique being chosen. The case of many techniques brings to light another reason why labour values do not convey correct information about the social relations of production: they do not pertain to the actual production conditions when technical change is motivated by profit.

**The triumph of Sraffa**

A way out of the labour value deadlock is to give up equation (1) and stick with equation (2) as the sole correct representation of values. The double system approach to
value determination gives way to a single system approach: ‘There is only one economy, one system, not two. There is no “underlying”, hidden economy, which operates in “values”’ (Duménil & Foley, 2008, 9). In other words, the only solution to the transformation problem is its dissolution. Since Duménil (1980; 1983-4), Foley (1982) and Lipietz (1982) proposed the ‘new interpretation’ of Marx’s theory of value, many orthodox Marxists have made this move. More or less knowingly, they accepted Sraffa’s theory of value, which is a reformulation of Marx’s theory of production prices.

Following the above three contributions, many other authors proposed reinterpretations which are rather different from each other yet have in common the same device: the use of labour productivity as a numéraire. Thus, let us use the expression ‘new interpretation’ as a portmanteau concept referring to all of them.

On the ground of equation (6) the average productivity of labour is taken as a numéraire, \( y = \hat{p}(l - a)s/L = 1 \), and the rate of exploitation can be written as

\[
e_p = \frac{\hat{p}(l-a)s}{wpL} - 1 = \frac{L-wpL}{wpL} = \frac{1-wp}{wp}
\]

(11)

We can say that the rate of surplus value is a ratio between unpaid labour, \( L - wpL \), and paid labour, \( wpL \). The device consists in using \( y \) to define the net output as equal to ‘living labour’. However, these definitions of ‘unpaid labour’ and ‘paid labour’ do not coincide with Marx’s definitions of ‘surplus value’, \( L - vbL \), and ‘the value of labour power’, \( vbL \).\(^6\)

It seems that a re-reading or rather a re-writing of Marx is required. The ‘new interpretation’ works as a monetary theory of labour value. \( y \) is called ‘the monetary expression of value’ or ‘the monetary expression of labour time’, and \( 1/y \) is meant as the ‘labour expression of money’ or the ‘value of money’. In this approach ‘labour value’ is immediately represented by money, as one unit of labour is equivalent to one unit of money.

It is worth pointing out that the labour productivity standard is first put forward by Sraffa (1960, 11) when he makes the value of net output equal to unity, \( \hat{p}(l - a)s = 1 \). Since he has already posited \( L = 1 \) in the preceding page, this is equivalent to postulating equation (6), in other words, taking labour productivity as a numéraire – ‘exactly the postulate of the New Interpretation proposed by Duménil and Foley’ (Bellofiore, 2008, 84). Why does Sraffa propose this normalization? One reason is that it permits use of the ‘standard commodity’ to establish a linear relation between the wage share and the profit rate. But there might be more.

\(^6\) Once normalized with \( y \), the prices of capital goods could be interpreted as quantities of redistributed ‘labour values’ (Wolff, Roberts & Callari, 1982) or ‘labor time-equivalents of constant capital’ (Foley, 2000, 25). However, since they are determined with Sraffa’s price equation (Petri, 2012), they do not correspond to Marx’s ‘dead labours’, \( va = (a + a^2 + \cdots a^{n+1}) \), but to Sraffa’s ‘dated quantities of labour’, \( \hat{p}a = (a + (1 + r)a^2 + \cdots (1 + r)^n a^{n+1}) \).
As some new interpreters conjecture, Sraffa seems to suggest that the net output and the quantity of living labour are not just formally equivalent (Gattei & Gozzi, 2010, 87). This conjecture is corroborated by research into Sraffa’s unpublished manuscripts. The Old Moor’s savior re-reads *Capital* in 1940. Then, in a note of 1942 entitled ‘Exploitation’, he introduces the notion of a ‘Pool of Profits’ with the meaning of unpaid labour. Perhaps it is from this perspective that we should interpret ‘the normalization of the value of living labour set equal to the value of the net product’ (Carter, 2014, 5).

To be true, a single system approach could be developed without any reference to Sraffa’s equation, since the numeraire $y$ can be applied to any conceivable set of price systems (Mohun, 1994, 407; Duménil & Foley, 2008, 1). Equation (2) is the one that determines prices at the highest level of abstraction compatible with that of Marx’s analysis (Screpanti, 1993). At a lower level of abstraction, the labour productivity standard could be applied to a fix-price oligopolistic economy with differential profit rates, to a dynamic process of market price gravitation, and even to a temporary equilibrium model with historical cost accounting of capital goods.

A reason why some Marxists take their distance from Sraffa is the ‘anomaly’ by which equation (2) treats wages as being paid *post factum*. Now, from a formal viewpoint this is not a problem, since all the theoretical propositions above remain valid if wages are treated as paid in advance. Nor is it a problem from a philosophical point of view, as the new interpreters take it as axiomatically true that value added is produced by labour. To thus the reduction of net output to living labour is true whatever the profit rate formula, and the question ‘who pays whom in advance’ does not impinge on ‘who exploits whom’.

The fact, however, is that in a capitalist economy ‘the labourer is not paid until after he has expended his labour power […] He has therefore produced […], before it flows back to him in the shape of wages, the fund out of which he himself is paid’ (Marx, 1867-93, 35, 567). In other words, ‘The capitalist, using the jargon of political economy, advances the capital laid down in wages […] But as a matter of fact the reverse takes place. It is the labourer who advances his labour to the capitalist for a week, a month, or three months’ (Marx, 1867-93, 36, 219).

Taking constant and variable capital, $K + V$, as a base for the profit rate calculation is the result of a distorted point of view: ‘the whole thing amounts to a capitalist *quid pro quo*, and the advance which the labourer gives to the capitalist in labour is turned into an advance of money given to the labourer by the capitalist’ (ibid). This is why business accountancy uses the formula $r = S/(K + V)$. Marx (1867-93, 35, 227) admits that – when he studies ‘the apparent degree of exploitation’ adopting this formula – he proceeds ‘according to the usual way of reckoning’ and, in fact, complies

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7 To produce is to bestow a certain amount of human labor on an ensemble of products […] Only human labor is productive […] It is necessary to postulate that this identification of value with labor incorporated holds for *any* product of any ensemble of productive processes’ (Duménil, 1983-84, 432-33).
with the jargon of political economy, i.e. that of the classical economists. Sraffa (1960, 10), instead, makes it clear to ‘assume that the wage is paid post factum […] thus abandoning the classical economists’ idea of a wage “advanced” from capital’.

Another reason why some new interpreters believe their approach to be at variance with Sraffa’s is that the latter takes as given the physical quantities of inputs and outputs whereas they take as given the overall amount of money anticipated to pay living labour and constant capital. Yet it is difficult to understand precisely what is at variance here. If you want to determine the prices of goods, you have to know the physical coefficients of production in any case. Then, if you wish to know the overall monetary value of transactions, just normalize prices with the quantity of money multiplied by its velocity of circulation. If you wish to know the overall money capital anticipated yearly, just express investments in terms of their monetary value. If you think that the monetary value of capital or net output should be determined before relative prices (Moseley, 1999), you can do so at the proper level of abstraction: that of macroeconomic theory. Not by chance Keynes quotes Marx’s ‘pregnant observation’ that production in a capitalist economy is a case of \( M-C-M \).

As for the wage, some new interpreters (e.g. Duménil, 1984; Duménil & Levy, 1991, Moseley, 1999) prefer to define it without specifying the wage goods basket, and treat it as a variable, not as a given. This is precisely how Sraffa (1960, 9-11) treats ‘the whole of the wage as a variable’ when dealing with a capitalist economy. As he remarks, the practice of regarding wages as ‘consisting of specified necessaries determined by physiological or social conditions […] loses much of its force […] when the wage is to be regarded as “given” in terms of a more or less abstract standard, and does not acquire a definite meaning until the prices of commodities are determined’ (ibid, 33).

Conclusions

Several Marxists have contested the ‘new interpretation’ from a methodological and a philological point of view. To mention just a few: Roemer (1990) observes that abandoning the dual system approach opens value determination to arbitrariness; Shaik & Tonak (1994) that it turns the whole relationship between surplus value and profit on its head; Fine, Lapavitsas & Saad-Filho (2004) that it wrongly assumes value to be immediately represented by money; and Petri (2012) that it adds nothing to the understanding of what determines profits.

More generally, it is hard to believe that Marx reasons in terms of a single system approach. But who can claim to have established ‘what Marx really said’, in this era?

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8 One might think that, when the length of the production process (e.g. one year) is longer than the length of the wage payment period (e.g. the month or the week), annual wages are actually anticipated by the capitalists, so that the full wage pre-payment formula approximates reality fairly well and, in any case, better than the full post-payment formula. This belief is not well founded, as proved by Steedman (1977, 103-5), and as shown in the appendix.
of hermeneutics? In any case, although philological concerns are understandable, it should be acknowledged that the ‘new interpretation’ has helped convince many Marxists that the labour theory of value can be abandoned without prejudicing the theory of exploitation. By the way, some new interpreters tend to use the phrase ‘labour theory of value’ in an ambiguous manner, giving the impression that their re-interpretation is talking about the same thing Marx is talking about. With this rhetoric device they might have misled some ingenuous Marxists. However – all’s well that ends well – they might have helped them not to feel like class enemies if they accept a theory that determines values correctly.

Marx seems possessed by a twofold self. He is an essentialist philosopher who believes value has a substance, and a social scientist who knows that value represents social relations. The labour theory of value is a fancy of the essentialist philosopher and a legacy he receives from Ricardo, the ‘naturalist’. It is a source of various analytical riddles and must be rejected by all Marxists who side with the social scientist. The analytical reasons for rejection add to those put forward by the anti-essentialist philosophers (Screpanti, 2008).

The only way to salvage Marx’s theory of exploitation is to abandon the labour theory of value and resort to the theory of production prices. Then you can choose the standard you like: it might be $w_P$, $y$ or something else. Sraffa, as reloaded by the new interpreters, would be happy anyway.
Appendix: When the period of wage payments and the length of the production process differ

Let us assume a production process lasting one year, and suppose the annual wage, $w_p$, is post-paid in $t$ sub-period instalments during the production period. The sub-period wage is $w_p/t$. The annual factor of profit is $1 + r = (1 + i)^t$, where $i$ is the sub-period rate of interest. Prices are determined as:

$$\hat{p} = (1 + r)\hat{p}a + \left[1 + (1 + i) + \cdots + (1 + i)^{t-1}\right]\frac{w_p}{t}l$$

Since $[1 + (1 + i) + \cdots + (1 + i)^{t-1}] = \left(\frac{(1+i)^t - 1}{i}\right) = \frac{r}{i}$,

$$\hat{p} = (1 + r)\hat{p}a + \frac{r}{ti}w_pl$$

As shown in figure 2, $r/ti$ (for $t = 12$ and $i \in [0.001, 0.1]$) is nearer to 1 than to $1 + r$. Thus the above formula approximates the full wage post-payment equation, $\hat{p} = (1 + r)\hat{p}a + w_pl$, better than the full wage pre-payment equation, $\hat{p} = (1 + r)\hat{p}a + (1 + r)w_pl$.

![Figure 2](image)

More generally, the difference between $[(1 + r) - r/ti]$ and $(r/ti - 1)$ is

$$x_t = (1 + i)^t - 2\frac{(1 + i)^t - 1}{ti} + 1$$

This equation has many solutions, but only one is economically meaningful, i.e. has real values corresponding to positive $i$'s. The meaningful section of $x_{12}$ is drawn in figure 3, together with that of a similar function, $x_{52}$, representing the case of weekly wage payments. Both curves show that the equation with full wage post-payment is better approximated than that with full pre-payment.

![Figure 3](image)
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References


