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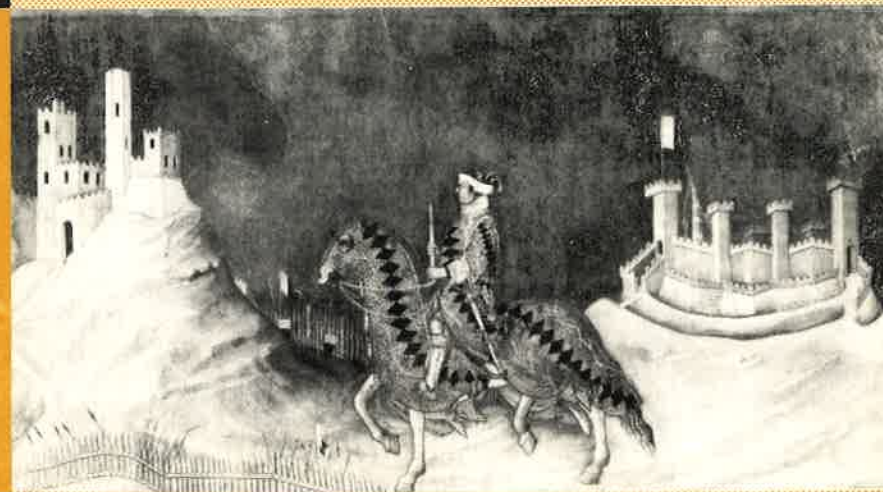
QUADERNI DELL'ISTITUTO DI ECONOMIA

Silvano Vicarelli

IMPATIENCE TO CONSUME, THE UTILITY OF WEALTH AND THE RATE OF INTEREST



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1. Introduction

The theories of interest based on "abstinence", "time-agios" and "impatience to consume" have a long history. The works of Böhm-Bawerk and Fisher, by integrating them with the "productivity" aspects and by providing them with a more adequate analytical apparatus, have contributed to their subsequent and successful reformulations. Fisher's work, in particular, because of its rigorous separation of the analysis in terms of flows from the analysis in terms of stocks, its definition of the "rate of time preference", of the "transformation frontier" and of the related concept of "marginal rate of return", is usually considered free of the fallacies attributed by the "Cambridge critics" to the Walrasian and Böhm-Bawerkian-type theories of capital and interest. Also the later models of Debreu, Allais, Arrow and others, directly inspired by the Fisherian tradition, have been criticized because of their rather abstract assumptions and because of the concept of "equilibrium" implicitly assumed, not because of any fault with the logic. This notwithstanding, there are several aspects of time-preference theories that may be considered unsatisfactory, even within the apparatus of the theories themselves.

It will be shown, in the first half of this essay, that a Fisherian-type general equilibrium theory of interest, in a world with more than one consumption good, does not allow to affirm, as usually happens, that the rate of interest is "determined" by the two *independent* forces of "impatience" and "productivity".

We shall then argue that the concept of "time preference" itself is ambiguous and cannot be identified with "impatience" proper; so that it is not correct to speak of an "impatience" theory of the interest rate.

Finally, we shall emphasize the fact that all the time-preference theories of the distribution of resources through time rests crucially on the assumption, be it explicit or implicit, that consumption is the sole end of economic activity and that, therefore, savings have the mere purpose of changing the time-pattern of the consumption flows. We shall show that there are good reasons to believe that the holding of wealth may be desired for its own sake and that this fact invalidates certain familiar results of time-preference theories.

2. A Fisherian general equilibrium model

In the exposition of his theory of interest Irving Fisher assumes a one- commodity world or, better, a single consumption commodity economy.[1] Capital goods, in fact, are supposed to be numerous, even though they do not appear explicitly in the analysis: they are inherited from the past, so that at the beginning of the period they are considered among the data of the problem. Subsequently, they are supposed to adapt themselves to the desired production of the stream of the consumption good over time. In this kind of world, the problem of relative prices and of their interaction with the interest rate does not arise.

Fisher is well aware that all the economic variables are interdependent and that in particular the system of relative prices interacts with the level of the interest rate; but he believes that "for practical purposes" the problem of relative prices can be handled separately from the problem of interest, so that prices can be taken as given in the explanation of the mechanisms leading to the equilibrium rate of interest.[2] The two steps of the analysis, the theory of interest and the theory of relative prices, can be

[1] The consumption commodity is considered by Fisher, as a matter of fact, as a "composite" commodity; that is as a basket composed of consumption goods and services in given and fixed proportions. But, as is well known, when the relative prices of its components change, this basket behaves as a single commodity, so that we are put back into a one commodity world.

subsequently reconciled, according to him, in a final and global explanation of the general equilibrium of the economic system in which "all the principles remain valid".

As examples of general theories of prices and interest, Fisher cites Walra's and Pareto's works; he also cites his own *Mathematical Investigations in the Theory of Value and Prices*. But all these models are single period models and do not take account of one of the fundamental aspects of the Fisherian theory of interest: the choice between present and future consumption . We have to wait till 1959 , with Debreu's *Theory of Value*, to have a rigorous formulation of a general equilibrium system containing the problem of inter-temporal choice (even though Debreu does not mention Fisher's works). But Debreu's analytical apparatus is rather different from the one used by Fisher; it presupposes, as is known, the existence of a "futures economy", that is of an economic system wherein perfect forward markets for all commodities, services and financial claims exist and where equilibrium is established simultaneously in all "spot" and

[2] Fisher's conviction is clearly expressed in the following passage: "It is, of course, realized that the principles of price determination involve interest just as the principles determining interest involve prices. A complete picture of economic equilibrium includes every possible variables, each acting and reacting on the others... Theoretically any analysis of one part of the economic organism must include an analysis of the whole, so that a complete interest theory would have to include also price theory, wage theory and, in fact, all other economic theory. But it is convenient to isolate a particular element by assuming the other elements to have been determined. So this book *The Theory of Interest* is a monograph restricted, so far as may be, to the theory of interest and excluding price-theory, wage theory and all other economic theory. Afterward it will be easy to dovetail together this interest theory, which assumes prices predetermined, with price theory which assumes interest predetermined thus reaching a synthesis in which the previous assumed constants become variables. But all the principles remain valid." (Fisher, I. *The Theory of Interest*, (1930), A.M. Kelley, New York, 1965, p. 131. For an analogous statement see also section 4 of Chapt. IV of the same book.)

"forward" markets. In this kind of world, each individual makes his consumption and production plans for all periods within the horizon (all individuals having the same horizon) and tries to obtain spot and forward contracts according to those plans. Spot and forward prices are determined therefore by the clearing of all markets, spot and forward: the equilibrium of the economy being determined not only for the current period, but for all future periods. The rate of interest does not appear explicitly in the analysis: it emerges simply as a derived variable, namely as the ratio between forward prices and spot prices.

We prefer not to adopt this kind of science-fiction economy but to remain in a more Fisherian world in which forward markets exist only for a limited number of commodities and the rate of interest enters explicitly into the economic decisions of the individuals. The better way to "dovetail" together Fisher's interest theory as exposed in *The Theory of Interest* with his price theory as exposed in the *Mathematical Investigations* seems, therefore, to adopt the Hicksian method of "temporary equilibrium" analysis.

Fisher's starting point is the assumption that each individual, at the beginning of the period taken into consideration, is endowed with a certain amount of *physical wealth*. This may include consumption goods, productive instruments (or capital goods), financial assets, "human capital", etc. In the current period, or "year", this stock of physical items can be used in two different ways: a) to produce a flow of services in a form that can satisfy immediately the human needs; b) to produce a flow of services that are used to produce "intermediate" goods to be used in the production of future consumption services. The only aim of production is, in other words, consumption: present consumption if the original stock is transformed directly into consumption services, and future consumption if the original stock is partly transformed into a different set of productive instruments which

are again transformed, in the following years, into more consumption services. The only use that can be made of the existing stock of resources and of "human capital" is therefore to devote it to the satisfaction of human wants: the only alternative is whether to devote it to present satisfaction or to future satisfaction. The choice that each individual must face (and society as a whole must face too) is then between present consumption and future consumption.

Without any loss of generality, we shall assume that the consumption services, or "consumption goods", that can be produced are of two kinds: A and B; and that the time horizon of every individual is of two periods: this "year" and the next "year". The number of individuals composing the economic system will be indicated by n.

Each individual, i, derives his satisfaction from the potential consumption streams according to a preference ordering that may be represented by the utility function:

$$U^i = f^i(a_1^i, b_1^i, a_2^i, b_2^i), \quad (i=1,\dots,n)$$

(where U^i represents the individula's level of utility, a_1^i and b_1^i his current consumption flows of goods A and B, and a_2^i and b_2^i his future consumption flows of the same goods). The *marginal rate of time preference*, in Fisher's words, is "the percentage excess of the present marginal want for one more unit of *present* goods over the *present* marginal want for one more unit of *future* goods."[3] In practice, the marginal rate of time preference is nothing more than the special name given to the marginal rate of substitution, minus one, in the case of goods consumed in two different periods of time.

From the above utility function we cannot derive a unique rate

[3] *Fisher, I. The Theory of Interest*, op. cit., p. 62.

of time preference as in the case of a single composite commodity; in fact, both present consumption and future consumption are a mixture of two goods that may be consumed in different proportions. We can therefore obtain four such rates: the rate between the present and the future consumption of good A, ρ_a ; the rate between the present and the future consumption of good B, ρ_b; the rate between the present consumption of good B and the future consumption of good A, $\rho_{a,b}$; and the rate between the present consumption of good A and the future consumption of good B, $\rho_{b,a}$. These rates can be analytically defined, for each individual, as follows:

$$1+\rho_a^i = \frac{\partial U^i/\partial a_1^i}{\partial U^i/\partial a_2^i}$$

$$1+\rho_b^i = \frac{\partial U^i/\partial b_1^i}{\partial U^i/\partial b_2^i}$$

$$1+\rho_{a,b}^i = \frac{\partial U^i/\partial b_1^i}{\partial U^i/\partial a_2^i}$$

$$1+\rho_{b,a}^i = \frac{\partial U^i/\partial a_1^i}{\partial U^i/\partial b_2^i}$$

As a matter of fact, only three of these four rates of time preference are independent of each other; the fourth can be derived from the other ones and can therefore be omitted in the analysis. $\rho_{b,a}^i$, for example, can be obtained from the other rates in the following way:

$$1+\rho_{b,a}^i = \frac{(1+\rho_a^i)(1+\rho_b^i)}{(1+\rho_{a,b}^i)}$$

Besides the rates of time preference just defined, we can derive

from the utility function the two traditional rates of substitution between goods to be consumed within the same year: that is the marginal rates of substitution between good A and good B in the first year and in the second year. But also in this case we need not introduce them explicitly into the analysis, because they are not independent of the rates of time preference and can be easily derived from them. For example, the marginal rate of substitution between A and B within the first year is given by:

$$\frac{(1+\rho_{b,a}^i)}{(1+\rho_b^i)} \cdot$$

In what follows, as we are mainly interested in time preference, we shall make use only of the first three independent rates of substitution defined above: ρ_a, ρ_b, and $\rho_{a,b}$.

These rates depend clearly on the quantities of goods consumed in the two years and can therefore be expressed as functions of the latter:

(1) $\rho_a^i = F_a^i(a_1^i, b_1^i, a_2^i, b_2^i), \quad (i = 1,\dots, n)$

(2) $\rho_b^i = F_b^i(a_1^i, b_1^i, a_2^i, b_2^i), \quad (i = 1,\dots, n)$

(3) $\rho_{a,b}^i = F_{a,b}^i(a_1^i, b_1^i, a_2^i, b_2^i), \quad (i = 1,\dots, n)$

The income of each individual is given by the value of the consumption goods he will plan to produce with his initial endowments. So the first year's income is given by:

$$A_1^i\, p_a + B_1^i\, p_b$$

and the second year's income is given by:

$$A_2^i\, p_a(1+\pi_a^i) + B_2^i\, p_b(1+\pi_b^i) ,$$

where A^i and B^i are the physical flows of commodities that the individual will decide to "produce" in the two years with his initial resource endowments; p_a and p_b are this year's "spot" prices of the two commodities, and π^i_a and π^i_b are the *expected* rates of variation (or "rates of inflation") of the two prices in the passage from the first to the second year. We assume that expectations are given but not necessarily equal for all individuals.

If we initially suppose, as in Fisher's "first approximation", that the two income streams are already determined in some way or other, each individual can only try to adjust them to his intertemporal consumption preferences by the process of borrowing and lending. That is, each individual faces only *exchange opportunities* or *market opportunities*, in the sense that the only way he can modify his income streams is by trading with other members of society. If this year's preferred stream of consumption is, in value, greater than this year's stream of income, the individual will be a borrower. In this case he will have to repay in the following year his debt plus an interest on it; so that his second year's consumption will be smaller than his second year's income. Just the opposite happens if the first year's preferred consumption is smaller than the first year's income: the individual will lend money at the market interest rate and will be reimbursed in the following year. So if we define the first year's savings, s^i_1, and the second year's savings, s^i_2, respectively as:

$$(4) \qquad s^i_1 = A^i_1 p_a + B^i_1 p_b - a^i_1 p_a - b^i_1 p_b, \quad (i = 1,\dots,n)$$

$$(5) \qquad s^i_2 = A^i_2 p_a(1+\pi^i_a) + B^i_2 p_b(1+\pi^i_b) - a^i_2 p_a(1+\pi^i_a) - b^i_2 p_b(1+\pi^i_b), \quad (i = 1,\dots,n)$$

the following constraint must hold for each individual:

$$(6) \qquad s^i_1(1+r) = s^i_2 , \quad (i = 1,\dots,n)$$

(where r stands for the market money rate of interest). That is, the savings (or dis-savings) of the current period plus interest must be equal to the dissavings (or savings) of the next period.

In order to maximize his total utility deriving from the flows of consumption of the two periods, each individual will borrow or lend money until the marginal factor of time preference for each commodity is made equal to the interest factor discounted by the "factor of inflation" of the same commodity, that is :

$$(7) \qquad (1+\rho^i_a) = \frac{(1+r)}{(1+\pi^i_a)}, \quad (i = 1, \dots, n)$$

$$(8) \qquad (1+\rho^i_b) = \frac{(1+r)}{(1+\pi^i_b)}, \quad (i = 1, \dots, n)$$

Furthermore, the marginal factor of time preference between the present consumption of good B and the future consumption of good A must be equated to the factor of interest discounted by the rate of inflation of good A, taking account of the relative prices of the two goods:

$$(9) \qquad (1+\rho^i_{a,b}) = \frac{p_b}{p_a}\frac{(1+r)}{(1+\pi^i_a)}, \quad (i = 1,\dots,n).$$ [4]

The ratio between the interest factor and the expected factor of variation of the price of a commodity can be defined as the *expected real interest factor* in terms of that commodity. As is known, the "real" rate of interest in terms of a commodity is usually defined by the following procedure: with one unit of money it is possible to buy, in the first year, $1/p_a$ units of commodity A;

[4] As we have seen, of the six rates of substitution that can be derived from the utility function, only three are independent of each other. The independent equilibrium conditions for utility maximization are therefore only three, the other ones being a consequence of the former.

if, in the second year, the price of *A* has changed at the rate π_a, with $(1+r)$ units of money it will be possible to buy $(1+r)/p_a(1+\pi_a)$ units of commodity *A*. So there is a definite quantity of *A* to be delivered a year hence which has the same exchange value as one unit of *A* to be delivered immediately; this quantity represents the "*A*-rate of interest" and is given by the ratio between $(1+r)/p_a(1+\pi_a)$ and $1/p_a$. In our case, the "rate of inflation" is not an *ex post* rate but an *expected* rate; so that also the "real" rate of interest in terms of *A* is an *expected* real rate. Furthermore, there is no *market* real interest rate, that is no *objective* and unique rate for all the members of the economy; on the contrary, there are as many *subjective* real rates as there are individuals (only under the assumption of "rational expectations" all individuals could be supposed to expect the *same* rate of inflation and therefore to act on the basis of the same real interest rates).

The *n* subjective "*A*-rates of interest" will therefore be given by the relations: $\frac{(1+r)}{(1+\pi_a^i)}$, (i=1,...,n). In an analogous way we can define the "real" rate of interest in terms of good *B* as: $\frac{(1+r)}{(1+\pi_b^i)}$, (i=1,...,n) .

The equilibrium conditions (7), (8) and (9) could then be expressed also by saying that, in order to maximize his utility, each individual must equalize his marginal rate of time preference for each commodity to the expected real rate of interest in terms of the same commodity.

In the Fisherian-type models with a single consumption commodity, the rate of interest is made equal for all individuals to a *unique* rate of time preference of "consumption in general"; here we have three rates of time preference interacting with the money rate of interest, the expected rates of inflation and relative prices. The simple Fisherian equivalence between interest and time preference is lost. As a matter of fact, it can be re-established, but

only *ex post*, that is only as the final consequence of the achievement of equilibrium by all individuals. To see this, we must define what the time preference of money spent on a particular good is. If an individual subtracts one unit of money to be spent on the immediate consumption of good *A*, he suffers a loss of satisfaction equivalent to : $\frac{1}{p_a}\partial U^i/\partial a_1$; in order to keep his utility at the same level as before the individual should spend a certain amount of money, say $(1+\rho_{ma})$, to increase his second year's consumption of good *A*. So, ρ_{ma} should be such as to satisfy the relation:

$$\frac{1}{p_a}\frac{\partial U^i}{\partial a_1^i} = \frac{(1+\rho_{ma})}{p_a(1+\pi_a^i)}\frac{\partial U^i}{\partial a_2^i}$$

From which we get:

$$1+\rho_{ma}^i = (1+\rho_a^i)(1+\pi_a^i)$$

In a similar way we can define the marginal rate of time preference of money spent on consumption of good *B*:

$$1+\rho_{mb}^i = (1+\rho_b^i)(1+\pi_b^i)$$

Finally, we can define the marginal rate of time preference of money subtracted from the present consumption of good *B* and devoted to the future consumption of good *A*:

$$1+\rho_{ma,b}^i = (1+\rho_{a,b}^i)(1+\pi_a^i)\frac{p_a}{p_b}$$

At this point it is easy to verify, from equations (7), (8) and (9), that in equilibrium:

$$(1+\rho_{ma}^i) = (1+\rho_{mb}^i) = (1+\rho_{ma,b}^i) = (1+r)$$

That is, once the general equilibrium situation has been achieved, the marginal rates of time preference for money spent on any kind of goods will be the same and will be equal to the money rate of interest. We could now speak, as in the traditional time preference models, of a unique rate of time preference for "consumption in general" which is equal to the interest rate and is the same for all individuals. But this unique rate does not exist out of the equilibrium situation: it can be defined and determined only *after* the rate of interest has been determined by the equilibrium of the economic system. So, in no way could we say that time preference for "consumption in general" "determines" r.

For the single individual acting in a competitive market, prices and the interest rate are a datum, in the sense that his actions affect them only infinitesimally. For the market as a whole, the order of causality is reversed: prices and the interest rate are variables which are the result of the community's demand and supply of goods and loans. Those people who, given their income flows, have a high marginal rate of time preference for present over future consumption will be borrowers, tending to raise the rate of interest. On the other hand, those people who start with a low time preference will be lenders, tending to lower the rate of interest. The resulting rate of interest will be such as to clear the market. This is what leads the Fisherian theories to affirm that the rate of interest is a measure of the "average" or "common" rate of preference for present over future consumption, as determined by the supply and demand of loans.

So, we must add the following condition that imposes the clearing of the loans market in the current year:

(10) $$\sum_{i=1}^{n} s_1^i = 0$$

Then, we must impose the condition for the clearing of the market of good A:

(11) $$\sum_{i=1}^{n} A_1^i = \sum_{i=1}^{n} a_1^i$$

The clearing of the market of good B cannot be imposed as an additional equilibrium condition; as is known from Walras' law, if all markets are cleared but one, then also this one will automatically be cleared.[5]

The clearing of next year's markets cannot obviously be imposed in a temporary equilibrium analysis; this is because there is no mechanism assuring that the plans of all individuals, made in the current year according to both current and *expected* prices, will be compatible in the future. The market mechanism will only make compatible the current year's plans.

If we continue to adopt the hypothesis that the physical flows of income A_1^i, B_1^i, A_2^i, B_2^i are given, the number of unknowns so far enumerated exceed by one the number of equations: the unknowns are, in fact, 9n+3 (that is a_1^i, b_1^i, a_2^i, b_2^i, ρ_a^i, ρ_b^i, $\rho_{a,b}^i$, s_1^i, s_2^i, for i=1,...,n, and p_a, p_b, i); while the independent equations are 9n+2 (that is the set of equations (1)-(11)). We could take, as usual, one of the two goods as the *numeraire*, thus eliminating its price from the unknowns; but we prefer, to remain in the Fisherian spirit and to be able to speak of monetary prices and of a monetary rate of interest, to add the *equation of exchange*:

[5] Analytically, this is implied by the fact that the condition

$$\sum_{i=1}^{n} B_1^i = \sum_{i=1} b_1^i$$

is not independent of the other ones: in fact, it can be derived from the equilibrium conditions (10) and (11) and from the definition of savings given in equation (4).

(12) $$M\,V = \sum_{i=1}^{n} a_1^i p_a + \sum_{i=1}^{n} b_1^i p_b$$

(where M stands for the given quantity of money and V stands for the given velocity of circulation of money). In any case, the substance of the subjects at issue does not rely on the above assumption.

The set of equations (1)-(12) will thus determine the absolute prices of goods A and B, the money rate of interest and, for each individual, the marginal rates of time preference, the quantities of goods consumed in the current period and planned for consumption in the next period and the net borrowings. All this represents Fisher's "first approximation" of the theory of interest: a theory that, because of the hypothesis that the flows of income are given and unmodifiable, leads to a purely *psychological* explanation of interest. This aspect of the time preference models of interest is the one that has never been questioned: even the "Cambridge controversies" on the theory of capital and the nature of interest have never denied that in a world of pure exchange the Fisherian-type explanations of interest could be sensible and coherent.

As a matter of fact, the introduction of more than one consumption good reveals that the correlation between the psychological element of time preference and the phenomenon of interest is not so simple and neat as the traditional theory would maintain. We refer, first of all, to the fact that the money rate of interest depends also on price expectations (as is shown by equations (7), (8) and (9)). As expectations belong, at least in part, to the sphere of psychology, they can interact with preferences for present and future consumption; thus, an increase of the expected rate of inflation could, for example, raise the desire for immediate consumption, causing a cumulative effect on the money rate of interest while leaving unaffected the "real" rates of interest. But we refer mainly to the fact that the money rate of

interest depends also on relative prices, as is shown by equation (9). This implies, for example, that the preferences of individuals for the immediate consumption of good B against the future consumption of good A , that is the functions giving $\rho_{a,b}$, might change, causing, at the general level, a variation in the relative prices of the two goods such as to leave unaffected the rate of interest (both monetary and "real").

3. Production opportunities

The physical income streams A_1, A_2, B_1, B_2 cannot obviously be considered as given. Each individual can utilize his particular stock of material wealth, (included "human capital") in different ways, within the limits imposed by the current technology: he can transform it into immediate consumption goods or into "intermediate", or capital, goods to be used in the production of future consumption goods. The quantities of produced consumption goods must therefore be considered as variables to be determined by the maximizing behaviour of the various individuals: in addition to the exchange opportunities each individual faces a set of *productive opportunities*.

From the technical relations among the given set of items of wealth initially owned and the possible flows of "intermediate" commodities and of final consumption goods, we can obtain the *opportunity frontier* facing each individual:

(13) $$T^i(A_1^i, B_1^i, A_2^i, B_2^i) = 0\ , \quad (i = 1,\dots,n)$$

The implicit function (13) shows the different and alternative physical income streams that can be technically (and efficiently) obtained in the two years. It should be stressed that the location of the production possibility frontier will, in general, depend on the stock of physical resources available at the beginning of the first year, among which capital goods are included. One of the characteristics of the Fisherian approach is that neither the initial capital goods, nor the ones that are produced during the first year and utilized in the second year, appear explicitly in the exposition of the theory: they exist but are simply lurking in the background. One of the fundamental consequences of this fact is that the "cost of production", in the traditional sense, disappears: the existing resources have already been produced in the past, so that there is no cost to sustain in the current period to use them. The only alternative is to use them or to leave them idle; and, in the first case, to use them in one direction or in a different direction. As the final aim of any kind of production is consumption, the "cost" of using resources in one direction in alternative to a different one is given by the loss of consumption involved by choosing the first direction. Analogously, the "returns" of using resources in one direction in alternative to a different one is given by the increase in consumption that can be obtained by choosing the first direction.

Thus, each individual is constantly faced with the opportunity to produce a certain set of consumption streams rather than a different one; and the choice between the two options is based on the comparison of the *disadvantages*, or *costs*, measured in terms of consumption losses and the *advantages*, or *returns*, measured in terms of consumption gains implied by the two alternatives. Without the existence of a range of *options* , that is of *possible* consumption streams open to an individual, the concept of cost could not exist, nor could the concept of return exist. Costs and returns are not *absolute* concepts: any cost and any return can be expressed only in comparative terms between two alternative options.

The Fisherian *rate of return over cost* is defined, as a consequence, as the percentage variation of consumption due to

the passage from one option to another. In this sense, it is always a *marginal* rate of return.

From the opportunity frontier expressed by equation (13), we can derive six such rates of return expressed in *physical* terms: these rates are obtained from the usual *marginal rates of transformation* of one consumption good into another one, less unity. But, analogously to what we have seen in the case of the utility function, only three of these six rates of transformation are independent of each other. Therefore, as we are interested in transformations over time more than in transformations within the same year, we shall use in the analysis only the following three marginal rates of return: the marginal rate obtained by shifting the resources from the production of A_1 in the current year to the production of A_2 in the following year, σ_a^i ; the marginal rate obtained by shifting the resources from the production of B_1 to the production of B_2, σ_b^i ; and the marginal rate obtained by shifting the resources from the production of B_1 to the production of A_2, $\sigma_{a,b}^i$. Analytically, these rates are given, for each individual's transformation function, by:

$$(1+\sigma_a^i) = \frac{\partial T^i/\partial A_1^i}{\partial T^i/\partial A_2^i}, \quad (i = 1,\dots,n)$$

$$(1+\sigma_b^i) = \frac{\partial T^i/\partial B_1^i}{\partial T^i/\partial B_2^i}, \quad (i = 1,\dots,n)$$

$$(1+\sigma_{a,b}^i) = \frac{\partial T^i/\partial B_1^i}{\partial T^i/\partial A_2^i}, \quad (i = 1,\dots,n)$$

The physical rates of return are obviously a function of all the physical flows of present and future production; they can therefore be expressed as:

(14) $\sigma^i_a = T^i_a(A^i_1, B^i_1, A^i_2, B^i_2)$, $(i = 1,...,n)$

(15) $\sigma^i_b = T^i_a(A^i_1, B^i_1, A^i_2, B^i_2)$, $(i = 1,...,n)$

(16) $\sigma^i_{a,b} = T^i_a(A^i_1, B^i_1, A^i_2, B^i_2)$, $(i = 1,...,n)$

As is known, out of all possible options open to an individual, that particular one will be selected which maximizes the present value of the flow of production, that is that maximizes the present value of wealth. To do this, each individual must equate the marginal factor of return of each good to the "real" factor of interest in terms of the same good:

(17) $1+ \sigma^i_a = \dfrac{1+r}{1+\pi^i_a}$, $(i = 1,...,n)$

(18) $1+ \sigma^i_b = \dfrac{1+r}{1+\pi^i_b}$, $(i = 1,...,n)$

(19) $1+ \sigma^i_{a,b} = \dfrac{p_b}{p_a} \dfrac{(1+r)}{1+\pi^i_a}$, $(i = 1,...,n)$

The general equilibrium system is now complete: we have added $7n$ new unknowns (the $4n$ quantities to be produced and the $3n$ marginal rates of tranformation of all individuals), and $7n$ new equations (the set (13)-(19)).

Analogously to what we have seen in the case of utility maximization, the Fisherian equivalence between the market money rate of interest and a unique rate of return on "monetary investment" is lost. We can obtain this relationship only *ex post*, after the system has reached the equilibrium position. To see this, we must define what the marginal rate of return on one unit of money invested is. To invest one unit of money means to reduce, for example, the production of $1/p_a$ units of good A in the first

year in order to obtain a greater amount of the same good in the second year. Given the physical marginal rate of return of good A, σ^i_a , the above decrease of production in the first year will allow the following increase of production in the second year:

$$\frac{1}{p_a} (1+\sigma^i_a)$$

which in monetary terms will give the rate of return of monetary investment in good A, σ^i_{ma} :

$$(1+\sigma^i_{ma}) = (1+\sigma^i_a)\,(1+\pi^i_a)$$

With a similar procedure it is possible to show that the marginal rate of return of monetary investment in good B is given by:

$$(1+\sigma^i_{mb}) = (1+\sigma^i_b)\,(1+\pi^i_b)$$

and that the marginal rate of return of monetary investment in the production of good A through the reduction of the production of good B is given by:

$$(1+\sigma^i_{ma,b}) = (1+\sigma^i_{a,b})\,(1+\pi^i_a)\,\frac{p_a}{p_b}$$

But from equations (17), (18) and (19) it is easy to verify that, *in equilibrium*,

$$(1+\sigma^i_{ma}) = (1+\sigma^i_{mb}) = (1+\sigma^i_{ma,b}) = (1+r)$$

That is, the rate of return on money invested in any kind of production is equal, for all individuals, to the money rate of interest.

So, *in equilibrium*, both the rate of time preference of monetary consumption and the rate of return on monetary investment are equal to the money rate of interest; but we must remember again that this is not an equilibrium condition, as in the traditional Fisherian models, but simply an *ex post* identity verified only by the equilibrium values of prices and interest. Which means that there are no "causal" relationships among the above three variables.

4. The order of causality

The fact that the equality among the money rate of interest, the marginal rate of time preference of monetary consumption and the marginal rate of return on monetary investment can be established only *ex post* does not totally disrupt the Fisherian insight according to which among the forces that have an influence on the rate of interest the psychological element of time preference and the technological element of the rate of return have some role to play. But it disrupts the straight and simple order of causality usually established by time-preference theories between the variations of preferences and of technical conditions on one side and the corresponding variations of the interest rate on the other side.

Actually, in a general equilibrium system it is meaningless to establish an order of causality among the variables, simply because "everything depends on everything else". As Samuelson pointed out several decades ago, "within the framework of any system the relationships between variables are strictly those of mutual interdependence. It is sterile and misleading to speak of one variable as causing or determining another. Once the conditions of equilibrium are imposed, all variables are simultaneously determined."[6] There is nevertheless a sense in which it is possible to establish an order of "causality" among the various elements of

the analysis: as is usually maintained "exogenous" variables, or "parameters", *determine* "endogenous" variables. Again in Samuelson's words: "the only sense in which the use of the term causation is admissible is in respect to changes in external data or parameters. As a figure of speech, it may be said that changes in these *cause* changes in the variables of our system."[7] If we supposed that technical conditions and individual preferences are exogenously given and are not influenced by the other variables during the analysis, it makes sense to affirm that the former "determine" in some way the latter. But this represents too vague a concept of causation; what the time-preference theories mean when they say that the rate of time preference and the rate of return "determine" the interest rate is that an increase (or a decrease) of the former cause an increase (or a decrease) of the latter. What we are going to show is that this stronger kind of causality is not, in general, valid.

The Fisherian position on the theory of interest is an eclectic one: interest is not a completely psychological phenomenon, as the "subjectivists" claim, nor is it a completely technical phenomenon, as the "productivity theory" supporters sustain.[8] In his words: "Impatience is impatience to spend, while opportunity is opportunity to invest. The more we invest and postpone our gratification, the

[6] Samuelson, P.A. *Foundations of Economic Analysis*, Harward University Press, Cambridge, Mass., 1947, p. 9.

[7] *Ibidem.*

[8] As is known, a long and harsh debate on the "true" cause of interest took place during the first decades of this century between the "subjective" school and the "productivity" school of the rate of interest. On this debate see, e.g., F. Fetter, *Interest Theories , Old and New*, "American Economic Review", 1914; and *Interest Theory and Price Movements*, "American Economic Review", 1927; Seager, H. R. *The Impatience Theory of Interest*, "American Economic Review", 1912; Brown, H.G., *The Marginal Productivity versus the Impatience Theory of Interest*, "Quarterly Journal of Economics", 1913.

lower the investment opportunity rate becomes, but the greater the impatience rate; the more we spend and hasten our gratification, the lower the impatience rate becomes but the higher the opportunity rate... Between these two extremes lies the equilibrium point which clears the market, and clears it at a rate of interest registering (in a perfect market) all impatience rates and all opportunity rates."[9] Therefore, "the opportunity line cannot be dispensed with in the theory of the rate of interest. It is something distinct from and in addition to the impatience lines as well as to the market lines...To adapt a simile of Alfred Marshall's, both blades of a pair of scissors are needed to make the scissors work".[10]

From the idea that *time preference* and *time productivity* are the two forces the working of which "determine" the interest rate a long series of corollaries have been derived:

A) first of all, since Böhm-Bawerk's times, it has been remarked that if all individuals had a zero rate of time preference, that is if in the equations (1), (2) and (3) of our model the dependent variables were zero for all possible values of the consumption streams, the phenomenon of "real" rates of interest could not exist: only the money rate of interest could be positive if the expected rate of inflation were positive. As a consequence, whatever the production opportunities, the investments would be pushed to such a level as to give zero "real" rates of return.

B) Conversely, if the production opportunities could only guarantee the tranformation of any quantity of present goods into an equal quantity of future goods, that is if the rates of transformation were always equal to one, the "real" rates of interest could never be positive: as a consequence, whatever the structure of the consumers' intertemporal preferences, their

[9] Fisher, I. *The Theory of Interest*, op. cit., p.177.

[10] *Ibidem*, p.282.

present consumption would be pushed to such a level as to make time preference vanish.

C) A general rise of the individuals' time preference, or any local rise in the neighborhood of the initial equilibrium point, would raise the equilibrium money rate of interest and, provided that the expectations of the future level of prices do not change, also the "real" rates of interest. A similar effect would produce in the case of a shift in wealth distribution in favour of individuals with relatively high marginal time preferences. The cause of the wealth redistribution has no relevance: it could be due either to a redistribution of the resources initially owned by individuals or to a change of the distribution of productive opportunities among them.

D) In the neighborhood of the initial equilibrium situation the technical progress has usually the effect of raising the marginal rate of transformation; so that its final effect is to produce a rise of the rate of interest. Fisher underlines very strongly this phenomenon: "The range of man's investment opportunities widens as his knowledge extends and his utilization of the forces and materials of Nature grows. With each advance in knowledge come new opportunities to invest. The rate of return over costs rises. With the investments come distortions of the investor's income stream. These distortions are softened through loans, so far as the individual is concerned, the distortion being thus transmitted from borrower to lender and so spread over society generally. This distortion means relative abstinence from consumption during the period of producing and exploiting the new devices, followed by greater consumption later. In the meantime human impatience is increased."[11]

[11] Fisher, I. *The Theory of Interest*. op. cit., p.341. See also Hirshleifer, J. *Investment, Interest and Capital*, Prentice-Hall, Englewood Cliffs, N. J., 1970, Chapter 4.

Furthermore, although the theory of interest based on time preference and productivity relies on a number of rather abstract assumptions, its supporters have never considered it completely empty. They derive from its theoretical apparatus a number of implications that are considered relevant to the understanding of the real world and endowed with a substantial predictive power.

So, a nation characterized by individuals with strong forethought and family affection is said to experience low interest rates: this is because the individuals are said to possess a relatively low time preference (the Scotch and the Jews are examples often cited, while the old Romans in the declining period of the Empire are cited as a counter-example). Again, a low time preference is attributed to communities in which the average level of the member's income is relatively high; while poor communities, like poor people in general, are said to have a high rate of time preference and hence to experience high rates of interest.

On the other hand, a nation with better productive opportunities than another is said to experience a higher rate of interest: this is because individuals, attracted by the high rates of return, tend to be borrowers. (The generally higher rates of interest of America in comparison with England is often cited as an example.)

As far as the time shape of the income stream is concerned, it is maintained that when in a community the income streams of its members are increasing, the rate of interest will be high, while when they are decreasing, the rate of interest will be low. This is because the increase of the prospective consumption with respect to the present one, tends to raise the marginal rate of substitution between the two. The case of the new developing countries is cited as an example: the Americans of the last century and of the beginning of this century, being constantly under the influence of "great expectations" on their future levels of production and income, have been always ready to pay a relatively large part of

their future consumption for a relatively small addition to their present consumption.

An economic system struck by a sudden catastrophe or an economic system facing new "great expectations" for the future is said to tend to display high rates of interest: in the first case the members of the community will suffer a shortage of present consumption in comparison with relatively unchanged prospects of future consumption; they will attempt, therefore, to borrow one from another, driving up the rate of interest. In the second case, the members of the community see a phase of abundance predominantly in the future and try therefore to realize more of this abundance in the form of current consumption.

Finally, the causes of "stagnation" of economic systems are attributed alternatively to a strong rate of time preference of population or to a very poor situation of the productive opportunities. In the first case the general desire to anticipate consumption will tend to generate high rates of interest, causing the rates of return to be high too: the level of present consumption will be high and the levels of saving and of investment will be low. In the second case the rates of return, being structurally low, will tend to generate low rates of interest; this, in turn, will induce people to increase consumption in order to lower their time preference. Again, the level of savings and of investment will be low, causing a low rate of growth of the economic system.

As a matter of fact, many of the above "causal" relations and empirical corollaries deriving from the Böhm-Bawerk-Fisherian tradition are questionable. We have already seen that in a world with more than one consumption good, the relationships among the phenomena of interest, time preference and time productivity are not so simple and neat. These relationships become even more obscure if we introduce some sensible considerations relating to the concept of "impatience" and to the fact that consumption may not be the only end of economic activity.

5. Time preference and "impatience"

We have so far defined time preference as the percentage excess of the present marginal want for one more unit of ***present*** consumption goods over the present marginal want for one more unit of *future* consumption goods. This percentage rate is directly identified by Fisher with "human impatience to consume": "I shall treat the two terms (impatience and time preference) as synonimous. Henceforth, the term impatience will be the one chiefly used partly because its meaning is more self evident, partly because it is shorter and partly because it does carry a presumption as to the *usual* direction of the time preference."[12] Following Fisher, most of the subsequent literature has adopted the same terminology.

It is nevertheless questionable whether the marginal rate of substitution between physical quantities of present and future consumption represents the true concept and the true measure of "impatience to consume". The latter, in fact, should refer to the *general* attitude of an individual towards the *advanced timing* of consumption, more than to his specific attitude towards the mere substitution, at the margin, of certain quantities of present consumption goods against other quantities of future consumption goods. This distinction has never been clear in the various versions of the theory of interest based on time-preference, from Rae to Fetter, Böhm-Bawerk, Fisher and their present followers. Only Böhm-Bawerk makes intuitively the distinction even if he does not specify it in analytical terms. While speaking of the three fundamental causes of interest, he suggests that the difference

[12] Fisher, I. *The Theory of Interest*, op. cit., p. 66.

between the relation of income to consumption as it exists at one point in time and the relation of income to consumption as it exists at another point in time is a cause of interest different from the fact that "we feel less concerned about future sensations of joy and sorrow simply because they do lie in the future."[13] Thus Böhm-Bawerk suggests that the first cause of interest is linked to the fact that the degree of substitution of present for future consumption depends on the *absolute* level of present consumption in comparison with the *absolute* level of future consumption: if the former is inferior to the latter, then an individual will generally place a higher value on present goods than on future goods. So he will be open to exchange one marginal unit of present goods for a greater amount of the same goods available in the future: "If a person suffers in the present from appreciable lack of certain goods, or of goods in general, but has reason to hope to be more generously provided for at a future time, then that person will always place a higher value on a given quantity of immediately available goods than on the same quantity of future goods... in other words, present goods would command a moderate premium, or agio."[14] But at the same time, Böhm-Bawerk suggests that, apart from the relative magnitudes of present and perspective consumption, a separate cause of preference for present goods exists: the fact that human beings systematically undervalue their future wants. The reasons for this are specifically psychological and are attributed by Böhm-Bawerk to three different causes: a) the lack of a complete and clear picture of the future state of wants; b) the lack of will power that causes many people to choose a lesser present pleasure against a stronger future pleasure; c) the consideration of the brevity and uncertainty of

[13] Böhm-Bawerk, E. *Capital and Interest*, Vol.II, *Positive Theory of Capital*, South Holland, Illinois, 1959, p.268.

[14] *Ibidem*, p.266.

human life. In all these cases, the preference for present goods is not due to their relative shortage with respect to future goods, but to the mere fact that they can be consumed *now* instead of *tomorrow*. So, as Böhm-Bawerk concludes, "even the persons whose present and future are approximately well provided for, and who thus would otherwise value present and future goods as approximately equal...are now drawn over to the group which places a higher value on present than on future goods."[15,16]

In order to give these hints an analytical form, that is in order to analyze the effect of timing alone on preferences, let us suppose that an individual's utility function, so far written in its most general form, can actually be written as:

$$U^i = F^i[u_1^i(a_1^i, b_1^i),\ u_2^i(a_2^i, b_2^i)]$$

where $u_1^i(a_1^i, b_1^i)$ represents the utility that the individual derives from consumption in the current year, or "immediate" utility, and $u_2^i(a_2^i, b_2^i)$ represents the utility that the individual hopes to derive from consumption in the next year, or "prospective" utility. Total, or "aggregate" utility, U^i, is a function of immediate and prospective utility.[17]

We can in this case drow "indifference curves" between immediate and prospective utility, showing all the couple of values that would keep unchanged the individual's aggregate utility. These

[15] *Ibidem*, p. 272.

[16] In his excellent survey of interest theories, also Conard recognizes "the need to distinguish between three terms that are often confused: Fisher's time preference, myopia, and Böhm-Bawerk's agio for present over future goods... The most serious but widespread error is to identify myopia with time preference." (Conard, J.W., *An Introduction to the Theory of Interest*, University of California Press, Berkeley, 1966, pp.40-41.) Conard, however, does not elaborate any convincing analytical distinction among these concepts.

curves refer to the abstract concept of utility, not to the physical quantities of the goods that are consumed in the two periods; each point on them can be associated to an infinite set of combinations of a_1 and b_1 (all the combinations that give a certain level of

[17] To express the utility function in this manner does not imply to revert to the concept of *cardinal* utility, with all its limiting assumptions. As is known, it is possible to derive the existence of *ordinal* utility functions (that is functions that retain their meaning under any monotonic, increasing transformation) from axioms about preferences: a complete and final solution to this problem has been given in a series of works by H. Wold (*A Synthesis of Pure Demand Analysis*, "Skandinavisk Aktuarietidskrift", 1943-1944), G. Debreu (*Representation of a Preference Ordering by a Numerical Function*, in Thrall, Coombs and Davis, eds., *Decision Processes*, Wiley, New York 1954), T. Rader (*The Existence of a Utility Function to Represent Preferences*, "Review of Economic Studies", 1963) and R. Bowen (*A New Proof of a Theorem in Utility Theory*, "International Economic Review", 1968).

Furthermore, from the basic axioms that are usually imposed on the preference structure of a "rational" consumer (*reflexivity, transitivity, completeness* and *continuity*), it derives that the ordinal utility functions are continuous. Differentiability has been subsequently demonstrated by G. Debreu in the case of ordinal utility functions associated to monotonic and convex preference orderings (*Smooth Preferences*, "Econometrica" 1972 and *Smooth Preferences. A Corrigendum*, "Econometrica" 1976).

From all this, it derives that the first order derivatives of an ordinal utility function with respect to its variables exist and may be called *marginal utilities*. The difference with respect to a *cardinal* utility function is that the marginal utilities are *not* invariant under any increasing transformation of the ordinal utility function; only if we adopted *increasing linear transformations*, an ordinal utility function would behave like a cardinal one. In any case, the *marginal rates of substitution*, being the ratio between a pair of marginal utilities, are clearly invariant with respect to any increasing transformation.

To write the utility function in the form: $U = F[u_1(a_1, b_1), u_2(a_2, b_2)]$, means to adopt the hypothesis of "weak separability"; that is to suppose that the composition of the particular bundle of goods that are consumed in the first period has no effect on the composition of the particular bundle that will be consumed in the following period, and conversely. What affects total utility are simply the levels of utility deriving from present consumption and future consumption.

immediate utility) and an infinite set of combinations of a_2 and b_2 (all the combinations that give the corresponding level of prospective utility). That is, they refer to the *timing* of the satisfaction deriving from consumption, not to the particular physical composition of present and future consumption.

Let us now begin by asking what we exactly mean when we say that an individual is *not* impatient to consume, that is what we mean when we say that he has *no* preference for the advanced timing of consumption. In any *given* situation the individual is faced with a *given* level of immediate utility, $\bar{u}_1^i$, and a *given* level of prospective utility, $\bar{u}_2^i$ to which a *given* level of aggregate utility, $\bar{U}^i$, is associated. We may ask, therefore, what would happen to the individual's total satisfaction if time were abolished, that is if both flows of satisfaction could be enjoyed in the present. If the individual's aggregate utility remains the same, that is if:

$$F^i[\bar{u}_1^i, \bar{u}_2^i] = F^i[(\bar{u}_1^i + \bar{u}_2^i), 0]$$

then he clearly puts the two levels of satisfaction (the present and the future one) on the same stage. In other words, he is not influenced by the time-shape of those particular flows of satisfaction, but only by their aggregate magnitude; he does not care "when", but only "how much"; he is not impatient.

If this is so for any pair of u_1^i and u_2^i (that is if the indifference curves between immediate and prospective utility are straight lines with a negative slope of 45-degrees), then the individual can be said to be *never* impatient.

In the case in which the anticipation of satisfaction from the future to the present makes the individual's aggregate satisfaction increase, that is in the case in which:

$$F^i[\bar{u}_1^i, \bar{u}_2^i] < F^i[(\bar{u}_1^i + \bar{u}_2^i), 0] ,$$

the individual clearly shows a bias towards immediate enjoyment. Future satisfaction has for him a lesser psychological value than present satisfaction, so that if time were abolished and he could enjoy both flows of satisfaction immediately, his total utility level would increase. Which implies that to compensate the renounce to the whole future satisfaction a smaller amount of present satisfaction is needed and, viceversa, to renounce to the present flow of satisfaction, a greater amount of future satisfaction is needed.

In order to find a measure of this bias towards present satisfaction, that is a measure of the individual's degree of impatience, we must consider that in any given situation, it is possible to find the increase of the flow of immediate utility which would exactly be equivalent to the whole flow of prospective utility. In fact, we could always make the indifferent curves between immediate and prospective utility intersect the horizontal axis by a linear transition, if necessary. This increase is obviously given by $\bar{U}^i - \bar{u}_1^i$; so that the ratio between the given amount of prospective utility, $\bar{u}_2^i$, and its equivalent amount in terms of immediate utility, minus one, represents the percentage rate by which future satisfaction must be discounted to make it comparable with present satisfaction. We shall call it the *rate of impatience* and indicate it by ι^i:

$$\iota^i = \frac{\bar{u}_2^i}{\bar{U}^i - \bar{u}_1^i} - 1$$

Which amounts to say that:

$$\bar{U}^i = \bar{u}_1^i + \frac{\bar{u}_2^i}{(1 + \iota^i)}$$

ι^t, in other terms, is the rate for which:

$$F^t[\bar{u}_1^t, \bar{u}_2^t] = F^t[\bar{u}_1^t(1+\iota^t), 0]$$

If the rate of impatience is positive and constant for any couple of u_1^t and u_2^t, the individual may be said to be *always impatient*, that is to discount the future at a constant rate, independently of the relative level of his present satisfaction (this is the case in which the indifference curves are straight lines with a negative slope greater than 45-degrees). If, on the contrary, the rate of impatience is negative and constant for any couple of u_1^t and u_2^t, the individual may be said to be *always patient* (this is the case in which the indifference curves are straight lines with a negative slope less than 45-degrees).

In general, however, it is more probable that the rate of impatience be influenced by the *relative* magnitudes of u_1^t and u_2^t; precisely, on the basis of traditional psychological hypothesis, it is probable that the rate of impatience tend to increase as immediate utility decreases in favour of prospective utility. So, an individual could be patient at relatively high levels of immediate satisfaction and become gradually impatient at relatively low values of immediate satisfaction.

As ι^t varies with the levels of immediate and prospective utility, it can be expressed as a function of the latter:

$$\iota^t = \iota^t(u_1^t, u_2^t) \ , \ \ (\text{with } \frac{\partial \iota^t}{\partial u_1^t} < 0 \text{ and } \frac{\partial \iota^t}{\partial u_1^t} > 0)$$

Aggregate utility, in its turn, can be expressed as a function of the rate of impatience:

$$U^t = u_1^t + \frac{u_2^t}{1+\iota^t(u_1^t, u_2^t)}$$

By differentiating this function for any given level of U^t, we get the *marginal rate of substitution between prospective and present utility* in the form:

$$\frac{\partial F^t/\partial u_1^t}{\partial F^t/\partial u_2^t} = -\frac{(1+\iota^t)^2 - u_2^t \frac{\partial \iota^t}{u_1^t}}{(1+\iota^t) - u_2^t \frac{\partial \iota^t}{\partial u_2^t}}$$

If we now calculate the rate of time preference between the present and the future *physical* consumption of a specified good, e.g. good A, we get:

$$1+\rho_a^t = \frac{\frac{\partial F^t}{\partial u_1^t} \frac{\partial u_1^t}{\partial a_1^t}}{\frac{\partial F^t}{\partial u_2^t} \frac{\partial u_2^t}{\partial a_2^t}}$$

Which, on the basis of the above relationship between the marginal rate of substitution of prospective for immediate utility and impatience, can be written as:

$$1+\rho_a^t = \frac{(1+\iota^t)^2 - u_2^t \frac{\partial \iota^t}{\partial u_1^t}}{(1+\iota^t) - u_2^t \frac{\partial \iota^t}{\partial u_2^t}} \ \frac{\frac{\partial u_1^t}{\partial a_1^t}}{\frac{\partial u_2^t}{\partial a_2^t}}$$

The time preference relative to good A appears now as a function of two psychological elements: *impatience*, that is the individual's general attitude towards the anticipation (or the posticipation) of the whole flows of satisfaction, and the marginal rate of substitution between the present and the future utility deriving from consumption of good A.

In the simpler cases, in which $\frac{\partial \iota^t}{\partial u_1^t} = \frac{\partial \iota^t}{\partial u_2^t} = 0$, that is in which impatience is constant, we can easily separate the influence of the two forces on the rate of time preference. In these cases, in fact:

$$1+\rho_a^i=(1+\iota^i)\,\frac{\dfrac{\partial u_1^i}{\partial a_1^i}}{\dfrac{\partial u_2^i}{\partial a_2^i}}$$

that is, time preference is simply the product of the impatience factor by a rate of substitution that could be interpreted as a "pure rate" of substitution. In the general cases, the relationship among the three variables is more complex and the rate of "pure substitution" is more hard to isolate. Nevertheless, we may continue to affirm that, as Böhm-Bawerk had rightly perceived, the rate of time preference is the result of two distinct psychological forces. The rate of impatience could be zero and yet the rate of time preference could be positive; impatience could even become negative, that is it could transform itself into "patience", and yet the rate of time preference could remain positive. A high level of time preference does not necessarily imply a high rate of impatience: it could be simply due to a situation of great scarsity of a_1 relatively to a_2, not to the bias of the individual towards immediate satisfaction. Conversely, a high level of impatience does not necessarily imply a high level of time preference: the strong attitude of the individual towards immediate satisfaction could be counterbalanced by a situation of great abundance of a_1 relatively to a_2.

These subtle distinctions are not merely terminological; nor are they devoid of relevance: in fact, as we shall point out later, the economic, institutional and sociological factors that may affect the degree of impatience of the people living in a given country in a given epoch, are in general different from the factors affecting the "pure substitution" of physical quantities of present and future goods.

If we now apply the concept of "impatience proper" to our equilibrium model, the equations (1), (2) and (3) of p. 7 must be replaced by the following ones:

(1') $$U^i=F(u_1^i,\ u_2^i),\quad (i=1,\dots,n)$$

(2') $$u_1^i=u_1^i(a_1^i,\ b_1^i),\quad (i=1,\dots,n)$$

(3') $$u_2^i=u_2^i(a_2^i,b_2^i),\quad (i=1,\dots,n)$$

(4') $$\iota^i=\iota^i(u_1^i,\ u_2^i),\quad (i=1,\dots,n)$$

(5') $$1+\rho_a^i=\frac{(1+\iota^i)^2-u_2^i\dfrac{\partial\iota^i}{\partial u_1^i}}{(1+\iota^i)-u_2^i\dfrac{\partial\iota^i}{\partial u_2^i}}\ \frac{\dfrac{\partial u_1^i}{\partial a_1^i}}{\dfrac{\partial u_2^i}{\partial a_2^i}},\quad (i=1,\dots,n)$$

(6') $$1+\rho_b^i=\frac{(1+\iota^i)^2-u_2^i\dfrac{\partial\iota^i}{\partial u_1^i}}{(1+\iota^i)-u_2^i\dfrac{\partial\iota^i}{\partial u_2^i}}\ \frac{\dfrac{\partial u_1^i}{\partial b_1^i}}{\dfrac{\partial u_2^i}{\partial b_2^i}},\quad (i=1,\dots,n)$$

(7') $$1+\rho_{a,b}^i=\frac{(1+\iota^i)^2-u_2^i\dfrac{\partial\iota^i}{\partial u_1^i}}{(1+\iota^i)-u_2^i\dfrac{\partial\iota^i}{\partial u_2^i}}\ \frac{\dfrac{\partial u_1^i}{\partial b_1^i}}{\dfrac{\partial u_2^i}{\partial a_2^i}},\quad (i=1,\dots,n)$$

And the equilibrium conditions (7), (8) and (9), of p. 9 must be replaced by the following ones :

(8') $$\frac{(1+\iota^i)^2-u_2^i\dfrac{\partial\iota^i}{\partial u_1^i}}{(1+\iota^i)-u_2^i\dfrac{\partial\iota^i}{\partial u_2^i}}\ \frac{\dfrac{\partial u_1^i}{\partial a_1^i}}{\dfrac{\partial u_2^i}{\partial a_2^i}}\,(1+\pi_a^i)=(1+r),\quad (i=1,\dots,n)$$

(9') $$\frac{(1+\iota^i)^2-u_2^i\dfrac{\partial\iota^i}{\partial u_1^i}}{(1+\iota^i)-u_2^i\dfrac{\partial\iota^i}{\partial u_2^i}}\ \frac{\dfrac{\partial u_1^i}{\partial b_1^i}}{\dfrac{\partial u_2^i}{\partial b_2^i}}\,(1+\pi_b^i)=(1+r),\quad (i=1,\dots,n)$$

$$(10') \qquad \frac{(1+\iota^i)^2 - u_2^i \dfrac{\partial \iota^i}{\partial u_1^i}}{(1+\iota^i) - u_2^i \dfrac{\partial \iota^i}{\partial u_2^i}} \; \frac{\dfrac{\partial u_1^i}{\partial b_1^i}}{\dfrac{\partial u_2^i}{\partial a_2^i}} \, (1+\pi_a^i) \, \frac{p_a}{p_b} = (1+r), \quad (i = 1,\dots,n)$$

The total number of equations is again equal to the total number of unknowns; but from the equilibrium conditions we see that it would be misleading to affirm, as is usually done, that the rate of interest is "determined" by impatience to consume. Impatience could even be absent; and yet the rate of interest could be positive and equal to the "pure rate of substitution" (taking account of expected inflation and of relative prices). In this case the influence of psychological factors on the rate of interest would be determined not by the so much emphasized human attitude towards the anticipation of consumption, but by the simpler fact that the quantity of present consumption is relatively scarce with respect to prospective consumption.

Furthermore, as the causes that may influence ι^i are different from the causes that may influence "pure substitution", a variation of the former is not necessarily accompanied by a compensatory variation of the latter, rendering the influence of psychological factors on the interest rate rather complex. We may also note that each individual will experience a unique rate of impatience in the equilibrium situation, but that the rates of impatience will not necessarily be the same for all individuals, even in a situation of general equilibrium.

6. Wealth and utility

It is a general attitude of many economic writings to assume, explicitly or implicitly, that consumption is the sole end of economic activity. The theories of interest based on the concept of time preference rest even more crucially on this assumption.

Savings and wealth are considered, in fact, as means for changing the time pattern of the consumption flows; that is as means for postponing present consumption to the future (or for anticipating future consumption to the present, if savings are negative). It is supposed, therefore, that the holding of wealth and the increase of it are not desired for their own sake and that individuals do not obtain any particular satisfaction, or utility, by the mere fact of "property". Wealth is considered not as a *direct* source of utility but only as an *indirect* one, through the consumption services it will render in the future: to save and to invest the savings in any kind of assets is a "sacrifice" that people make in order to be happier in the future.

As is well known, this point of view in the explanation of the saving behaviour and the rate of interest is very old: from Nassau Senior and Rae, to Böhm-Bawerk and Fisher, from Frank Ramsey to the "life-cycle" theorists, the economic analysis of the consumption and saving behaviour has been conducted in various contexts but within the psychological axiom that consumption is "pleasure" and saving, being the renunciation of it , is "pain".

Senior was the first to put the stress on the "pain" and "sacrifice"" aspects of saving[13]: he defined saving as *abstinence* from consumption, in the hypothesis that the natural impulse of every human being is to devote the whole of his resources to the immediate satisfaction of present needs and that only the existence of a reward, that is of an interest, can convince him to renounce a part of immediate consumption. Abstinence is a cost and interest is its reward: this is the consequence of Senior's postulates.

Almost contemporaneously, John Rae, considered by both Böhm-Bawerk and Fisher as the real founder of the modern psychological

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[13] Senior, N. *Political Economy*, (1st edition 1836) Griffin and Co., London, 1850.

theory of interest, introduced the more subtle concept of the "weakness" of future pleasures in comparison with immediate pleasures (a concept akin to what we have called "impatience proper"): "The actual presence of the immediate object of desire in the mind, by exciting the attention, seems to rouse all the faculties, as it were, to fix their view on it, and leads them to a very lively conception of the enjoyment which it offers to their instant possession. The prospects of a future good, which future years may hold out to us, seem at such a moment dull and dubious, and are apt to be slighted, for objects on which the daylight is falling strongly, and showing us in all their freshness just within our grasp. There is no man, perhaps, to whom a good to be enjoyed today would not seem of very different importance from one exactly similar to be enjoyed twelve years hence, even though the arrival of both were equally certain."[19]

This general principle is applied not only to consumption goods, but to capital goods as well. In Fisher's equally clear words: "capital wealth, or capital property of like kind, available early is preferred to the capital wealth, or capital property of like kind, available at a more remote time simply because the *income* from the former is available earlier than the *income* from the latter. Thus all time preference resolves itself, in the end, into the preference for comparatively early *income* over comparatively remote, or deferred, *income*."[20] Income, in the Fisherian terminology, is any flow of services that come from the stock of capital; but its utility derives from the fact that it can be resolved

[19] Rae, J. *The Sociological Theory of Capital*, (1834), The Macmillan Co., New York, 1905, p. 54. Rae's original work completely escaped the notice of his contemporaries and so did not have any particular influence on them; it became a sort of literary rediscovery after the ideas presented in it had been included in the works of other later writers.

[20] Fisher, I. *The Theory of Interest*, op. cit., p. 63.

into enjoyment income, that is into a flow of services directed to the satisfaction of human wants. The means of production and any form of stock are simply the necessary and helpful preliminaries to "psychic income", that is human enjoyment. And the reason for this is that any income item which consists merely of an intermediate or preparatory service is desired only for the sake of *enjoyment* income to which it paves the way: " the consumer prefers the service of milling flour in the present to milling flour in the future because the enjoyment of the resulting bread is available earlier in the one case than in the other." [21]

From this point of view it follows that consuming and investing differ only in degree, not in substance: their difference "depends on the length of time elapsing between the expenditure and the enjoyment. To spend is to pay money for enjoyments which come very soon. To invest is to pay money for enjoyments which are deferred to a later time."[22] Analogously, consumption and saving differ only in degree, the former giving an immediate enjoyment and the latter a deferred enjoyment. In conclusion, what generates utility is a flow of consumption be it present or deferred, not the stock of capital, or wealth: "income is the alpha and omega of economics." [23]

The same sort of statements can be found in Böhm-Bawerk's works and especially in the works of the leader of the "purely psychological" school of interest, F. Fetter. But, as everybody has probably had the oppurtunity to notice, points of view similar to the ones discussed above are scattered all along the history of economic thought and still prevail in most of the contemporaneous theoretical works.

[21] *Ibidem* , p. 64.

[22] *Ibidem*, p. 9.

[23] *Ibidem*, p. 13.

There are, nevertheless, several exceptions to this general rule: some groups of economists have in fact pointed out, in different contexts and at different times, that the property of a "stock" may be a source of satisfaction in itself, besides the flow of consumption services it can produce. But the hints in this direction have not been explored deeply enough and the consequences deriving from the change of the axiom that consumption is the only end of economic activity have not been completely drawn.

Senior and Rae themselves admitted that the "wish for power", the "wish for distinction", the "desire of personal and family aggrandizement", the wish to "rank high in the estimation of the world" may induce many people to accumulate a stock of wealth not in view of future consumption but as an immediate object of pleasure.[24] So does Fisher when, in a footnote, recognizes that "...a man may include in the benefits of his wealth the fun of running the business, or the social standing he thinks it gives him, or political or other power and influence, or the mere miserly sense of possession or the satisfaction in the mere process of further accumulation."[25] But by "benefits" Fisher means the flow of income expected from wealth; that is he seems to consider the particular kind of "enjoyments" deriving from the "social standing", the "sense of possession", etc. as particular kinds of consumption flows to be added to the usual flows of direct or indirect consumption services. He concludes, in fact, that "...however indirect, unusual or bizarre the benefit, the principle still holds that the value of any capital good or goods is derived solely from the prospect of future benefits."[26] But Fisher does

[24] See, e.g., Senior, N. *Political Economy*, op. cit. p.27 and Rae, J. *The Sociological Theory of Capital*, op. cit., p.125 and p. 287.

[25] Fisher, I. *The Theory of Interest*, op. cit., p. 17, n.5.

[26] *Ibidem*.

not take into account the fact that the "bizarre" benefits he is speaking of are not traded in a market and therefore cannot be considered as expected monetary values to be discounted in the calculation of the present value of wealth. The relevance of these "bizarre" psychic enjoyments is that they derive directly from the mere fact that wealth exists and that, for this reason, they must be included, in some way or the other, in the utility function. Utility, in other words, must be expressed as a function not only of the flows of consumption but also of the stock of wealth. And this, in its turn, implies that present utility increases both with the present consumption and the present increase of wealth, that is with present saving. But how is it possible, in a theory of impatience, to consider savings both as a means to increase deferred enjoyments and as a means to increase (through the increase of present wealth) present enjoyments? Fisher, confining this subject to a footnote, never allowed this doubt to interfere with his analysis of the role of impatience in the theory of interest.

We could add more names to the list of authors who explicitly recognize that the mere holding of assets yields satisfaction;[27] while most of them do not derive from this fact any particular consequence, some realize that it can cast some doubt on the theories of interest based on the concept of "impatience to consume". Henry Simons, for example, contends that "to assume that all economic behaviour is motivated by desire for consumption goods, present and future, is to introduce a teleology which is both useless and false...In a world where capital

[27] See on this point the quotations of Smith, A., Ricardo, D. McCulloch, J., Marx, K., Stuart Mill, J., Marshall, A., Veblen, T., Cassel, G. Knight, F. and Keynes, J.M. contained in Steedman, I. *Time Preference, the Rate of Interest and Abstinence from Accumulation*, "Australian Economic Papers", Dec. 1981. See also S. Vicarelli, *Ricchezza, Ammortamento e Reddito*, Universita' di Siena, 1984.

accumulation proceeds as it does now, there is something sadly inadequate about the idea of saving as postponed consumption." [28]

Pigou, on the other hand, clearly envisages the analytical consequences of the hypothesis that utility can derive also from wealth holding: " people are led to save in part by a desire actually to hold wealth for the amenity, so to speak, from holding it"; thus, "a thorough equilibrium requires, not that the rate of interest in terms of consumption goods shall be equal to the rate of time-preference, but that it shall be less than the rate of time-preference by some quantity that represents the rate of amenity return from marginal saving."[29]

We must remember, finally, that most of the monetary theorists of the last forty years have explicitly introduced the stock of money into the individuals' utility functions. The motivations have been of different kinds and the psychological justifications have often been taken for granted or omitted; but this is not really relevant for economic analysis. Transaction motives, "uncertainty" and liquidity considerations cannot be discussed here; what matters is the admission of the fact that the possession of an average amount of money yields a real service, which can be compared with the direct utility deriving from the consumption of other commodities.

In all these cases, however, the context of the analysis does not take into consideration the problem of intertemporal choice and of allocation of resources over time. As a consequence, no particular

[28] Simons, H. *Personal Income Taxation*, The University of Chicago Press, Chicago, 1938, pp. 94-99.

[29] Pigou, A.C. *Economic Progress in a Stable Environment*, "Economica", 1947, pp. 246-47. Pigou believes that this kind of psychological considerations can explain why people may save even if the real rate of interest is zero or negative; and this, in turn, can explain the excess of savings over investments even at zero interest rates and the existence of situations of short-term unemployment equilibrium.

effect on the theory of interest is drawn from the hypothesis that wealth yields utility.

In other cases (especially in the more recent literature on the subject), wealth has been explicitly introduced into utility functions in the context of intertemporal analysis; but also in these cases the final consequences for the theory of interest have not been explored, mainly because the analysis has been conducted for single individuals or firms and not in the context of general equilibrium.[30] Furthermore, since the times of Ramsey's famous article[31], most of the analysis have been based on very restricted forms of utility functions: namely, intertemporal utility functions represented by a discounted sum (or integral) of a stream of instantaneous utility levels, where future utilities are discounted by a rate which is held constant independently of the time profile of the utility stream associated with each consumption schedule. The making of too strong and often irrealistic

[30] See, for example, Uzawa, H. *Time Preference, the Consumption Function and Optimum Asset Holdings*, in Wolfe, J.N. (ed.), *Value, Capital and Growth. Papers in Honour of Sir John Hicks*, Edinburgh University Press, Edinburgh 1968; Clower, R. and Johnson, B. *Income, Wealth and the Theory of Consumption*, in Wolfe, J.N. (ed.), *Value, Capital and Growth*, op. cit.; Steedman, I. *Time Preference, the Rate of Interest and Abstinence from Accumulation*, "Australian Economic Papers", Dec. 1981.

A number of recent work on macroeconomic theory have taken into account, in various ways, of the influence of wealth on consumption decisions; but also in these cases the context of the analysis is different from the one proper of the interst theory. See, for example, Spiro, A. *Wealth and the Consumption Function*, "Journal of Political Economy", 1962; Ball, R. and Drake, P. *The Relationship between Aggregate Consumption and Wealth*, "International Economic Review", Jan. 1964; Houthakker, H. and Taylor, L. *Consumer Demand in the United States, 1929-1970*, Cambridge University Press, Cambridge, Mass. 1970; Dornbusch, R. and Mussa, M. *Consumption, Real Balances and the Hoarding Function*, "International Economic Review", 1975.

[31] Ramsey, F. *A Mathematical Theory of Savings*, "The Economic Journal", 1928.

assumptions on the nature of the individuals' preferences is not a good method for achieving results that may aim at a general validity.

7. Analytical consequences

To admit that wealth yields utility is to admit that an individual takes pleasure in becoming richer; that is that every increase of wealth yields a positive satisfaction and any decrease of it yields a negative satisfaction. The wealth already owned is certainly important for the psychology of an individual, but even more important may be the satisfaction obtained from the *increase* of wealth (or the pain suffered for a *decrease* of it). So, we could represent in the utility function not only wealth but also its variations. In our analysis we shall adopt the second alternative, because it contrasts more clearly with the traditional view that savings are only a means of transferring consumption in time: a positive net saving, in fact, means not only a renunciation of present consumption in favour of future consumption, but also an increase in net wealth and therefore an increase in the level of present satisfaction. We shall continue to adopt the hypothesis that the time horizon is of two periods and that within the end of the second year all resources will be consumed, that is net wealth will be zero. We shall also assume that savings will accrue at the end of the period, so that the second year's savings are not relevant to utility and therefore are not included in the utility function. The latter can then be written as:

$$U^i = f^i(a_1^i,\ b_1^i,\ a_2^i,\ b_2^i,\ s^i\ ), \quad \text{with} \quad \partial U^i/\partial s^i > 0\ ,$$

where s^i is net saving, or the increase of wealth between the first and the second year.

The individual has now the task of adjusting the two income flows to his intertemporal preferences of both consumption and wealth increase. To the old "conflict" between present and future consumption we have now added the new "conflict" between consumption and wealth increase: a greater amount of present consumption will be associated, in fact, with a slower increase of wealth. The quantities of consumption goods and the net variation of wealth are linked , as we have seen, by the constraints (4), (5) and (6) of p. 7. As we are now interested only in the first year's savings, these constraints can be reduced to two and rewritten as follows:

$$(6") \qquad (A_1^i p_a + B_1^i p_b) - (a_1^i p_a + b_1^i p_b) = s^i, \quad (i=1,\dots,n)$$

$$(7") \qquad s^i(1+r) + [A_2^i p_a(1+\pi_a^i) + B_2^i p_b(1+\pi_b^i)] - \\ -[a_2^i p_a(1+\pi_a^i) + b_2^i p_b(1+\pi_b^i)] = 0, \quad (i=1,\dots,n)$$

The equations of our general equilibrium system must now be changed in the following way: the marginal rates of time preference defined by the equations (1), (2) and (3) of p. 7 are now a function also of the amount of savings:

$$(1") \qquad \rho_a^i = F_a^i(a_1^i, b_1^i,\ a_2^i,\ b_2^i, s^i), \quad (i = 1,\dots,n)$$

$$(2") \qquad \rho_b^i = F_b^i(a_1^i, b_1^i,\ a_2^i,\ b_2^i, s^i), \quad (i = 1,\dots,n)$$

$$(3") \qquad \rho_{a,b}^i = F_{a,b}^i(a_1^i, b_1^i,\ a_2^i,\ b_2^i,\ s^i), \quad (i = 1,\dots,n)$$

To these rates of time preference we must now add two more independent rates: the rate of time preference between savings and the future consumption of good *A* and the rate of time preference between saving and the future consumption of good *B*, analitically defined as follows:[32]

$$(1+\rho^i_{a,s}) = \frac{\partial U^i/\partial s^i}{\partial U^i/\partial a^i_2}\,, \quad (i = 1,..,n)$$

$$(1+\rho^i_{b,s}) = \frac{\partial U^i/\partial s^i}{\partial U^i/\partial b^i_2}\,, \quad (i = 1,..,n)$$

As the three previous ones, also these rates will be functions of the quantities of consumption goods and of savings:

$$(4'') \qquad \rho^i_{a,s} = F^i_{a,s}\,(a^i_1, b^i_1, a^i_2, b^i_2, s^i)\,, \quad (i = 1,...,n)$$

$$(5'') \qquad \rho^i_{b,s} = F^i_{b,s}\,(a^i_1, b^i_1, a^i_2, b^i_2, s^i)\,, \quad (i = 1,...,n)$$

In order to maximize utility, the individual must now behave not in the way indicated by the equations (7), (8) and (9) of p. 9, but in such a way as to equalize:

$$(8'') \qquad \frac{(1+r)}{(1+\pi^i_a)} = (1+\rho^i_a) - (1+\rho^i_{a,s})p_a\,, \quad (i=1,...,n)$$

$$(9'') \qquad \frac{(1+r)}{(1+\pi^i_b)} = (1+\rho^i_b) - (1+\rho^i_{b,s})p_b\,, \quad (i=1,...,n)$$

$$(10'') \qquad \frac{(1+r)}{(1+\pi^i_a)}\frac{p_b}{p_a} = (1+\rho^i_{a,b}) - (1+\rho^i_{a,s})p_b\,, \quad (i=1,...,n)$$

The meaning of the first equilibrium condition is rather obvious: what now must be equated to the "real" rate of interest in terms of good A is *not* the old marginal rate of time preference between the present and the future consumption of good A; we must take into

[32] The other two rates of substitution between the saving and the consumption of the two goods in the current year can be derived from the equations (1")- (5"); as they are not independent, they must be omitted from the analysis.

account, in fact, that the renunciation of the immediate consumption of 1 unit of good A in favour of a certain future quantity of the same good, makes savings increase by p_a and therefore makes utility increase by $\partial U^i/\partial s^i.p_a$. To calculate the reduction of future consumption which would be sufficient to keep unchanged the total level of satisfaction, we must divide the above increase of utility by the marginal utility of a_2:

$$\frac{\partial U^i/\partial s^i\;p_a}{\partial U^i/\partial a^i_2} = (1+\rho^i_{a,s})\;p_a$$

This expression must therefore be subtracted from $(1+\rho^i_a)$ in order to obtain the *net* increase of future consumption which is just about sufficient to compensate for the loss of immediate consumption. In other words, the old rate of substitution, ρ_a , must be corrected to take account of the fact that the renounce to present consumption produces two distinct effects on utility: the one linked to the increase of future consumption and the one linked to the increase of saving. An analogous interpretation can be applied to equilibrium conditions (9") and (10").

One of the consequences of all this is that savings will be, *coeteris paribus*, greater than before: according to the traditional view, if the rate of time preference is greater than the rate of interest , the individual will *dissave* by increasing present consumption; but if utility depends also on savings and if the marginal utility of savings is relatively high, the individual might not reduce his savings notwithstanding the fact that his rate of time preference is greater than the rate of interest.

The causal relationship between the time preference of consumption and the rate of interest appears now weaker than ever: first of all, the level of utility and the marginal rates of substitution are functions not only of physical quantities of goods but also of a magnitute expressed in terms of *value*, s. This means that the marginal rates of time preference are now functions of the price system: there is no measure of time preference independent of prices and the money rate of interest. In the second place, the time preference for consumption could be positive and yet the rate of interest could be zero or negative. Thirdly, a change in the individuals' psychological attitudes towards consumption could cause a change in the rates of time preference not necessarily leading to a change in the interest rate.

Furthermore, the formal equivalence between the money rate of interest and a unique rate of time preference of "consumption in general" cannot be established even in the equilibrium situation. Recalling the definition of the marginal rate of time preference of money spent in buying good A:

$$(1+\rho_{ma})=(1+\rho_a)\ (1+\pi_a)$$

we now obtain, from the equilibrium condition (8"), that:

$$(1+\rho_{ma}^i)=(1+r)+(1+\rho_{a,s}^i)\ p_a(1+\pi_a^i)$$

Analogously, the marginal rate of time preference of money spent in buying good B will be, in equilibrium, such that:

$$(1+\rho_{mb}^i)\ =\ (1+r)+(1+\rho_{b,s}^i)\ p_b(1+\pi_b^i)$$

As before, the marginal rate of time preference of money spent on any kind of consumption good is unique, because in the equilibrium situation we have that:[33]

[33] It is easy to show that also the rate of time preference of money subtracted to the present consumption of good B and devoted to the future consumption of good A is, in equilibrium, equal to $(1+r)+\rho_{a,s}\ p_a(1+\pi_a)$.

$$(1+\rho_{a,s}^i)\ p_a(1+\pi_a^i)\ =\ (1+\rho_{b,s}^i)\ p_b(1+\pi_b^i)$$

But in this case the money rate of interest cannot be identified with the rate of time preference of consumption in general: the latter is equal to the former plus "something else". Time preference could be, in the solution of the general equilibrium system, equal to zero and yet the money rate of interest could be positive. Furthermore, if any of the parameters changed, we could get a solution with a higher r and a lower ρ_m, because of the possible changes of relative prices and of the value of s.

At this point, if we introduce the subtle distinction between "impatience proper" and "pure substitution", the "impatience theory of interest" becomes even more confusing. The equilibrium conditions (8"), (9") and (10"), in fact, now become:

$$(8''')\quad \frac{(1+r)}{(1+\pi_a^i)} = \frac{(1+\iota^i)^2-u_2^i\frac{\partial\iota^i}{\partial u_1^i}}{(1+\iota^i)-u_2^i\frac{\partial\iota^i}{\partial u_2^i}}\ \frac{(\frac{\partial u_1^i}{\partial a_1^i}-\frac{\partial u_1^i}{\partial s^i})p_a}{\frac{\partial u_2^i}{\partial a_2^i}},\quad (i=1,\dots,n)$$

$$(9''')\quad \frac{(1+r)}{(1+\pi_b^i)} = \frac{(1+\iota^i)^2-u_2^i\frac{\partial\iota^i}{\partial u_1^i}}{(1+\iota^i)-u_2^i\frac{\partial\iota^i}{\partial u_2^i}}\ \frac{(\frac{\partial u_1^i}{\partial b_1^i}-\frac{\partial u_1^i}{\partial s^i})p_b}{\frac{\partial u_2^i}{\partial b_2^i}},\quad (i=1,\dots,n)$$

$$(10''')\quad \frac{(1+r)}{(1+\pi_a^i)}\ \frac{p_b}{p_a} = \frac{(1+\iota^i)^2-u_2^i\frac{\partial\iota^i}{\partial u_1^i}}{(1+\iota^i)-u_2^i\frac{\partial\iota^i}{\partial u_2^i}}.$$

$$(\frac{\partial u_1^i/\partial a_1^i}{\partial u_1^i/\partial b_1^i}-\frac{\partial u_1^i/\partial s^i}{\partial u_2^i/\partial a_2^i}p_b),\quad (i=1,\dots,n)$$

Impatience, "pure substitution" between goods and between goods and savings, relative prices and the interest rate are so intermingled that it is really difficult to establish an order of causality or to predict the effect of changes of individuals' preferences on the rate of interest. How could we infer from the above relations that impatience is one of the major forces determining the rate of interest?

8. The order of causality again

What in the end emerges is that the "causes" the working of which determine the rate of interest and its movements cannot be simply reduced to *time preference* and *time productivity*. The existence of *positive* real rates of interest is not linked to the mere existence of time preference for consumption goods, as Böhm-Bawerk argued in his famous "three causes"; it depends in a more subtle way on the relation between "true" impatience and "pure" subsitution and on preferences for consumption and savings. But as savings are measured in monetary units, the rate of time preference between consumption and savings depends also on the absolute level of prices; which means that the rate of interest cannot be "explained" in terms of forces independent of the price system and of interest itself.

As can be clearly seen from equations (8"), (9") and (10") of p. 46, the rates of time preference between present and future consumption could change but, if this change produces adequate movements of relative prices, the rate of interest could not change in the same proportion or could even change in an opposite direction . Furthermore, the rate of interest could change not because people become more or less eager to consume but because they become more concerned about the holding of wealth. The pleasure of holding wealth weakens the desire to anticipate consumption: if the former did not exist, the interest rate would probably be pushed to very high levels by the desire for immediate consumption. On the other hand, a low rate of time preference for

consumption would generate negative *real* rates of interest if the desire for wealth were sufficiently strong. In any case, the absence of the phenomenon of interest would not be linked to the absence of time preference for consumption, but to the equilibrating action of the psychological forces pushing towards anticipated consumption and of those pushing towards the accumulation of wealth.

More arguments against the traditional results of the theory of interest may be derived from the fact that time preference is in reality the result of impatience proper and of "pure substitution". As we have already noted, the rate of interest could exist even if people were not "impatient": from equations (8'"), (9'") and (10'") of page 49, we clearly see that even if ι were zero for all individuals, the "real" rates of interest could be positive. Viceversa, the rate of interest could vanish even in a situation in which people were impatient, if the flows of physical consumption were particularly abundant during the current period.

Moving to the understanding of the real world, we might then argue that people or nations characterized by high levels of income do not necessarily experience low rates of time preference and therefore low rates of interest. The abundance of present consumption goods tends to lower "pure substitution" between present and future consumption; but because of historical and sociological factors, the desire to enjoy life in the present more than in the future could generate a high rate of impatience and, *ceteris paribus*, a high rate of interest. Conversely, poor people or poor nations do not necessarily experience high rates of interest: if for cultural reasons their degree of foresight, self-control and regard for posterity is relatively high, their rate of impatience could be low, pushing the interest rate down. Fisher himself admitted that: "not only do we find examples of high rates of preference for present over future goods among the prodigally rich, but often we find low rates of preference for present over future goods among the thrifty poor. Examples are especially frequent among the Scotch, the French peasants and the Jews, whose propensity to accumulate and to lend money even in the face of misfortune is well known."[34] The distinction between impatience proper and "pure substitution" is not useless if we consider that the forces influencing these two psychological factors may be different and may change for different reasons.

As far as the effect of technical progress is concerned, we may note that its impact on the rate of interest is not so certain as it is usually argued. The opportunity frontier can change, in fact, in different ways, according to the kind and path of technological progress. There might be, for example, changes in the productivity of one or more factors shifting the entire opportunity frontier outwards: in this case, with the initial endowment, each individual would be able to produce more of both present and future consumption goods. But this kind of progress could simply represent an improvement in the *average* productivity of investment, that is a higher future production for any level of present production; not necessarily an improvement in the *marginal* productivity, that is a higher increase of future production for any decrease of present production. In this case, the improvement in investment opportunities could produce any kind of effects on the interest rate: the latter could increase or decrease according to whether the interaction of technical progress with all the variables of the economic system will lead to higher or lower equilibrium marginal rates of return. Furthermore, if the improvement of technology had to influence only the transformation of the present production of good B into the future production of good A, that is only $\sigma_{a,b}$, the movement of relative

[34] Fisher, I. *The Theory of Interest*, op. cit. p.376.

prices could produce any kind of change in the interest rate.

An economic system struck by a sudden catastrophe could not tend to display higher rates of interest; if the catastrophe involves a reduction of the stock of wealth as well as a shortage of present consumption, both the time preference for consumption *and* the marginal utility of saving could increase. The total effect on the rate of interest could be, therefore, not as strong as usually assumed.

Finally, among the causes of "stagnation" of economic systems we must now enumerate not only the strong rate of time preference for consumption of the population and the poor situation of the productive opportunities, but also the weak pleasure that people derive from the holding of wealth or the fear of losing, for political or other reasons, the accumulated wealth. The latter elements would tend to raise the rate of interest and therefore to lower the level of investments. A rather strong concern about the satisfaction produced by saving and the certainty of not being deprived of the accumulated wealth could, on the contrary, counteract the effect of a strong preference for immediate consumption; it might also counteract the effects of the structurally low rates of return on investment, by contributing to raise the level of saving and to lower the rate of interest. Again, the distinction between the time preference for consumption and the pleasure deriving from wealth is not useless, because the factors influencing the former may differ from the factors influencing the latter.

The existence of a world with many consumption goods, a more precise analysis of the psychological elements that determine the inter-temporal decisions of consumption and the thought of wealth not simply as a proxy for future consumption but as an independent "object of desire", do significantly affect some of the conclusions of the traditional inter-temporal theory of resource allocation. It seems therefore proper to insist that, even within the structure of the Fisherian-type theories, it is not appropriate to consider the rate of interest as the simple result of the interaction of the "two blades of a scissor", that is of time preference and time productivity, but as the result of a more complex bundle of phenomena.

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