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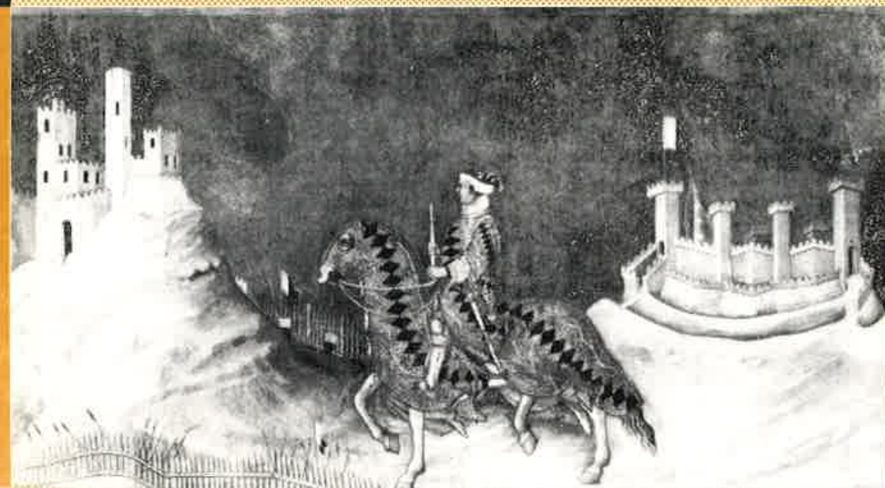
Omar Chisari

WORKERS' ASSETS, FOREIGN DEBT AND THE DYNAMICS OF AN ECONOMY



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Omar Chisari

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WORKERS' ASSETS, FOREIGN DEBT AND THE DYNAMICS OF AN ECONOMY (*)

1. Introduction and survey of relevant literature.

Involuntary unemployment is one of the most interesting phenomena attracting economists' attention. Its social and individual costs are a calamity, moreover when it has a chronical character.

However striking its existence can be and clear its effects many explanations of its origins are based on voluntary decisions or in some kind of economic agents' irrationality. Workers' refusal to reduce money wages or the rational expectations approaches that emphasize the role of time preference between consumption and leisure can be included under these categories -an interesting summary is provided by Negishi (1980)-.

The recent analyses of the random time patterns of the rate of unemployment, given the Phelps' hysteresis effect -see Phelps (1972)-, put their accent on the unpredictable character of the demand for labor while still admitting some kind of inflexibility of money wages -the insiders' resistence to accept a reduction under the existence of outsiders' unemployment-.

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Though accepting the universal unpleasant consequences of a high level of the rate of unemployment, we must admit that its undesirable effects are not homogeneous from the point of view of the development degree of different economies.

A given rate of unemployment has certainly more drammatic consequences in a low developed economy than in a developed economy, in which the accumulated assets can be used to live upon.

From this point of view, that is, the one that considers the degree of development, the best known explanations of the existence of unemployment are those presented, for example, in Eckaus' (1955) and Lewis' (1954) papers.

The first one emphasizes the role of technological inflexibilities, while the second one introduces the assumption of a perfectly elastic labor supply function at a socially accepted real wage rate. Both of them admit that the total labor supply is given and equal to the available labor force.

Keynesians explanations of unemployment have been advanced too. They give no important role to the supply of labor, the rate of unemployment being determined by the demand for labor. The supply of labor is completely inelastic, always to the right of the labor demand function.

But if it is recognized that the degree of development must be taken into account and that the level of accumulated assets owned by those that participate in the

labor market has to be considered in the analysis, then we must also include their level as an important variable in the assumptions on the shape and general features of the labor supply function.

Just assuming that it is perfectly inelastic and independent on the level of workers' assets amounts to neglecting a priori its possible influence on the dynamic path of the economy, an aspect which we intend to emphasize here.

Under the influence of a paper by Barzel and McDonald (1973), this is the direction to be followed in this study.

A minimum standard of living will be introduced -hence, a real wage rate inflexibility-, which has to be reached by any worker. It is not necessarily a "real" subsistence condition of minimum consumption, but it might be a "socially" determined minimum real level.

Now, it is clear that the labor supply decision will depend not only on the real wage rate, but also on the difference between the yield of workers' real assets and the value of this consumption basket, and therefore on the total capital available in the economy and on workers' share in it.

When workers' assets are reduced, or when their yield becomes negligible, if the wage rate is given, they will have to increase their labor supply in order to meet the consumption standards. Moreover, a reduction in the real wage rate might force them to increase the quantity of labor supplied, leading to a negative sloping labor supply function, a common phenomenon, according to the econometric evidence summarized by Martin and Neary (1980).

It is interesting to mention that the hypothesis of a negative sloping labor supply function has not been absent from economists' theories.

According to Douglas (1964), the majority of the English mercantilists of the seventeenth and eighteenth centuries, and the imperialists of this century -referring their assumption to the inhabitants of the tropics- accepted such hypothesis.

It seems that Marshall himself had accepted the idea of the existence of a subsistence level -see Walker (1975)-, though considering it not an effective constraint.

Joan Robinson (1953), while discussing a supply curve of effort rather than a supply curve of labor, stated this point in a very clear way in a paragraph which is worth to quoting here -see pp.120-121-:

"A rise in the level of real wages will have effects which tell in opposite directions. On the one hand it will raise the personal efficiency of the workers by improving their standard of life; on the other hand, as needs become less urgent, the choice between an increase in leisure and an increase in earnings is likely gradually to turn more and more in favour of leisure, in spite of the extra inducement to work represented by a higher rate of reward".

We are dealing with two simultaneous effects.

The first one might be called a Duesenberry's effect, since the minimum necessary consumption might be socially determined -see Duesenberry (1964)-.

Duesenberry intended to show that the hypotheses of independence and reversibility of every individual's consumption behavior are false. However, he only considered the possibility of reducing the stock of assets as a way of financing the "normal" consumption patterns, without referring to the consequences in the labor market. More recently, Akerlof (1980) raised again the point by discussing the case of a society whose members exhibit a utility function which depends on reputation, with these agents following or not social customs according to the associated costs of each alternative behavior. As he also did show, the outcome of these assumptions might be the existence of involuntary unemployment, though a rigid wage rate is introduced without a corresponding justification of its existence. We can say that Duesenberry's solution is inverted here; our agents are ready to change their behaviour in the labor market in order to preserve their current wealth level.

It is interesting to mention here that Ryder and Heal (1973) discussed the possibility of the existence of cycles in a model of growth when present preferences depend on past consumption patterns.

On the other hand, a wealth effect is introduced. Tobin (1980) has presented a very interesting account of the accademic discussions of the Pigou and Fisher effects during the depression of the thirties. Note that in no case the effects on the labor supply function were studied.

In our model, these minimum consumption and wealth effect are interacting, leading to the inexistence of equilibrium, to multiplicity of equilibria or to complicated dynamic behaviors, that include the possibility of limit cycles.

The economy we are going to discuss presents other characteristics which it is important to mention.

First of all, an external debt must be repaid, and hence a minimum level of the commercial balance must be reached. This will determine that a certain minimum level of the rate of exchange which must be achieved in order to fulfil the constraint.

Depending on the composition of workers' assets and consumption patterns, a high level of the rate of exchange might determine the supply of labor function to have a peculiar C-shape, and therefore the existence of the above mentioned phenomena.

Following Goodwin's model (1967) we are going to accept that prices are determined according to a mark-up rule over average costs and that the market for the only produced good is always in equilibrium. However, the investment function will depend on the rate of profits instead of introducing the assumption that investments are equal to total profits. In our second model the mark-up factor will be constant, as it is assumed in Goodwin's model.

A main difference with that model is that the role of the labor supply function is emphasized here, obtaining the cases of "discouraged" and "additional" worker as special outcomes, depending on the level of workers' wealth. Some aspects of the dynamics within Goodwin's model taking into account the properties of the labor supply function were examined by Di Matteo (1982).

It should be recalled that Goodwin's model was presented originally for a closed economy case; however, it provided also the basic structure for Korkman's (1980) analysis of the effects of a devaluation on the cyclical behavior of economic variables. He focused his study on the consequences of a requirement of external balance, but, as this author has accepted, the cycle is already present in Goodwin's paper and it was not necessarily related to the devalution.

Regarding devaluations, the empirical evidence seems to point out the existence of cyclical movements after a devaluation has occurred. The literature has elaborated on this aspect, and very recently, the role of the labor market has been also discussed.

In the cases of Bean (1987) and Svensson and Razin (1983), the Harberger-Laursen-Metzler effect is introduced again into the discussion of the effects of a devaluation, now by considering an intertemporal framework and a more accurate microeconomic model.

Svensson and Razin distinguish between permanent and transitory deteriorations in the terms of trade and their results depend heavily on the assumptions regarding to the movements of the rate of time preference with respect to the level of welfare.

Bean's model is very interesting to our aims, since he considers the influence of the wealth effect -that follows a terms of trade deterioration- on labor supply.

Recall that according to the Harberger-Laursen-Metzler effect, given investments and without government deficit, a terms of trade deterioration reduces (real) income, therefore savings, and the current account surplus.

Bean attributes importance to the effects of the terms of trade changes on labor supply, since, in general, a change in the terms of trade also modifies the relationship between the product and the consumption wage rates -relevant to firms and workers' decision processes -. As he discusses a two period model, his results are related to the rate of time preference too, as in Svensson and Razin's paper.

However, his analysis is not able to include the possibility of involuntary unemployment. The adjustment is obtained through changes in the real wage rate which in turn influence the labor supply decision -including wealth effects-. But unemployment is not taken into consideration, or at least, can be only of a frictional or voluntary character, accompanying movements in relative prices.

Hanson (1983) has recently studied the possibility of a contractionary devaluation, again in a model of flexible labor supply. In his model, if wages are determined in the

labor market then output and the balance of payment will be inversely related.

The role of unemployment is not fully discussed, though as the possibility of an indexed nominal wage rate is also considered it cannot be excluded.

This last paper refers the analysis to the impact of a devaluation on the level of activity, a discussion that began almost two decades ago, but that recently has been revitalized.

In effect, the impact of a devaluation on output and employment were already analyzed by authors like Belozerkovsky (1970) and Sidrauski (1969) in pioneering papers.

Belozerkovsky considered the -negative- effect of a devaluation on national output through the wealth effect corresponding to a change in the real value of the foreign debt. When a devaluation reduces the relative price of domestic non traded goods and moreover, that is the sector which holds the burden of the foreign debt, then the associated capital losses may be absorbed by a fall in output.

It should be noted the emphasis of this author in considering a wealth effect, stemming from a devaluation that alters the value of the debt in terms of home assets.

Sidrauski's paper illuminated the role of a restrictive monetary policy as a main cause of the contractionary effects of devaluations. The extreme importance that the monetary authorities attribute to the control of the monetary expansion in order to avoid inflation -after the devaluation- could lead to a fall in the level of employment.

In his model, the nominal wage rate is determined exogenously to the labor market, and the rate of exchange is settled by the monetary authority; hence, the analysis is a priori compatible with the existence of involuntary unemployment.

Note that a successful devaluation, that is, one that reduces the wage rate, will increase the level of employment only if: 1) the labor supply function is not bounding, or 2) the labor supply function is decreasing with respect to the real wage rate. These cases are not fully discussed by Sidrauski, though his analysis seems to rely on the first hypothesis.

This is an important aspect for the model to be presented. A successful devaluation, one that allows the economy to increase its commercial balance surplus, and then that reduces workers' consumption if they consume the exportable good, must be accompanied by a fall in the wage rate.

Then, if employment is not to fall, we must accept the existence of persistent disequilibrium in the labor market, or it will be necessary to allow the labor supply function to behave in an unexpected way: it should be negatively sloping, at least within some range.

It must be also mentioned that we are going to assume that the real wage rate is determined as the quotient of

the nominal wage rate and of a price index obtained as a linear combination of home and imported goods.

The hypothesis of a given wage rate, without any reference to the current state of the excess demand for labor has been introduced frequently in the literature.

In general, it has been used with the aim of studying the effects of government spending in different kinds of goods -traded and nontraded-, as is the case of Helpman (1976) and Helpman (1977). Several papers continued these pioneering studies by introducing the same hypothesis -see for example Rodseth (1979), Barry (1985) and Barry (1987)-. In the international economics general analysis, Brecher (1974) had already examined the effects of the existence of a minimum wage rate constraint.

None of these papers, however, discussed deeply the essence of the minimum wage rate constraint and most of the analyses were restricted to a static framework.

Though the problem we are dealing with here is different, since we are considering the case of a one good economy and therefore neglecting the influence of substitution effects, the structure that is provided for the labor market might be useful in the clarification of the nature of the minimum real wage constraint. As it will be seen in the next sections, the existence of persistence disequilibrium in the labor market -not solvable through the adjustment of the wage rate- introduces the role of government as critical in the determination of the real wage rate.

Other papers have also taken into account the possible contractionary effects of a devaluation; some of them have additionally considered the wealth effects associated to it.

A classical analysis has discussed the recessionary effects of a devaluation taking into account the different propensities to consume of wage and profits earners -Dìaz Alejandro (1963)-.

More recently, the so called J-curve has been introduced for understanding the existence of some contractionary interval in the dynamic path of the real product -though associated with a process that converges asymptotically to an equilibrium-; Casprini (1977) and Noman and Jones (1979) have elaborated on this point.

The wealth effects originated in a devaluation have been studied in Lapan and Enders (1978), and in a context of imperfect information -a difference between real wealth and the one that is perceived by economic agents- by Heymann (1984) for showing the possibility of cyclical movements in production.

Nonlinearities of the Phillips curve have been also introduced by Larrain (1986) in the framework of an open economy for discussing the possibility of the existence of Hopf or Poincarè limit cycles. Such nonlinearities are not clearly justified, a common feature to the models that have traditionally used that assumption for obtaining cyclical movements -see Gabisch and Lorenz (1987)-.

Regarding an important hypothesis to be used in our second model, some remarks are in order too.

It will be assumed thar labor productivity depends on the real wage rate, that is, an increase in the real wage rate also increases workers' productivity.

This assumption corresponds to the so called "efficiency theory of wages", and in the literature -though the recent renewed interest by authors like Solow (1979)- some previous analyses can be mentioned. In effect, and as it was mentioned above, Joan Robinson (1953) accepted that assumption; also Morishima (1969) and Leibenstein (1963) introduced it as an important characteristic of the market for labor, with special relevance for underdeveloped economies.

In our model, it has a main role in determining the existence of a Hopf bifurcation and hence of cyclical movements.

The paper is organized as follows.

Sections 2 and 3 will be devoted to the discussion of the general properties the labor supply function and of the main equations of the model.

Two versions of it will be obtained by assuming different behaviors of the mark-up factor.

In Section 4 the case of a variable -endogenous- mark-up factor is presented. Two equilibria are possible under this hypothesis; one of them will exhibit a Hopf bifurcation cycle when the appropriate parameter values are chosen.

This outcome will be illustrated by means of a computer's program now available in the literature -see Kocak (1986)-.

The solution arising when the mark-up factor is constant and the productivity of labor is an endogenous variable is discussed in Section 5. In this case a Hopf bifurcation cycle might appear too and it is also simulated with an example.

Appendix A is devoted to showing that a labor supply function that exhibits the features we will attribute to it in this paper produces chaotic movements of the real wage rate and of the level of employment. It is assumed a walrasian adjustment process, and most of the analysis is based on a modified version of the logistic curve.

Appendix B summarizes Hopf's bifurcation theorem.

2. The model.

In this section we are going to present the main equations of the model.

First of all, let's introduce the following definitions:

k, is the capital stock;

q, is the quantity of the only domestic Hicks-Leontief good;

L, is the quantity of labor used in production;

W, is the nominal wage rate;

w, is the real wage rate;

p, is the price per unit of the home good;

E, is the nominal rate of exchange, that is, the number of home currency units necessary to buy a unit of the foreign currency;

Z, is the mark-up factor;

G, is the rate of profit;

C, is the autonomous real expenditure -which however will be an endogenous variable-;

U, is the rate of unemployment ($0<U<1$);

M, is the total quantity of imported good;

F, is the commercial balance surplus;

and the following production coefficients:

$a = L/q$,

the quantity of labor necessary per unit of the good to be produced; also

$b = q/k$,

the average productivity of capital, and m, the quantity of the only imported good necessary per unit of the home good to be produced.

Some of these will be constants and others will be variables in order to determine an equilibrium for the equations of the model.

Let's assume that:

a1. The investments function has the form

(1) $k' = f(G) - Dbk$,

where f and D are positive constants; the last one gives the rate of depreciation which also depends on the intensity in the use of capital in production, given by b.

Recall that G is the rate of profit, that is

(2) $G = (pq-WL-Emq)/pk = (1-1/Z)b$,

i.e., total money profits over capital nominal value.

a2. Nominal prices are settled on a mark-up basis over average costs, which do not include the cost of capital

(3) $p = Z(Wa+mE)$.

Notice that some level of imports is therefore necessary, a traditionally accepted feature of the kind of economy we are studying here.

a3. The nominal wage rate is determined according to

(4) $W = w\langle rp+(1-r)E\rangle$,

where r is the share of the home good in workers' consumption basket.

a4. The real wage rate is determined by an equation of the form

(5) $w' = H(U^*-U)$,

where U^* stands for the minimum accepted level of the rate of unemployment. H is a positive constant and then the real wage rate is reduced if U is growing over its normal level.

Under walrasian assumptions U^* should be taken equal to zero. However, under our assumptions on the labor supply function that procedure would not necessarily lead to an equilibrium, simply because it might not exist at all -next section will elaborate more deeply on this aspect-.

a5. Exports are assumed to depend on its price in terms of the foreign currency, that is

$x = x(p/E)$.

Hence, total revenue will be given by

$R(p/E) = (p/E)x(p/E)-z(p/E)$,

where $z(p/E)$ corresponds to imports by home consumers. Therefore the commercial balance can be written as

(6) $R(p/E) - mq = F$,

where it has been assumed that the price of the imported good in terms of foreign currency is equal to one, and that capital inflows are impossible. F is immediately used to repay a debt, so that it has neither real no monetary effects. Note also that workers' income is not taken into account into the imports function; this assumption then implies that their consumption of the imported good -though perhaps being positive- can be neglected when put in relative terms with total consumption.

Additionally, we have the following equations:

(7) $L = aq$,

(8) $q = bk$,

(9) $v(wL,p/E) + C + x(p/E) + f(G) = q$,

(10) $U = L/S(w,gk,G)$

where $S(w,gk,G)$ stands for the supply of labor function which depends on the real wage rate, on the proportion of total capital belonging to workers and upon its yield as determined by G.

Equation (9) corresponds to the necessary equality in the market for the home good, between the quantity demanded and total production. The function v(wL,p/E) is the quantity of the home good demanded by workers, which depends on their current income from the labor market and on the relative prices of the goods. C will be considered as an unknowm which is determined endogenously in order to achieve the zero excess demand condition in the market for the home good. It might stand for government expenditure, for example, or for any other consumption source without actual consequences.

Therefore we have ten equations -when w' and k' are zero- and the following list of variables to determine, at least in principle:

k,b,W,w,L,Z,q,U,C and G.

Hence we have ten unknowns too; note that the following are considered as parameters:

a,v,F,p,E,U*,m,r and g.

Obviously, one of them might become a variable instead of one of the quoted above, probably leading to different dynamic results. To begin with, we are going to assume that p and E are given -model I- and hence that R is fixed.

It should be noted that g will be considered constant through out this paper. This is an extreme simplification; however, an important version of the model will be developed in a further paper, taking into account that g might be variable. For example, it might change over time following a rule such as :

(11) $g' = N(w-w^*)$,

where N is a positive constant, and w* is a threshold level, beyond which workers increase their share in total capital. This seems to be the simplest assumption regarding to the change of g over time.

But beyond this simple formulation, there are very clear reasons for understanding why the equilibrium tends to be "more" unstable when g is allowed to move through time.

The change of workers' share in total capital depends almost certainly on their current wealth, since a higher level of it stimulates both consumption and savings. An explosive path can arise under these conditions, with g growing -or falling- continuously up to becoming one -or zero-.

Increasing the number of dimensions also creates several new problems in the mathematical analysis of the differential system, including structural instability. They are beyond the scope of this introductory paper.

We are going to devote next section to the discussion of the properties of the labor supply function, since they have important consequences for the dynamic analysis of the model.

3. The labor supply decision.

Assume that the representative worker's utility function can be written as

$$u = u(C_h, C_m, s, T-S),$$

where u is differentiable up to the second order at least and monotonically increasing in all of its arguments, and where s stands for total savings.

Here C_h and C_m are the consumed quantities of the home and of the imported good respectively. T stands for total available time and S for the quantity of labor units made available in the market. Therefore, total utility is an increasing function of leisure.

We also include the following two constraints:

(c1) $C_h^{e} C_m^{(1-e)} > A$,

where A is a positive constant, and

(c2) $WS + (1+G)pgk - ps - pC_h - EC_m = 0$.

The first one corresponds to a minimum level of consumption, with some degree of sustitutability allowed between the home produced good and the imported one. Note that fixed A, the equality gives a strictly convex curve in the (C_h, C_m) space. A similar assumption regarding to the consumption basket has been previously introduced by Miconi (1986).

The second one is the budget constraint, equalizing the sources of income -from the labor market and from the accumulated assets ownership- with its possible uses: consumption of both goods and savings.

It should be noticed that it is assumed here that the representative worker does not hold money balances, neither in the home currency nor in the foreign one. This is an extreme assumption which can be relaxed; however, in order to keep our results still valid it must be admitted that

the agent is not capable of adjusting the composition of his portfolio with an infinite velocity. In other case, the devaluation would not have effects of the supply of labor function, through the wealth effect.

Consider now the cheapest basket of goods which fulfils the constraint (c1). Both goods shares in it are obtained from minimizing

$pC_h + EC_m$

subject to (11).

So the Lagrangian function is

$pC_h + EC_m + Y<C_h^{e} C_m^{(1-e)} - A>$,

where Y is the Lagrange multiplier, and hence the first order conditions become

$p + YeC_h^{e-1} C_m^{1-e} = 0$,

$E + Y(1-e)C_h^{e} C_m^{-e} = 0$,

and (c1); they determine the optimal values of C_h and of C_m given the relative price p/E:

$C_h^* = A<(1-e)p/eE>^{e-1}$,

$C_m^* = A<(1-e)p/eE>^{e}$.

Hence, since $0<e<1$, an increase in p/E reduces C_h and increases C_m.

Let's consider next (c2).

It can be written now as

$S = <ps + pC_h + EC_m - pgk(1+G)>/W$,

and assume also that

$pC_h^* + EC_m^* > (1+G)pgk$,

that is, the value of the cheapest necessary consumption basket is higher than the value of accumulated assets held by the worker plus its current yield -given by G-.

Since

$$s > -gk,$$

S must be positive, its minimum level given by the expression

$$(12)\ S_n = <pC_h^* + EC_m^* - (1+G)pgk>/W.$$

The maximization problem does not imply that the optimal level of S will be given by S_n. However, (12) gives an idea of the parameters that determine the minimum quantity of labor supplied in the market.

It can be noticed that S_n is a homogeneous function of zero degree in the nominal variables p, W and E, since they are included into the argument of the function through their quotients.

Also it is now possible to show that S_n might be a decreasing function of the real wage rate, when it is measured through w.

Taking into account the definitions introduced in the previous section, S_n can be written as

$$S_n = <pC_h^*+EC_m^*-pgk-pgaL(Z_0-Z_1w)>/w<rp+(1-r)E>,$$

where

$$Z_0 = (p-mE)/p,$$

$$Z_1 = <rp+(1-r)E>/p.$$

Hence

$$)S_n/)w = (pC_h^*+EC_m^*-pgk-pgaLZ_0)/(-w^2pZ_1).$$

Therefore when g or the total capital stock are small enough, S_n is a decreasing function of w. Note that the only possible way through which this derivative might become positive is through the influence of w on G, the rate of profit; when G is a constant, the sign is clearly negative:

$$)S_n/)w = -S_n/w.$$

Also the following effects can be computed:

$$)S_n/)k = -g/wZ_1,$$

$$)S_n/)g = -<k+aL(Z_0-Z_1w)>/wZ_1,$$

that is, an increase in the total capital stock of the economy -given the real wage rate and g- or alternatively, an increase in workers' share in it, reduces necessary working hours. This conclusion is not surprising, and in Figure I we depict a supply curve of labor that summarizes these results.

As it can be seen, at the higher levels of w, S is an increasing function of the real wage rate; on the other hand, at its lower values the individuals tend to increase the quantity of labor they would accept to work following a decrease in the current real wage rate. In this last case, necessary hours have a main influence in the decision.

The movement from the curve labeled 1 to curve 2 is the consequence of an increase in gk, the stock of capital held by workers.

Most of our results can be illustrated using a labor supply function of the form

$$(13)\ S = Aw + B/gwk,$$

which shape is presented in Figure II.

When B is zero, that is, when it is not necessary to work, the labor supply function is reduced to a line through the origin; but as long as B is increased, the function is pushed rightwards and adquires the shape of a hyperbola.

The minimum quantity of labor supplied is given by

$S^{min} = 2(AB/gk)^{1/2}$,

when

$w^{min} = w = (B/Agk)^{1/2}$.

S^{min} is a decreasing function of gk.

This is an interesting example that may lead to very different outcomes regarding the existence of equilibria and to their stability.

In Figure III we have a case where, given the quantity of labor required in the market, the equilibrium does not exist.

Under these conditions, the origin of the involuntary unemployment can be explained using different -and not necessarily contradictory- theories.

On one hand, it might be argued that the unemployment problem rests on a failure of effective demand, since L is completely to the left of S. On the other hand, Lewis (1954) argument might be raised since the labor supply function is perfectly elastic at "any" wage rate.

Alternatively, since the coefficients of production are included in the determination of L, we are not able to rule out an analysis based upon factors missallocations and

limited technical flexibilities only on a priori grounds. Notice, also, that it is always possible to draw a negatively sloping labor demand function which does not intersect our labor supply function.

However, all of these explanations of the unemployment phenomenum depend critically on the fact that S is always to the right of L. In effect, if it were drawn as it is done in Figure IV, the involuntary unemployment would disappear.

Henceforth, the aspect that will be analyzed is not the demand side but the behavior of the supply of labor function.

Let's go back for a moment to Figure III. It is easy to see that a walrasian mechanism is not able to guide the system to an equilibrium since it does not exist.

If it is also admitted that L cannot be increased -as a hypothesis- then the problem of determining the real wage rate becomes a problem that cannot be solved through the endogenous forces of the market for labor. Under a walrasian adjustment process, the real wage rate would fall persistently, to the point of becoming zero.

Under these conditions, the action of an external agent to the market is necessary to stabilize the system, or at least to stop the continuous fall of w.

Since the demand for labor is the short side it could be argued that the market is monopsonic, and hence that the real wage rate is settled by the firms. This aspect was already studied in Chisari (1984).

Assume instead that it is the government who determines a lower bound for w. Therefore, the problem of the government's own aims becomes relevant and should be considered in the discussion.

Though different functions that the government is intending to maximize could be postulated, we are going to assume that it is able to understand the chronic character of the registered involuntary unemployment. Thus it will try to keep the rate of unemployment within an acceptable range or at a "normal" level, say U* -see our assumption a4 too-.

The government might search for the minimum possible U*, given by

$$U^{*min} = 1 - L/S^{min}.$$

In that case, as we have already seen, U* will depend on gk since S^{min} depends on it.

When U* is bigger than U^{*min} two equilibria exist, as it is shown in Figure V.

Now, depending on the adjustment equations, the stability of these equilibria must be studied. This is the analysis to be carried on in next section.

4.ModelI: fixed prices and a flexible mark-up factor.

As we have already mentioned we are going to assume that g is given, and therefore that workers' share in the capital stock is a constant.

In that case the model is reduced to the following pair of differential equations

$$k' = <f(Z_0-Z_1w)-Dk>L/ak,$$

$$w' = H<U^*-1+Lwgk/(gkAw^2+B)>,$$

when the labor.supply function is defined by (13).

The curve k'=0 is determined by the pairs (k,w) such that

$$Z_0-Z_1w = kD/f$$

with a slope -see Figure I.1-

$$dk/dw = -fZ_1/D < 0.$$

If the real wage rate is too high (small) a reduction (an increase) will be necessary in the stock of capital to meet the condition of zero net investments.

The locus of the points where w'=0 is determined by the equation

$$k = (1-U^*)B/gw<L-(1-U^*)Aw>,$$

with the slope

$$dk/dw = -(1-U^*)B<L-2w(1-U^*)A>/g<w(L-(1-U^*)Aw)>^2.$$

Figure I.2 shows that this is a strictly convex function which reaches its minimum value at

$$w = L/2(1-U^*)A,$$

when it is defined in the interval (0,L/A(1-U*)).

It also shows that two regions do exist. Given the real wage rate, an increase in the capital stock -from a point laying on the curve- leads the economy to an increase in the real wage rate up to meeting again the equilibrium set. On the other hand, a reduction in the capital stock must be followed by a reduction of w through time. In Figure I.3 both curves are depicted together for a two stationary states case.

It can be seen that the equilibrium labeled 2 is locally stable, while equilibrium 1 is unstable.

In order to guarantee the existence of equilibria it will be assumed that

$fZ_0/D > 4AB/gL^2$,

$L/A(1-U^*) > Z_0/Z_1 > L/2A(1-U^*)$.

The first inequality implies that the abscisa of k'=0 is bigger than the one of w'=0. The second group of inequalities guarantee that two equilibria do exist.

The jacobian matrix is

J1	J3
J4	J2

where

$J1 =)w'/)w = HLgk(B-gkAw^2)/(gkAw^2+B)^2$,

$J3 =)w'/)k = BHLwg/(gkAw^2+B)^2 > 0$,

$J4 =)k'/)w = -LfZ_1/ak < 0$,

$J2 =)k'/)k = -fL(Z_0-Z_1w)/ak^2 < 0$.

It can be seen that J1 is negative at equilibrium 2 and positive at equilibrium 1. In effect from w'=0 we have

$dk/dw = -()w'/)w)/()w'/)k) = -Lkg(B-gkAw^2)/B$,

and consequently taking into account that J3 is positive,)w'/)w has the opposite sign of dk/dw, which we know is positive at equilibrium 2 and negative at equilibrium 1.

Since)w'/)w is negative, a necessary condition for the existence of a Hopf cycle is not verified at equilibrium 2, for the trace of the matrix is always negative.

However, a cycle can be obtained if some parameters in the model are correctly specified.

Let's assume that A = 0. Recalling that our labor supply function is

$S = Aw + B/gkw$,

this assumption implies that 'necessary' hours are the only important in workers' decisions. Additional hours, as represented by the term Aw, have been neglected as an explanatory variable in workers' labor supply.

Under this new hypothesis the system of differential equations becomes

$k' = <f(Z_0 - Z_1w) - Dk>L/ak$,

$w' = H(U^*-1+Lwgk/B)$.

In this case the elements of the jacobian matrix will have the following sign pattern:

$)k'/)k = -DL/ak < 0$,

$)k'/)w = -fLZ_1/ak < 0$,

$)w'/)k = HLwg/B > 0$,

$)w'/)w = HLgk/B > 0$,

where w and k are calculated at their equilibrium values.

The eigenvalues, Y_1 and Y_2, will be obtained from solving the equation:

$Y^2 + L(D/ak-Hgk/B)Y + HgL^2(fZ_1w/k-D)/aB = 0$;

if the trace is zero,i.e. if

$H = DB/gak^2$,

they will be

$Y_{1,2} = +- (1/2)<-4HgL^2(fwZ_1/k-D)/aB>^{1/2}$.

When

$fwZ_1 > Dk$

they will be a pair of pure imaginary numbers and the equilibrium will look like a center -though it is necessary to study the other terms in Taylor's expansion for determining its real nature-. If the sign is reversed we will have a saddle point, since one of the roots will be a real positive number, and the other a negative one.

A numerical example will show that appropriate parameter values do generate a Hopf bifurcation and give rise to a family of closed trajectories in a neighborhood of one of the equilibria.

In Figure I.4 a numerical example developed using Kocak's (1986) PHASER program is presented.

The translation of the system of equations to PHASER language is shown in Figure I.4. Figure I.5 also gives the direction field of the pair of differential equations involved, where (the letters on the right hand correspond to PHASER's language):

f:= F	Z_0:= P
Z_1:= p	D:= D
L:= h	1/a:= a
H:= G	U*:= U
g:= R	A:= A
B:= C	x1:= k
x2:= w,	

with s = 0.

The numerical example has been computed using Runge-Kutta's algorithm -see Braun (1975)- with the following parameters values:

U* = 0.5,	Lg = 10x0.095 = 0.95,
Z_0 = 5,	Z_1 = 1,
a = 1,	B = 5,
f = 0.1,	A = 0,
D = 0.1,	H = 2.7560889.

The equilibria will be

E1	w1 = 1.3819	k1 = 3.6181,
E2	w2 = 3.6181	k2 = 1.3819.

A neighbourhood of E2 is shown in Figure I.6.

The same neighborhood is presented in Figures I.7 up to I.11. In Figure I.7 the equilibrium and the initial condition

k(0) = 1.5,

w(0) = 3.4,

give rise to an orbit, which has been depicted after a hundred iterations.

Figure I.8 shows the previus orbit and an additional one by setting the initial condition

k(1) = 1.4,

w(1) = 3.3.

Figures I.9 up to I.11 show the trajectories beginnig at k(0) and w(0) and the direction field for different values of H. The first one, which corresponds to a stable focus is obtained when H = 1. The second one is the closed orbit that appears when H is chosen so that the trace becomes zero, and the last one corresponds to an unstable focus, when H =4.

To end with, Figure I.12 shows a neighborhood of E2, when the initial condition is

$k(2) = 1.382,$

$w(2) = 3.619.$

It can be seen that though being very near to the stationary point, a closed trajectory is still observed.

This result might point out the existence of a multiplicity of orbits, an outcome that is not forbidden by Hopf's theorem.

It must be also mentioned that the second condition of Hopf's theorem, i.e., that the real part of the roots has a non vanishing derivative with respect to H -see Appendix B- is also met with since

$dReY/dH = gkL/2B > 0,$

when it is computed for a zero trace at the stationary point.

5.Model II: Price flexibility, constant mark-up factor and endogenous labor productivity.

In our previous model money prices were considered as constants, while allowing the mark-up factor, Z, to be a variable. These hypotheses will be modified now by exchanging the role of these variables though still maintaining the equality between the number of equations and that of the unknowns.

Those unknowns are now

k,b,w,W,p,L,q,U,C,G and a.

The last one is the inverse of labor productivity, that is, the quantity of labor required per unit of the final good. It will be assumed that it is a variable, more specifically

b1. a is a function of the real wage rate, i.e.

$a = a(w),$

with a negative derivative. That is, a reduction in the real wage rate tends to reduce workers' productivity. Moreover, it is assumed that

$a = a_0 w^y,$

with $y<-1$.

Note that b.1 implies that

$a + w(da/dw) = a_0 w^y(1+y) < 0,$

the elasticity of the labor coefficient with respect to the real wage rate is negative and bigger than one in absolute value.

When prices are flexible, the known relation between the external constraint and national product must be modified. In effect, recalling equation (6) we have

$R(p/E) - mq = F,$

and taking into account that -from (3) and (4)-:

$p/E = Z\langle aw(1-r)+m\rangle/(1-Zawr),$

we arrive to

$q = Q(w) = \langle R(w)-F\rangle/m.$

Under b.1 we obtain also

$d(p/E)/dw = Z(1-r+Zrm)(a+wda/dw)/(1-Zwra) < 0;$

notice that since p/E must be a positive number it is necessary that

$$1 > Zwra.$$

Therefore the following inequality must be verified:

$$w > (Za_o r)^{1/(1+y)}.$$

Thus:

$$dQ/dw = <dR/d(p/E)><d(p/E)/dw>/m > 0.$$

Moreover, it will be assumed from now on that

b.2. $Q(w) = Qo/(J-w)$,

where Qo and J are positive constants.

Hence

$$dQ/dw = Qo/(J-w)^2 > 0.$$

We are going to use the following assumption too

b.3. The labor supply function can be written in the form

$$S(w,gk) = Aw + (T-sgk)/w,$$

with A, T and s positive constants.

It can be seen from b.3 that depending on the level of gk -workers' share in the stock of capital- and consequently on k when g is a constant, the labor supply function might have one of the alternative shapes presented in Figure II.1.

Since under the new conditions prices are flexible, the real value of the "subsistence" basket might be changing when the real wage rate is modified and hence, the expression corresponding to necessary hours should also

consider this dependence. It will be assumed that this effect is already included in our labor supply function.

Regarding to the demand for labor, it can be written as

$$L = a(w)Q(w)$$

from the definition of a.

We are ready now to formulate the assumptions referred to the dynamic behavior of the model.

b.4. The real wage rate is adjusted to the excess demand for labor, i.e.

$$w' = H<a(w)Q(w) - Aw-(T-sgk)/w>.$$

b.5. Total investments depend of the rate of profit, and of the intensity in the use of capital in production, as measured by b (=q/k). That is

$$k' = f(G)-Dkb^2.$$

In this case the use of capital in production affects depreciation in a quadratic way.

Since the mark-up factor is a constant, and assuming that f is also fixed, the equation in b.5 determines the level of the real wage rate at the stationary state. It can be put as

$$k' = <f(1-1/Z)-Dkb>b = 0,$$

and therefore it is required that

$$Q(w) = Qo/(J-w) = f(1-1/Z)/D,$$

which determines

$$w* = J-<f(1-1/Z)/DQo>,$$

that will be positive if J is big enough.

Given w^*, it is possible to determine k^* from the differential equation in b.4

$$w' = 0 \implies k^* = <T - w(Qoa_o w^y/(J-w) - Aw)>/sg,$$

where $w = w^*$.

Therefore the equilibrium is unique. The respective curves are presented in Figures II.2, II.3, and II.4. It can be seen that at least in principle a cyclical behavior cannot be ruled out.

The Jacobian matrix is -at the point (k^*, w^*)-:

J1	J2
J3	J4

where

$J1 =)k'/)k = 0,$

$J2 =)k'/)w = -DQo^2/k^*(J-w)^3 < 0,$

$J3 =)w'/)k = Hsg/w^* > 0,$

$J4 =)w'/)w \doteq H<Qoa_o w^{y-1}(y(J-w)+w)/(J-w)^2 - A + (T - sgk^*)/w^2>.$

If $J4 = 0$ then the characteristic roots will be

$$Y = \pm <-4J2J3>^{1/2} i,$$

that is, pure immaginary numbers; moreover their real parts depend on the parameters values. In effect, the conditions of the Hopf bifurcation are met and henceforth a limit cycle will exist for certain parameters values.

After some manipulations, it can be shown that the real part of the roots is zero if

$$X = Aw^*(w^* - 2J) + <J(1+y) - yw^*>Qoa_o w^{*y}/(J - w^*) = 0,$$

and also that

$$dX/dy = Qoa_o w^{y+1}<J - w^* + \log(J(1+y) - yw^*)>/(J - w^*)$$

is different from zero if

$$J - w^* + \log w^{*J(1+y)-yw^*}$$

is different from zero.

The period of the orbit will be -see Appendix B-:

$$T(E) = \ /<DQ_o^2 Hsg/k^* w^*(J - w^*)^3>^{1/2}.$$

From this expression we can obtain the change in T(E) associated with a movement in $\dot{g}$, for example. However, taking into account the long run level of k, it can be seen that T(E) does not depend on g -see above the expression for k^*-.

(*) A numerical example.

Using the PHASER program we are going to show that the above presented conditions can be met, and consequently that in a numerical example an orbit arises.

The pair of differential equations translated in PHASER's language is presented in Figure II.5.

The definitions of the variables are the following (the letters on the right hand correspond to PHASER's language):

D:= D	f(1-1/Z):= Z
Qo:= Q	a_o:= A
y:= a	J:= b
T:= B	sg:= T
H:= H	A:= C

and where the symbol * stands for a multiplication operation.

In Figure II.6 the location of the equilibrium and of a limit cycle are shown, when the parameters values are

$D = 0.1$, $f(1-1/Z) = 0.1$,

$Q_o = 1$, $a_o = 1$,

$J = 2$, $y = -1.6$,

$sg = 0.2$, $T = 2$,

$H = 1$, $A = 1$.

The equilibrium is

$w^* = x_2^* = 1$,

$k^* = x_1^* = 1.2$.

The direction field of the system is also shown. The initial conditions are

$x1 = k(0) = 1.21$,

$x2 = w(0) = 1$.

It can be seen that the form of our function Q(w) determines a frontier or threshold beyond which the system changes its trajectories -see Figures II.7 and II.8-.

In Figure II.9 the trajectory of the capital stock and its last values can be observed after two hundred iterations using the Runge-Kutta's algorithm -on this algorithm Braun (1975) provides a good explanation-.

Figures II.10 up to II.13 show the behavior of the system with different initial conditions and window sizes. The last one seems to indicate the possibility of multiplicity of closed trajectories -it should be recalled that Hopf's theorem does not guarantee neither unicity nor stability-.

On Figure II.14 two trajectories are depicted when the initial conditions are

$k(0) = 1.3$,

$w(0) = 0.9$,

and

$k(1) = 1.23$,

$w(1) = 0.99$.

The trajectory starting at $\langle k(0),w(0)\rangle$ moves away from the equilibrium, while the one starting at $\langle k(1),w(1)\rangle$ converges to a cyclical path.

Appendix A: Chaotic trajectories and subsistence effect.

It will be shown in this appendix that the dynamic path of the real wage rate and of the demand for labor might exhibit chaotic trajectories in the sense of Li and York (1975) -see also Gabisch and Lorenz (1987)- when the labor supply function is the one discussed in the main body of this paper and when a walrasian adjustment process is also assumed.

That is, the movement of the real wage rate -given by a walrasian adjustment difference equation- shows irregularities such that it will be impossible to distinguish its trajectory from a pure random path.

Of course our results depend heavily on the values of the parameters since we are going to work almost by analogy with a logistic difference equation.

It can be recalled that the logistic difference equation

$x_{t+1} = Mx_t(1-x_t)$

with $x_t \subset \langle 0,1\rangle$ and $M \subset \langle 0,4\rangle$, exhibits chaotic movements when $M > 3.575$ since an accumulation of fixed points of period 2^n does exist. When $M = 3.65$ the graph of the trajectory in the (x_{t+1},x_t) space covers an entire region without converging to the equilibrium -see also Kocak (1986)-. This is depicted in Figure A.1.

By continuity, it can be shown that the chaotic movements are also possible when the logistic equation becomes

$$x_{t+1} = Mx_t(1-x_t)+N,$$

N being a small enough positive number. In effect, assume that Li and York's theorem is verified; consequently there is a point A in the interval <0,1> such that for the continuous function F

$$F^3(A) < A < F(A) < F^2(A),$$

where the superscript indicates the number of iterations, and hence that a chaotic movement has been found.

By continuity, when N is small enough it will be also true that

$$F<F(F(A)+N)+N>+N < A < F(A)+N < F(F(A)+N)+N$$

and therefore chaos will be also present in the redefined equation.

The interval where x_t is defined must be correspondingly altered to

$$<1/2-(M^2+4NM)^{1/2}/2M,1/2+(M^2+4NM)^{1/2}/2M>$$

which includes the interval <0,1> and therefore the point A. It is also assumed that

$$1/2-(M^2+4NM)^{1/2}/2M < M/4 + N < 1/2-(M^2+4NM)^{1/2}/2M,$$

and inequality that is verified when, for example, M=3.65 and N=0.001 and where M/4 + N is the maximum value of x_{t+1}.

Note that for any x_t in this interval, x_{t+1} also belongs to it.

Figure A.2 illustrates this point; the trajectory of the system when N = 0.001 and M is still equal to 3.65 is shown after two thousand iterations. It can be observed that the graph of the path followed by x_t is quite similar to the one obtained when N = 0.

On Figures A.3 and A.4 the path followed by x_t is computed in both cases. The corresponding graphs look like key edges which have been depicted randomly. The corresponding program in PHASER's language is presented in Figure A.5.

Our next step consists in showing that the trajectory of the wage rate as determined by a walrasian adjustment process can be put in a form similar to the logistic equation, when the subsistence effect is introduced into the labor supply function.

Let's write our labor supply function as

$$S(w) = 3B + (D-3B)w + Bw^2,$$

where D and B are positive constants that verify the following conditions:

$$3B > D,$$

$$12B^2 > (3B-D)^2.$$

The shape of this function is shown in Figure A.6. It should be noted that the slope of the function is negative when

$$w < (3B-D)/2B.$$

The demand for labor function will be written as

$$L(w) = a - bw,$$

with a and b positive constants.

Let's consider now the adjustment equation, given by

$$w_{t+1} - w_t = m<L(w_t)-S(w_t)>,$$

which after some steps can be put as

$$w_{t+1} = mBw_t<1-mb-(D-3B)m/Bm - w_t> + (a-3B)m.$$

When b tends to zero, and taking Bm = 3.65 it is necessary that

$$1-bm -(D-3B)m = Bm,$$

$$(a - 3B)m = 0.001.$$

In that case our equation is almost equal to the logistic one, with the exception of the constant (a-3B)m, which, however can be taken as small as it is necessary -by reducing the magnitude of m, for example-.

This outcome illustrates one of the admissible consequences of assuming that a subsistence effect does exist. It is not necessarily true that this will be the path followed by the real wage rate, but the fact that a chaotic movement of the real wage rate and of the demand for labor -obtained from a chaotic path through a linear transformation- can be obtained without random variables seems to be interesting enough.

Very recently, Blanchard and Summers (1987) -see also Cross (1987)- have presented a model based upon Phelps' hysteresis effect for understanding random movements in the rate of unemployment. However, in their model the random movements are originated in random shocks exogenous to the system and without an economic justification.

In their case, history matters in the sense that the entire path of the rate of unemployment over time affects its current level -through the lost abilities after a long duration unemployment period, for example-.

History is important in our model too, since it has determined the present level of workers' assets and, therefore, the main properties of the labor supply function. The difference between both treatments is that in our case the forces that determine a random walk are endogenous, and hence, it is possible to give an economic interpretation of their existence.

Appendix B: Some remarks on Hopf's bifurcation theorem.

In the main body of this paper, Hopf's bifurcation theorem has been used for showing the existence of cyclical movements in the real wage rate and in the stock of capital.

Some numerical examples illustrated the possibility of observing a cycle around the only equilibrium in our fixed mark-up case and in a neighborhood of the equilibrium with the highest real wage rate in the case of an endogenous -variable- mark-up factor.

The Hopf bifurcation has been already used in economics for showing the existence of closed trajectories in a neighborhood of stationary points -by Benhabib and Nishimura (1979), for example-.

The main assumptions and results of Hopf's theorem are summarized here.

To begin with, we can follow Chow and Hale (1982) for defining a bifurcation.

Let X,Z and V be real open sets, and assume that

M:XxV --> Z

has continuous derivatives up to the first order at least.

Let's consider the equation

M(x,v) = 0,

where v C V and x C X. Let's also take S VxX, be the set of solutions of this equation and S(v) = <x C X: (v,x) C S>.

Let's also assume that U is an open set in X.

S(v) is said to be "equivalent" to S(w) if S(v) U is homeomorphic to S(w) U.

It is said that v_o is a "bifurcation point" for this relation if for any neighborhood V_o there is an x_o C $S(v_o)$ and a pair of numbers v_1 and v_2 that belong to that neighborhood and with the property that $S(v_1)$ is not "equivalent" to $S(v_2)$.

Hopf's theorem shows the bifurcation of a periodic orbit from the equilibrium position of a n^{th} order differential equation. Different versions of this theorem can be presented, involving or not analyticity for example.

According to Hassard et al. (1981), we can formulate the main ideas as follows.

Let's consider an autonomous system of differential equations

dx/dt = f(x,v),

where x C R^n and v is a real parameter defined in an interval V.

Let's assume that x = x*(v) is an isolated point and that the Jacobian matrix:

A(v) = D_xf(x*(v),v)

has a pair of complex conjugate eigenvalues Y_1 and Y_2:

Y_1 = Y_2 = a(v) + ib(v),

such that for some number v = v_o C V:

$b(v_o)$ = b_o > 0,

$a(v_o)$ = 0,

$da(v_o)/dv$ = 0.

Then v_o is a "critical value" of v. If the eigenvalues of $A(v_o)$ -other than +-ib_o- all have strictly negative real parts, the above assumption implies that a loss of linear stability of the stationary point x* as v crosses the value v_o does exist.

Also, a family of periodic solutions appears out of the equilibrium -the Hopf bifurcation-. This family can be written as

x = $p_E(t)$ (0< E <Eo)

where Eo measures the amplitude

max_t $p_E(t)$-x*(v_o)

and Eo is sufficiently small.

Moreover, T(E), the period of the orbit $p_E(t)$ is approximately equal to 2 /b_o.

It should be noticed that any system of autonomous differential equations can be redefined by taking the new variables as the deviations from the solution values, that is, it can be written as

$$dX/dt = F(X,m),$$

where now we have $X^* = 0$ and the critical value of m is

$$m = v - v_o.$$

Under these new definitions, if $da(0)/dm > 0$, the periodic solutions which arise for $m > 0$ are called "supercritical", while those appearing when $m < 0$ are "subcritical". The defitions lack agreement when $da(0)/dm <$ 0 according to Hassard et al..

Interesting applications of this theorem -with emphasis in biological examples- can be found in Marsden and MacCracken (1976). Hassard et al. present a list of steps to be performed for computing the existence of a Hopf bifurcation and for studying the stability of the orbits that arise. Unfortunately they require the computation of the third order derivatives -at least- of the system. This procedure was followed in Chisari (1987).

In our models two critical parameters were chosen. In the first one, H, the velocity of adjustment of the real wage rate to the deviations from the rate of unemployment of its normal value was chosen; in the second one, the elasticity of labor productivity to the real wage rate, y, was considered as the critical parameter among the possible candidates.

6. Concluding remarks.

Two different models with a main common feature have been discussed here.

Both of them take into account a labor supply function that exhibits a negative slope -at least in some interval- with respect to the real wage rate due to the simultaneous influence of a wealth and a 'Duesenberry' effects.

The reduced magnitude of workers' assets influences workers' decisions, who chose to increase the number of hours they would accept to work when a decrease in the real wage rate is observed with the aim of sustaining their consumption patterns.

It is assumed here that workers' share in total capital is constant; the case of a variable one, which produces very complicated dynamics will be discussed in another paper.

The first model considers the case of a flexible - endogenous- mark-up factor. Two equilibria are possible under this assumption, and it is shown that a limit cycle might exist in a neighborhood of the stationary point with the highest real wage rate -the smallest level of capital-. Hopf's bifurcation cycle is used for showing the existence of a closed trajectory, and it is illustrated by means of a simulation of an example with a computer's program.

This same procedure is also used for showing the existence of a cycle in our second model. In this case, the mark-up factor is assumed to be constant -exogenous- and the productivity of labor is considered as a function of the real wage rate.

Some other hypotheses that are introduced in the paper are worth mentioning.

First of all, we are discussing the behavior of an economy which is constrained to obtain a given surplus of its commercial balance for repaying its foreign debt; since we also assume that a constant coefficient of imports in production does exist, this constraint determines the level of output.

Secondly, total investments are assumed to depend on the rate of profit and on a depreciation factor that is variable with the intensity in the use of total capital. This assumption tries to take into account the empirically observed variability of the intensity in the use of capital over cycles.

OCTOBER, 1987.

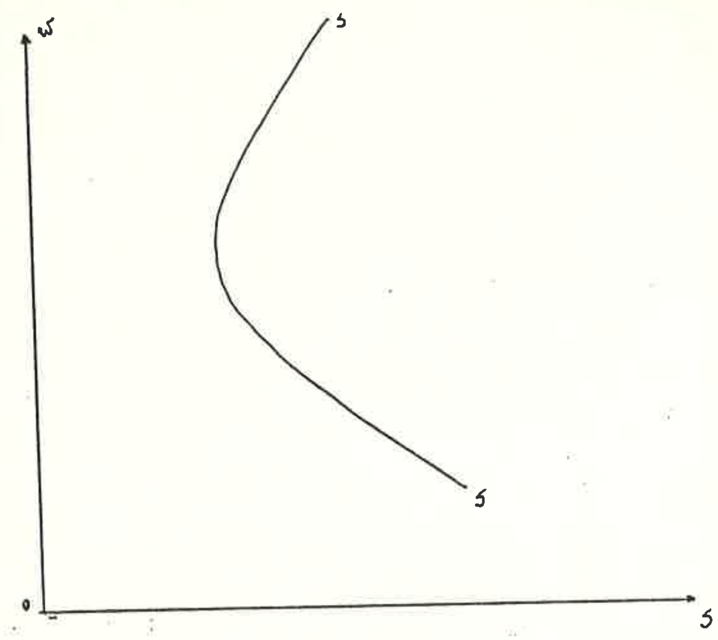


Fig. I

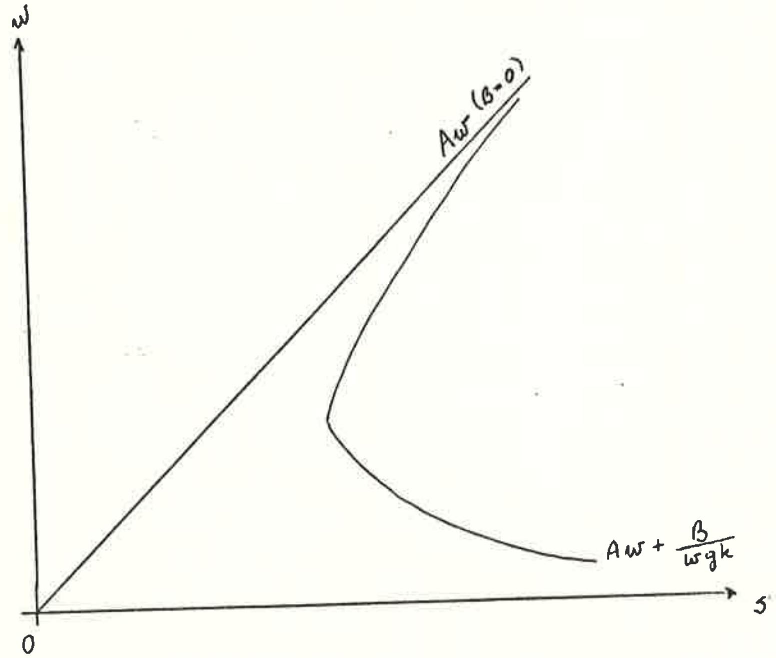


Fig. II

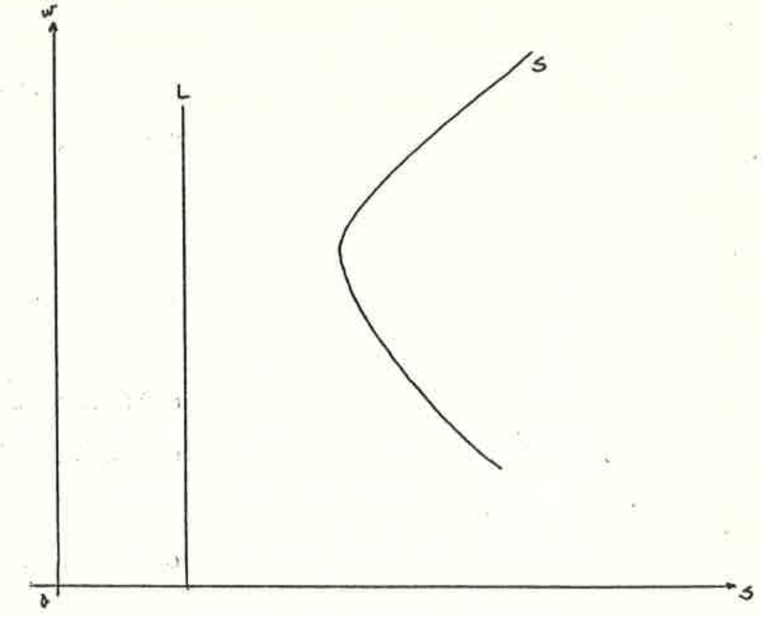


Fig. III

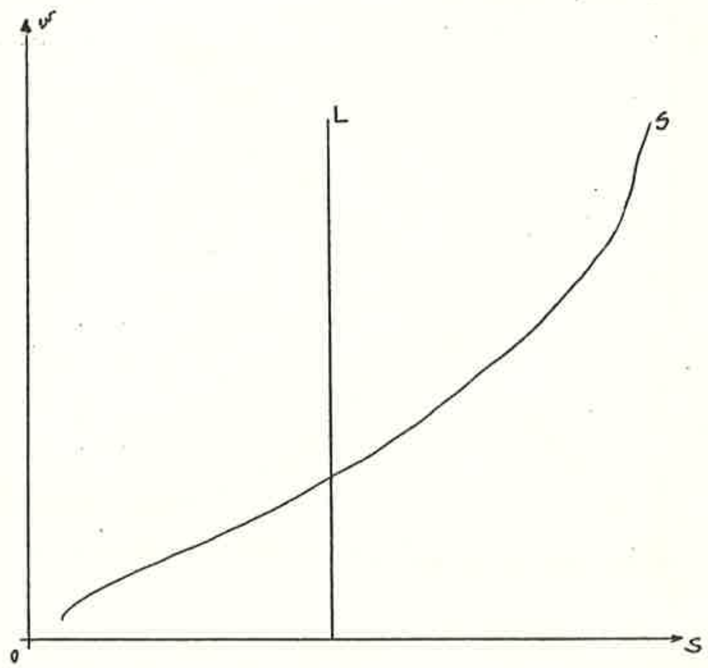


Fig. IV

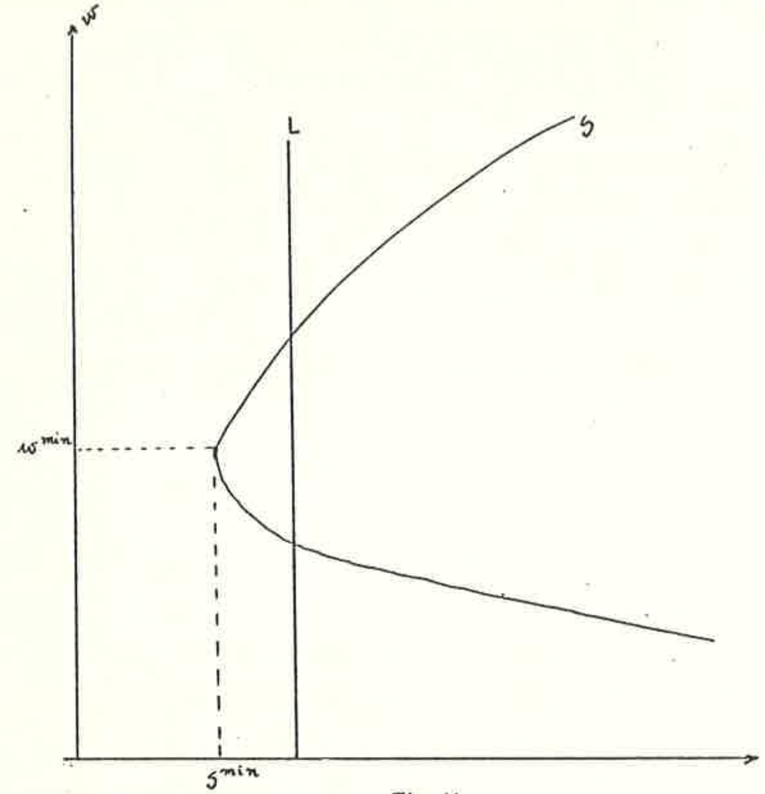


Fig. V

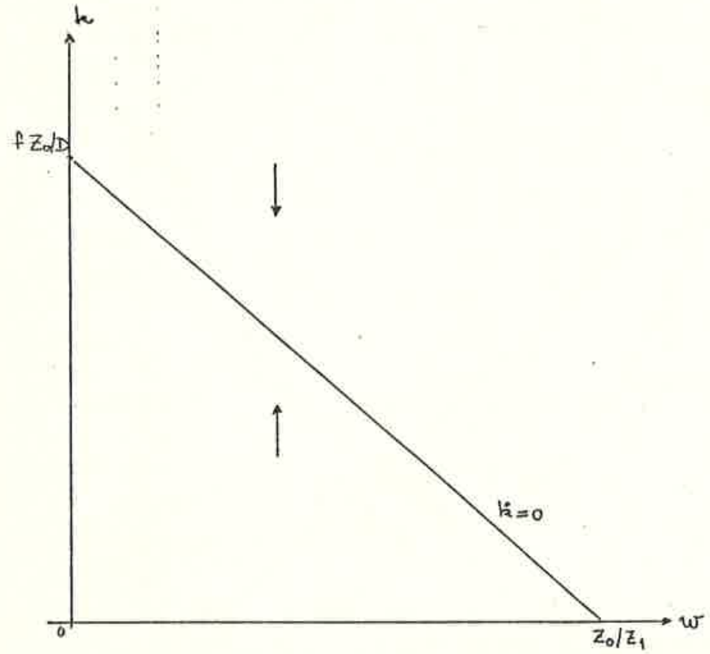


Fig. I.1.

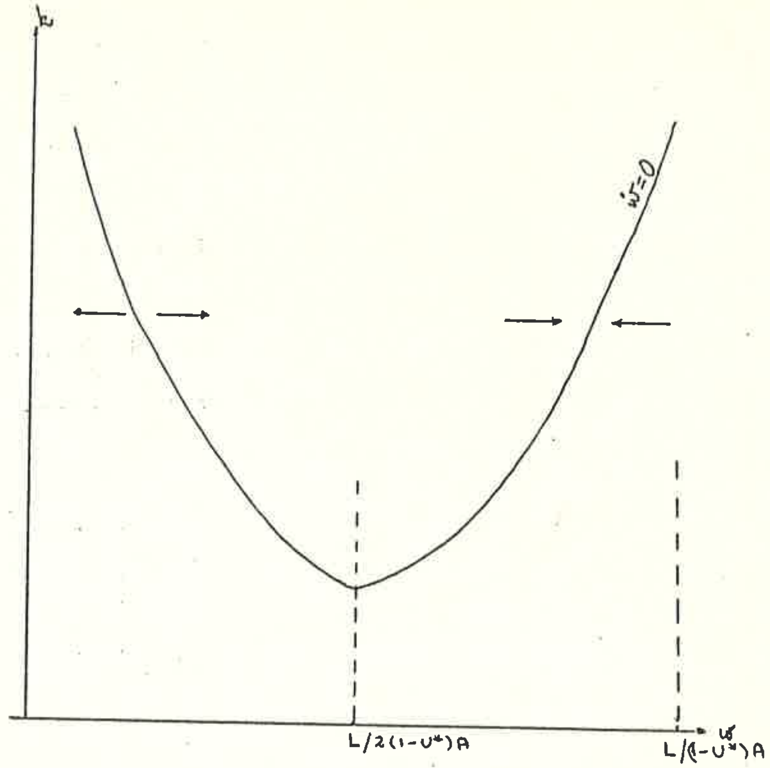


Fig. 1.2.

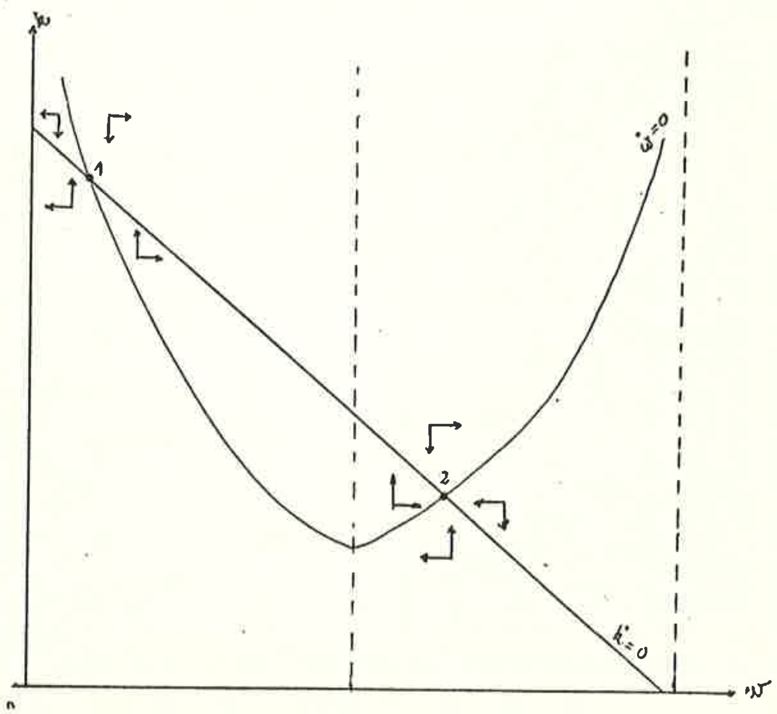


Fig. 1.3.

```
                 e19
x1'  =  ((F/x1)*(P-(p*x2))-D)*a
*(h-(s*x2))
x2'  =  (G*(U-1.000 ))+(G*(h-(s
*x2))*R*x1*x2/((A*(x2^2.000 )
*R*x1)+C))
```

Fig. 1.4.

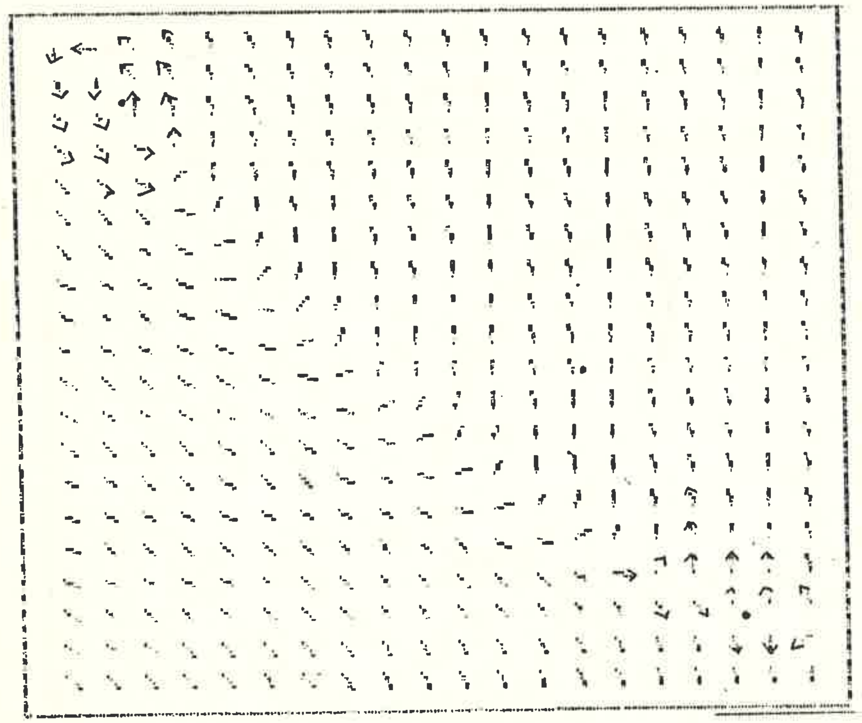
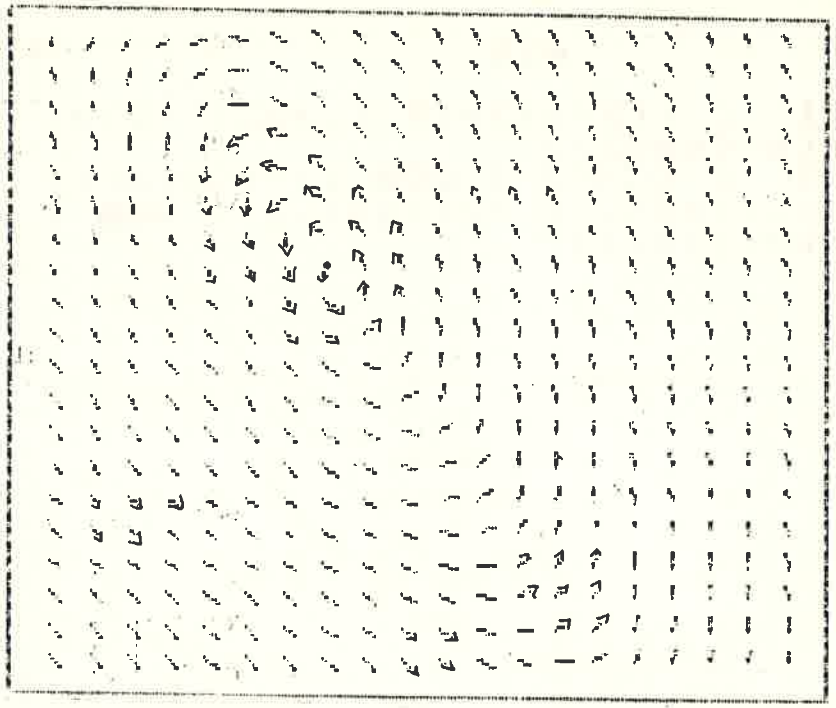


Fig. 1.5.



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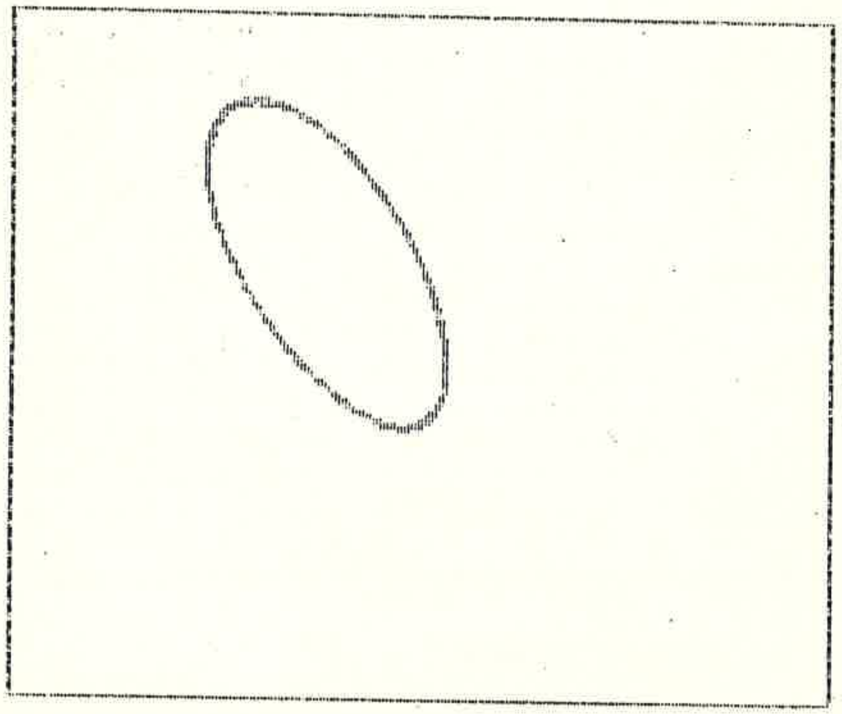

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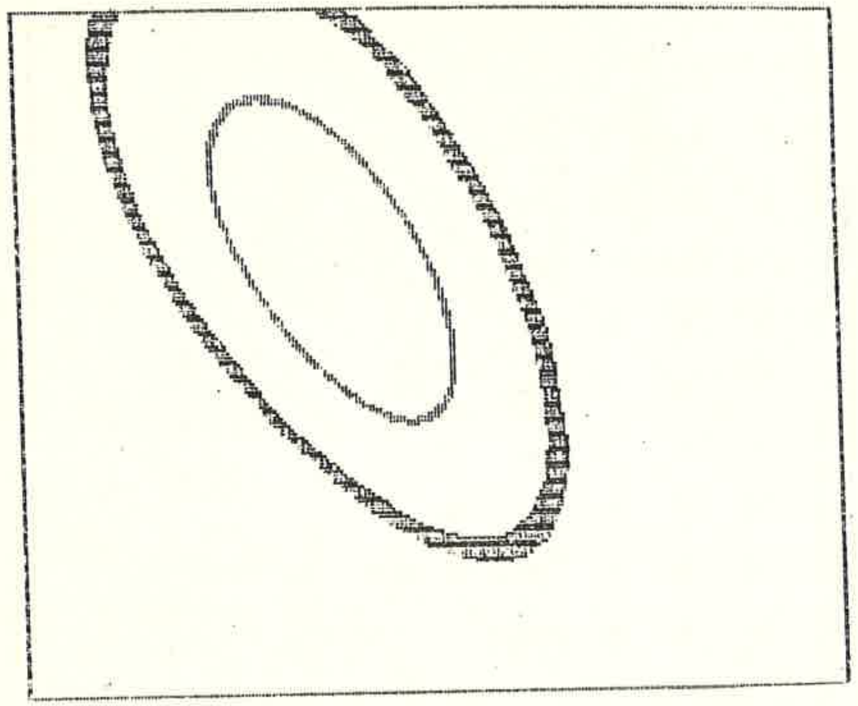

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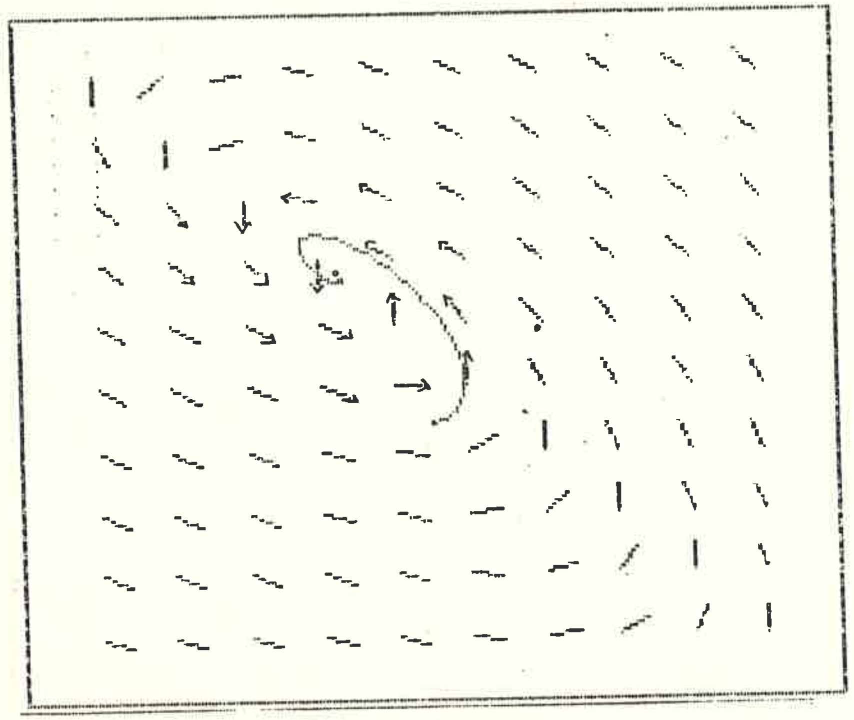

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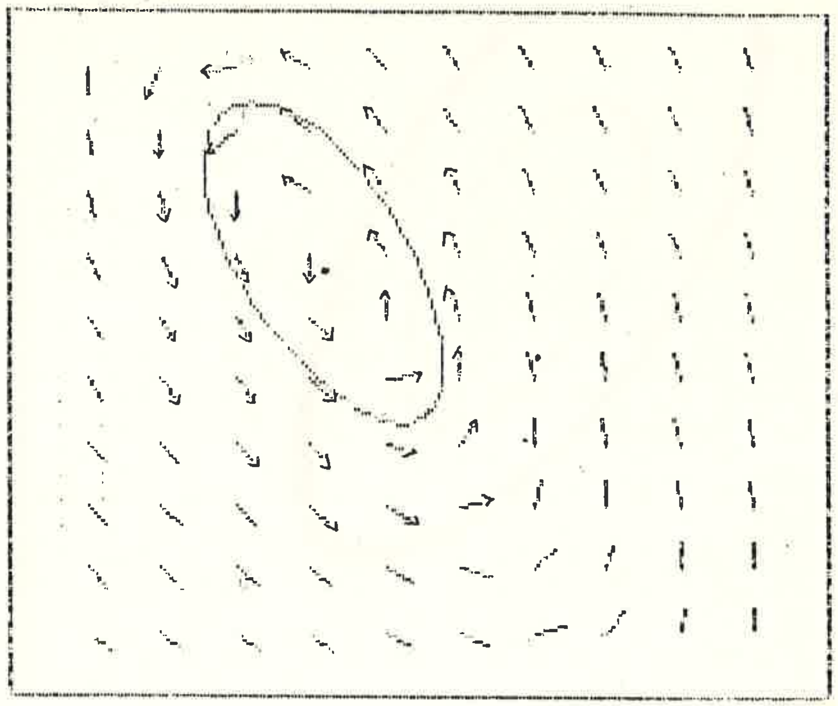

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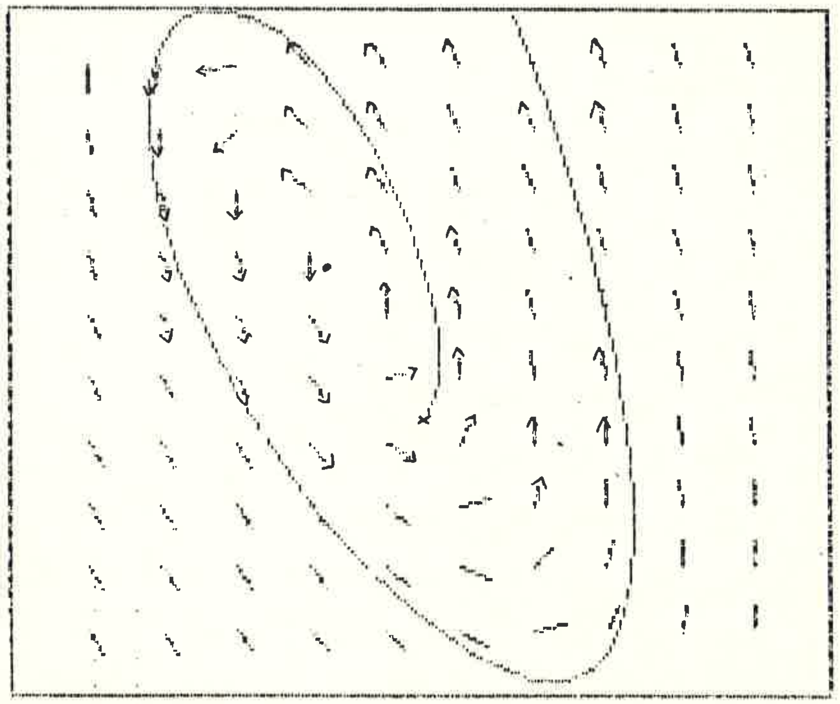

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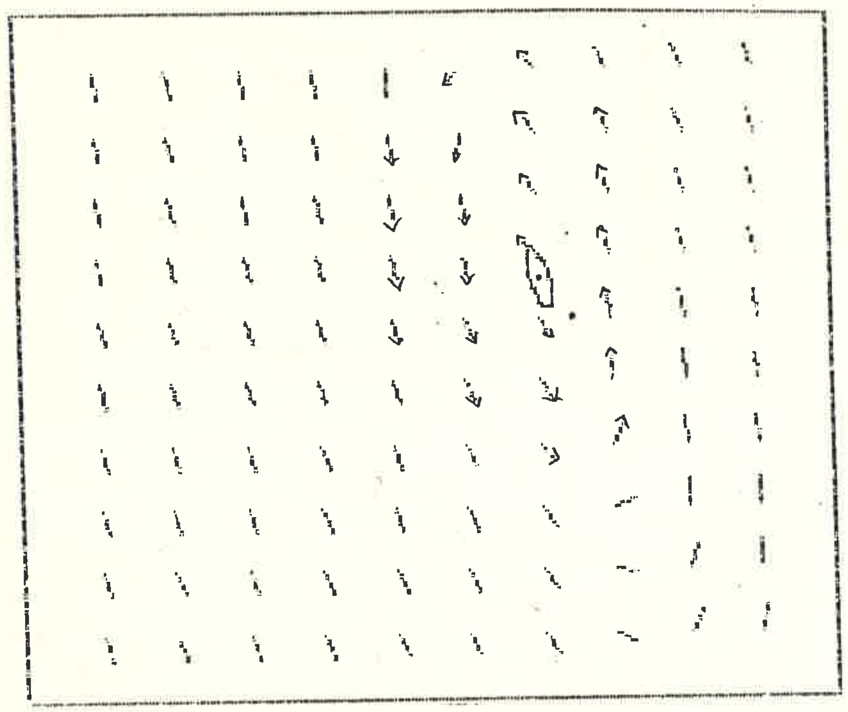

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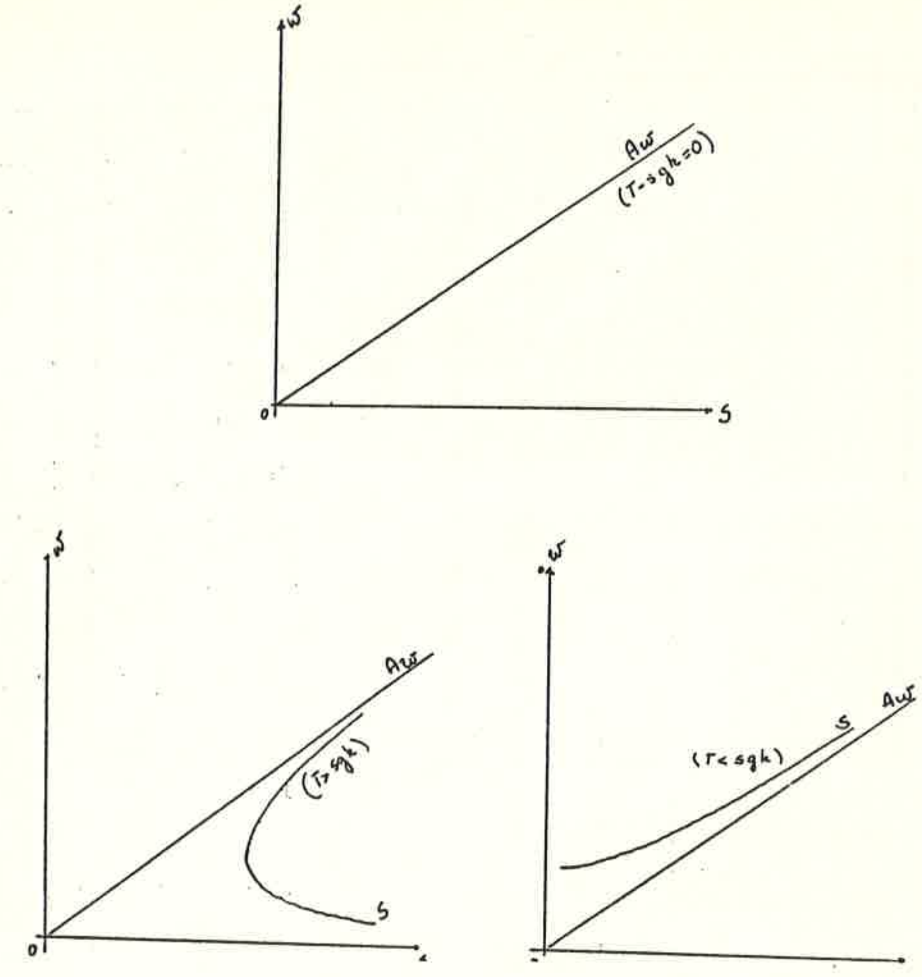
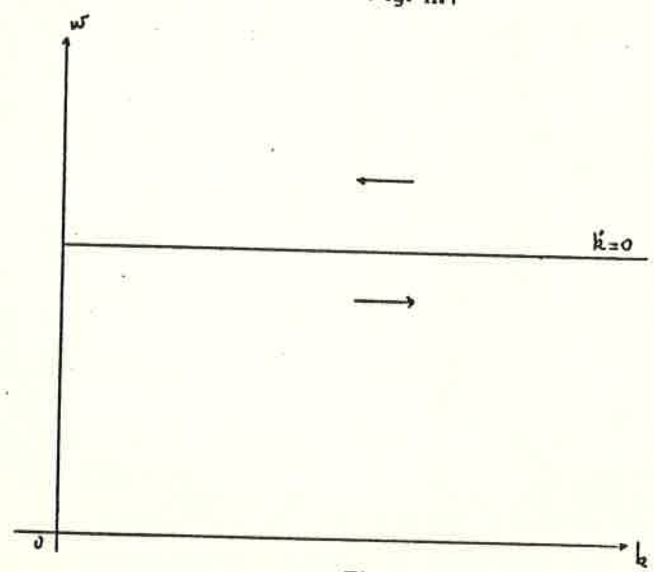


Fig. II.1



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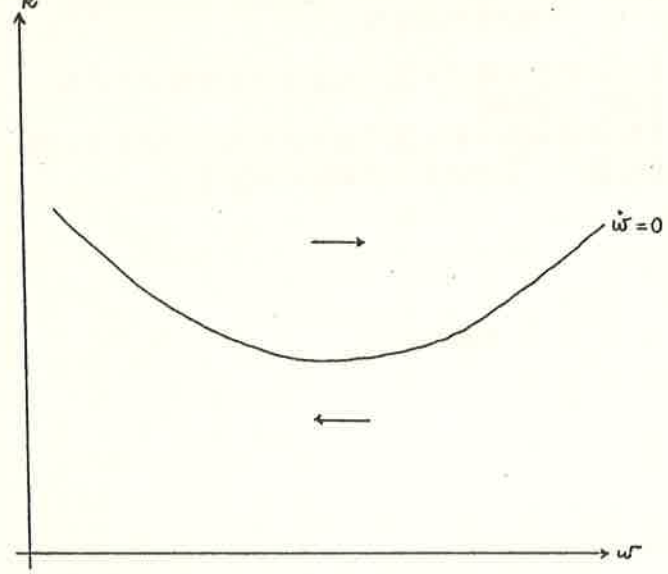

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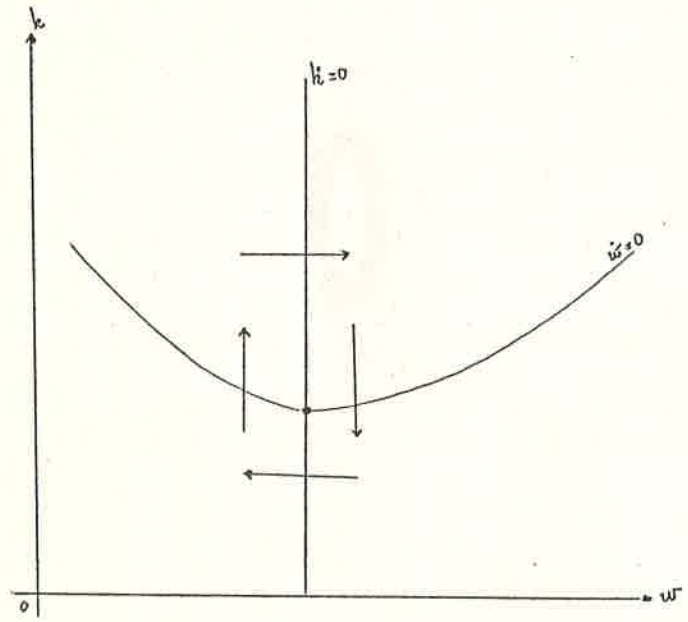
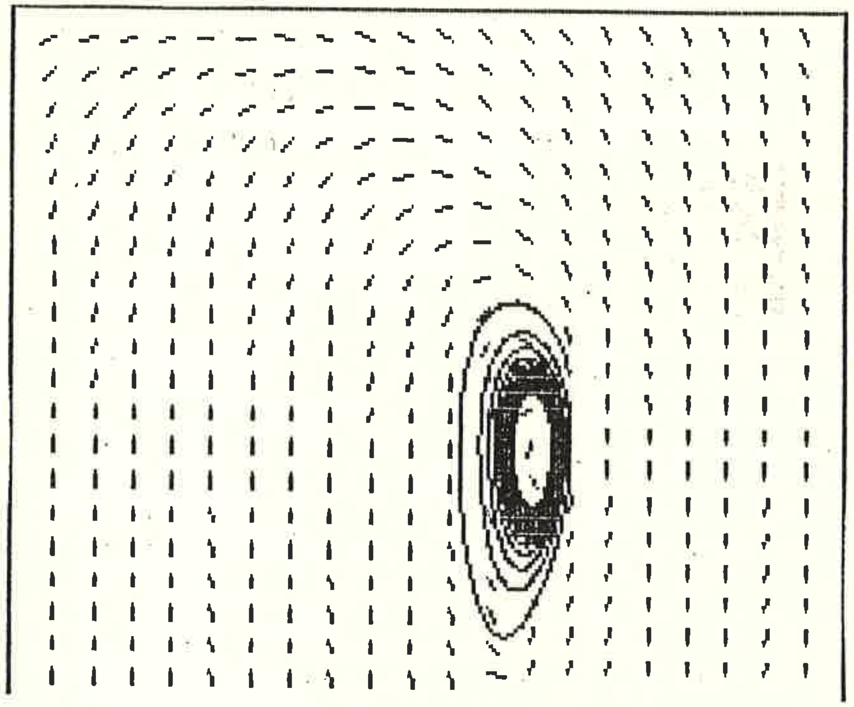


Fig. II.4.

```
unomas4

x1' = (Z-(D*Q/(b-x2)))*Q/((b-
x2)*x1)*1.000
x2' = (H*Q*A*(x2^a)/(b-x2))-H
*C*x2-((B-(T*x1))*H/x2)
```

Fig. II.5.



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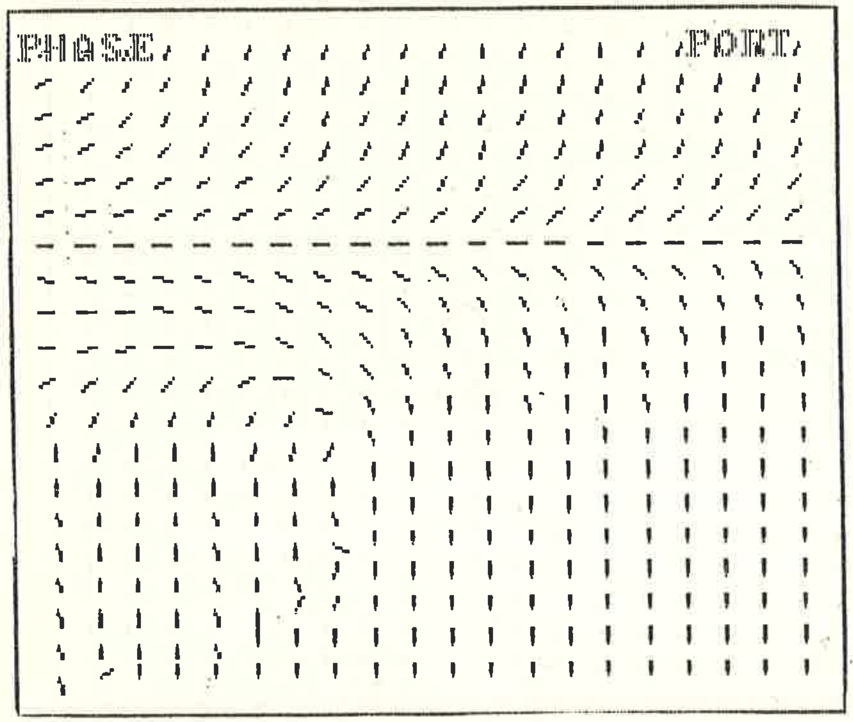

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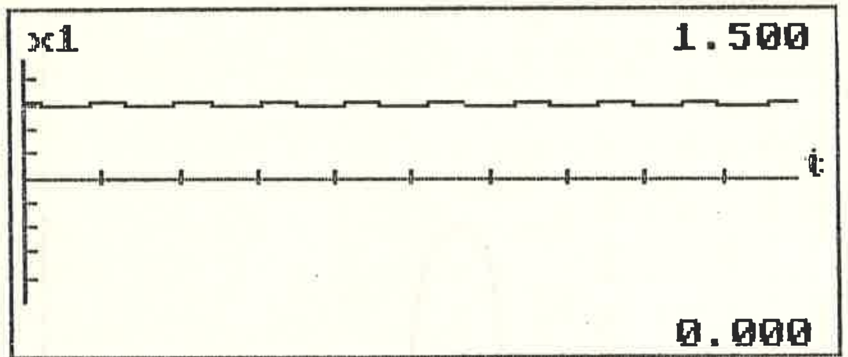


LAST Xi

IC	Time	X	Y	Z
1	199.30	1.2056	1.0252	
1	199.40	1.2053	1.0258	
1	199.50	1.2051	1.0264	
1	199.60	1.2049	1.0269	
1	199.70	1.2046	1.0274	
1	199.80	1.2044	1.0279	
1	199.90	1.2042	1.0284	
1	200.00	1.2039	1.0289	

Fig. II.8.

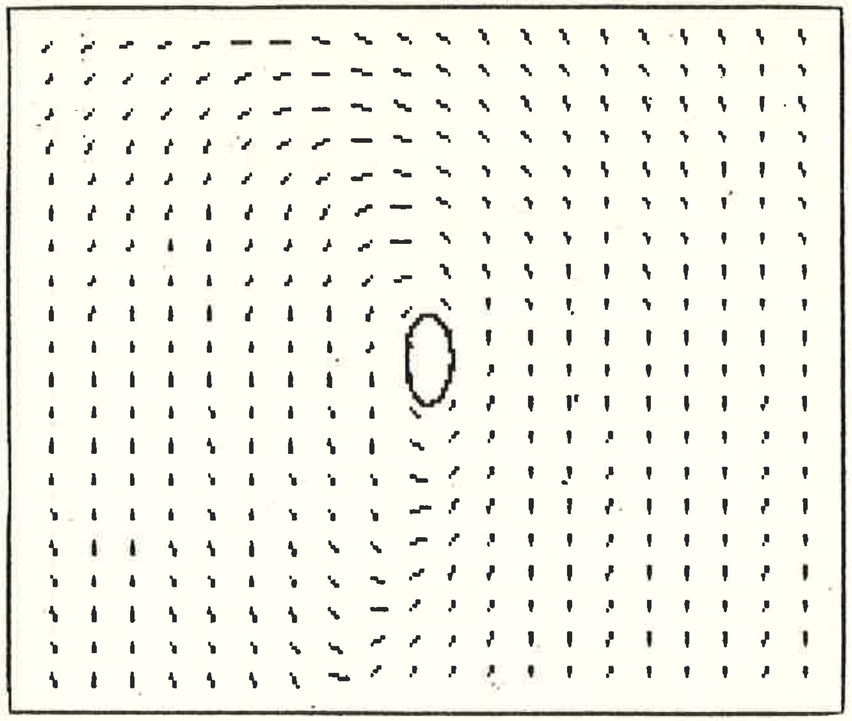


Fig. II.9.

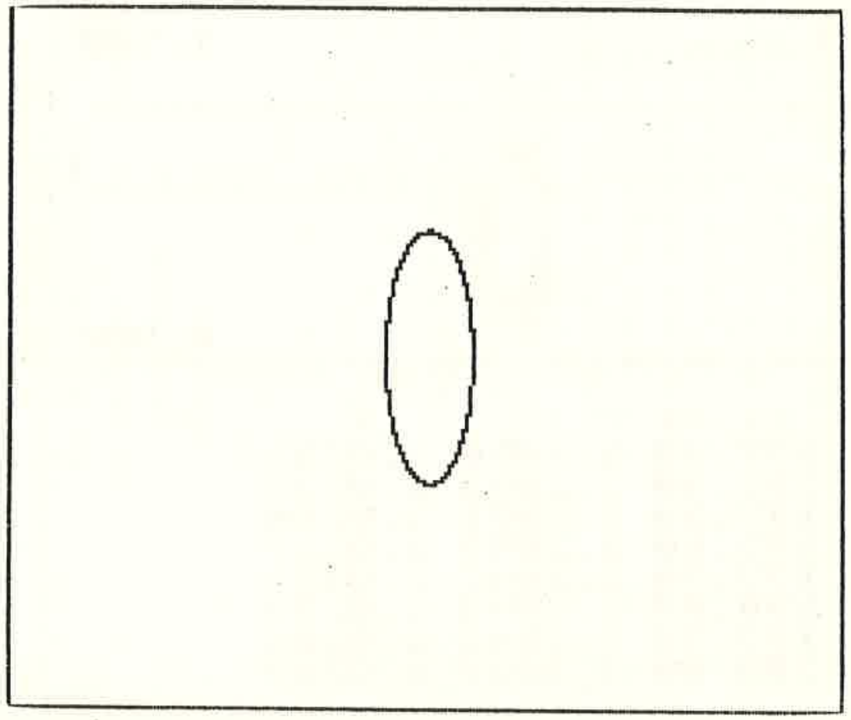


Fig. II.10

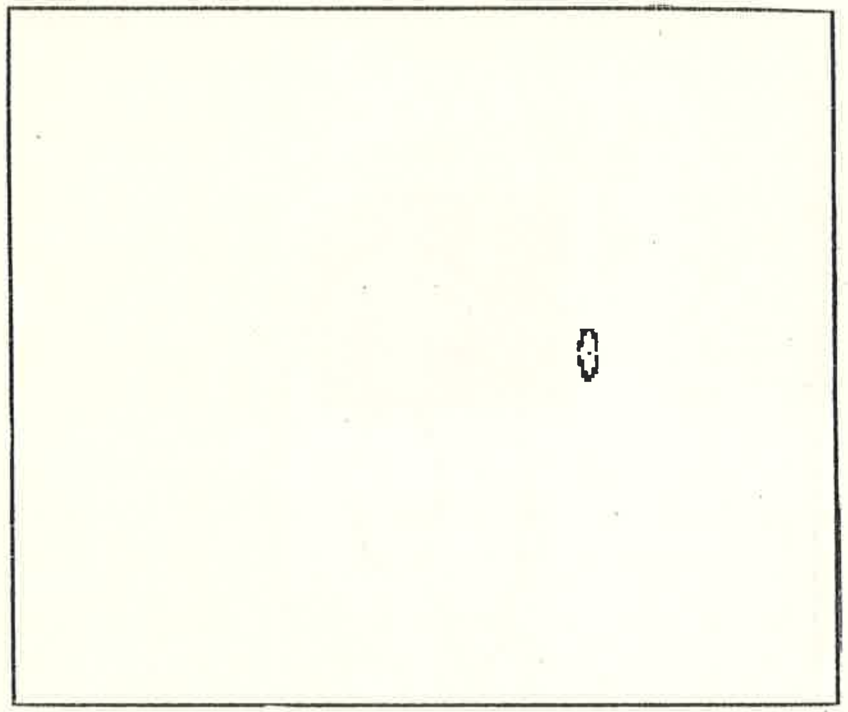
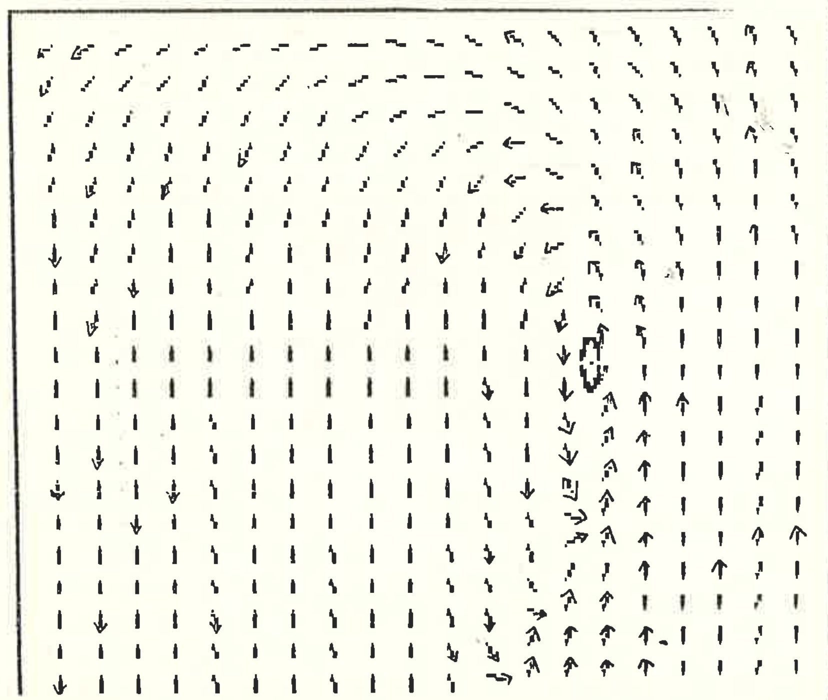


Fig. II.11.



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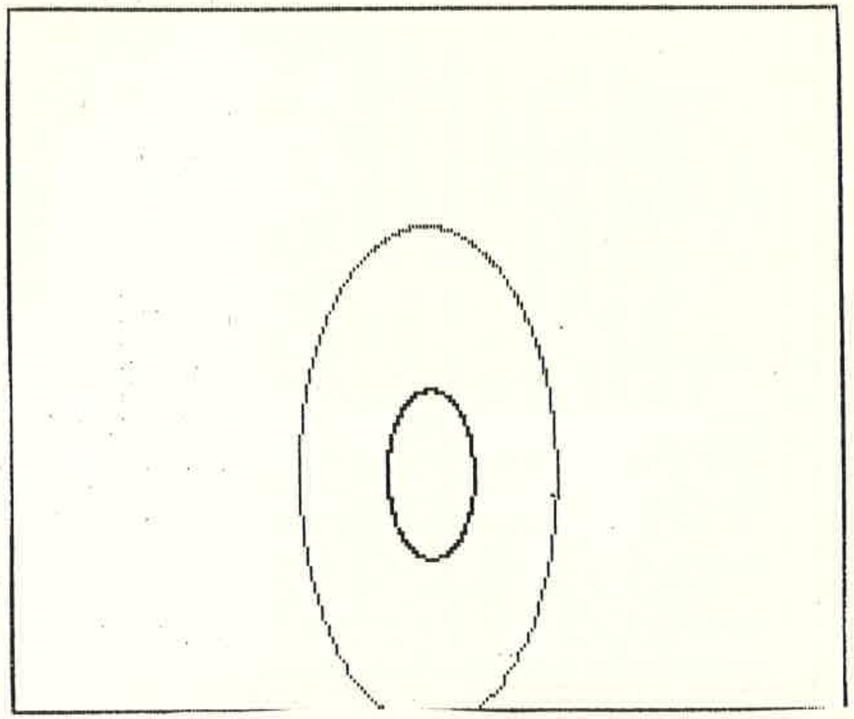

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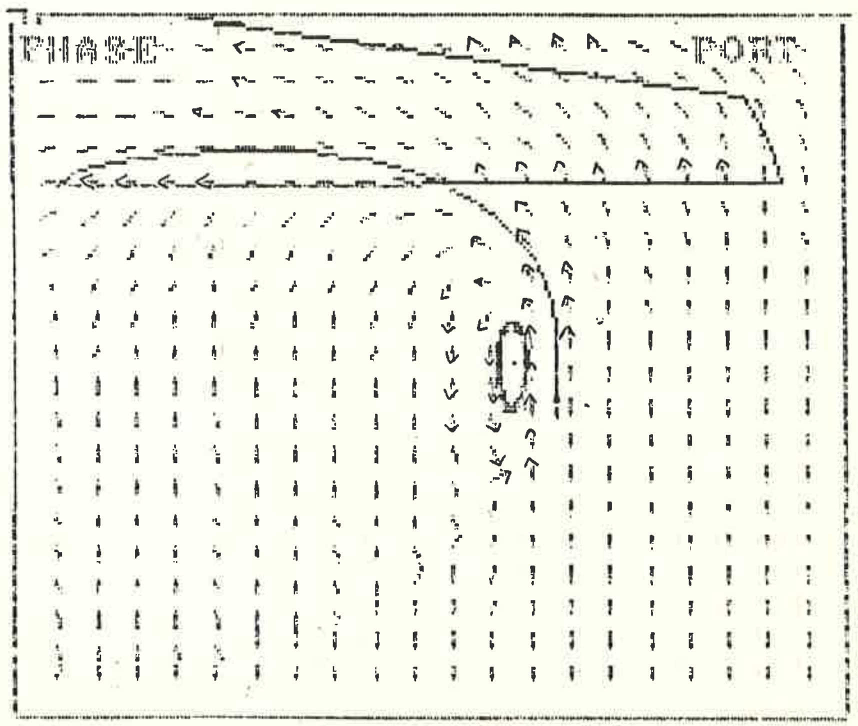

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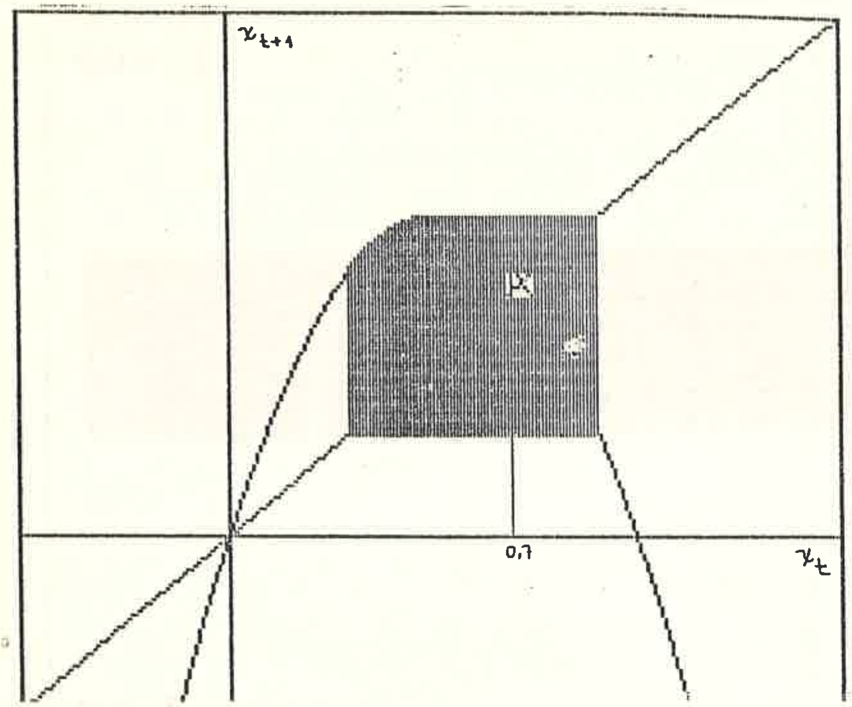
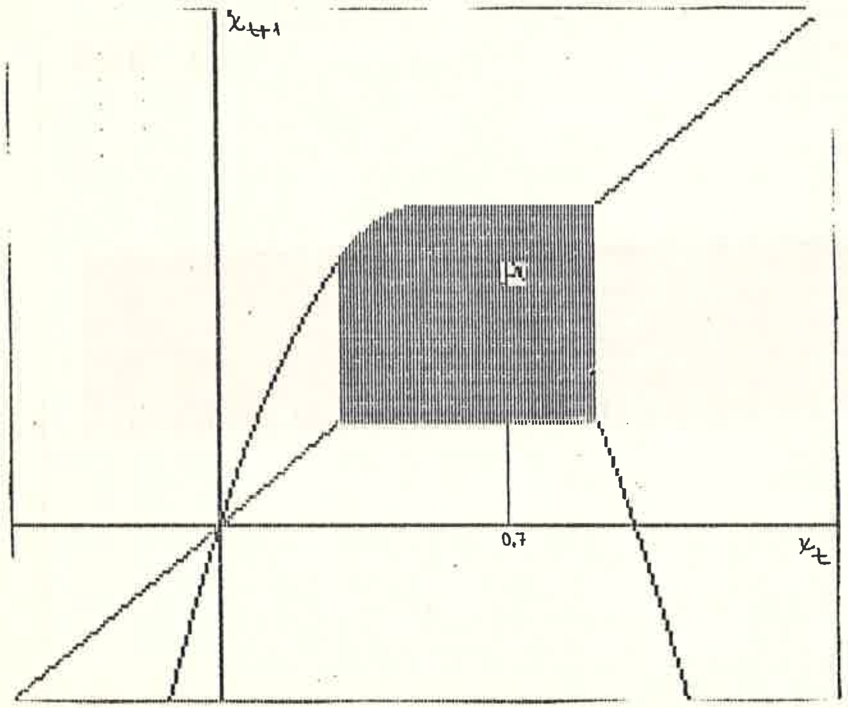
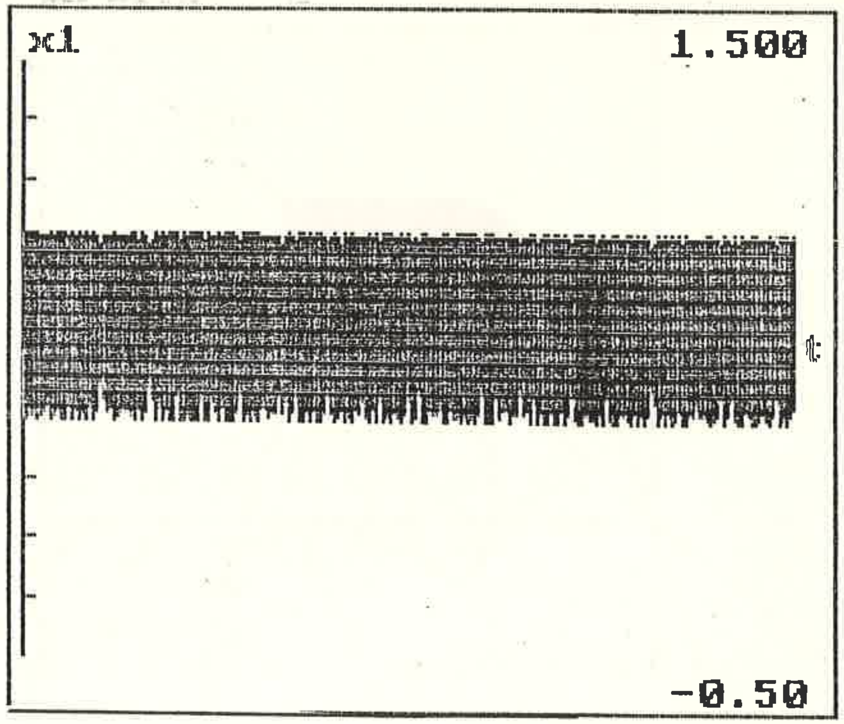


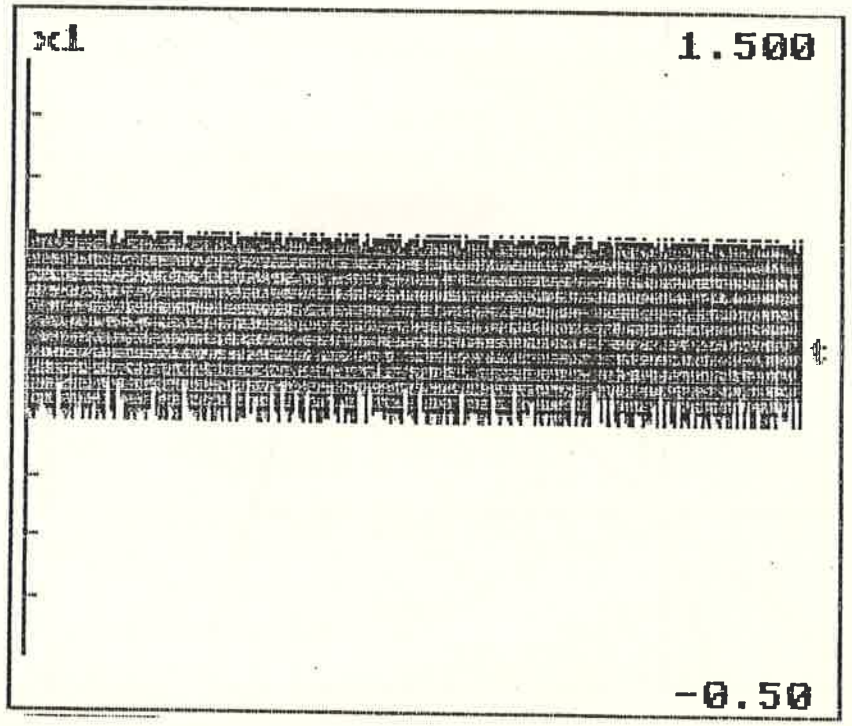
b = 0 Fig. A.1.



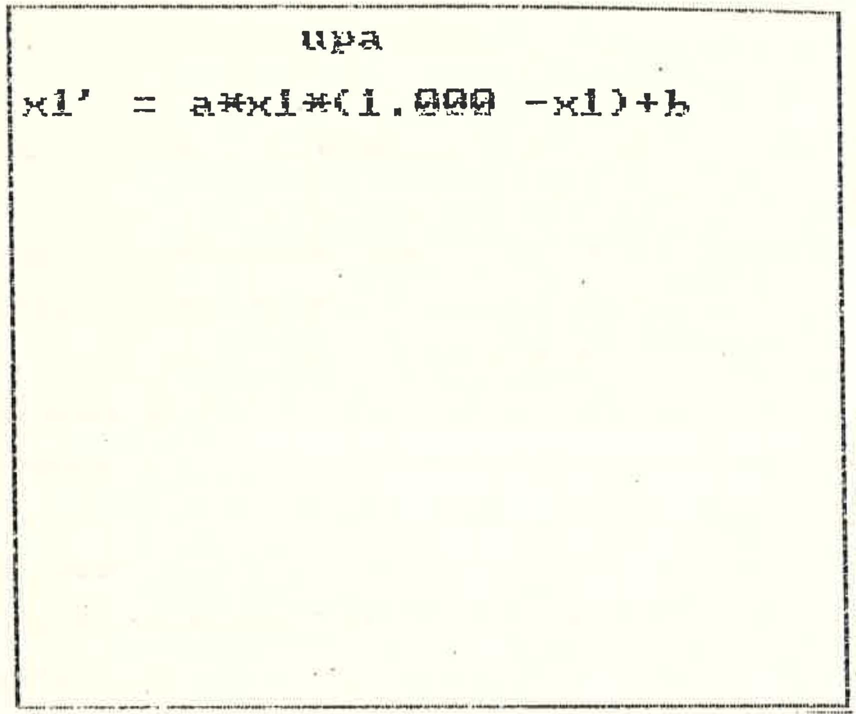
b = 0.001 Fig. A.2.



b = 0 Fig. A.3.



b = 0.001 Fig. A.4.



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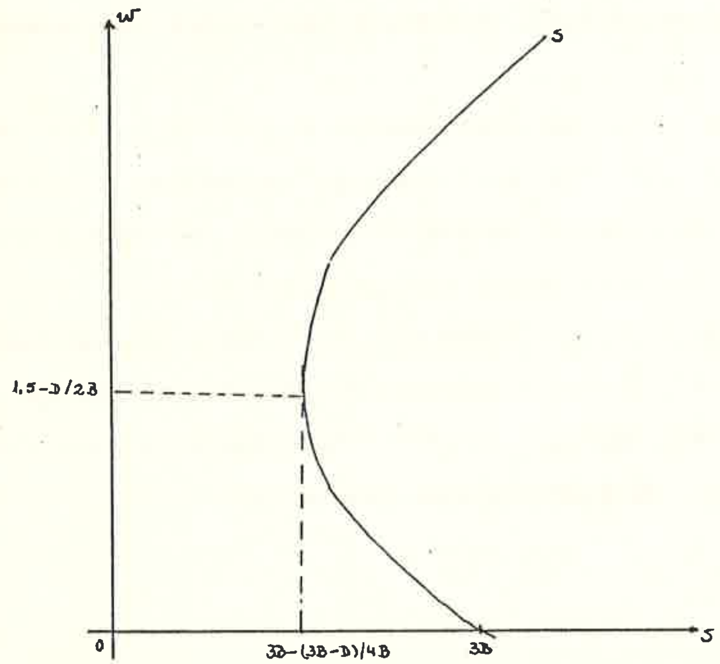

Fig. A.6.

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