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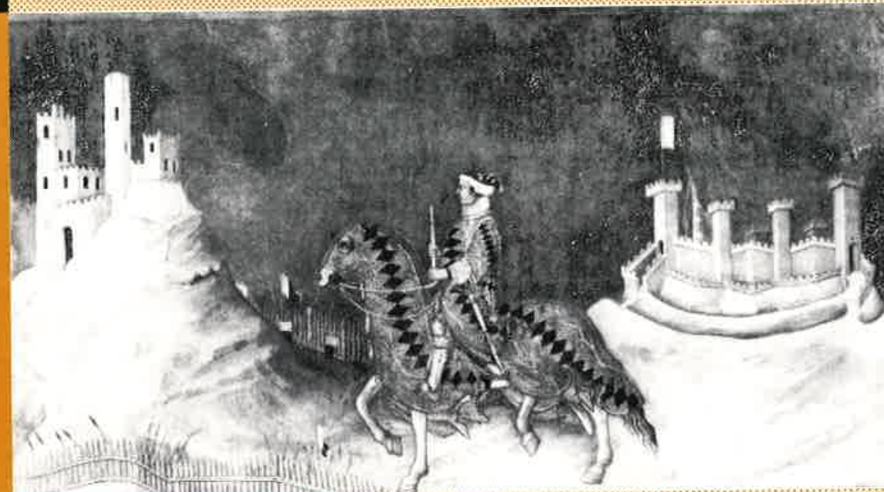
UNIVERSITA' DEGLI STUDI DI SIENA

QUADERNI DEL DIPARTIMENTO
DI ECONOMIA POLITICA

Stefano Vannucci

ON EXACTLY STRONG-NASH-IMPLEMENTABLE POWER ALLOCATIONS



QUADERNI DEL DIPARTIMENTO DI ECONOMIA POLITICA

• Redazione: **Dipartimento di Economia Politica**, Piazza S. Francesco, 17 -53100 Siena- tel. 0577/298620

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Stefano Vannucci

ON EXACTLY STRONG-NASH-IMPLEMENTABLE POWER ALLOCATIONS

Siena, febbraio 1988

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1. *Introduction**

In this paper a plausible notion of "implementable power allocation" among n agents is proposed. This notion appears to be a rather natural one from a broad power-indices perspective, when emphasizing a special kind of game forms, namely social choice functions (SCFs).

Roughly speaking, a non-null R^n_+-vector p is an exactly strong-Nash-implementable power allocation (ESNIPA for short) -w.r.t. a suitably fixed "power" index" F defined over R^{2^n-1}- whenever a pair $(r,v) \in R_{++} \times R_+^{2^n-1}$ exists such that:

i) v is the veto function of a (neutral) SCF which turns out to be an exactly stable SCF, w.r.t. strong Nash equilibrium (SNE) - a requirement which amounts to robustness of outcomes against strategic manipulation by completely informed and cooperatively playing agents, for any prevailing linear preference profile;

ii) $\bar{F}(v) = rp$, where $\bar{F}(v)$ is the multi-projection of F(v) on the coordinates corresponding to singletons in $\mathcal{P}(N)$.

Thus, an ESNIPA w.r.t. F is a F-valued power allocation which a "social planner" can implement by carefully designing a (SNE) stable decision scheme.

We show that, whenever the "power index" F is a homogeneous projective operator, every semipositive vector belonging to a lattice half-line in R^n is an ESNIPA.

This proposition may be seen as a consistency result between game theoretic solution concepts.

Hence, it may be noticed the sharp contrast with recent results showing the inherent inconsistency of 'value-like' functions with any typical cooperative stability concept- e.g., the core- if the latter is directly applied to the power allocation itself, viewed as a payoff vector.

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* A preliminary version of this paper was presented by the author at the Fourth Italian Meeting on Game Theory and its Applications (Bergamo, June 15-17, 1987).

The present paper expands on this point, as well as on some possible applications of the 'positive' result mentioned above.

2. *Preliminaries*

Let $N = \{1,\dots,n\}$ be a finite set, the set of 'agents'. This set will remain fixed throughout this paper. We denote by A another-not fixed-finite set, the set of relevant 'outcomes'. We assume $n = |N| \geqslant 2$, $m = |A| \geqslant 2$.

Let L(A) be the set of all linear orderings defined over A (in the sequel, the relevant set A will be unambiguously identifiable from the context. Hence, we shall write simply L for L(A)).

For any coalition $S \in P(N)\setminus\{\emptyset\}$, a *linear S-profile* is a function $R^S: S \longrightarrow L$, where L is defined over the relevant A. We shall denote by L^S the set of all such (linear) S-profiles. An A-valued *social choice function* (SCF) is a function $f: L^N \longrightarrow A$. An(A-valued) SCF f is *neutral* iff for all permutations π of A and for all profiles $R^N \in L^N$, $f(\pi R^N) = \pi f(R^N)$, where $\pi R^N = (\pi R_1,\dots,\pi R_n)$, and πR_i, $i=1,\dots,n$, is defined in the 'obvious' way. We shall be exclusively concerned with neutral SCFs.

A (linear) profile $T^N = (T_1,\dots,T_n)$ is a strong Nash equilibrium (SNE) of the SCF f at R^N iff for no $S \in P(N)\setminus\{\emptyset\}$ and $Q^S \in L^S$ s.t. $f(Q^S, T^{N\setminus S}) \neq f(T^N)$, $(f(Q^S,T^{N\setminus S}), f(T^N)) \in \bigcap_{i\in S} R_i$ holds true.

For any $R^N \in L^N$, we denote by $SNE(f,R^N)$ the-possibly empty- set of all SNEs of the SCF f at R^N. A SCF f is *SNE-stable* iff for all $R^N \in L^N$, $SNE(f,R^N) \neq \emptyset$. A SCF f is *exactly SNE-stable* (or ESNE-stable, for short) iff it is SNE-stable and, moreover, for all $R^N \in L^N$, a $Q^N \in L^N$ exists s.t. $Q^N \in SNE(f,R^N)$ and $f(Q^N) = f(R^N)$ (the concept of an ESNE-stable SCF is due to Peleg: see Peleg's seminal paper(12), and Peleg's monograph(13) for subsequent related results).

An *effectivity relation* (ER) between N and A is a binary relation $E \subseteq P(N) \times P(A)$ s.t. for all $S \in P(N)$, $B \in P(A)$:

i) if $(S,B) \in E$ then $\emptyset \notin \{S,B\}$;

ii) if $S \in P(N)\setminus\{\emptyset\}$ then $(S,A) \in E$;

iii) if $B \in P(A)\setminus\{\emptyset\}$ then $(N,B) \in E$.

The *star-effectivity relation* of an A-valued SCF is a binary relation $E^*(f) \subseteq P(N) \times P(A)$ defined as follows:

$(S,B) \in E^*(f)$ iff $S \neq \emptyset$, $B \neq \emptyset$ and moreover-for any $R^N \in L^N$- $(x,y) \in \bigcap_{i\in S} R_i$ for all $x \in B$ and $y \in A\setminus B$ entails $f(R^N) \in B$.

It can be easily verified that the star-effectivity relation of any unanimity-respecting SCF is an ER.

The *(star)-veto function* $V^*(f)$ of a neutral A-valued SCF f is defined as follows: for all $S \in P(N)\setminus\{\emptyset\}$,

$$(V^*(f))(S) = \sup\{|B| \,/\, B \subseteq A \text{ and } (S, A\setminus B) \in E^*(f)\}.$$

For any neutral SCF f, $E^*(f)$ and $V^*(f)$ convey the same information about f. Hence, they are essentially equivalent for all our analytic purposes. $V^*(f)$ is, of course, a $\{0,1,\dots,m-1\}$-valued function. We can immediately define its normalized version, $V_1^*(f)$, as follows:

for all $S \in P(N)\setminus\{\emptyset\}$, $(V_1^*(f))(S) = (V^*(f))(S)/(m-1)$.

A n-person *(monotonic) simple game (SG)* is a (monotonic) function $v: \langle P(N)\setminus\{\emptyset\}, \subseteq\rangle \rightarrow \langle\{0,1\}, \leqslant\rangle$. A SG v is *proper* iff for all $S \in P(N)\setminus\{\emptyset\}$, $v(S) = 1$ entails $v(N\setminus S) = 0$. A SG v is *strong* iff for all $S \in P(N)\setminus\{\emptyset\}$, $v(S) = 0$ entails $v(N\setminus S) = 1$.

We are ready now to define the *(star)-simple game* of a neutral A-valued SCF f, $w^*(f)$, by the following rule:

for all $S \in P(N)\setminus\{\emptyset\}$,

$$(w^*(f))(S) = \begin{cases} 1 \text{ if for all } R^N \in L^N,\ x \in A \text{ and } y \in A\setminus\{x\}: \\ \qquad (x,y) \in \bigcap_{i\in S} R_i \text{ entails } f(R^N) = x \\ 0 \text{ otherwise} \end{cases}$$

We emphasize the following

Fact: Let v be a proper and strong SG s.t. $v = w^*(f)$ for some neutral SCF f (for instance, the simple majority SCF, when n is an odd number). Then, $v = V_1^*(f)$

-that is, v may be safely identified with the normalized (star) veto function of f.

Finally, a n-person *game in characteristic function form with transferable utility* is a function

$$v: P(N) \rightarrow R \quad \text{s.t.} \quad v(\emptyset)=0.$$

It is said to be an *additive* game whenever for all $S \in P(N)\setminus\{\emptyset\}$, $v(S) = \sum_{i \in s} v(\{i\})$.

We can think of R^{2^n-1} as the t.u. game space. Simple games and veto functions are, of course, contained therein.

The index set of the coordinates of our t.u. game space is obviously bijective to the set of non-null coalitions in $P(N)$. We are assuming that the relevant bijection $g: P(N)\setminus\{\emptyset\} \rightarrow \{1,\ldots,2^n-1\}$ induces a monotonic indexation of non-null coalitions (i.e., for any $i,j \in \{1,\ldots,2^n-1\}$, $i \leqslant j$ implies $g^{-1}(i) \leqslant g^{-1}(j)$).

The following additional notation will be used. We denote by $\bar{I}$ the set of all coordinates corresponding to a singleton in $P(N)$. Moreover, for any given operator $F: R^{2^n-1} \rightarrow R^{2^n-1}$, $\bar{F}$ is the multiprojection of F on the set of singleton-coordinates (i.e. $\bar{F}=(F_i)_{i \in \bar{I}}$).

3. *Exactly strong-Nash-implementable power allocations*

Let us consider the following version of the principal-multiagent problem. The principal - a (team of) social planner(s) - has a method $M(n)$ to compute the "fair" allocations of power among n agents for any environment in a given class X - where X is a space of relevant characteristics: say, polls, electoral outcomes, or share distributions in a firm. The task of the principal is the implementation of these "fair" power allocations by carefully designing collective decision schemes- namely, neutral social choice functions whose domain is the set of all n-tuples of linear orderings defined over the relevant set of alternatives.

Moreover, it is assumed that the principal:

i) is quite confident that strong Nash equilibrium is the most appropriate solution concept in order to predict the behaviour of agents, and ii) wishes to minimize deviations from "sincere" outcomes due to strategic manipulations.

Therefore, the principal will restrict his/her choice to the class E(n) of (n-person, neutral) ESNE-stable SCFs.

Clearly enough, any SCF in this class induces a power allocation among coalitions. Such a power allocation may be summarized by a simple game or, more generally, by a (normalized) veto function.

Hence, it makes sense the following assumption:

iii) the principal evaluates the amount of (relative) power allocated to each agent by a given homogeneous (see the definition below) power index $F: R^{2^n-1} \rightarrow R^{2^n-1}$.

Thus, the principal faces the following problem: find the set of all implementable fair power allocations, that is -assuming for the sake of simplicity that M(n) is a surjective function- the subset A of R^n s.t. for some choice of $h: \tilde{X} \rightarrow E(n)$ the diagram represented below commutes:

$$\begin{tikzcd} \tilde{X} \arrow[r, "\tilde{M}(n)"] \arrow[d, "h"'] & R^n \\ E(n) \arrow[r, "V_1^*"'] & R^{2^n-1} \arrow[u, "\bar{F}"'] \end{tikzcd}$$

(Where $\tilde{X} = M^{-1}(n)$ (A), h is a function representing a 'method' to select neutral ESNE-stable SCFs, V_1^* is the normalized veto function implicitly defined in sect. 2, $\tilde{M}(n)$ is the restriction to $\tilde{X}$ of the "fair allocation" method defined above in this sect., $\bar{F}$ is the multiprojection of F, as defined in sect. 2).

In other words, a power allocation is "implementable" whenever it ultimately coincides with the power allocation induced by a neutral ESNE-stable SCF, as evaluated through the value-like function F. Obviously, the larger the set of "implementable" power allocations, the wider the scope for effective institution design on the part of the principal.

The problem formulated above motivates the following.

Definition. Let $F: R^{2^n-1} \to R^{2^n-1}$ be a homogeneous operator - i.e. a function s.t. for any $v \in R^{2^n-1}$ and $c \in R$, $F(cv)=cF(v)$. Then, a semi-positive R^n-vector $p=(p_1,\dots,p_n)$ is an *exactly strong-Nash-implementable power allocation* (ESNIPA) w.r.t. F whenever a pair $(r,v) \in R_{++} x R_+^{2^n-1}$ exists s.t.

i) $v=V_1*(f)$ for some neutral ESNE-stable SCF f;

ii) $\bar{F}(v)=rp$, where $\bar{F}(v)=(F_i(v))_{i \in \bar{I}}$, and $\bar{I}$ is the set of "singleton-coordinates" defined above.

4. Main result

We need the following additional definitions.

An operator $F:R^{2^n-1} \to R^{2^n-1}$ is said to be *projective* iff for all additive $v \in R^{2^n-1}$, $F(v)=v$.

A *lattice half-line in* R^n is a half-line having a non-null lattice point-i.e. a non-null vector with integer components- as a generator.

We are, finally, ready to state and demonstrate the main results of this paper:

Theorem. Let $F:R^{2^n-1} \to R^{2^n-1}$ be a homogeneous projective operator. Then, any semi-positive point $p=(p_1,\dots,p_n)$ belonging to a lattice half-line in R^n is an ESNIPA w.r.t. F.

Proof. By hypothesis $p=cz$, for some $c \in R_{++}$ and $z \in Z_+^n \setminus \{0\}$. Let us now associate to any $a \in R^n$ an additive t.u. game in c.f.f., $\tilde{a} \in R^{2^n-1}$, by the following rule:

for any coalition $g^{-1}(i)$, $\tilde{a}_{g^{-1}(i)} = \sum_{\substack{h \in \bar{I} \\ g^{-1}(h) \subseteq g^{-1}(i)}} a_{g^{-1}(h)}$, $i = 1,\dots,2^n-1$.

(It should be recalled here that g is the given bijection between coordinates of R^{2^n-1} and non-null coalitions).

Hence, $F(\tilde{z})$ is well defined and, by projectivity, $F(\tilde{z})=\tilde{z}$. Moreover, if $\tilde{z}=V*(f)$ for some A-valued SCF f, and $|A|=m$, then -by homogeneity of F- $\bar{F}(V_1*(f))=(m-1)^{-1}\bar{F}(\tilde{z})=(m-1)^{-1}z=(m-1)^{-1}c^{-1}p$.

Thus, it remains to be shown that $\tilde{z}=V*(f)$ for some ESNE-stable SCF f. We shall just give a sketch of the proof.

First, we choose a set A s.t. $m=|A|=(\sum_{i=1}^{n} z_i)+1$.

Next, we define the following ER $E_z \subseteq P(N) \times P(A)$: for all $S \in P(N)$, and $B \in P(A)$, $(S,B) \in E_z$ iff $\emptyset \notin \{S,B\}$ and $\sum_{i \in s} z_i \geq m-|B|$. Therefore, for any $S \in P(N) \setminus \{\emptyset\}$ $\sup\{|B| / (S, A \setminus B) \in E_z\} = \sup\{|B| / \sum_{i \in s} z_i \geq |B|\} = \sum_{i \in s} z_i$, so that, in fact, $\tilde{z}=V*(f)$ for any SCF f s.t. $E*(f)=E_z$. On the other hand, it is easily seen that $E_z=E*(f)$ where $f:L^N \longrightarrow A$ is the voting-by-veto SCF with (integer) 'weights' $z=(z_1,\dots z_n)$, and $(1_1,\dots,1_m)$ (see Peleg(12,13), Moulin(7); or Moulin,Peleg(8)).

Now, f is ESNE-stable, by the main theorem in Peleg(12). Hence, $\tilde{z}=V*(f)$, for some ESNE-stable SCF f. Q.E.D.

Definition - A semivalue is an operator $F:R^{2^n-1} \to R^{2^n-1}$ satisfying the following properties: i) *projectivity* (see the definition above); ii) *(range) additivity:* for all $v \in R^{2^n-1}$, F(v) is an additive game; iii) *linearity:* for all $r_1, r_2 \in R$ and $v_1, v_2 \in R^{2^n-1}$:

$$F(r_1v_1+r_2v_2) = r_1F(v_1) + r_2F(v_2);$$

iv) *symmetry:* for any permutation σ of N, $F(\sigma\ v) = \sigma(F(v))$; v) *monotonicity:* for all $v \in R^{2^n-1}$, if v is monotonic-i.e. under our assumptions $i \leq j$ entails $v_i \leq v_j$, $i,j=1,\dots,2^n-1$, whenever $g^{-1}(i) \subseteq g^{-1}(j)$ - then F(v) is monotonic.

The concept of a semivalue is due to Dubey, Neyman, Weber (2).

A semivalue may be thought of as a (mildly) generalized power index. In fact, the Shapley value and the Banzhaf value are outstanding examples of a semivalue.

On the other hand, the set of lattice points in R^n becomes of special interest when related to proportional representation applications and, more generally, to any fair apportionment problem.

In fact, a proportional representation problem amounts to the issue of dividing a certain integer number of indivisible objects-e.g. seats - among a given set of 'parties' or 'populations' according to their 'weights' (see Balinski, Young(1), or Lucas(5) for a full fledged discussion- and a general resolution- of apportionment problems).

Thus, the main features of a *proportional representation scheme* (PRS) are the following: i) a PRS allocates an integer number-or a set of admissible integer numbers - to any relevant 'party', or 'agent'; ii) these integer numbers should reflect as faithfully as possible an exogenously given set of -not necessarily integer-'weights'; iii) the 'weights'- and the resulting PRS-allocations- are meant to represent the 'fair' allocations of power in the given situation. Therefore, even from a prescriptive point of view, any pro-PRSs argument cannot be really convincing, without the explicit proof that the allocations of power which are aimed at are in fact implementable.

The following corollary is a straightforward consequence of the result presented above, and supplies a somewhat positive answer.

Corollary 1. Let p be a semi-positive lattice point in R^n. Then p is an ESNIPA -by $(1,\tilde{p})$ - w.r.t. any semivalue.

Finally, one may consider another problem. Let us suppose that the principal is concerned with the task of preventing distortions of the "fair" power allocations due to lack of perfect communication among agents. In order to deal with this problem, we adopt here the following model (see Myerson(9) and Owen(11)). Let us introduce a class of graphs called *communication graphs* (c.g.s). A c.g. G(N) is defined as follows. The nodes of G(N) are the elements of N, i.e. the agents.

An arc $\{i,j\} \subseteq N$ of G(N) denotes "feasibility of direct communication-and cooperation- between the agents i and j".

Any communication graph G(N) induces a canonical linear operator $G':R^{2^n-1} \to R^{2^n-1}$ defined by the following rule:
for all $v \in R^{2^n-1}$, and $S \in P(N)\setminus\{\emptyset\}$

$$(G'(v))(S) = \begin{cases} v(S) \text{ if } S \text{ is connected in } G(N) \\ \sum_{i \in s} v(\{i\}) \quad \text{otherwise} \end{cases}$$

Definition. Let CG(N) denote the class of all communication graphs defined over N. Then, an ESNIPA - by (c,v) - $p \in R^n$ (see the definition above) is *robust* iff for any $G(N) \in CG(N)$ a $r \in R_+$ exists s.t. $\bar{F} \circ G'(v) = rp$. Moreover, p is *uniformly robust* iff a $r \in R_+$ exists s.t. for any $G(N) \in CG(N)$, $\bar{F} \circ G'(v) = rp$.

It can be easily proved the following

Lemma - Let v belong to R^{2^n-1}. Then, v is a fixed point of G' - for any $G(N) \in CG(N)$- iff v is additive.

Proof - Sufficiency: it follows immediately from definitions.

Necessity: let us consider-for any $S \in P(N)\setminus\{\emptyset\}$-a G(N) s.t. S is disconnected in G(N).

Then, $v(S)=(G'(N))(S)= \sum_{i \in s} v(\{i\})$.

It can be easily verified that this lemma and the additivity property enjoyed by the veto function $\tilde{z}$ used in the proof of the main result entail the following

Corollary 2 - Let $F:R^{2^n-1} \to R^{2^n-1}$ be a homogeneous projective operator. Then, any semi-positive point $p = (p_1,\dots,p_n)$ belonging to a lattice half-line in R^n is an uniformly robust ESNIPA w.r.t. F.

5. Discussion

It should be stressed the sharp contrast existing between the 'possibility' result presented above and two major (classes of) strictly related 'impossibility' results.

Firstly, there are 'impossibility' results arising from social choice problems. It is well known that in this area limitative theorems are ubiquitous, while 'positive' results are rare and relatively difficult to obtain. At a closer scrutiny, our 'positive' result is not a somewhat mysterious exception. From a technical point of view, our finding is a straightforward consequence of some major 'possibility' results of social choice theory. Moreover, its relatively non-restrictive nature arises from the supplementary 'degree of freedom' allowed by our problem. This supplementary 'degree of freedom' is, obviously, the endogenous determination of the outcome set's cardinality. This amounts, in practice, to an implicit restriction on the decision scheme.

Under the most natural interpretation of that restriction, one may assume that the agents are facing an outcome set consisting of their own proposals- a proposal for each weight-unit - plus a 'conflict outcome', or 'status quo outcome'.

This confirms the crucial role of the 'conflict outcome''s determination process. Incidentally, 'random dictator' methods could be appealing here, from a prescriptive point of view.

Let us turn now to a second class of related 'impossibility' results. In recent years much effort has been devoted to the study of the following problem: find a Shapley value-like function $F{:}D \rightrightarrows R^{2^n-1}$ -where D is a suitably large class of games: say the class of proper simple games, or R^{2^n-1}, or even some class of n.t.u. games- s.t.
for any $v \in D$, if Core $(v) \neq \emptyset$ then $\bar{F}(v) \in$ Core(v) - or $\bar{F}(v) \subseteq$ Core (v) - (of course, $\bar{F}$ denotes the multiprojection of F on $\bar{I}$: see notation above, sect. 3).

This is perhaps the simplest kind of consistency problem between solution concepts one may conceive of.

Generally speaking, the answer has been in the negative. In fact, it is well-known that, for finite economies, the n.t.u. -Shapley value allocations- a generalization of the Shapley value-allocations devised in order to manage the general case of non transferable utility economies- need not be in the core (see Yannelis(14) for a recent extensive analysis).

Moreover, Nurmi(10) shows, by some interesting examples, that whenever the relevant power index is the Shapley value or the Banzhaf value the power index -valued payoff vector of a game need to belong to the (non-empty) core of the game itself- nor to its own bargaining set.

In a more formal vein, Kleinberg and Weiss(4) introduce the concept of a weak value, that is a range additive, linear and symmetric operator defined over R^{2^n-1} (hence, a generalization of the concept of a semivalue).

They, then, show that for $n \geqslant 3$ no efficient weak value is consistent with the core, in the sense mentioned above (it should be recalled here that the Shapley value *is* efficient). Kleinberg and Weiss comment upon their own result as follows: "From this we see the inherent irreconciliability (at least for finite-player games) of 'coalition power', as embodied in the core, and the notion of fair division that is implicit in the structure of symmetric values" (Kleinberg, Weiss(4)).

The results presented in this paper show that this alleged 'inconsistency' vanishes when the relevant concept of cooperative stability refers to the game form underlying the given 'game' in R^{2^n-1}, rather than to the 'game' itself. In fact, it should be clear that the problem analyzed in this paper may be seen as a kind of consistency problem between any value-like solution concept and strong Nash equilibrium, which is a refinement of the core solution concept.

Our own problem may be reformulated as follows:
find a value-like function F s.t. a suitably "rich" range of F-values turns out to be a set of power allocations -as measured by F- which may be induced by some ESNE-stable neutral SCF.

In this framework the stability concept acts as a filtering device on the domain of the value-function F. Hence, 'consistency' means that the set of F-values which survive this filtering process is fairly large.

This makes sense if and only if the 'games' in R^{2^n-1} under consideration are not games in the ordinary intuitive sense -i.e. preference-depending objects.

They are, rather, game-form like entities - i.e. non preference - depending objects. The latter interpretation seems to be the most efficient one, in order to distinguish between "a priori power" and "satisfaction", and to analyze the former concept (see Miller(6) for some penetrating observations on this point).

The main result of this paper entails that any value-like solution concept is consistent with strong Nash equilibrium - according to the kind of consistency problem outlined above - provided that the 'relevant' F-values are integer values, as in standard apportionment problems.

Moreover, if we are prepared to make use of mixture effectivity functions, then it can be easily shown that ESNIPAs span the entire allocation space R^n_+.

Let us conclude this paper with the formulation of two open related problems:

1) Find the set of all ESNIPAs, within the deterministic framework used in this paper (we suspect that an appropriate extension and adaptation of the results recently obtained by Holzman - see Holzman(3)- might be instrumental to that end).

2) Find the set of all ESNIPAs having a non-additive 'game' as their own inverse image.

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