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The Passage of Time, Capital, and Investment in Traditional and in Recent Neoclassical Value Theory

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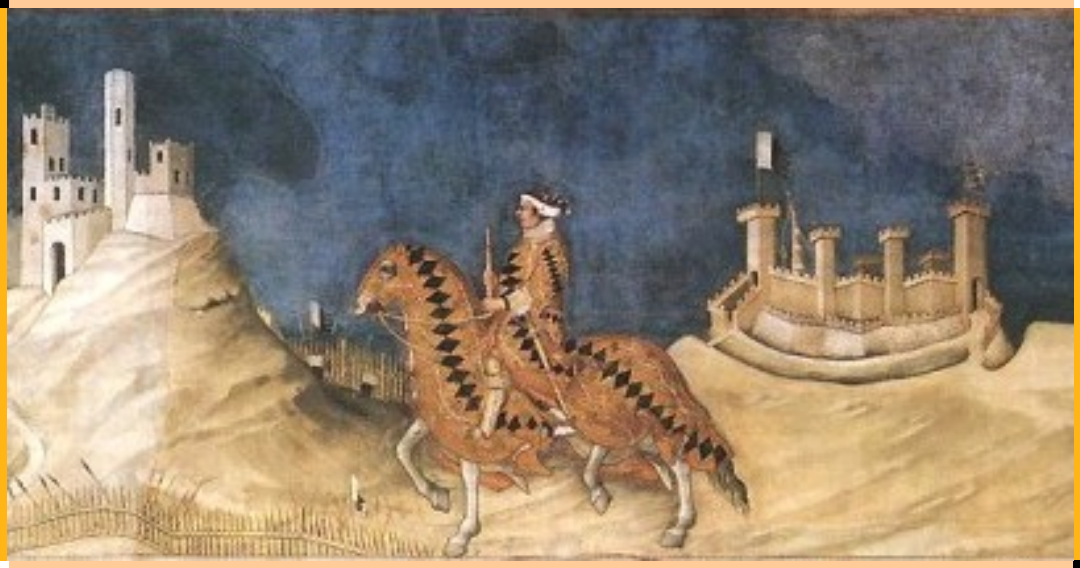

Fabio Petri[1]

THE PASSAGE OF TIME, CAPITAL, AND INVESTMENT IN TRADITIONAL AND IN RECENT NEOCLASSICAL VALUE THEORY

Abstract

With the shift from traditional analyses where capital is a single value factor of variable 'form' to the neo-Walrasian versions, general equilibrium theory has encountered new problems pointed out by P. Garegnani (1976, 1990): impermanence problem, price-change problem, substitutability problem radically question the right to consider neo-Walrasian equilibria as approximating the actual path of real economies. The paper briefly summarizes these problems and then concentrates on a fourth problem, the savings-investment problem, arguing that neo-Walrasian general equilibrium models assume that investment is adjusted to full-employment savings but cannot justify this assumption. The treatment of investment in intertemporal general equilibrium is subjected to a new criticism: it is shown that the tâtonnement assumes Says' Law all along the adjustments, and determines investment in a way that would crumble if it were not assumed that consumers determine their demands for consumption goods on the basis of an assumption of full employment incomes, which is not justified outside equilibrium, and was not assumed in traditional analyses. This reinforces the absence of reasons to view neo-Walrasian equilibrium paths as sufficiently approaching actual paths. It is concluded that behind the reference to intertemporal equilibrium as the microfoundation of macro analyses there is a continuing faith in traditional neoclassical time-consuming adjustment mechanisms, based on the old and untenable conception of capital that the shift to neo-Walrasian equilibria intended to do without.

Keywords: investment, intertemporal general equilibrium, capital, Garegnani

JEL classification: D50, E22, B21

§1. *Purpose of the paper, and a brief historical introduction.*

The way the marginalist/neoclassical theory of value proposes to deal with the passage of time has been drastically altered by a change undergone by the treatment of

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capital in this theory, a change that started in the early 1930s and had become practically universal among neoclassical value theorists by the end of the 1960s. The change consisted of a generalized adoption of Walras's specification of the endowment of capital as a given *vector* in the formulation of the general equilibrium equations, a specification that had remained decidedly minoritarian until then: the great majority of marginalist economists had treated the capital endowment as a scalar, a quantity of a single factor of variable 'form'. In a previous paper (Petri, 2014) I have argued that this change is responsible for the charge of sterility moved against modern general equilibrium theory by the late Professor Mark Blaug. In this diagnosis I relied on the line of criticism of modern general equilibrium theory started no less than forty years ago by Pierangelo Garegnani (1976), then taken up and developed by several other contributions[2], and yet so far never discussed by neoclassical theorists. Even as widely read a scholar as Blaug appears to have been unfamiliar with it. So one more stimulus to discuss this line of criticism does not seem useless, especially if accompanied – as in the present case – by a new criticism of the treatment of investment in intertemporal equilibria, that further strengthens the argument.

Why did the shift to a vectorial specification of the given capital endowment drastically alter the role of the passage of time in the neoclassical theory of value and distribution? Because it was irreconcilable with the traditional concentration of the theory of value, classical as well as marginalist/neoclassical, on the determination of normal, long-period, product prices describing the centres of gravitation of *time-consuming* disequilibrium adjustments. Adam Smith's distinction between *market* price and *natural* price reappears in all traditional marginalist authors as the distinction between moment-by-moment price and normal, equilibrium (long-period) price, the latter being the centre of gravitation of the former, and therefore indicating its average. The aim to determine such normal prices requires that the data determining them are sufficiently persistent and sufficiently unaffected by disequilibrium actions. In the marginal/neoclassical approach, these data include the factor endowments, because normal product prices are the ones associated with the equilibrium income distribution, whose determination requires given factor endowments. This explains why, with the sole exception of Walras, all founders of the marginal or neoclassical approach, when attempting to determine an economy-wide (and not a partial) equilibrium, treated the amounts of the several capital goods as variables, endogenously determined by the equilibrium itself: an equilibrium resulting from time-consuming adjustments could not include, among its data, quantities of capital goods that disequilibrium productions

[2] Two very important subsequent contributions by Garegnani on this topic are Garegnani (1990; 2012). References to contributions by Ciccone, Eatwell, Gehrke, Kurz, Milgate, Mongiovi, Panico, Schefold are supplied in Petri (2004); some more recent contributions will be mentioned later.

would inevitably alter[3]. The reconciliation with the conception of distribution as resulting from an equilibrium between demand for, and given endowments of, the several factors of production was achieved by conceiving the several capital goods as transient embodiments of a single factor 'capital', a factor measured as an amount of value and capable of changing 'form' without changing in 'quantity'. The resulting general equilibrium, although fully disaggregated (as clearly shown e.g. by Wicksell's *Lectures* (1934)), was as persistent as the equilibrium of an economy where the only factors were labour and land, because the total quantity of 'capital' so conceived was as persistent as the endowment of labour.

Indeed the endowment of labour – a traditional marginalist economist could have argued – is of course not strictly constant, but it changes with a speed which is of a lower order of magnitude than the speed with which productions adjust to demands, production methods adjust to relative factor prices, and product and factor prices adjust in response to excess demands; therefore one can legitimately treat the labour endowment as given when determining the equilibrium toward which those adjustments push prices and quantities. Now – the traditional marginalist economist would have continued – the same reasoning holds for the *total endowment* of 'capital', which is altered by net savings with a speed of an order of magnitude similar to that of the speed of change of population, a much lower speed than the potential speed of change of the *composition* of 'capital'.

This general equilibrium determined the composition of 'capital' endogenously, as the one associated with a uniform rate of return on the supply prices of the several capital goods. Equilibrium product prices were therefore, from this point of view, totally analogous to the classical natural prices of Smith or Ricardo, or to the prices of production of Marx, although of course the income distribution behind them was determined on the basis of a very different theory. Adopting Marshallian terminology, we can call this equilibrium a *long-period general equilibrium*. It aimed at describing the situation around and toward which actual day-by-day prices and quantities gravitate, owing to the tendency of investment to go where the rate of return is higher, and in this way to bring about a uniform rate of return on the supply price of capital goods.

Let me insist here on a point which is still often misunderstood. The presence in these traditional marginalist economists of a 'quantity of capital' in no way meant that they were assuming an 'aggregate production function'. The given 'quantity of capital' was indispensable in spite of a completely disaggregated treatment of both capital, consumption, and production methods, because the endowments of the several capital

[3] In Marshall's short-period equilibrium only the most durable capital goods are given, the quantities of circulating capital goods (work-in-progress) are endogenously determined; and the entire composition of capital becomes endogenous when Marshall discusses the role of the total quantity of capital, see Dvoskin and Petri (2016, Section 2).

goods were variables that the equilibrium had to determine endogenously; then the full employment of 'original' factors left the equilibrium indeterminate until a final equation established equality between demand for, and the given endowment of, the total 'capital' the single factor, to be then 'allocated' among the several capital goods (so as to yield the same rate of return everywhere) and constraining therefore their availabilities.

This is not the place to repeat the analytical demonstration of this point (see Garegnani, 1990). We are interested here in the role of the passage of time in this conception of equilibrium. This role was, from the point of view of method, essentially the same as in the classical authors: the passage of time allowed the gravitation toward the persistent normal (or long-period) position to operate. This allowed neglecting the indeterminable effects of the myriad accidental and transitory influences on moment-by-moment prices and quantities, and to concentrate on the centres of gravitation reflecting the action of more persistent forces. These normal prices and quantities indicated the averages of moment-by-moment prices and quantities over sufficiently long periods, and their changes indicated the *trend* of these averages.

This 'method of long-period positions', as it has been called, was universally accepted[4] from Adam Smith up to the shift to a vectorial specification of the capital endowment started by Lindahl, Hayek, Hicks. What motivated the shift? In all three authors one finds a growing awareness of the inconsistencies of the notion of 'capital' as a single factor, necessarily a quantity of exchange value, therefore not measurable independently of what its quantity was supposed to contribute to determine, relative prices[5]. The three authors believed that the shift avoided the indefensible conception of capital as a single factor without entailing a need to abandon the marginalist/ neoclassical conclusions on the tendencies operating in market economies (Petri 2014, p. 472). Subsequent neoclassical theory has accepted their view. However, serious doubts on the defensibility of this view are raised by the fact, that the shift radically changed the way general equilibrium theory deals with the passage of time, rendering the new notions of equilibrium hardly reconcilable with time-consuming disequilibria.

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[4] Even, although with contradictions, by Walras, see Petri (2016). The defining element of a long-period position is that, for all capital goods in use, the supply price (equal to minimum average cost) equals the demand price (the capitalization of future rental rates). The adjusted composition of capital allows treating relative prices as sufficiently close to constant, then normal product prices are the uniform-profit-rate prices whose determination was the object of the theory of value not only of Ricardo or Wicksell or Sraffa, but also of Walras. The common acceptance of the method of long-period positions and the deep differences nonetheless existing between classical and marginal/neoclassical approach are illustrated in Garegnani (2007).

[5] The problem had been openly admitted by Wicksell (1934, p. 202); this made it difficult to go on treating capital as a single factor.

This makes it very difficult to connect the theoretically determined general equilibrium with the explanation and prediction of actual economic events.

Sections 3 to 5 summarize the three aspects of this difficulty pointed out by Garegnani; these sections do not pretend to novelty except for a few remarks (e.g. the last paragraph of section 3, the first paragraph and the comments on Radner in section 4), but will be useful to readers not yet thoroughly familiar with this line of criticism. Sections 6 to 9 expand on a fourth aspect: the arbitrariness of assuming that investment is adjusted to full-employment savings in intertemporal equilibria. This arbitrariness is highlighted via a new criticism, based on the need to admit that consumers' incomes must derive from factor supplies that find purchasers. Section 10 concludes that the current reliance on intertemporal equilibrium theory, as the foundation of the neoclassical approach to value distribution and growth, implicitly rests on a continuing belief in the old time-consuming neoclassical disequilibrium adjustments. So the traditional method has not really been abandoned; but these time-consuming adjustments rely upon the untenable conception of capital as a single factor.

§2. *Why intertemporal general equilibrium.*

There are indications (Petri, 2016) that Walras aimed at determining long-period prices: not only his equations assumed a uniform rate of return on supply price, but also he treated the prices determined by his system of equilibrium equations as so persistent that he could determine the purchase price of land by dividing land rent by the rate of interest, the capitalization formula appropriate to a rate of rent and a rate of interest constant for the indefinite future. When in the 1930s Lindahl, Hayek and Hicks proposed to do without the conception of capital as a single 'fund' and to turn to Walras's specification of the capital endowment as a given vector, differently from Walras they were clear that long-period prices require an *endogenously* determined capital composition. So they were clear that they were abandoning the attempt to determine a long-period position, and that their neo-Walrasian equilibria could entail quick changes of relative equilibrium prices over time (because the arbitrary initial composition of capital could easily undergo quick changes). So the agents' equilibrium decisions had to embody an awareness of the fact that even *equilibrium* relative prices might be far from constant. Two possible solutions were explored.

The first one was that of *intertemporal equilibria*, where future prices are determined simultaneously with relative current prices, through an assumption either of existence, already in the period when equilibrium is established, of markets for all future goods, or of perfect foresight. The second solution, the one preferred by Lindahl and Hicks, was that of *temporary equilibria* (without perfect foresight), where agents

take their decisions in the initial ('current') period on the basis of (possibly mistaken) *expectations* of future prices. But the latter approach appears to have been abandoned. The attempts, carried out in the 1970s and 1980s, to arrive at a formalized theory of temporary equilibria came to a complete halt, owing to grave difficulties with the formalization, and with the existence, of temporary equilibria under sufficiently general assumptions. Recent treatises in general equilibrium and recent textbooks in advanced microeconomics omit all mention of temporary equilibria; applied contemporary neoclassical theorists, for example macroeconomists and growth theorists, mention only intertemporal equilibrium theory as the microfoundation of their models. For this reason, below I discuss almost only intertemporal equilibria.

The notion of intertemporal equilibrium was formalized by Arrow and Debreu as a *reinterpretation* of the exchange-and-production non-capitalistic model without a rate of interest; the variables and equations of this model were simply reinterpreted as referring to dated commodities and discounted prices. The intertemporal general equilibrium model had then to cover a finite number of periods, because the non-capitalistic model has a finite number of commodities. Nowadays, frontier research is on models extending into the infinite future, but the finite-horizon model already shows the aspects that I wish to discuss. So most of what follows will refer to the finite-horizon model.

§3. *The impermanence problem.*

A first difficulty with connecting intertemporal general equilibrium theory with the explanation and prediction of actual economic events is the following. The given initial endowments of the several capital goods imply that the equilibrium cannot be conceived as a centre of gravitation of time-consuming adjustments: these adjustments would alter the quantities of the several capital goods present in the economy, and would alter them differently from how the equilibrium, if perfectly and instantaneously reached, would have them change with the passage of time: so the equilibrium (or equilibria, if there isn't uniqueness) corresponding to the initial data would no longer be there to be reached. The equilibrium lacks the persistence, and independence from disequilibrium adjustments, of the old notion of equilibrium based on endowments of labour, land, and 'capital' the single factor. It is to prevent changes in the vector of capital endowments that, in order to study how equilibrium is reached, the assumption is made of the fairy-tale auctioneer, who congeals the economy until, through a tâtonnement that mysteriously takes no time, equilibrium is achieved. Indeed no need was felt for the auctioneer before the shift to neo-Walrasian equilibria (except, not by chance, by Walras!). In neo-Walrasian equilibria if the patently unrealistic auctioneer

assumption is not made, we have the problem admitted by a highly esteemed neoclassical microeconomist:

> "In a real economy, however, trading, as well as production and consumption, goes on out of equilibrium ... in the course of convergence to equilibrium (assuming that occurs), endowments change. In turn this changes the set of equilibria. Put more succinctly, the set of equilibria is path dependent ... [This path dependence] makes the calculation of equilibria corresponding to the initial state of the system essentially irrelevant" (Franklin M. Fisher, 1983, p. 14).

Nor can one try to minimize the problem by arguing that adjustment to equilibrium, although not instantaneous, is still quite fast and therefore 'wrong' changes in capital endowments can be presumed negligible. Without the auctioneer, adjustments cannot but require considerable time. For example, after some labour immigration, production of many consumption goods must be started before knowing the demand for them when they will come out, because one cannot know in advance how much real wages will have changed by then, nor how that will affect choices. Mistakes are inevitable, and their correction will require new productions, new mistakes, etcetera.

Therefore, supposing the once-for-all immigration to be at the beginning of period 0, the corresponding intertemporal equilibrium cannot be trusted to give a good indication of the behaviour of the economy during period 0. Then at date 1 (the beginning of period 1) the capital endowments will be different from the ones predicted by the equilibrium path for that date, so the economy would not be able to behave during period 1 as the original equilibrium predicted even if at date 1 adjustments were instantaneous; the equilibrium path has been altered. In fact, at date 1 too there will be disequilibrium decisions and disequilibrium productions, so the danger arises of a further deviation from the original equilibrium path, and of a cumulation of deviations as time proceeds. Therefore the intertemporal equilibrium path corresponding to the initial capital endowments can be trusted to give a good indication *neither* of the initial behaviour of the economy, *nor* of the path over a sequence of periods; and this, for reasons logically prior to whether the auctioneer-guided tâtonnement is stable or not, and depending simply on the fact that realistic adjustments shift the equilibrium itself, and in directions that the theory is unable to indicate, because dependent on the unpredictable details of disequilibrium.

It follows that trusting intertemporal equilibria as good indicators of the path of actual economies would require a convincing theory of the *actual* path, that showed that the equilibrium path yields a good approximation to it. But with modern general equilibrium theory the traditional marginalist way to reach a theory of the actual path in spite of the impossibility to specify the accidental events of disequilibrium – by arguing

that, through error correction or compensation, the actual path gravitates around and toward *a persistent equilibrium defined independenty of the accidents of disequilibrium* – is lost: neo-Walrasian equilibria are not unaffected by disequilibrium actions, and equilibrium theory cannot indicate how they shift; this renders the tendency of the actual economy impossible to establish.

The sole neoclassical attempt so far to face the problem (that is, to study whether an economy *with heterogeneous capital goods* gravitates toward a definite state if one admits implementation of disequilibrium exchanges and productions) is the one by Franklin Fisher (1983), and it does not surmount the problem, because the outcome of the adjustments remains indeterminate (Petri, 2004, pp. 67-71). Apart from Fisher, when one finds an admission that disequilibrium decisions alter endowments, the economy under consideration is the market fair of an exchange economy, where the endowments are of consumption goods and therefore are consumed at the end of the fair. In a single such market fair, disequilibrium exchanges alter endowments and therefore the final result is to an extent indefinite. But this problem can be surmounted by assuming – as done in the experiments surveyed in Bryant, 2010, ch. 8 – a *repetition* of market fairs, *all starting with the same data*: the same agents come to each market fair with the same initial endowments and preferences, and with the memory of what went on in previous fairs. Then the experiments suggest that learning and error correction over the succession of market fairs (note the traditional role of the passage of time!) generally ensure a tendency toward an approximate general equilibrium. This result is taken as supportive of general equilibrium theory, without noticing that the absence of change of data from one market fair to the next cannot be assumed for an economy with production and heterogeneous capital goods, as admitted by Fisher. There is one paper, Gintis (2007), that considers a production economy and admits disequilibrium productions in an agent-based model where, the author claims, there is capital; but the factor Gintis calls capital is physically homogeneous, does not depreciate, and is not produced, so it is surprising that the author calls it capital, it is a land. Whenever heterogeneous capital is admitted, one finds either the tâtonnement, or, more and more frequently, *no mention at all* of the stability question, even—and it is truly a scandal—in advanced textbooks (e.g. Jehle and Reny (2011), Kreps (2013)). Thus, if one leaves the 'Sraffians' and Franklin Fisher aside, there seems to be either little awareness of the impermanence problem[6], or a strong unwillingness to face it – hardly a praiseworthy attitude.

[6] Frank Hahn in one instance admitted that "equilibria which cannot be reached from historically given initial conditions by an acceptable process of learning should, I contend, be ruled out. What that means is that the equilibrium definition should include the requirement of reachability ... economic theory should deal with equilibria which are stable under some acceptable process." (Hahn 1991, pp. 70-71). But Ariel Dvoskin, who must be thanked for

§4. *The price-change problem.*

A second difficulty of modern general equilibrium theory in connection with the passage of time is the following. Differently from long-period equilibria that determine persistent prices, now the theory must admit an awareness of decision makers, in the initial period, that equilibrium prices cannot be expected to remain unchanged or nearly unchanged as time unfolds. The *price-change problem* that thus arises is the difficulty with determining the influence of this fact on initial-period decisions. I leave aside the form the price-change problem takes in temporary equilibria[7]. For the determination of intertemporal equilibria, one must assume *either* complete futures markets existing already at the initial moment, *or* a sequential Radner equilibrium (an Equilibrium of Plans, Prices and Price Expectations), which assumes perfect price foresight. An immediate criticism then is, that to arrive at *defining* the equilibrium the theory must assume *the presence of elements with no correspondence with the real world*: neither complete futures markets, nor perfect price foresight are found in reality. Equilibrium cannot even be *defined* unless one assumes the world to be different from what it is!

And the difference is radical – it takes us to fairy-tale worlds. The absence of complete futures markets in the actual world is evident, and their impossibility is obvious: not-yet-born consumers cannot be present at the initial date to announce whether they will wish to buy apples or pears. This is widely admitted; but the alternative – perfect foresight – means to fall out of the frying-pan into the fire. Let me remember just two out of the many difficulties.

First, many things *cannot* be predicted, as a matter of logic: a perfect foresight assumption excludes all true *novelties*, in particular unexpected discoveries (e.g. pennicillin, new bacteria) and all new results of human creativity, such as new scientific theories and theorems (in economic science too), new technological inventions, new

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bringing these forgotten lines back to the attention of the profession, rightly notes that "any 'acceptable process' of adjustment must allow for the implementation of *actual*, i.e. disequilibrium, activities and therefore, the position to which the adjustment converges cannot be defined and considered 'stable' on the basis of a given vector of capital endowments known before the adjustment has been completed": therefore, he concludes, Hahn should have admitted "that the neo-Walrasian treatment of capital is simply incompatible with the 'requirement of reachability' " (Dvoskin, 2016, pp. 220-21). Such an explicit admission is absent in Hahn's writings.

[7] See Garegnani (1976; 1990, pp. 53-57), and Petri (2004, appendix 5A3). One aspect of this problem not stressed by Garegnani is the possibility of bankruptcies in the current period, due to mistaken expectations entertained in the past as to current-period prices: this can cause discontinuities in excess demands, which endanger the existence of an equilibrium.

fashions, new political ideas, new ways of organizing things (e.g. just-in-time), new music: all things that often have profound economic impact.

Second, one is neglecting the question, how was correct foresight learned? it cannot be a magical gift, so it must have been learned through trial and error; but then during the trial-and-error period the economy was not in equilibrium: hence, the economy is not always in equilibrium, and when not, we do not know how it behaves. So when does one have the right to assume that the perfect foresight assumption is not far removed from real conditions?

We can refer to Roy Radner for an authoritative answer to this last question:

> Although it is capable of describing a richer set of institutions and behaviour than is the Arrow-Debreu model, the perfect foresight approach...is contrary to the spirit of much of competitive market theory in that it postulates that individual traders must be able to forecast, in some sense, the equilibrium prices that will prevail....[this] seems to require of the traders a capacity for imagination and computation far beyond what is realistic...An equilibrium of plans and price expectations might be appropriate as a conceptualization of the ideal goal of indicative planning, or of a long-run steady state toward which the economy might tend in a stationary environment. (Majumdar & Radner, 2008, p. 444)

Radner admits in these lines[8] that, as part of a descriptive theory, the perfect foresight assumption is legitimate only for the determination of situations where relative prices *have no reason to change*, and where therefore past prices are an excellent guide to future prices. However, a situation of unchanging relative prices requires an *endogenously* determined composition of capital. Therefore, Radner is implicitly admitting here that the sequential reinterpretation of Arrow-Debreu equilibria proposed by him *is vitiated by an internal contradiction,* because the perfect foresight assumption is incompatible with the arbitrary given vectorial capital endowment of Arrow-Debreu equilibria. What is implicitly admitted here by Radner is that—contrary to the usual view—complete futures markets, and perfect foresight, are far from being alternative but equivalent assumptions. Complete futures markets allow the determination of the equilibria corresponding to any *arbitrary* vector of capital endowments observed at a given instant. Perfect foresight makes sense only for situations where relative prices are constant or nearly so, which requires an endogenously determined composition of capital, as indeed the reference to a steady

[8] These lines can be attributed to Radner alone, because taken almost verbatim from Radner (1982, p. 942). They were already quoted in Petri (2015), but the comment here is new. See Grandmont (1982, pp. 879-880) for an evalutation of the perfect foresight assumption strikingly similar to Radner's, but that concludes to a need to *reject* the assumption (Grandmont turns to the temporary equilibrium approach).

state makes clear. So it makes no sense to assume perfect foresight for economies where the initial composition of capital is given.

Furthermore, since the sole situation in which a perfect-foresight equilibrium might reasonably describe the actual behaviour of an economy is a “long-run steady state ... in a stationary environment”, then the question, whether the economy *gravitates* toward a situation for which one could assume perfect foresight, would require a theory *not* based on perfect foresight, that intertemporal equilibrium theory does not provide. So one is left with no help for the determination of the actual path of the economy.

§5. *The substitutability problem.*

In long-period equilibria, the treatment of capital as a single factor of endogenously determined ‘form’ not only made it possible to admit time-consuming adjustment, it was also indispensable for the marginalist factor substitution mechanisms to operate. Different production methods mostly require different capital goods, not the same capital goods in different proportions; once the capital goods are the ones requested by a certain method, generally input proportions are rigid; thus the change in factor proportions required by the neoclassical substitution mechanisms cannot operate if the ‘form’ of capital is given. For example a car factory, in order to change labour utilization per car produced, must change the capital goods forming the assembly line. The factor substitution mechanism based on consumer choice is blocked too, if increase in the production of a consumption good requires more of specialized intermediate goods whose endowments are given. This was clear to traditional marginalist/neoclassical economists, several of them admitted that only the treatment of capital as a single factor of variable 'form' could give plausibility to the assumption of variability of factor proportions.

For this reason, in the initial period of a neo-Walrasian equilibrium the demands for inputs will be very rigid, with the risk of an implausible equilibrium income distribution. This can be called the *substitutability problem*.

For example, Hicks admits in *Value and Capital* that in the first ‘week’ of a sequence of temporary equilibria and in ‘weeks’ in the near future the level of output of most firms will be dictated by the amount of intermediate goods ('work-in-progress') already in the pipeline, and therefore “The additional output which can be produced in the current week, or planned for weeks in the near future, will usually be quite small” (1946, p. 206), and for the same reason the variation in inputs can only be very small (*ibid*., p. 211). Hicks’ admission is in the context of temporary equilibria but clearly it applies to the first periods of an intertemporal equilibrium too. Then in the first periods of an intertemporal equilibrium a real wage ensuring equality between supply and

demand for labour might easily be so low that workers would prefer to turn to looting and revolts, or conversely so high as to absorb nearly the entire product (because of difficulties with the full employment of the other factors). In *Value and Capital* Hicks gets round this difficulty by noticing that wages are generally sticky, and change only gradually in the direction indicated by the excess demand for labour. He is ready to admit it, because it helps him surmount a danger of instability of temporary equilibrium due to expectations[9]; but in this way he admits that if the composition of the capital endowment is arbitrary then for at least some periods the economy *cannot be assumed to have real wages corresponding to what the equilibrium would determine*. This is a third reason why the intertemporal equilibrium path (or the sequence of temporary equilibria) cannot be trusted to give correct indications on the actual path.

§6. *Garegnani's claim.*

The impermanence problem, the price-change problem and the substitutability problem arise in intertemporal equilibria because of the abandonment of the specification of the capital endowment as that of a single factor of variable 'form'. The latter specification of the capital endowment avoided these problems. The impermanence problem did not arise, because the quantity of capital, being only slowly altered by net savings, was as persistent as the endowment of labour, as pointed out in section 1. The price-change problem did not arise, because the persistence of the long-period equilibrium allowed neglecting the very slow changes that equilibrium prices might undergo as a result of endogenous gradual changes in the data of equilibrium owing to population growth or net savings[10]. And the possibility to change the 'form' of capital, by allowing changes in the types of capital goods, avoided or reduced the substitutability problem.

We owe these clarifications to the late Pierangelo Garegnani, who in this way made it clear that in order to avoid the indefensible conception of capital as a single factor of variable 'form' the neoclassical approach had to pay a very high price: enormous difficulties with arguing that its neo-Walrasian versions determine equilibrium paths that sufficiently indicate the actual path of real economies. But Garegnani did not stop there; he further claimed that that price had been paid to no avail, because *the old conception of 'capital', and of 'well-behaved' substitution mechanisms between 'capital' and labour, are still needed by the approach: without*

[9] See Dvoskin and Petri (2016).

[10] Technical progress or other sudden data changes would be treated through comparative statics.

that conception, the approach cannot argue the adjustment of investment to savings, which is indispensable to the approach.

The arguments Garegnani advanced to prove this claim (Garegnani 2000) have been criticized, and unfortunately his illness, and then departure in 2011, prevented him from fully replying to the criticisms, so it is still unclear whether he was right or not. But his claim (the italicized sentence) can be sustained through a different argument.

§7. *The savings-investment problem.*

We meet here a fourth problem of intertemporal equilibria, the *savings-investment problem*. Differently from the three problems discussed in sections 3 to 5, this problem was already present in long-period equilibria, but it takes now a new form. I will argue that the theory of intertemporal equilibria (which includes not only the formalization of equilibrium and the analysis of its existence and uniqueness, but also the discussion of its stability) assumes but does not justify the equality between investment and full-employment savings.

In traditional marginalist analyses based on capital conceived as a single factor of variable 'form', the adjustment of investment to savings was based on the assumed capacity of the rate of interest to adjust firms' demands for capital to its supply, and therefore also to adjust changes in the demand for capital (net investment) to changes in its supply (net savings), by altering long-period technical choices of firms and consumption choices of consumers. The basis for the stability of the adjustment process was the decreasing long-period demand curve for capital, that now we know to be an unwarranted notion because of reverse capital deepening[11].

What particularly interests us here is that traditional marginalist analyses admitted the possibility of inequality between investment and full-employment savings, or between aggregate demand and full-employment aggregate supply, and discussed the time-consuming adjustments that would tend to correct it. An approximate equality between investment and full-employment savings was argued to hold only as an average over booms and busts, and because of the existence of those time-consuming adjustments. There is nothing analogous in the theory of intertemporal equilibria. In this theory, when the stability of equilibrium is discussed (via tâtonnement adjustments, of course, because no implementation of disequilibrium production decisions can be tolerated), the equality between (discounted) value of overall investment (sum, over the ensemble of periods, of the discounted values of each period's purchase of capital

[11] See Garegnani (1978, 1990), Dvoskin and Petri (2016).

goods produced inside the equilibrium[12]) and (discounted) value of overall full-employment savings is assumed not only in the definition of equilibrium but *also all during the disequilibrium adjustments* that if stable bring equilibrium about. The possibility of an aggregate demand (over the ensemble of equilibrium periods) different from the value of full-employment aggregate supply is excluded by assumption, so the issue does not arise of what might bring the two to equality. At disequilibrium prices intended savings in a period may differ from intended investment in that period, but on the ensemble of periods the differences compensate one another *by assumption*, not because of a stable adjustment process. This is obtained thanks to an assumption about consumer incomes in disequilibrium, that I will argue is indefensble; its removal undermines the tâtonnement process, which cannot proceed because some quantities demanded become indeterminate; a theory of investment (otherwise unnecessary, because—thanks to that assumption—demands for consumption goods are well defined, and investment is determined by the assumed equality of quantities produced to quantities demanded) becomes indispensable.

These statements will be now clarified and supported. The discussion will refer to the only developed analysis of the stability of the intertemporal equilibrium of a production economy: the auctioneer-guided 'factor tâtonnement' studied by Michael Mandler (2005). It is the standard tâtonnement for the atemporal non-capitalistic production economy, formulated so as to admit constant-returns-to-scale industries, and reinterpreted as applying to the dated commodities of a finite-horizon intertemporal equilibrium.

For the sake of argument, let us accept two assumptions implicit in this tâtonnement. First, at the initial date there are complete futures markets for all the periods covered by the equilibrium: the reinterpretation with perfect foresight in place of complete futures markets does not seem possible, because of grave difficulties with reconciling perfect foresight with a need to *find* the equilibrium by some form of groping[13]. Second, quantities produced adapt to quantities demanded at each round of the tâtonnement. This assumption deserves a brief comment because it is the usual one when constant-returns-to-scale industries are admitted, and yet it is highly problematical. At each 'round' of the tâtonnement, a certain vector of 'original' factor prices having been announced, the assumption of firm-level constant returns to scale

[12] Savings of each period is analogously defined as income accruing to factors utilized inside the period, and not used to purchase consumption goods produced during the period. Note that at disequilibrium prices investment *in a period* can be different from *full-employment* savings of that period, but the difference will be compensated by differences of opposite sign in other periods; see below, section 8. Space reasons prevent discussing the different definitions in Garegnani (2000), where investment includes the purchase of initial endowments of circulating capital goods.

[13] See Petri (2011a, p. 65).

and/or free entry (an inevitable assumption if the analysis is to pretend to any generality) obliges the auctioneer to propose product prices equal to minimum average costs, in order to avoid infinite or zero supplies; but then, in order to avoid indeterminate industry supply decisions, one must assume that the auctioneer *acts as a central planner*, who at each 'round' of the tâtonnement *imposes* to each industry a supply equal to demand. Mandler does not seem conscious of the need for the planner-auctioneer; but without it, there is no reason why the firms in an industry, indifferent as to whether to enter or not and to how much to produce since profits are zero, and anyway each one deciding on its own, should come out to produce exactly the quantity demanded. But with a planner-auctioneer, are we still in a market economy? This difficulty (that derives from the illegitimate importation into the neo-Walrasian tâtonnement of traditional reasonings based on assuming that disequilibrium productions are actually carried out[14]) will be neglected in what follows, but only in order to make it possible to perceive a further unacceptable aspect of the analysis.

At each round of the tâtonnement, because of the assumption of quantities produced equal to quantities demanded, disequilibria can arise only in the markets of 'original' resources (i.e. not produced, or produced before the equilibrium's beginning and inherited from the past: labour of the several periods, land of the several periods, initial endowments of capital goods, and possibly initial inventories of consumption goods, here however assumed absent for simplicity). For the same reason, all along the tâtonnement, investment decisions are very easy: *all* production decisions are *to order*, because decided on the basis of known demands; and therefore all investment decisions too – decisions to buy newly produced capital goods and to use them for production – can be seen as *to order*, because those capital goods will produce goods for which sale is guaranteed, and at a known price. There never is a decision to buy and utilize a capital good except based on a certainty as to what that decision will earn.

But this would not be enough to make investment decisions determinate, without a *further assumption*: at each 'round' of the tâtonnement, the demands for consumption goods are assumed to be based on *consumer incomes corresponding to the value, at that round's prices, of their factor supplies*, independently of whether there is or not a demand for those supplies[15]. Consumers are assumed to determine their demands on the basis of the assumption that their factor supplies will find purchasers. Thus, at each

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[14] See Petri (2004, pp. 190-197).

[15] Completeness would require admitting that the endowments of consumers may also include initial inventories of consumption goods, whose supplies could not be called 'factor supplies'. I assume – it makes things simpler without affecting my argument – that equilibrium starts at date 0 after the consumption goods produced before date 0 have all been consumed. I assume further that newly produced capital goods are bought directly by firms, so all along the tâtonnement, consumer endowments only include 'original' factors.

'round', once the auctioneer announces the complete price vector, consumer decisions are fully determined, independently of what firms decide. From these given demands for produced consumption goods, by assuming outputs are produced with the cost-minimizing factor proportions one obtains derived demands for inputs. If these inputs are capital goods to be produced inside the equilibrium, their production implies further derived demands, and by proceeding in this way one finally obtains the demands for 'original' factors, whose aggregate value (in discounted terms) equals the value of the demands for consumption goods from which they are derived, because firms' profits are zero. Given the assumption of productions adjusted to demands, disequilibria only arise in the 'original' factor markets. Now, the aggregate (discounted) value of the consumer supply of 'original' factors to firms translates by assumption into demand for produced consumption goods, to whose (discounted) value the aggregate (discounted) value of derived 'original' factor demands is equal. Therefore, in disequilibrium there can be excess supply of some 'original' factors, but counterbalanced by excess demand for other 'original' factors (possibly of another date): the problem is only one of non coincidence of (intraperiod and intertemporal) *composition* of factor demand and *composition* of factor supply. On the ensemble of the periods (*not* for each single period), there cannot be aggregate demand problems: Say's Law, redefined for intertemporal complete markets to mean an aggregate (discounted) value of demand for factors always equal to the aggregate (discounted) value of factor supply, holds *by assumption* all during the tâtonnement.

This means, at each 'round', an equality by assumption between (the discounted values of) aggregate net savings and aggregate net investment over the ensemble of periods; in fact, both will be negative and equal to the value of the initial capital endowment, which in the last period(s) is not renewed because the economy is approaching its end. In disequilibrium, in each period there need not be equality between desired gross savings (excess of the period's gross income from intended factor supplies to firms, over the period's desired expenditure on consumption goods) and desired gross investment (planned purchase of new capital goods produced in the period), but the inequality will be compensated by inequalities of opposite sign in other periods.

The factor tâtonnement need not be stable; but Mandler (2005) shows that it is stable if the economy-wide vectorial consumer excess demand function satisfies the Weak Axiom of Revealed Preference. If one accepts the factor tâtonnement, instability can only arise from violations of the Weak Axiom due to heterogeneous consumers. This is argued by Mandler to contradict Garegnani's claim: if consumer choice poses no obstacle, the presence of heterogeneous capital goods appears to cause no impediment to the convergence to (an essentially unique) equilibrium; no recourse to the conception

of capital as a single factor nor to 'well-behaved' substitution between capital and labour appears necessary.

I do not wish to discuss here Garegnani's own argument, that needs further study. Rather, I want to point out the indefensibility of what I have called the *'further assumption'*.

If one wants the tâtonnement to try and mimic, however remotely, the functioning of markets, then one cannot assume that the incomes on which consumers can count in formulating their demands for consumption goods are the incomes corresponding to the value of their *intended* supplies of factors; one would be then depicting a process of *ex ante* coordination of production decisions and consumer desires, a utopian planned economy. In real market economies, only factor supplies that find purchasers generate income for their owners. This should be part of any enquiry into their stability.

To see why, imagine you observe the following simple neoclassical competitive constant-returns-to-scale stationary production economy. Homogeneous labour and homogeneous land, the sole factors, produce a single output, 'food', in yearly production cycles according to a standard differentiable production function common to all producers; factors are paid their marginal products; there are no savings: every period the income earned by factors is spent entirely on the food produced in that period. Land is fully employed; a real wage fixed by law and higher than the equilibrium level causes some labour unemployment; the unemployed workers have no income, hence cannot demand the product (they survive because helped by their relatives). The realistic fact that only employed factors earn an income and can demand the product implies that in this economy there is disequilibrium only in the labour market: demand for output equals supply. This is the *necessary* way to analyze, in the neoclassical approach, the effect of a real wage kept persistently above its equilibrium level, by trade unions or by law[16]. But if indispensable in this case, the same procedure must be admitted to be indispensable for each round of the tâtonnement: not only the latter would be otherwise depicting a planned economy, but also it would be unable to study the stability of equilibrium with a rigid real wage, a realistic case that the theory must be able to analyze.

Having grasped the reasonableness and indeed necessity of the assumption that only employed factors earn an income and can demand produced goods, let us now

[16] "[W]hen, in a stationary closed community, the general level of real wages is raised, and maintained, at a height inconsistent with normal employment ... a final position must be reached which is precisely the same as that which would have occurred if there had been a direct reduction in the number of labourers available ... The final position thus reached is one of equilibrium, if the existence of the unemployed is left out of account." Hicks (1932 [1963]), pp. 198-199.

assume that, in the same economy, each consumer supplies one unit of only one factor, either labour, or land. Now suppose relative factor rentals are the equilibrium ones, but firms produce one half of the full-employment output. Half the supply of labour and half the supply of land remain unemployed. But again there is no disequilibrium in the output market: earnings and hence the demand for the product still equal the value of the product. This example makes it even more evident that consumer incomes cannot be assumed given before firms decide factor employments. *It is firms' decisions as to input use that determine consumer incomes and hence consumer demands for produced goods*.[17] Therefore, if one wants to understand disequilibrium processes, one has no right to take consumer incomes as equal to full-employment incomes, to derive from these incomes their demands for consumption goods, and to derive demands for factors from these demands.

The moment this is accepted, the announcement of prices does not suffice to determine consumer demands; at least part of firms' decisions as to productions, and hence as to factor demands, must be determined first, and will determine the income consumers have at their disposal for consumption purchases or savings. In an intertemporal economy this implies that investment decisions are necessarily at least partly indeterminate. An example will illustrate.

§8. *An example.*

8.1. *Standard tâtonnement.* Assume a three-dates intertemporal competitive economy where only one good, corn, is produced by many small firms, with labour and corn-capital as inputs, and with free entry. Corn is the numéraire. The economy starts at instant or date t=0 (the beginning of period 0), and distinguishes goods and services according to the three dates t=0, t=1 and t=2, where the economy ends (actually one can imagine the economy ends a little afterwards, to give consumers the time to consume

[17] This admission did not create problems in traditional marginalist analyses of the stability of equilibrium. Consider again the labour-land economy producing the single output 'food'. Suppose initially the supply of land is rigid and fully employed, while labour is partly unemployed because the real wage is above its full-employment level. Income derives only from employed factors; the output market is in equilibrium. If now the real wage decreases, each price-taking firm can be assumed to increase the amount of labour it employs, in the expectation of being able to sell an increased amount of output at a more or less unchanged price. This expectation will not go disappointed, because the increased output raises the total incomes of factor owners by the same amount, so that the value of aggregate demand (assuming all income translates into expenditure) increases by exactly as much as the value of output. The admission that consumers' incomes are created by the firms' decisions to employ factors does not prevent the tendency toward equilibrium. But if there are savings and capital goods, there must be a mechanism adjusting investment to savings.

the consumption goods that come out at t=2). Production processes that take one period are started at t=0 and at t=1. Thus markets for labour, land and output exist at the two dates 0 and 1, while at date 2 there is only the output market. At date 0 the economy starts with a given initial endowment of corn G_0 produced the previous period; for simplicity I assume a rigid labour supply and a rigid propensity to *gross* savings s out of each date's real income (except at the last date, of course), hence sG_0 is offered as corn-capital for period-0 production, and $(1-s)G_0$ is eaten. So the economy can also be viewed as starting with endowments of corn-capital sG_0 and of labour L_0.

Gross output at date 1 is $G_1=F(K_0,N_0)$, at date 2 it is $G_2=F(K_1,N_1)$, where K_t, N_t are respectively the corn-capital demanded (and utilized) by the aggregate of firms, and aggregate labour demand (and employment), at date t=0,1 to be used during period t=0,1; they need not be equal to the respective supplies.

Supplies of corn-capital at the two relevant dates are sG_0 and sG_1; labour supplies are L_0 and L_1. Corn supply at t=2, G_2, is only for consumption.

The gross production function G=F(K,N), common to all firms and periods, is a standard differentiable one with Constant Returns to Scale (CRS) and strictly convex isoquants; (neoclassical or pure) profits must be zero, so because of CRS the undiscounted real wages w_1, w_2 (quantities of corn to be paid at t=1 to each unit of N_0 and at t=2 to each unit of N_1) univocally determine factor proportions and the undiscounted gross rental rates earned by each unit of corn-capital $\rho_1(w_1)$, $\rho_2(w_2)$, so as to have real wages equal to the marginal products of labour, and gross rental rates equal to gross marginal products of corn-capital. For example if $F(K,N) = K^{1-\alpha}N^{\alpha}$, and $\alpha=1/2$, it is $\rho=1/(4w)$.

In the standard tâtonnement, in this economy at each round the auctioneer need decide only w_1 and w_2 and call them together with the associated $\rho_1(w_1)$, $\rho_2(w_2)$. Output G_2 adjusts[18] to consumer demand for date-2 corn; output G_1 adjusts to the sum of consumer demand for date-1 corn and capital demand K_1 derived from date-2 consumer demand. Consumer demands are derived from incomes equal to the value of their factor supplies. Demands are, in undiscounted terms, the right-hand sides of these supply-equals-demand equations:

$$(1) \qquad G_1 = (1-s)(w_1L_0+\rho_1sG_0) + K_1$$
$$(2) \qquad G_2 = w_2L_1 + \rho_2s(w_1L_0+\rho_1sG_0).$$

In equation (1), the first term on the right-hand side is the demand for date-1 corn for consumption purposes, while K_1 is the demand for investment purposes. K_1 is determined as the amount of corn-capital of date 1 needed to produce the quantity

[18] We must assume the planner-auctioneer.

demanded of G_2 with the optimal factor proportions determined by w_2, and the same holds for the other demands for factors. In equations (3) to (6) below, the right-hand sides are the functions that determine the quantity of a factor needed for its marginal product to be the one determined by the given real wages when the output to be produced is the indicated one:

(3) $K_0 = F_K^{-1}(G_1,w_1)$
(4) $N_0 = F_L^{-1}(G_1,w_1)$
(5) $K_1 = F_K^{-1}(G_2,w_2)$
(6) $N_1 = F_L^{-1}(G_2,w_2)$.

Once w_1 and w_2 are announced by the auctioneer, $\rho_1(w_1)$ and $\rho_2(w_2)$ are determined, and equation (2) determines G_2; then equation (5) determines K_1, and equation (1) determines G_1. Then the excess demands for the three 'original' factors, $K_0–sG_0$, $N_0–L_0$, $N_1–L_1$, can be determined, and on their basis the auctioneer can determine the direction in which to change w_1 and w_2 for the next 'round'.

Note that, out of equilibrium, it is generally $K_1 \neq s(w_1L_0+\rho_1sG_0)$; flukes apart, the optimal factor proportions in the production of G_2 will not be such as to cause firms to desire to invest as much as consumers desire to save at date 1. If $K_1 < s(w_1L_0+\rho_1sG_0)$, then it is possible that there be excess supply on both date-0 factor markets, compensated by excess demand for period-1 labour.

The above was in terms of undiscounted prices, which in my view makes things clearer. But it is easy to pass to the discounted or present-value prices generally preferred in the intertemporal equilibrium literature. I will discount to date 1, but it suffices to multiply the expressions below by ρ_1^{-1} to obtain their value discounted to date zero. The value of consumers' factor supplies, that is their aggregate income, discounted to date one, is

$$w_1L_0+\rho_1sG_0 + w_2L_1\rho_2^{-1}.$$

The discounted cost of production of G_2 is

$$[w_2N_1+\rho_2K_1]\ \rho_2^{-1} = w_2N_1\rho_2^{-1}+K_1;$$

the discounted cost of production of the part of G_1 demanded for consumption is

$$w_1N_0+\rho_1K_0–K_1;$$

hence the discounted total cost of production of final output (consumption goods) is

$$w_1N_0+\rho_1K_0+w_2N_1\rho_2^{-1},$$

which is the discounted value of the demand for factors. The discounted aggregate income of consumers goes to demand G_2, and the consumption part of G_1; from equations (1) and (2) we obtain a discounted total value of the demand for final output (consumption goods)

$$(1\text{-}s)(w_1L_0+\rho_1sG_0) + [w_2L_1 + \rho_2s(w_1L_0+\rho_1sG_0)]\rho_2^{-1} =$$

$= w_1L_0+\rho_1sG_0 + w_2L_1\rho_2^{-1}$.

Since production equals demand and average cost of production equals price, we obtain

$w_1N_0+\rho_1K_0+w_2N_1\rho_2^{-1} = w_1L_0+\rho_1sG_0 + w_2L_1\rho_2^{-1}$.

This confirms that Say's Law, reinterpreted for intertemporal markets, holds by assumption: the (discounted) aggregate value of factor supply is necessarily equal to the (discounted) aggregate value of factor demand, or, the (discounted) aggregate value of production is necessarily equal to the (discounted) aggregate value of demand.

8.2. *Consumer income only from demanded factors.* If instead one assumes that consumer incomes correspond to the value of the factor supplies *that find purchasers,* then these incomes are no longer determined once w_1 and w_2 are given. Equations (3) to (6) continue to determine demands for factors if G_1 and G_2 are additionally given, but now the consumer incomes that determine them depend in turn on factor demands. Therefore (still assuming a given gross propensity to save s out of G_0 and of G_1) the supply=demand conditions for output at dates 1 and 2 are the following (where the demands for G_1 and G_2 are the right-hand sides):

(7) $G_1 [= F(K_0,N_0)] = (1-s)(w_1N_0+\rho_1K_0) + K_1$

(8) $G_2 [= F(K_1,N_1)] = w_2N_1+\rho_2K_1$.

Here K_0, N_0, K_1, N_1 are the quantities *demanded* of corn-capital and of labour. Equation (8) may raise perplexities, because it appears to express simply the necessary equality between value and cost of G_2, and therefore it may appear not to indicate what determines the demand for G_2. But this is precisely the point. G_2 *is indeed indeterminate*. For each level of G_2 taken as given, the given w_2 determines K_1 and N_1, and then the fixed propensity to save determines the level of G_1 required for equality between supply and demand for corn-capital at date 1,

$K_1 = sG_1 = s(w_1N_0+\rho_1K_0)$,

which is simply a rewriting of equation (7), thus also determining N_0 and K_0 once w_1 is given[19]. And for each level of G_2 and connected level of G_1, the demand for G_2 is

[19] It is possible that, if G_2 is somehow given, demand for a factor at date 0 or at date 1 exceeds supply; since we are still assuming a tâtonnement albeit a non-standard one, we can assume that, in such a case, the auctioneer calculates herself what the incomes and decisions of consumers would be if, hypothetically, factor supplies were equal to demands (on my assumptions on consumer preferences this is easy); by comparing these demands with actual supplies the auctioneer can derive the excess demands for 'original' factors so as to proceed to determine the wages to be proposed in the next round. (This tâtonnement does not pretend to be fully realistic, it aims at being no more than an *initial* unveiling of the complications that the standard tâtonnement hides under the carpet. I am still excluding actual disequilibrium productions and exchanges.)

precisely the right-hand side of equation (8), because those are the incomes that go to demand G_2: previous incomes go to previous consumption expenditure, except for the savings sG_1 that correspond to K_1 and at date 2 earn an income $\rho_2 K_1$ which, together with $w_2 N_1$, is the income employed in demanding G_2. So demand for G_2 is always equal to G_2, *it is determined by* G_2; something else than the demand for G_2 is needed to determine G_2.

This means that the announcement of factor rentals and prices does not suffice to determine excess factor demands, and the tâtonnement *cannot proceed*, unless some theory is supplied of what determines G_2 at each 'round'.

This shows the importance of the '*further assumption*' that at each 'round' of the tâtonnement the spendable income of consumers equals the value of their endowments indipendently of the demand for them. This assumption hides the indeterminateness of disequilibrium incomes, due to their being in fact created by factor utilization decisions. Its removal makes a theory of investment obligatory. G_2 depends on firms' decisions at t=1 as to how much corn to produce for t=2, that is, *it depends on investment decisions* at t=1.

Once the 'further assumption' is dropped, these decisions are not to order. Their autonomy implies that there is no guarantee any more that they will be the ones required for the full employment of resources. If they are not, it will be savings that will adjust to investment via variations of quantities produced. Indeed, this is exactly what is indicated by equations (7) and (8) once G_2 is given: G_1 adjusts so that sG_1 equals the investment K_1 motivated by the expectation that future sales will be G_2.

We can conclude that in the existing theory of intertemporal equilibria the adaptation of (overall) investment to (overall) full-employment savings is an assumption for which no support is provided. No proof is given that this adaptation will result from some adjustment mechanism: the adaptation is assumed from the start, and is assumed to hold all along the tâtonnement, on the basis of a fairy-tale assumption about consumer incomes that flatly contradicts how these incomes are determined in market economies.

It might be countered that neoclassical theory does have arguments, in support of a tendency toward full employment, that accept that incomes are created by firms' production decisions: namely, the neoclassical-synthesis arguments that, in the debates on Keynes in the years 1950s and 1960s, allowed concluding (on the basis of the so-called Keynes effect) that *there is* a tendency toward full employment if money wages are downward flexible in the presence of unemployment, their decrease reduces the price level and hence the demand for money, the money supply does not decrease, and this reduces the rate of interest. These arguments relied on the theory that investment is a decreasing function of the rate of interest, a theory derived from the traditional decreasing demand curve for capital the single value factor. But if one appeals to this

theory, one relies on the traditional conception of capital that the shift to neo-Walrasian equilibria was intended to avoid; furthermore, one relies on time-consuming disequilibrium adjustments that neo-Walrasian theory cannot admit (the traditional decreasing demand curve for capital is a long-period notion, derived from long-period choice of techniques, that takes time to be imposed by competition); and finally, this theory of investment, besides being notoriously contradicted by the empirical evidence, is undermined by reverse capital deepening[20].

§9. *Infinite-horizon equilibria.*

Let us return for a while to the imaginary world of intertemporal equilibrium theory. In recent decades, more and more this theory has abandoned the assumption that the equilibrium extends over a finite number of periods, implicitly recognizing the absurdity of assuming that the economy ends at a certain precise (and correctly forecasted) future date. By assuming equilibrium over the infinite future, the neoclassical theorist avoids the need to assume no production of capital goods in the last period of the finite-periods intertemporal equilibrium: this production would have to be based on expectations of what will happen in subsequent periods, introducing the complications of temporary equilibria without perfect foresight, and in particular renouncing the right to treat all investment as if to order. With equilibrium over the infinite future, investment can again be treated as if to order.

The theory of infinite-horizon disaggregated intertemporal equilibria is not yet settled: with heterogeneous consumers there can be problems of equilibrium multiplicity and possibly non-convergence to a steady state, and with overlapping generations there can be problems of indeterminateness (a continuum of equilibria). If, as the profession seems inclined to do (with unclear justification, it must be said), one decides to consider these problems as of secondary importance, then the general picture derivable from these equilibria seems to add very little to the picture obtained from a Ramsey one-good model with a single representative consumer. In these equilibria there is continuous full employment of resources, labour earns its marginal value product and capital earns the remainder (if we neglect land), growth is supply-determined and hence a faster growth rate requires more savings and less consumption: all messages derivable already from Solow's 1956 growth model. Each period, the available factors (with the capital endowment now a heterogeneous vector) imply a production possibility frontier, PPF, that because of the full employment of resources entails a tradeoff between more

[20] See Petri (2004, chs. 4 and 7), and Petri (2015) that also points out an inevitable role of the accelerator (even accepting neoclassical capital-labour substitution) which can easily overpower any influence of the rate of interest. These contributions also refute other attempts to derive a negatively interest-elastic investment function without relying on traditional capital-labour substitution.

consumption that period, or more capital goods (and hence a PPF shifted outwards) the next period. Assuming, as is done for the Ramsey model, a benevolent planner intent on maximizing social welfare, the need to choose also the composition of consumption and the composition of investment every period does not introduce relevant additional problems (except the need to assume even more computational capacity); and again – under certain assumptions – it is possible to show that the outcome of the maximization can be supported as an infinite-horizon intertemporal equilibrium[21].

It is this disaggregated intertemporal equilibrium over the infinite future that is claimed as the rigorous microfoundation of current mainstream DSGE (Dynamic Stochastic General Equilibrium) macroeconomic models, or more generally, DGE models (the acronym proposed by Wickens, 2008, to include the models without stochastic elements, e.g. the Ramsey model): "it is now widely agreed that macroeconomic analysis should employ models with coherent intertemporal general-equilibrium foundations" (Woodford, 2009, p. 269). The premise of these models is therefore that infinite-horizon intertemporal general equilibrium theory is a robust starting point for a descriptive theory.

But the extension of the equilibrium to cover an infinite number of future periods does not eliminate the impermanence problem, the substitutability problem, the price-change problem, and the savings-investment problem. The first three arise in essentially the same terms as for finite-horizon intertemporal equilibria (however, I cannot help being amazed at the little resistance met by the absurd assumption of correct foresight over the infinite future). The savings-investment problem arises too, and generates the same indeterminateness, as I proceed to point out.

The dependence of consumer incomes on firms' decisions holds over infinite horizons too. To view this fact clearly, imagine a full-employment equilibrium over the infinite future, in an economy with many types of consumers, each type including identical consumers each one supplying only one unit of one 'original' factor. Now imagine that, at the same prices, firms produce of each good one half of the equilibrium quantity and employ one half of the equilibrium inputs, leaving one half of factor supplies unemployed. One half of each type of consumers is unemployed and with no income. Product markets are in equilibrium; investment equals savings. If firms decided to produce more, factor unemployment would decrease in spite of no change in relative prices. The equality of savings and investment *at equilibrium prices* does not determine production levels, because production decisions of firms determine incomes and hence

[21] This needs, of course, the planner-auctioneer in order to surmount the indeterminateness of supply decisions at equilibrium prices. It deserves notice that, to the best of my knowledge, in the literature on infinite-horizon intertemporal equilibria the stability issue is not studied nor mentioned; the term itself 'stability of equilibrium' tends to be used in the very different sense of convergence of the equilibrium path to a steady state.

savings, which are therefore *determined by* investment decisions. The general equilibrium requires an *additional* assumption that investment be at the level required for the full employment of resources, an assumption for which the theory does not and cannot provide support: even if one attempted a tâtonnement-based defense of the stability of equilibrium, one would run against the illegitimacy of assuming full-employment consumer incomes when the full employment of factor supplies should result from the adjustments.

§10. *The real foundation of the attribution of descriptive validity to intertemporal equilibria: Garegnani confirmed.*

The three problems of contemporary equilibrium theory stressed in sections 3 to 5 would suffice to conclude that, by itself, the intertemporal equilibrium path cannot prove that it indicates or approximates the actual path followed by a market economy. The fourth problem, illustrated in sections 6 to 9, strengthens this conclusion by showing that the absence of aggregate demand difficulties (that is, the legitimacy of the full employment assumption against Keynesian objections) is simply assumed.

And yet, as pointed out in §9, intertemporal equilibria *are* attributed descriptive, explicative value by many neoclassical economists, that is, are considered sufficiently indicative of actual paths. This can only be due to a *prior* belief that the qualitative characteristics of the equilibrium path traced by intertemporal equilibria are similar to those traced by actual paths. This belief cannot be based on the theory of the equilibrium path itself, because the latter says nothing on the actual path. So I suggest that this belief is based on the following, possibly not even fully conscious, reasoning:

"The traditional time-consuming marginalist/neoclassical disequilibrium adjustments in the labour and capital markets do exist and operate, and they maintain the economy close, on average, to the path traditionally traced by *long-period* neoclassical analyses, the path nowadays represented in simplified form by Solow-type growth models; now, disaggregated intertemporal equilibrium paths are qualitatively similar to Solow paths: they too trace a full-employment path, with income distribution determined by marginal products; it is therefore possible to argue that intertemporal equilibria too sufficiently correctly indicate the qualitative trend of the economy."

But then, what allows modern general equilibrium theory not to be laughed at as totally ridiculous as a positive theory is a continuing faith in traditional (time-consuming) neoclassical adjustments based on capital-labour substitution where capital is the single value factor, in other words, a continuing faith in the capacity of old

neoclassical analyses based on long-period tendencies and on capital the single factor to indicate the trend of a market economy[22].

Therefore the usual characterization of the aggregative neoclassical growth models used in mainstream macro literature, as simplified versions of the 'rigorous' disaggregated infinite-horizon intertemporal equilibrium model and deriving their legitimacy from the latter model, appears to be *the opposite of the truth*. The neoclassical analyses based on the capacity of traditional capital-labour substitution to cause the economy to gravitate, *through time-consuming adjustments*[23], toward full employment are the real microfoundation of the claimed validity of intertemporal equilibrium theory as a positive theory, not the reverse. Without a faith in those analyses the implausible assumptions needed by neo-Walrasian equilibria would make it impossible to attribute descriptive relevance to these equilibria.

In section 3 I stated that a theory of the *actual* path would be needed to assess whether the intertemporal equilibrium path gives an acceptable approximation to the behaviour of real economies. What has emerged here is that behind the current belief, at least among some theorists, in the descriptive validity of intertemporal equilibria *there is* an implicit theory of the actual path, and it is the traditional neoclassical theory based on gravitation toward long-period equilibria, because the faith persists in capital the single factor and in traditional capital-labour substitution. To put it in an expressive although imprecise way[24], it is Solow's growth model that, if accepted, allows assigning some descriptive value to Arrow-Debreu equilibria, not the reverse.

This permits a conclusion essentially coinciding with the claim by Garegnani summarized in section 6: those contemporary neoclassical theorists who attribute

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[22] Evidence in support of the above reconstruction of the implicit reasoning of these neoclassical theorists is supplied in Dvoskin (2016) and Dvoskin and Petri (2016).

[23] The legitimacy of the assumption of continuous equilibrium for a Solow-type model does not derive from intertemporal equilibrium theory, where this assumption is an unfortunate necessity, but from the long-period nature of the 'momentary' equilibrium of a Solow-type model, whose data, in spite of this 'momentary' adjective, are in fact as persistent as those of traditional long-period neoclassical equilibria (neglecting the problems with their treatment of capital), so that this 'momentary' equilibrium is in fact a long-period equilibrium, aiming to represent the central message of those traditional equilibria in simplified form. The persistence of the 'momentary' equilibrium allows admitting time-consuming disequilibria, corrected or compensated on such a time scale as to leave the very-long-period trend essentially unchanged.

[24] Imprecise, because it is not really the Solow model but rather long-period marginalist analysis that one is accepting, and Solow's model is only accepted as representing the latter in simplified form, neglecting consumer and consumption goods heterogeneity, and land. Capital in Solow's model is not really a single good, it is – as made evident by the empirical applications of the model – a summary measure of the economy's total stock of heterogeneous capital goods, that is, it is the traditional single factor 'capital' of variable 'form', embodied in the heterogeneous capital goods, necessarily measured as an amount of exchange value (which is why it is in the same units as aggregate output).

explanatory, predictive value to neo-Walrasian general equilibria[25] have only apparently abandoned the notion of 'capital' the single value factor and the belief in 'well-behaved' substitution between 'capital' and labour. This belief is seldom explicitly admitted, probably because it is known that the conception of 'capital' as a single factor is actually indefensible, as the phenomenon of reswitching has proved; but the belief is there, it is what allows believing that a lower real wage raises the demand for labour and a lower real interest rate raises investment, and that, because of the adjustment mechanisms based on these elasticities, Solow-type growth paths do sufficiently approximate actual growth and distribution paths. This allows considering the absurdity of instantaneous adjustments and complete futures markets or perfect foresight not to be a cause of fundamentally mistaken (qualitative) predictions, because the resulting equilibrium path is qualitatively similar to a Solow growth path. But then the criticisms, based on capital theory, of traditional neoclassical adjustments undermine the reliance on intertemporal general equilibria too.

So, behind the smokescreen of the reference to Arrow-Debreu and of the assumption of continuous equilibrium, neoclassical theory has not really abandoned the traditional method, that admits that the economy is always in disequilibrium and that the theoretical model can only aim at describing the trend which time-consuming stabilizing adjustment mechanisms acting in disequilibrium tend to re-establish when the economy diverges from it. But the impossibility clearly to refer to the discredited notion of capital the single value factor makes the smokescreen necessary, to hide the clay feet of the entire construction.

The implication suggested by all this is that the traditional method—based on the idea that the passage of time allows market prices and quantities to gravitate around and toward normal values—is impossible to abandon, but it must be combined with non-neoclassical theories of income distribution and of investment and employment.

REFERENCES

Bryant, W. D. A. (2010). *General Equilibrium. Theory and Evidence*. Singapore: World Scientific.

Dvoskin, A. (2016). "An unpleasant dilemma for contemporary general equilibrium theory". *The European Journal of the History of Economic Thought*, 23(2), pp 198-225.

Dvoskin, A., and F. Petri (2016), "Again on the relevance of reswitching and reverse capital deepening", *Metroeconomica*, doi: 10.1111/meca.12137.

[25] One must distinguish these theorists, mostly macroeconomists, from the specialists in general equilibrium theory who on the contrary seem more and more skeptical on general equilibrium theory as a descriptive theory.

Fisher, F. M. (1983). *Disequilibrium Foundations of Equilibrium Economics*. Cambridge: Cambridge University Press.

Garegnani, P. (1983(1976)), "On a change in the notion of equilibrium in recent work on value and distribution", in M. Brown, K. Sato and P. Zarembka, eds., *Essays in Modern Capital Theory*, Amsterdam: North-Holland Publishing Company, 1976, pp. 25–45; as reprinted in John Eatwell and Murray Milgate (eds), *Keynes's Economics and the Theory of Value and Distribution*, London: Duckworth, and New York: Oxford University Press, 1983, pp. 129–45.

Garegnani, P. (1978). "Notes on Consumption, Investment and Effective Demand: I"; *Cambridge Journal of Economics*, Vol. 2, pp. 335-353.

Garegnani, P. (1990). "Quantity of capital", in J. Eatwell, M. Milgate, P. Newman (eds.), *The New Palgrave: Capital Theory*. London: Macmillan, pp. 1-73.

Garegnani, P. (2000). " Savings, Investment and Capital in a System of General Intertemporal Equilibrium ." In H. D. Kurz , ed., *Critical Essays on Piero Sraffa's Legacy in Economics*. Cambridge : Cambridge University Press , pp. 392–445.

Garegnani, P. (2007). "Professor Samuelson on Sraffa and the Classical Economists". *European Journal of the History of Economic Thought* 14:2, pp. 181–242.

Garegnani , P. (2012) . " On the Present State of the Capital Controversy ." *Cambridge Journal of Economics* 36 : 1417–1432 .

Gintis, H. (2007). "The dynamics of general equilibrium". *Economic Journal*, 117, pp. 1280–1309.

Grandmont, J. M. (1982) "Temporary general equilibrium theory", in: K. J. Arrow & M. D. Intriligator (Eds) *Handbook of Mathematical Economics*, Vol. II (Amsterdam: North-Holland).

Hahn, F. H. (1991). "History and Economic Theory". In K. Arrow (Ed.), *Market and Welfare*. Basingstoke: Macmillan.

Hicks, J.R. (1932 [1963]), *The Theory of Wages*, London: Macmillan; as reprinted with additions, 2nd edn., 1963.

Hicks, J.R. (1946). *Value and Capital*, 2nd edition. Oxford: Clarendon Press (first edition, 1939).

Jehle, G. A., and P. J. Reny (2011). *Advanced Microeconomic Theory*. Harley, England: Financial Times Prentice Hall (Pearson).

Kreps, D. M., (2013). *Microeconomic Foundations I*, Princeton: Princeton University Press.

Majumdar, M. & Radner, R. (2008). "Uncertainty and general equilibrium", in: S. Durlauf & L. Blume (Eds) *The New Palgrave Dictionary of Economics*, 2nd edn, Vol. 8 (London: Palgrave Macmillan).

Mandler, M. (2005). "Well-behaved production economies", *Metroeconomica* 56:4, 477-494.

Petri, F. (2004). *General Equilibrium, Capital and Macroeconomics: A Key to Recent Controversies in Equilibrium Theory*. Cheltenham: Edward Elgar.

Petri, F. (2011a). “On the recent debate on capital theory and general equilibrium”, in: V. Caspari (Ed.) *The Evolution of Economic Theory*. *Essays in Honour of Bertram Schefold* (Abingdon, UK, and New York: Routledge).

Petri, F. (2011b). “On the likelihood and relevance of reswitching and reverse capital deepening” in Gehrke, C. and Salvadori, N. (eds.) *Keynes, Sraffa and the criticism of neoclassical theory. Essays in Honour of Heinz D. Kurz*. London: Routledge, pp. 380-418.

Petri, F. (2014). “Blaug versus Garegnani on the ‘Formalist Revolution’ and the Evolution of Neoclassical Capital Theory”. *Journal of the History of Economic Thought*, 36, pp 455-478 doi:10.1017/S105383721400056X

Petri, F. (2015). "Neglected Implications of Neoclassical Capital-Labour Substitution for Investment Theory: Another Criticism of Say's Law", *Review of Political Economy*, 27:3, pp. 308-340 DOI: 10.1080/09538259.2015.1067367

Petri, F. (2016). “Walras on Capital. Interpretative Insights from a Review by Bortkiewicz”, *Contributions to Political Economy*, 35.

Radner, R., (1982). “Equilibrium Under Uncertainty”, ch. 20 in K. J. Arrow and M. D. Intriligator, eds., *Handbook of Mathematical Economics*, vol. II, North-Holland, 993-1006.

Robertson, D. H. (1931). “Wage-Grumbles”, in *id.*, *Economic Fragments*. London: P. S. King & Son.

Wickens, M. (2008). *Macroeconomic Theory. A General Equilibrium Approach*. Princeton, NJ: Princeton University Press.

Wicksell, K., (1934). *Lectures on Political Economy*, vol. 1. London: Routledge.

Woodford, M., (2009). “Convergence in macroeconomics: elements of the new synthesis”. *American Economic Journal: Macroeconomics*, 1(1), pp. 267–279.