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n. 784 – Settembre 2018
Attitudes Toward Climate Policies in a Macrodynanmic Model of the Economy*

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September 2018

Abstract

In a recent article published in *Ecological Economics*, Guarini and Porcile (2016) expanded the Balance-of-Payments Constraint (BoPC) growth model in order to address the challenges posed by greenhouse gas emissions suggesting a way in which environmental variables can be included in the structure of this family of models. Building on their set up, we incorporate how people with different environmental attitudes or sentiments influence each other and contribute to the design of environmental policies. We detail the concept of transition probabilities for the agent’s switching from pro- to anti-environmental positions and vice-versa and discuss the macroeconomic results that follow. Numerical simulations allow us to investigate in more detail the implications of the validity of Porter’s hypothesis as well as decoupling conditions.

**Keywords:** Sustainability, Open economy, Environmental innovation, Porter’s hypothesis, Thirwall’s Law.

**JEL:** E12; F43; Q55; Q56; Q57

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*We are grateful to Chandni Dwarkasing and James Newell for their careful reading and helpful suggestions. The usual caveats apply.*
1 Introduction

Climate change is a major challenge to contemporary societies, with related effects likely to be extensive and potentially devastating (IPCC, 2013; 2014). From a macroeconomic perspective, there is a fundamental contradiction between the way we currently organise production and any reference to environmental sustainability. On the one hand, we have the well documented positive correspondence between the growth rate of output and greenhouse gas (GHG) emissions (e.g. Itkonen, 2012; Tapia-Granados et al., 2012; Asici, 2013; Bassetti et al., 2013). On the other hand, with a constant labour force, the economy needs to expand at the same rate as labour productivity in order to obtain a stable rate of employment. This means that, under current conditions, any attempt to reduce GHG emissions threatens employment while it is not possible to pursue full-employment without increasing the pressure on natural resources.

The aforementioned relationship has received considerable attention in the literature on ecological economics and a range of alternatives have been proposed to overcome this dilemma. For instance, a low-growth or slow-growth regime has been discussed in detail by Victor (2008), Jackson and Victor (2011), and Jackson (2016). These scholars basically proposed a reduction in working hours and structural shifts towards low productivity growth sectors as a way of breaking the link between employment and economic growth making it possible to build the foundations of a green economy.

Decoupling emerges as an obvious alternative to allow for economic growth without the corresponding increases in environmental pressure. At least some degree of decoupling has been documented in the literature (e.g. Raupach et al., 2007; Brinkley, 2014; Naqvi and Zwickl, 2017). Still, to the extent that there is little evidence in favour of absolute decoupling, a combination of moderate growth and relative decoupling appears as a conciliatory option (for a scenario analysis see, for example, Victor, 2012).

The adoption of environmental friendly policies capable of producing changes in growth regimes have been formalised over the past decades from different theoretical traditions. Such efforts have made possible a better understanding of the interfaces and interplay between “nature’s household” and “humanity’s household” or, in other words, the ecosystem and the (macro)economy. Among alternative theories of growth and distribution, the perception of the production process as one in which there is little room for substitution between factors, the emphasis on irreversibility and path dependence, and the relevance of considering various social actors instead of assuming a single rational agent, have allowed a fruitful convergence with ecological economics (see Kronenberg, 2010; Fontana and Sawyer, 2013; 2016; Rezai et al., 2013; Taylor et al., 2016; Kemp-Benedict, 2018; Rezai et al., 2018).

It must be noted, however, that most existing contributions have been based on a closed economy set up. Needless to say, in the real world, economies are open to international trade and there are complications involved in applying analytical results based on the assumption of a closed economy. When studying macroeconomic dynamics in open economies an important problem arises that we consider deserving of careful analysis. The reason for this is that one of the most influential empirical regularities in the Kaldorian growth literature, namely, Thirlwall’s law, states that, in the long-run, growth is subject to the Balance-of-Payments Constraint (BoPC). The fact that countries cannot finance increasing balance-of-payments imbalances forever implies there is an adjustment in aggregate demand that constrains growth (Thirlwall, 1979; 2011).

A large empirical literature gives support to Thirlwall’s law both in its aggregate and multisectoral versions (see, for example, Cimoli et al., 2010; Gouvea and Lima, 2013; Romero and McCombie, 2016; forthcoming). In a recent article published in Ecological Economics, Guar-
ini and Porcile (2016, hereafter G&P) expanded the BoPC growth model in order to address
the challenges posed by greenhouse gas emissions suggesting a way in which environmental
variables can be included in the structure of this family of models. Demand and productivity
regimes were modified to take into account Porter’s hypothesis according to which environ-
mental innovations, spurred by environmental policies, can foster competitiveness (Porter,

Even though G&P provided important insights on the interaction between the external
constraint on the one hand, and ecological concerns on the other, the dynamic system proposed
is extremely simple and incapable of representing the irreversibility and path dependence of
environmental trajectories. Furthermore, the discussion provided on public policy does not
consider nor formalise the various actors that interact to form the social conventions which
ultimately guide policy itself.

A number of studies have pointed out that one of the major barriers to realising a transition
to a low-carbon economy lies in a lack of broad public support (e.g. Pietsch and McAllister,
2010; Wiseman et al, 2013) while a change in individual behaviours and lifestyles is considered
to be of vital importance for making the transition to a sustainable society (see Leiserowitz
et al, 2006; Steg and Vlek, 2009). Hence, attitudes or sentiments towards the environment
become a crucial component to explain the adoption and effectiveness of climate change policies
(Tjernström and Tietenberg, 2008; Hurst et al 2013; Witt et al 2014; Ratliff et al 2017; Aasena
and Vatn, 2018).

This paper aims to make a contribution to the literature on macrodynamics and ecological
economics by expanding G&P in order to incorporate how people with different environmental
attitudes or sentiments influence each other and contribute to the design of environmental
policies. Useful groundwork for setting up an elementary and rigorous sentiment dynamics
can be found in Lux (1995) with applications especially in macroeconomic and stock market
interactions (e.g. Franke 2012; Flaschel et al, 2018). The main novelty of our exercise consists
in dividing the population between supporters and opponents of environmentally friendly
policies with the composition changing endogenously over time. The macroeconomic and
environmental implications are studied in more detail by means of numerical simulations
which further suggest that the system exhibits sensitivity to initial conditions.

The remainder of the paper is organized as follows. In the next section we present our the-
eoretical model building on G&P’s extension of the BoPC approach. We introduce the concept
of transition probabilities for the likelihood of an agent switching from support for, to oppo-
tion to, environmentally friendly policies and vice-versa, and discuss the main macroeconomic
results. Section 3 considers the analytical properties of the system. In section 4, we present a
numerical simulation exercise that allows us to investigate further the implications of Porter’s
hypothesis as well as decoupling conditions. Some final considerations follow.

2 The model

One of the main contributions of G&P was to modify demand and productivity regimes in
the BoPC growth model in order to take into account Porter’s hypothesis. In two well known
articles Porter (1991) and Porter and van der Linde (1995) challenged the view that the
relationship between environmental goals and industrial competitiveness involves a trade-
off between social benefits and private costs. They argued that by stimulating innovation,
strict environmental regulations can actually enhance competitiveness. The main principle
consists in understanding pollution, and for our purposes GHG emissions, as a manifestation

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of economic waste that involves inefficient or incomplete utilisation of resources and highlights the opportunity costs of pollution instead of its actual costs.

In this section we present a macrodynamic model built upon G&P to study the macroeconomic/environmental effects of Porter’s hypothesis as well as decoupling conditions, in a context in which individuals with different opinions towards the environment interact and ultimately determine environmental policies. The model consists of four basic blocks of equations – (i) demand conditions, (ii) supply conditions, (iii) sentiment dynamics, and (iv) remaining behavioural relations – resulting in a three dimensional dynamic system.

2.1 Demand conditions

In an open economy with a central government, the expenditure identity is given by:

\[ Y = C + I + G + X - M \]

where \( Y \) is output, \( C \) stands for consumption, \( I \) is investment, \( G \) corresponds to government expenditures, \( X \) are exports, and \( M \) represents imports. Since we are abstracting from any price considerations, the real exchange rate is held constant and for simplicity we assume is equal to one. For simplicity, it is also assumed that all trade consists in the exchange of final goods.

Define \( A = C + I + G \) as domestic absorption. Hence, we can rewrite the expenditures identity as:

\[ Y = A + X - M \] \hfill (1)

Even though behavioural relations constitute a single block of equations, for expositional purposes it is useful to introduce the following traditional function for imports:

\[ M = M(Y); \quad M_Y > 0 \] \hfill (2)

Substituting equation (2) in (1), taking logarithms and time derivatives, we obtain:\footnote{For any variable \( x \), \( \dot{x} \) indicates its time derivative (\( dx/dt \)), while \( \ddot{x} \) indicates its growth rate (\( \dot{x}/x \)).}

\[ \frac{\dot{Y}}{Y} = \frac{\alpha \ddot{A}/A + \beta_1 \ddot{X}/X}{1 + \beta_2 \pi} \] \hfill (3)

where, following G&P notation, \( \alpha = A/Y \) is the share of domestic absorption on income, \( \beta_1 = X/Y \) corresponds to the share of exports, \( \beta_2 = M/Y \) is the share of imports, and \( \pi = (\partial M/\partial Y)(Y/M) \) is the income elasticity of imports which for simplicity is assumed to be constant. The expression above separates the growth rate of output into two demand components, a domestic and a foreign one.

2.2 Supply conditions

Consider the following Leontief production function:

\[ Y = \min \{ K/\vartheta; qNe \} \]

where \( K \) stands for capital, \( \vartheta \) corresponds to the capital-output ratio, \( q \) is labour productivity, \( N \) is total labour force, and \( e \) is the employment rate. The employment rate is given by \( L/N \), where \( L \) is the level of employment. We depart from G&P since we explicitly include capital
in the production function instead of the level of GHG emissions. The reason for this is that firms when producing do use a certain amount of machinery and equipment as inputs while emissions are actually a secondary output. G&P do not provide a more detailed explanation for their modelling choice nor empirical evidence to support the chosen specification.

Notice that this function is in a sense an accounting identity because \( Y = K (Y/K) = (Y/L) N (L/N) \). For a constant capital-output ratio, the Leontief dynamic efficiency condition states that:

\[
\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{q}}{q} + \frac{\dot{N}}{N} + \frac{\dot{e}}{e}
\]  

(4)

Suppose, as a simplifying hypothesis, that the size of the labour force is constant. Hence, it follows that:

\[
\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y}
\]  

(5)

\[
\frac{\dot{e}}{e} = \frac{\dot{Y}}{Y} - \frac{\dot{q}}{q}
\]  

(6)

Capital accumulation strictly follows output’s growth rate, which in the model is determined by aggregate demand. The reason for this is that firms adjust their stock of machinery and equipment in order to match the expansion of demand. On the other hand, employment rates fundamentally depend on the difference between the growth rate of output and labour productivity. That is, if output grows faster than the increase in the productivity of workers, employment will expand. However, if labour productivity growth rates are above the rate of growth of output, then employment will be reduced.

Even though we do not explicitly include the environment in the production function, this does not mean that no treatment of greenhouse gas emissions is provided. GHG emissions, \( P \), are a subproduct of production and here are treated as such. Define \( Z \) as a measure of environmental efficiency. The higher \( Z \) is the lower are emissions per unit of output such that \( P = \frac{Y}{Z} \). Taking logarithms and time derivatives:

\[
\frac{\dot{P}}{P} = \frac{\dot{Y}}{Y} - \frac{\dot{Z}}{Z}
\]  

(7)

In this way, we correct and recover the G&P expression for the relation between the growth rates of pollutant emissions, output, and environmental efficiency. We will come to this last expression later to evaluate decoupled conditions.

### 2.3 Sentiment dynamics

Lux (1995) formalised a mechanism of mutual mimetic contagion in speculative markets that has been intensively used to assess macroeconomic and stock market interactions (e.g. Franke, 2012; Flaschel et al, 2018). The main idea is that traders who do not have access to information about the fundamentals of the economy necessarily have to rely on what can be observed in the markets to take decisions concerning their actions. Without entering into the issue of what kinds of behaviour can be designated as rational, he considers that following others’ opinions is not necessarily irrational. For example, a speculator will be more willing to sell if s/he sees most traders selling. On the other hand, with a high proportion of optimistic traders, it will be very probable that the few remaining pessimistic ones will change their attitude and buy.

A similar reasoning can be applied to ecological thinking. As briefly discussed in the introduction to this paper, one of the major barriers to realising a transition to a low-carbon
economy lies in a lack of broad public support (see Pietsch and McAllister, 2010; Wiseman et al, 2013). People disagree on their degree of support for environmental policies for different reasons. An immediate one could simply be the fact that changes in consolidated individual or collective behaviours and lifestyles are not an easy task. This is particularly threatening given that those changes are of vital importance for making the transition to a sustainable society (Leiserowitz et al, 2006; Steg and Vlek, 2009).

The literature on environmental psychology has documented the existence of a relationship between materialistic values and environmental attitudes or behaviours (e.g. Kaiser and Byrka, 2011; Kasser, 2011; Hurst et al, 2013). Individuals pursuing intrinsic goals such as close family relationships and community well-being have been found to be more likely to engage in less harmful environmental behaviour in contrast with those who pursue financial success or image and fame. There is also considerable evidence pointing to the influence of political views on environmentally friendly attitudes and sentiments (Drews and van der Bergh, 2016; McCright et al, 2016; Aasena and Vatn, 2018) which ultimately impact the adoption of climate change policies as shown by Tjernström and Tietenberg (2008).

To the extent that there is still a significant degree of asymmetric information regarding climate change and people have different views about the environment, it is important to understand how they interact. The sum of individual sentiments and attitudes generates what we refer to as collective opinion, and the later determines the explicit and implicit rules that influence our own beliefs. It is reasonable to suppose that with a high proportion of people with pro-environment attitudes or sentiments, those with the opposing attitudes or sentiments will be likely to change their views. A simple and domestic example, though not directly related to GHG, is selective waste collection. If everybody in the neighbourhood does it, new tenants are more likely to do so. On the other hand, a high proportion of people with materialist values make it more likely that those who are initially pro-environment change their positions or at least start to adopt less environmentally-friendly attitudes.

Suppose the population equals the labour force and is divided between those who have environmentally friendly, \( N^+ \), and non-environmentally friendly, \( N^- \), attitudes:

\[
N = N^+ + N^-
\]

while the difference between these two groups, \( n \), can be written as:

\[
n = N^+ - N^-
\]

Defining:

\[
\Phi = \frac{n}{N} = \frac{N^+ - N^-}{N^+ + N^-}
\]  

we have that \( \Phi \in [-1, 1] \) is an index describing the average sentiment of the population towards environmental issues. If all citizens of this society are pro-environment, then \( \Phi = 1 \). At the other extreme, a complete prevalence of materialistic values delivers \( \Phi = -1 \). For an equal division of the population between these two groups we have \( \Phi = 0 \).

Recall that as a simplifying hypothesis we are assuming that the labour force and population do not change in time, i.e. \( \dot{N}/N = 0 \). Taking time derivatives of equation (8) and making use of the respective definitions, we have that:

\[
\Phi = \frac{\dot{n}}{N} = \frac{\dot{N}^+ - \dot{N}^-}{N^+ + N^-}
\]  

\[\frac{9}{9}\]
For a constant \( N \), changes in the sentiments index fundamentally depend on the difference between variations in the two groups that form the population. Hence, we need to specify the behaviour of \( \dot{N}^+ \) and \( \dot{N}^- \) taking into account that people might change their own views on the topic. In mathematical terms we write:

\[
\begin{align*}
\dot{N}^+ &= N^- p^{+-} - N^+ p^{+-} \\
\dot{N}^- &= N^+ p^{+-} - N^- p^{+-}
\end{align*}
\]

where \( p^{+-} \) is the probability of someone who is opposed to environmentally-friendly policies changing their mind and \( p^{+-} \) stands for the probability of the opposite case.

Following the discussion provided in the beginning of this subsection, \( p^{+-} \) and \( p^{+-} \) depend on the distribution of the population between the two groups. The higher the share of pro-environment citizens in the economy the higher the probability of someone with the opposing attitudes changing his/her views. Similar reasoning in the opposite direction applies. Therefore, consider:

\[
\begin{align*}
p^{-+} &= v^+(\Phi), \quad v_\Phi^+ > 0 \\
p^{+-} &= v^-(\Phi), \quad v_\Phi^- < 0
\end{align*}
\]

Some scholars have argued that there might be feedbacks from macroeconomic conditions to public support for environmental measures (see, for example, the literature reviewed by Hurst et al, 2013). In fact, it is reasonable to assume that during an economic crisis, when employment rates are low, people care less about topics such as climate change. Facing more urgent survival decisions, environmental concerns become secondary. Such line of argument has some similarities with the so called “basic needs hierarchy” according to which once basic physiological needs, such as access to food and physical safety, have been taken care of, humans begin to pursue other goals. This assumption has been subject to much debate and critique over the past decade (for a discussion, see Tjernström and Tietenberg, 2008). In any case, we do not tackle these issues here as they go beyond the scope of the paper. If we manage to convince the reader of the importance of the mechanism so far described, future research should be done to incorporate additional interaction channels.

Substitute equations (12) and (13) in (10) and (11) so that changes in \( \dot{N}^+ \) and \( \dot{N}^- \) are a function of the sentiments index. Further inserting the resulting expressions in (9) and making use of the definitions of \( N \) and \( n \), we obtain the dynamic relation that governs sentiment dynamics towards the environment:

\[
\dot{\Phi} = (1 - \Phi) v^+(\Phi) - (1 + \Phi) v^-(\Phi) = \theta(\Phi)
\]

where \( \theta_\Phi \rightarrow 0 \).

### 2.4 Remaining behavioural relations

As mentioned several times throughout the paper, one of the main contributions of G&P was to modify demand and productivity regimes in the BoPC growth model in order to take into account Porter’s hypothesis according to which strict environmental regulations can actually enhance competitiveness by stimulating innovation. Empirical evidence on the topic is ambiguous. There is some evidence of a positive impact of environmental regulation on innovation activity though the same cannot be said about productivity growth (Lanoie et al,
If competitiveness gains are measured in terms of the performance of exports, Costantini and Mazzanti (2012) found some support for a Porter-like mechanism for a sample of European countries. Still, the literature is far from a consensus.

In what follows and in line with G&P, we present the remaining behavioural relations necessary to close the model. Even though we recognise the absence of a consensus regarding Porter’s hypothesis, we adopt a friendly position towards it.

2.4.1 Exports function

Porter’s hypothesis is investigated by adopting export and labour productivity growth functions that are non-neutral to environmental regulations. Strictly in accordance with G&P, we consider the following function of exports:

\[ X = X(Y^*, Z), \quad X_{Y^*} > 0, \quad X_Z > 0 \]  \tag{15}

where \( Y^* \) corresponds to World Gross Domestic Product (GDP).

Taking logarithms and time derivatives, we have:

\[ \frac{\dot{X}}{X} = \varepsilon \frac{\dot{Y}^*}{Y^*} + \xi \frac{\dot{Z}}{Z} \]  \tag{16}

where \( (\partial X / \partial Y^*)(Y^*/X) = \varepsilon \) is the income elasticity of exports and \( (\partial X / \partial Z)(Z/X) = \xi \) stands as a “green elasticity” parameter.

Furthermore, we also make the following assumption about the dynamics of \( Z \):

\[ \frac{\dot{Z}}{Z} = F(\lambda), \quad F_\lambda \geq 0 \]  \tag{17}

with \( \lambda \) capturing the existence of environmental regulations. The effect of \( \lambda \) on \( \dot{Z}/Z \) is dubious depending on the validity of Porter’s hypothesis. In this way, public policy produces changes along the curve while the adoption of green practices that enhance competitiveness which do not necessarily derive from environmental regulation leads to shifts in the curve (for an investigation on “whether it pays to be green” see Antonietti and Marzucchi, 2014; Zeriti et al, 2014).

2.4.2 Environmental policy

Considering the empirical evidence revisited in the previous subsection pointing out that environmental policy depends on environmentally friendly attitudes and sentiments of individuals, consider:

\[ \lambda = \lambda(\Phi), \quad \lambda_\Phi > 0, \quad \lambda(0) = 0 \]  \tag{18}

Environmental regulation depends on the design of public policies which ultimately are the result of people’s attitudes and sentiments towards the environment. Existing rules are supposed to capture a given collective opinion on a subject, in this case, the environment. They can be changed only if the composition of the population is not the same any longer. Since the sentiment index is such that \( \Phi \in [-1, 1] \), an equal distribution of citizens between those for and against environmentally friendly policies, i.e. \( \Phi = 0 \), is supposed to produce no environmental policy at all.
2.4.3 Aggregate demand adjustment

Equilibrium in the current account requires that exports and imports grow at the same rate. Recall that from equation (2) we have $M/M = \pi Y/Y$. Hence, making use of equations (16) and (17), the growth rate of output that guarantees equilibrium in the balance-of-payments, $y_{bp}$, is given by:

$$y_{bp} = \frac{\varepsilon \dot{Y}*/Y* + \xi F(\lambda)}{\pi}$$

that is, an extended environmental version of Thirlwall’s law. For developing economies, convergence strongly depends on the ratio between income elasticities, $\varepsilon/\pi$, which in turn depends on the patterns of specialisation of the productive structure (see Thirlwall, 2011; Dávila-Fernández et al, 2018).

According to G&P, the adjustment of the rate of growth of output to the external constraint takes places through changes in the rate of domestic absorption, more specifically, through expansionary or contractionary fiscal policy. We adopt the same behavioural rule so that, when the economy exceeds the BoPC rate of growth, i.e. $Y/Y > y_{bp}$, and hence a current account deficit emerges, the government adopts contractionary fiscal policy to correct the external deficit. This fall is associated with a perception that there will be a crisis at some point in the near future if the government fails to curb growth of imports. G&P make reference to instability in the exchange rate market and outflows of foreign capital that follow this perception. It is argued that crowding in effects of government expenditures may also induce a similar fall of private expenditure. Inversely, for $Y/Y < y_{bp}$, the government has space for a more expansionary fiscal policy.

In order to simplify notation, define $\dot{a} = \dot{A}/A$ as the growth rate of domestic absorption. Therefore, the adjustment of aggregate demand follows:

$$\dot{a} = \psi \left( y_{bp} - \frac{\dot{Y}}{Y} \right)$$

where $\psi > 0$ is a parameter that captures the speed of adjustment of output to the external constraint.

Substituting equation (17) in (16), and the result in (3) we obtain the rate of growth of output as a function of domestic absorption and changes in environmental regulation:

$$\frac{\dot{Y}}{Y} = \frac{\alpha a + \beta_1 \left[ \varepsilon \dot{Y}*/Y* + \xi F(\lambda) \right]}{1 + \beta_2 \pi}$$

A quick look at the macroeconomic data shows that in general $\beta_1 \approx \beta_2$. Hence, substituting equations (19) and (21) in equation (20), and assuming as a simplification hypothesis that $\beta_1 = \beta_2$, we have that:

$$\dot{a} = \psi \left[ \frac{\varepsilon \dot{Y}*/Y* + \xi F(\lambda) - \alpha \pi a}{\pi (1 + \beta_2 \pi)} \right]$$

2.4.4 Labour productivity growth

We allow environmental regulation to affect $\dot{q}/q$. Furthermore, alternative theories of growth and distribution have extensively explored the relationship between factor productivity growth and cost shares. For instance, the wages share of income is a measure of the cost of labour...
weighted by its productivity and, as such, can potentially affect the growth rate of labour productivity. This is because firms facing higher labour costs have incentives to adopt labour saving production techniques (Hicks, 1932; Duménil and Levy, 1995; Acemoglu, 2003; Hein and Tarassow, 2010).

In this model, we are abstracting from income distribution considerations but we can still take into account the aforementioned relationship through employment rates. The reason for this is that as the labour market tightens and the labour shortage becomes clearer, there is an increase in the bargaining power of workers which exerts upward pressure on wages, leading firms to adopt labour-saving production techniques (see, for example, Sasaki, 2013; for a review of the literature, see Tavani and Zamparelli, 2017).

Hence, we make:

\[
\frac{\dot{q}}{q} = G(\lambda, e), \quad G_\lambda \geq 0, \quad G_e > 0
\]  

(23)

where it is assumed that \( \lambda \) may or may not affect the growth rate of labour productivity.

### 2.5 Dynamic system

Our dynamic system consists of three differential equations in the employment rate, domestic absorption, and sentiments or attitudes towards the environment. Substituting equations (18), (21), and (23) in (6), we obtain the behaviour of employment rates. The dynamic equations for domestic absorption and sentiment towards the environment were already reported in (22) and (14), respectively, and are rewritten here.

\[
\begin{align*}
\dot{e} &= \left(1 + \frac{\beta_1}{\beta_2 \pi} \right) \left[ \frac{\varepsilon Y^*/Y^* + \xi F(\lambda, \Phi)}{1 + \beta_2 \pi} \right] - G(\lambda, e, \Phi) \\
\dot{a} &= \psi \left[ \frac{\varepsilon Y^*/Y^* + \xi F(\lambda, \Phi) - a \pi a}{\pi (1 + \beta_2 \pi)} \right] \\
\dot{\Phi} &= \theta(\Phi)
\end{align*}
\]  

(24)

Notice that function \( \theta(\cdot) \) is highly non-linear which leaves the door open to the existence of multiple non-trivial equilibria. This is particularly interesting for the literature on ecological economics because it indicates the complexity of ecological problems and the possibility of path dependence. In order to provide a more concrete view of its structure and properties, we define functional forms for \( p^{-+} \) and \( p^{+-} \) following Lux (1995) and Franke (2012). The properties of the resulting expression for sentiment dynamics have been extensively studied and provide solid ground on which to stand. Hence, suppose:

\[
\begin{align*}
p^{-+} &= \zeta \exp(\mu \Phi) \\
p^{+-} &= \zeta \exp(-\mu \Phi)
\end{align*}
\]  

(25)

(26)

where \( \zeta > 0 \) captures the speed of change, and \( \mu > 0 \) is a measure of the “strength of infection” or “herd behaviour”. This last parameter is particularly important for the existence of a unique or multiple equilibria values as we will show in the next section.
Substituting equations (25) and (26) in (14) we can rewrite the dynamic system (24) as:

\[
\begin{align*}
\dot{e} &= \frac{\alpha a + \beta_1 \left[ \varepsilon \dot{Y}^*/Y^* + \xi F(\lambda(\Phi)) \right]}{1 + \beta_2 \pi} - G(\lambda(\Phi), e) \\
\dot{a} &= \psi \left[ \varepsilon \dot{Y}^*/Y^* + \xi F(\lambda(\Phi)) - \alpha \pi a \right]/\pi (1 + \beta_2 \pi) \\
\dot{\Phi} &= \zeta [(1 - \Phi) \exp (\mu \Phi) - (1 + \Phi) \exp (-\mu \Phi)]
\end{align*}
\]

(27)

The system depicts the dynamics of environmental sentiments which interact with the macro-economy through employment rates and domestic absorption.

3 Local stability analysis

In steady-state \( \dot{e}/e = \dot{a} = \dot{\Phi} = 0 \). This gives us the following equilibrium conditions:

\[
\begin{align*}
\alpha a + \beta_1 \left[ \varepsilon \dot{Y}^*/Y^* + \xi F(\lambda(\Phi)) \right]/(1 + \beta_2 \pi) &= G(\lambda(\Phi), e) \\
\varepsilon \dot{Y}^*/Y^* + \xi F(\lambda(\Phi)) &= \alpha \pi a \\
(1 - \Phi) \exp (\mu \Phi) &= (1 + \Phi) \exp (-\mu \Phi)
\end{align*}
\]

(28)

The growth rate of aggregate demand – given by the rate of growth of domestic absorption and exports – must equal the natural rate of growth – which in this case is simple labour productivity growth – to deliver a stable employment rate. Furthermore, GDP growth rates follow the external constraint which requires that the growth rate of domestic absorption weighted by the income elasticity of imports equals the rate of growth of exports. Finally, the sentiment index towards the environment can only stabilise when the probabilities of changing between groups equilibrate.

Given the equilibrium conditions (28), we can state and prove the following propositions regarding the existence and uniqueness of an internal equilibrium.

**Proposition 1** If the “strength of infection” regarding sentiments towards the environment is weak enough, i.e. \( \mu \leq 1 \), the dynamic system has a unique non-trivial equilibrium solution that satisfies:

\[
\begin{align*}
y_{bp}^E &= G(0, e^E) \\
a^E &= \frac{y_{bp}^E}{\alpha} \\
\Phi^E &= 0
\end{align*}
\]

where for any variable \( x \), \( x^E \) indicates its equilibrium value and \( y_{bp}^E = \left[ \varepsilon \dot{Y}^*/Y^* + \xi F(\lambda(\Phi^E)) \right]/\pi \) corresponds to the steady-state “green version” of Thirlwall’s law.

**Proof.** See Mathematical Appendix. ■

That is, if the individual’s opinions about the environment do not depend on what most people think about the subject, \( \mu \leq 1 \), we should expect an equal distribution between those
who support environmentally friendly policies and those who do not. In this context, it makes little sense to refer to Porter’s hypothesis, at least as far as the determination of equilibrium values is concerned. This is because an equal distribution between \( N^+ \) and \( N^- \) produces no response in terms of environmental policy, \( \lambda (0) = 0 \). We are addressing Porter’s hypothesis by looking at the impact of environmental policy on (i) export competitiveness through GHG emissions efficiency, \( F_\lambda \) and (ii) through the direct impact on labour productivity, \( G_\lambda \). However, for the determination of equilibrium, it does not matter if \( F_\lambda \) or \( G_\lambda \) are greater than zero because society is not able to produce any kind of environmental policy in the first place.

The determination of equilibrium follows a sequence that goes from sentiments to the macroeconomy. Once the distribution of the population between those for and against environmentally friendly policies is determined, the growth rate of exports stabilises. This allows the government to adjust fiscal policy in order to make the growth rate of domestic absorption, \( a \), match the external constraint. Finally, the employment rate adjusts so as to guarantee that (Harrod’s) natural growth rate equalises Thirlwall’s law.

\[
\Phi^E \Rightarrow a^E \Rightarrow e^E
\]

When collective sentiments do matter for how the individual feels and behaves in relation to the environment, there is a qualitative change in the nature of the system that now exhibits multiple equilibria, as stated in the following proposition.

**Proposition 2** If the “strength of infection” regarding sentiments towards the environment is strong enough, i.e. \( \mu > 1 \), the dynamic system has two additional non-trivial equilibrium solutions, \( \Phi^{E_1} > 0 \) and \( \Phi^{E_2} < 0 \), that satisfy:

\[
 y_{bp}^{E_i} = G \left( \lambda \left( \Phi^{E_i} \right), e^{E_i} \right) \\
 a^{E_i} = \frac{y_{bp}^{E_i}}{\alpha} \\
 (1 - \Phi^{E_i}) \exp(\mu \Phi^{E_i}) = (1 + \Phi^{E_i}) \exp(-\mu \Phi^{E_i})
\]

where \( i = [1, 2] \) stands for each additional equilibrium solution.

**Proof.** See Mathematical Appendix. ■

In the case in which the individual’s position is strongly influenced by the social context, there is a stronger interaction between macroeconomic variables and environmental attitudes that can lead the economy to new equilibrium situations. For each of them it makes sense to refer to Porter’s hypothesis since \( \Phi \neq 0 \) implies \( \lambda \neq 0 \). Three main macroeconomic variables are affected by environmental regulation, namely, the growth rate of output, the growth rate of labour productivity, and the employment rate. With regard to the first two, the effects are straightforward given that there is a positive relationship between the “green version” of Thirlwall’s law, labour productivity and \( \lambda(\Phi) \). In simple terms, \( \Phi^{E_1} > 0 \) corresponds to the equilibrium with higher output and productivity growth. Looking to equilibrium employment rates, however, the final effect is undetermined. This is already obvious from equation (6) from which we have \( \dot{e}/e = \dot{Y}/Y - \dot{q}/q \). It will pay to be green also in terms of employment if and only if \( \partial y_{bp}/\partial \Phi > \partial (\dot{q}/q)/\partial \Phi \).

Concentrating on the dynamic equation of environmental sentiments it is easy to see that for \( \mu \leq 1 \) then \( \theta_\Phi (0, e^E) < 0 \). On the other hand, for \( \mu > 1 \) we have (i) \( \theta_\Phi (0, e^E) > 0 \) while (ii) \( \theta_\Phi (\Phi^E, e^{E_i}) < 0 \). Hence, we are able to state and prove the following propositions regarding the local stability of equilibria.
Proposition 3 When the “strength of infection” regarding sentiments towards the environment is weak enough, i.e. $\mu \leq 1$, the unique internal equilibrium point of the dynamic system is locally stable.

Proof. See Mathematical Appendix. ■

In a scenario where the interaction between citizens with different opinions on environmental policy is low, there is a balance between the two population groups that results in no environmental regulations at all. This equilibrium is always stable and society basically ignores climate change because it is unable to produce environmental policies in the first place. Things will continue the way they are until natural resources are exhausted. The story changes if there is sufficient interaction between environmental sentiments and attitudes.

Proposition 4 When the “strength of infection” regarding sentiments towards the environment is strong enough, i.e. $\mu > 1$, the internal equilibrium solution with an equal distribution between sentiments for and against environmentally-friendly policies is locally unstable.

Proof. See Mathematical Appendix. ■

Proposition 5 When the “strength of infection” regarding sentiments towards the environment is strong enough, i.e. $\mu > 1$, the two additional non-trivial equilibrium solutions with $\Phi^{E_1} > 0$ and $\Phi^{E_2} < 0$ are locally stable.

Proof. See Mathematical Appendix. ■

These last two propositions indicate that, for $\mu > 1$, being indifferent towards the environment is not an option anymore. The intensity of interactions is such that society converges either to a situation in which the majority of the population adopts environmentally-friendly attitudes or becomes openly hostile to them. Both cases are stable and initial conditions which become crucial to understand different trajectories. One could think of the United States as a textbook example of a materialistic society with a more aggressive posture against the environment while Europe has historically lead international pro-environment efforts.

At this point it is important to notice that $G_\lambda > 0$ is not crucial for the local stability properties of the model. Hence, for $G_\lambda < 0$, propositions 3-5 remain the same. The only difference concerns the determination of equilibrium employment rates. Since for $G_\lambda < 0$ we have $\partial (q/q) / \partial \Phi < 0$, environmental regulation always delivers higher steady-state employment. In other words, it always pays to be green.

4 Numerical simulations

We are ready to perform a numerical exercise to evaluate the macroeconomic effects of Porter’s hypothesis. For this purpose, we need to define functional forms for $F(\cdot)$, $G(\cdot)$, and $\lambda(\cdot)$. Our chosen specifications are linear so as to keep the exercise as simple as possible and to emphasise that the dynamics obtained do not rely on specific non-linearities in the behavioural relations, with the exception of the very natural non-linearity in the switching process already introduced.

\[
\begin{align*}
F(\lambda) & = f\lambda \\
G(\lambda, \pi) & = g_1\lambda + g_2\pi \\
\lambda(\Phi) & = \Phi
\end{align*}
\] (29)
where $f$, $g_1$, and $g_2$ are positive structural parameters. Also notice that environmental regulation simply reflects attitudes or sentiments of the population towards the environment.

The main result presented in the analytical part of this paper was that depending on the “strength of infection” regarding sentiments towards the environment, we might have a unique stable equilibrium in which the population is equally divided between those for and against environmentally friendly policies, or multiple equilibria with the majority of the population supporting environmental regulation or opposing this kind of intervention. In this section, we further investigate the properties of this outcome.

In order to choose plausible parameter values, we have considered the evidence provided in a number of empirical studies and well known macroeconomic regularities. In particular with regard to Porter’s hypothesis, given the absence of a consensus in the literature on its validity, we adopted sufficiently low values so as to provide trajectories with economic meaning.

$$\alpha = 1, \beta_1 = 0.5, \beta_2 = 0.5, \varepsilon = 1.25, \bar{Y}^*/Y^* = 0.03, \xi = 0.15$$

$$\pi = 1.5, \psi = 0.5, \zeta = 0.1, f = 0.1, g_1 = 0.0075, g_2 = 0.0275$$

The crucial parameter $\mu$ is supposed to capture the “strength of infection” of the switching process between environmental sentiments or attitudes. For $\mu \leq 1$ we have simple convergence to the unique equilibrium solution. This equilibrium is stable and society ignores climate change because it is unable to produce environmental policies in the first place. Setting $\mu > 1$ corresponds to the case with multiple equilibria and is more interesting for several reasons. First, because it corresponds to a representation of the statement “history matters”. Different initial conditions can potentially lead to very different equilibrium points. Secondly, because a sufficiently high $\mu$ indicates that people do care about other people’s opinions on environmental issues and there is an interaction between individual and collective beliefs with one influencing the other. Last but not least, we have that environmental sentiments and attitudes have important macroeconomic implications that may or may not be desirable.

Hence, it what follows we adopt $\mu = 1.1$. Figure 1 depicts trajectories for different initial conditions that indicate convergence to two different equilibrium points, $(e^{E_1}, a^{E_1}, \Phi^{E_1}) = (0.95, 0.03, 0.5)$ and $(e^{E_2}, a^{E_2}, \Phi^{E_2}) = (0.86, 0.02, -0.5)$, the first one with the majority of the population being in favour of pro-environment governmental intervention (in green), and the other against these kinds of policies (in red). Given that in both cases the macroeconomy is non-neutral to $\Phi$, we also have different equilibrium values for employment rates and output growth. In the scenarios reported it always pays to be green, both in terms of employment and growth.

Furthermore, fixing initial conditions of environmental sentiments, $\Phi_0 = 0$, we identify a “corridor of stability” with trajectories converging to $(e^{E}, a^{E}, \Phi^{E}) = (0.91, 0.025, 0)$. We would like to emphasise that any small deviation from $\Phi = 0$ immediately falls into one of the cases previously discussed. The mathematical properties of such corridor are interesting but go beyond the scope of this paper. This is mainly because a case in which changes in the micro level are always balanced at the macro level in a 1:1 proportion is extremely unlikely to happen in reality. Still, we report in Figure 2 the trajectories for a set of different initial conditions with $\Phi_0 = 0$ showing the dynamics briefly discussed.
4.1 A note on decoupling conditions

With these results in mind we can also bring some considerations on decoupling conditions. G&P showed that, once we incorporate Porter’s hypothesis into the BoPC framework, a policy that aims at decreasing GHG emissions can be potentially harmful to the environment in a sort of macroeconomic rebound effect. Substitute the extended environmental version of Thirlwall’s law, see equation (19), and (17) in (7). This gives us:

\[ \frac{\dot{P}}{P} = \frac{\varepsilon \dot{Y}^*/Y^* + \xi F(\lambda)}{\pi} - F(\lambda) \]  

(30)

Derivating (30) with respect to environment regulation, \( \lambda \), we have that the validity of Porter’s hypothesis does not generate a rebound effect as long as:

\[ \xi < \pi \]

which is easily satisfied since empirical evidence suggests \( \pi > 1 \) (see, for example, Romero and McCombie, 2016; Dávila-Fernández and Sordi, 2018) and \( \xi \) is relatively close to zero.
This result already appeared in G&P, however, here it comes with an extra flavour. Recall $\lambda$ is a function of $\Phi$. Ceteris paribus, societies that are against environmentally friendly policies, $\Phi < 0$, will present less growth but much lower environmental efficiency. Since $\xi < \pi$ is likely to be satisfied, in this case, the growth rate of emissions will be significantly higher. In the numerical example presented in this section we have that, in steady-state, for $\Phi^E_1 = 0.5 \Rightarrow \dot{P}/P = -0.02$ while for $\Phi^E_2 = -0.5 \Rightarrow \dot{P}/P = 0.07$. Therefore, Porter’s hypothesis can potentially increase the growth rate of output while reducing the growth rate of GHG emissions.

Figure 3 plots the differences in trajectories of GHG emission levels and growth rates for the initial conditions $(\epsilon_0, a_0, \Phi_0) = (0.85, 0.02, 0.01)$ in green and $(\epsilon_0, a_0, \Phi_0) = (0.85, 0.02, -0.01)$ in red. In the first case, diagrams (a) and (c), we have that an initially positive growth rate of GHG emissions leads to an increase in pollution up to a certain point when environmental regulation effectively turns $\dot{P}/P < 0$ resulting in a reduction in $P$. In the second case, diagrams (b) and (d), society converges to an equilibrium in which the majority of the population is against environmental regulation. Hence, the government actually adopts policies that are harmful to the environment resulting in an acceleration of emissions.

Finally, we can go further and determine the conditions for absolute decoupling, that is, for obtaining $\dot{P}/P < 0$ with $\dot{Y}/Y > 0$. From equation (30) we have that this will be the case as long as:

$$\lambda > F^{-1} \left( \frac{\epsilon Y^*/Y^*}{\pi - \xi} \right)$$

How likely this inequality is to be satisfied is something to be investigated empirically. Our numerical simulations suggest it is feasible but further research is certainly needed.

5 Final Considerations

Climate changes is one of the most important challenges contemporary societies are facing, with related effects likely to be extensive and potentially devastating. In a recent article published in Ecological Economics, Guarini and Porcile (2016) expanded the BoPC growth model in order to address the challenges posed by greenhouse gas emissions suggesting a way...
in which environmental variables can be included in the structure of this family of models. Building in their set up, we incorporate how people with different environmental attitudes or sentiments influence each other and contribute to the design of environmental policies. We detailed the concept of transition probabilities for the agent’s switching from pro- to anti-environmentally friendly positions and vice-versa and discuss the macroeconomic results that follow. Numerical simulations allowed us to investigate in more detail the implications of the validity of Porter’s hypothesis as well as decoupling conditions.

If we manage to convince the reader of the importance of the mechanism so far described, future research should explore the possibility of feedback from the macroeconomy to environmental sentiments. Another natural extension consists in escape from the dicotomy pro- and anti-environmentally friendly attitudes to a more realistic set up that includes the possibility of neutrality. The existence of a corridor of stability suggested by our numerical simulations provided some initial insights in that direction but was still too preliminary. Franke and Westerhoff (2018) have recently included neutral agents when studying sentiment dynamics in the macroeconomy and is a useful reference in that sense.

Different studies have pointed out that one of the major barriers to the adoption of an open agenda against climate change lies in a lack of broad public support. In this context, changes of individual behaviours and lifestyles are of vital importance in making the transition to a sustainable society. Given that the sum of individual sentiments and attitudes generates what we refer to as collective opinion, and the later determines the explicit and implicit rules that influence our own beliefs, understanding the interaction between sentiments towards the environment and the macroeconomy becomes a crucial component to explain the adoption and effectiveness of climate change policies.

References


**Mathematical appendix**

**Proof of Proposition 1**

The determination of equilibrium follows a sequence that goes from sentiments towards the environment to the macroeconomy. Properties of equation (14) have been extensively discussed in the literature. Lux (1995) showed that for \(\mu \leq 1\) this equation has a unique equilibrium given by \(\Phi^E = 0\). Since \(\Phi^E\) is determined independently from the rest of the economy, Lux’s demonstration is also valid here. Graphically we have:

![Graphical illustration](image)

where we set \(\zeta = 1\) only for expositional reasons. Adopting different values of this parameter changes the scale of the vertical axis with no further implications.
From equation (22) we have that in steady state \(\dot{Y}^* / Y^* + \xi F(\lambda(\Phi)) = \alpha \pi a\). Recall that \(\lambda(0) = 0\). Making use of (18) we can rewrite that expression as \(\dot{Y}^* / Y^* + \xi F(0) = \alpha \pi a\). Notice that \(\dot{Y}^* / Y^* + \xi F(0)\) corresponds to the equilibrium growth rate of exports. Hence, we have that \(\left[\dot{Y}^* / Y^* + \xi F(0)\right] / \pi\) is the equilibrium “green version” of Thirwall’s law, \(y_{bp}^E\).

This means that the growth rate of domestic absorption is defined and given by \(a^E = y_{bp}^E / \alpha\).

Finally, \(\dot{e}/e = \alpha a + \beta_1 \left[\dot{Y}^* / Y^* + \xi F(\lambda(\Phi))\right] / (1 + \beta_2 \pi) - G(\lambda(\Phi), e)\). Recall that \(\Phi^E = 0\) and \(a^E = y_{bp}^E / \alpha\). Hence, we have that \((1 + \beta_1 \pi) y_{bp}^E / (1 + \beta_2 \pi) = G(0, e)\). By assumption, \(\beta_1 = \beta_2\). Therefore, \(y_{bp}^E = G(0, \infty)\). \(G : \mathbb{R} \rightarrow \mathbb{R}\) is a function monotonically increasing in \(\lambda\) and \(e\). It follows that the unique equilibrium for the employment rate, \(e^E\), exists and is such that \(y_{bp}^E = G(0, e^E)\).

**Proof of Proposition 2**

To prove Proposition 2 we follow the same sequence of steps as in Proposition 1. Lux (1995) showed that for \(\mu > 1\) this equation has a two additional equilibrium given by \(\Phi^{E_1} > 0\) and \(\Phi^{E_2} < 0\). Graphically we have:

![Graph showing the growth rate of domestic absorption and employment rate](image)

where once more we set \(\zeta = 1\) only for expositional reasons. Adopting different values of this parameter changes the scale of the vertical axis with no further implications.

Once \(\Phi^{E_1}\) is reached, we are able to determine the growth rate of domestic absorption, \(a^{E_1} = y_{bp}^{E_1} / \alpha\). Employment adjusts in order to allow the natural growth rate to equalise the external constraint, i.e. \(y_{bp}^{E_1} = G(\cdot)\). Since \(G : \mathbb{R} \rightarrow \mathbb{R}\) is a function monotonically increasing in \(\lambda\) and \(e\), it follows that the unique equilibrium for the employment rate, \(e^E\), exists and is such that \(y_{bp}^{E_1} = G \left[\lambda \left(\Phi^{E_1}\right), e^{E_1}\right]\).

**5.1 Proof of Proposition 3**

The Jacobian matrix that corresponds to our dynamic system is such that:

\[
J = \begin{bmatrix}
J_{11} & J_{12} & J_{13} \\
0 & J_{22} & J_{23} \\
0 & 0 & J_{33}
\end{bmatrix}
\]
where the elements are given by:

\[ J_{11} = -G_e e^E < 0 \]
\[ J_{12} = \frac{\alpha e^E}{1 + \beta_2^2} > 0 \]
\[ J_{13} = \left( \frac{\beta_1 \xi F \lambda \Phi}{1 + \beta_2^2} - G_e \lambda \Phi \right) e^E \approx 0 \]
\[ J_{22} = 0 \]
\[ J_{23} = \frac{\psi \alpha}{(1 + \beta_2^2)} < 0 \]
\[ J_{31} = 0 \]
\[ J_{32} = 0 \]
\[ J_{33} = \theta_\Phi > 0 \]

so that the characteristic equation can be written as:

\[ \rho^3 + b_1 \rho^2 + b_2 \rho + b_3 = 0 \]

where the coefficients are given by:

\[ b_1 = - \text{tr} \, J = -(J_{11} + J_{22} + J_{33}) \]

\[ b_2 = J_{22}J_{33} + J_{11}J_{13} + J_{11}J_{22} \]

\[ b_3 = - \text{det} \, J = -J_{11}J_{22}J_{33} \]

The necessary and sufficient conditions for the local stability of a given equilibrium point is that all roots of the characteristic equation have negative real parts, which from Routh-Hurwitz conditions, requires:

\[ b_1 > 0, \quad b_2 > 0, \quad b_3 > 0 \quad \text{and} \quad b_1b_2 - b_3 > 0 \]

When \( \mu \leq 1 \) we already proved that in equilibria \( \Phi = 0 \) and \( \theta_\Phi < 0 \). In this case it is easy to see that:

\[ b_1 = G_e e + \frac{\psi \alpha}{(1 + \beta_2^2)} - \theta_\Phi > 0 \]
\[ b_2 = - \left[ \frac{\psi \alpha}{(1 + \beta_2^2)} + G_e e \right] \theta_\Phi + \frac{G_e e \psi \alpha}{(1 + \beta_2^2)} > 0 \]
\[ b_3 = - \frac{G_e e \psi \alpha \theta_\Phi}{(1 + \beta_2^2)} > 0 \]
The crucial condition for local stability becomes the last one. Through direct computation we find that:

\[
\begin{align*}
b_1b_2 - b_3 &= \left[ G_e e + \frac{\psi_\alpha}{(1 + \beta_2 \pi)} - \theta_\Phi \right] \left[ G_e e \psi_\alpha \left( 1 + \frac{2 \pi}{\beta_2 \pi} \right) - \frac{\psi_\alpha \theta_\Phi}{(1 + \beta_2 \pi)} - G_e e \theta_\Phi \right] + \frac{G_e e \psi_\alpha \theta_\Phi}{(1 + \beta_2 \pi)} \\
&= \frac{G_e e \psi_\alpha}{(1 + \beta_2 \pi)} - \frac{\psi_\alpha \theta_\Phi}{(1 + \beta_2 \pi)} \left[ G_e e \psi_\alpha \left( 1 + \frac{2 \pi}{\beta_2 \pi} \right) - \frac{\psi_\alpha \theta_\Phi}{(1 + \beta_2 \pi)} - G_e e \theta_\Phi \right] \left( 1 + \frac{2 \pi}{\beta_2 \pi} \right) > 0
\end{align*}
\]

Therefore, the system is locally stable.

### 5.2 Proof of Proposition 4

When \( \mu > 1 \) is easy to see that for \( \Phi = 0 \) we have \( \theta_\Phi > 0 \). It immediately follows that:

\[
b_3 = -\frac{G_e e \psi_\alpha \theta_\Phi}{(1 + \beta_2 \pi)} < 0
\]

and the system is locally unstable.

### 5.3 Proof of Proposition 5

When \( \mu > 1 \) we already proved the emergence of two additional non-trivial equilibrium solutions, \( \Phi^{E_1} > 0 \) and \( \Phi^{E_2} < 0 \). For each of them the partial derivative \( \partial \Phi / \partial \Phi = \theta_\Phi < 0 \). Following the same sequence of steps as in Proposition 3 we have that all Routh-Hurwitz conditions are satisfied and, thus, the system is locally stable.