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Some new insights on the empirics of Goodwin's growth-cycle model

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Abstract

Vector Autoregressive (VAR) models have been used for a long time now to study profit-squeeze cycles, most of the time using problematic Hodrick-Prescott (HP) filtered time series. In a recent paper, Hamilton (2018) has provided a simple alternative that overcomes the main drawbacks of the HP procedure. In order to evaluate the empirical relevance of the profit-squeeze mechanism, we compare both methodologies using quarterly data for the United States from 1948-67 to 2016. Furthermore, we present an extension of Goodwin's (1967) growth-cycle model that includes employment rates, income distribution, and capacity utilisation as endogenous variables. We show analytically that the system always admits a family of periodic solutions. The model is estimated econometrically using the Autoregressive Distributed Lag (ARDL) approach. Through numerical simulations and making use of our estimations, we confirm that fluctuations are persistent and bounded.

Keywords: Distributive cycles; Growth-cycle; HP filter; VAR; ARDL.

JEL: E32, E64.

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1 Introduction

It is fair to say that the effort to understand business cycles within a growth framework is widespread in Economics nowadays. In this respect, the idea that growth and fluctuations are an intertwined phenomenon can be traced back to the seminal work of Goodwin (1967) who developed a model where investment comes from profits squeezed out of the income of the workforce, thus determining the pace of capital accumulation. The dynamic system obtained, which blends aspects of the Harrod–Domar growth set up with the Phillips curve, captures his profound insight that the trend and the cycle are indissolubly fused, and to the critical conclusion that distributional conflict produces endogenous cycles (see Harcourt, 2015).

Goodwin presented a theory of economic fluctuations whereby the economic variables interact with each other cyclically and endogenously, with the cycle emerging from deterministic interactions, and not as the outcome of exogenous aleatory shocks. Formally, the original growth-cycle model consists of two simultaneous non-linear dynamic equations, one for the employment rate and one for the wage-share. It corresponds to a Lotka-Volterra's predator-prey system stated in a Marxian form, in which income distribution is the predator and the employment rate as the prey.

Over the past decades, the model has been expanded in almost all possible directions. Initial extensions included Desai (1973) on inflation, Wolfstetter (1982) on fiscal policy, van der Ploeg (1983) on its relation with neoclassical growth, and Choi (1995) on efficiency wages. Modifications to study qualitatively different dynamics were developed by Pohjola (1981) in a discrete-time version of the model and Sportelli (1995) in a Kolmogoroff generalisation. Sato (1985) extended the system to a two-sector economy while Skott (1989a) introduced Keynesian effective demand aspects.

The study of the role played by induced technical change in the growth-cycle framework was initiated by Shah and Desai (1981) and further elaborated by van der Ploeg (1987). Other notable contributions include Foley (2003), Julius (2006), and Velupillai (2006), who also developed macrodynamic disequilibrium models with a technical progress function. An entrepreneur state that invests in infrastructure capital and finances publicly funded research was introduced only recently by Tavani and Zamparelli (2018).

To better understand the relationship between distributive cycles and the Minskyan financial instability hypothesis, Keen (1995, 2013) has built a family of Goodwin-Minksy models, which were shown to be structurally unstable. In the same way, even though with different features, Sordi and Vercelli (2006, 2014) have presented a series of macrodynamic models exploring the non-linear interactions between financial and distributive variables. Stockhammer and Michell (2017), on the other hand, have tried to put forward the concept of pseudo-Goodwin cycles, understood as a distributive cycle that could be generated by Minskyan dynamics.

The stability properties of the growth-cycle model within a Kaleckian accumulation function have been investigated by Mariolis (2013) and Rodousakis (2015). Distributive cycles with Kaldorian and Kaleckian features are also part of the contributions of von Arnim and Barrales (2015). Dávila-Fernández and Sordi (2018a, 2018b) have extended the framework to an open economy set up establishing a connection with Thirlwall's law. Inventory considerations have been introduced by Grasselli and Nguyen-Huu (2018). Furthermore, in what concerns the correspondence between Harrodian instability and the Keynesian effective demand principle, Sportelli (2000) and Schoder (2014) provided valuable exercises capable of generating endogenous cycles.

Sasaki (2013) also blends a Goodwinian and Marxian macrodynamic disequilibrium model of growth and fluctuations with Kaleckian features. The starting point of his analysis rests on the observation that investment in the Goodwin model has the same passive role played in the Neoclassical tradition since savings determine it. Sasaki then presents an extension to Goodwin's formulation in which in which he introduced an independent investment function in line with Post-Keynesian growth theory. The author obtained a system of three variables, namely employment rate, profit share and rate of capacity utilisation and demonstrated the existence of a limit cycle amongst these variables.

Last but not least, Chiarella et al. (2005) have been engaged in a major research program on (disequilibrium) macroeconomic analysis that has resulted in a massive series of papers and books with several other contributors including Asada et al. (2003, 2011) on the "Keynes-Meltzer-Goodwin" system. A similar effort is found in Flaschel (2015) with the extension of Goodwin's distributive cycle by introducing both effective demand forces and endogenous innovations, providing a platform of what Goodwin himself described as the "Marx-Keynes-Schumpeter" approach.

On the empirical front, the theory of endogenous distributive cycles has been examined in a significant number of studies. Qualitative support is found for different countries and time spans in Desai (1984), Harvie (2000), Mohun and Veneziani (2008), and Zipperer and Skott (2011), among others. On the other hand, scholars such as Goldstein (1999), Barbosa-Filho and Taylor (2006), Moura Jr. and Ribeiro (2013), Basu et al (2013), Kiefer and Rada (2015), Barbosa-Filho (2016), and more recently Grasselli and Maheshwari (2018) provided parametric quantitative evidence. Non-parametric treatments have also received some attention by Kauermann et al. (2012). Furthermore, Barrales and von Arnim (2017), using quarterly data for the United States, have decomposed employment, utilisation, and income distribution time series into wavelets of varying periodicity and showed the existence of cycles *a la* Goodwin.

VAR models have been used to study profit-squeeze cycles since at least Goldstein (1999). Barbosa-Filho and Taylor (2006) popularised this methodology using quarterly data for the US economy. However, it must be noted that most exercises rely on Hodrick-Prescott (HP) filtered time series. The problem is that the HP filter produces series with spurious dynamic relations. Although some of its drawbacks have been known for some time, the method continues to be widely adopted. Recently, Hamilton (2018) has strongly argued against its use showing this to be a serious mistake. More importantly, he provided a simple alternative that successfully separates trend and cycle with none of the HP drawbacks.

Hence, the main contribution of this paper lies in comparing both methodologies using quarterly data for the United States (US) between 1948-67 and 2016. We show that using the Hamilton detrending method significantly increases the magnitude as well as the time length of the response. In this way, we can evaluate the robustness of previous results found in the literature regarding the empirical relevance of the profit-squeeze mechanism.

Furthermore, we also developed an extension of the original growth-cycle model that includes the employment rate, wage-share, and capacity utilisation as endogenous variables. As our brief revision of the literature indicates, extensions along this lines have been already performed extensively over the past decades. The closest example is Sasaki (2013) who also deals with a 3-dimensional dynamic system. In our case, nevertheless, the main innovations are twofold.

First, we show that introducing an independent investment function that only relies on the accelerator effect, and allowing the growth rate of output to follow what we refer to as "Skott's rule", the system always admits a family of periodic solutions. This result holds independently of the functional forms and parameter values we chose. Secondly, we estimate the model econometrically using the Autoregressive Distributive Lag (ARDL) approach. Through numerical simulations and using our estimates, we confirm that fluctuations are persistent and bounded. The main contribution of this part lies in providing a simple baseline model to study distributive dynamics that deals with the interactions between labour and goods markets while has some empirical support.

The remaining of the paper is organised as follows. Section 2 revisits the original Goodwin's (1967) model showing the main mechanisms that are involved in the profit-squeeze dynamics. Section 3 provides our first econometric exercise that assesses the robustness of the profit-squeeze mechanism to the use of HP and Hamilton (2018) filters. In the next section, we present an extension of the growth-cycle model that includes the rate of capacity utilisation as a new endogenous variable. Section 5 brings our econometric estimation of the obtained dynamic system and the corresponding numerical simulations. Some final considerations follow.

2 The original growth-cycle model

In the original Goodwin (1967) model the endogenous variables are the employment rate, here represented by e, and the wage-share, ϖ. For expositional purposes, we can divide it into three blocks of equations: (i) supply conditions, (ii) distributive conditions, and (iii) behavioural relations.

2.1 Supply conditions

Consider a closed economy without government activity that uses capital, K, and labour, N, to produce output, Y, by using a fixed coefficient technology, given by:

$$Y = \min\{Ku; qNe\} \tag{1}$$

where $u = Y/K$ stands as a measure of capacity utilisation and $q = Y/L$ is labour productivity. Employment rates are defined by the ratio between the level of employment and the total labour force, $e = L/N$.

The Leontief dynamic efficiency condition states that:[1]

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} + \frac{\dot{u}}{u} = \frac{\dot{q}}{q} + \frac{\dot{N}}{N} + \frac{\dot{e}}{e} \tag{2}$$

For a constant capacity utilisation, such that $\dot{u}/u = 0$, an exogenous labour force growth rate, η, and exogenous labour productivity growth, α, it follows from Eq. (2) that the rate of growth of output equals the rate of capital accumulation while the growth rate of employment is given by the difference between actual and natural output growth rates:

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} \tag{3}$$

$$\frac{\dot{e}}{e} = \frac{\dot{Y}}{Y} - \alpha - \eta \tag{4}$$

[1]For any variable x, $\dot{x}$ indicates its time derivative. Notice that the Leontief production function is in a sense an accounting identity because of $Y = K(Y/K) = (Y/L)N(L/N)$.

2.2 Distributive conditions

In an economy with two factors of production and no government, the income identity is:

$$Y = wL + rK \tag{5}$$

where w stands for real wages and r is the rate of return on capital.

Hence, the wage-share is defined as $\varpi = wL/Y = w/q$, and taking logarithms and time derivatives, we have:

$$\frac{\dot{\varpi}}{\varpi} = \frac{\dot{w}}{w} - \alpha \tag{6}$$

so that the share of wages on income increases as long as real wages grow faster than labour productivity.

2.3 Behavioural relations

So far we have described dynamic relations directly obtained from the manipulation of accounting identities. Therefore, basically by definition, they are always true. Theory enters the story when we introduce the final set of equations. Assuming that all savings come from profits and that all profits are reinvested, we have that:

$$\frac{\dot{K}}{K} = (1 - \varpi)u \tag{7}$$

where u is held constant.

In order to take into account the labour market, Goodwin (1967) has considered a Marxian reserve army mechanism translated in terms of a Phillips Curve.[2]

$$\frac{\dot{w}}{w} = f(e), \;\; f_e > 0 \tag{8}$$

indicating that the bargaining power of workers increases as employment expands.

2.4 The dynamic system

Substituting Eqs. (3) and (7) on (4), we obtain a dynamic equation for the rate of employment as a function of income distribution. On the other hand, substituting Eq. (8) on (6), it is easy to see that variations of the wage-share are determined by the rate of employment:

$$\frac{\dot{e}}{e} = (1 - \varpi)u - \alpha - \eta$$
$$\frac{\dot{\varpi}}{\varpi} = f(e) - \alpha$$

The so-called "Goodwin cycle" works as follows: an increase in the employment level leads to an increase in the wage share, which decreases the profit share and thus capital accumulation. The outcome of lower capital accumulation is a decrease in the output and

[2] According to Franke et al. (2006, p.453) "Goodwin indicates that income distribution plays a crucial role in the dynamics of nominal and real variables. It is determined by the interplay of a wage as well as a price Phillips curve, and in turn impacts positively on aggregate demand via workers' consumption and negatively via profit-oriented investment."

consequently in the employment level, leading to a reduction in the wage share and an increase in the profit share. A higher profit share leads to faster capital accumulation and, hence, an increase in output growth and employment level. At this point the cycle restarts.

$$\uparrow e \Rightarrow \uparrow \varpi \Rightarrow \downarrow \dot{K}/K \Rightarrow \downarrow \dot{Y}/Y \Rightarrow \downarrow e$$
$$\downarrow e \Rightarrow \downarrow \varpi \Rightarrow \uparrow \dot{K}/K \Rightarrow \uparrow \dot{Y}/Y \Rightarrow \uparrow e$$

3 Hodrick-Prescott *vs* Hamilton filters

VAR models have been used to study profit-squeeze cycles since at least Goldstein (1999). This methodology that was later popularised by Barbosa-Filho and Taylor (2006) consists in evaluating the response of the rate of employment and the wage-share to exogenous shocks on each other. A positive shock on employment should lead to an increase in the wage-share through the real Phillips curve while a positive shock on the share of wages should reduce employment rates through capital accumulation. In this context, impulse response functions are simple but valuable tools of analysis. Furthermore, the VAR approach is supposed to control the inherent endogeneity of the system, given that all variables are simultaneously determined and influence each other. The main disadvantage is that it only corresponds to an indirect assessment of the theoretical model.

Previous exercises have extensively relied on HP-filtered data. As mentioned in the Introduction of this paper, the HP filter produces series with spurious dynamic relations. Although some of its drawbacks have been known for some time, recently, Hamilton (2018) has strongly argued against its use showing this to be a serious mistake. More importantly, he provided a simple alternative that successfully separates trend and cycle with none of the HP drawbacks. In this section, is our intention to verify the robustness of the profit-squeeze mechanism to the use of HP and Hamilton filters.

3.1 Data

Our dataset is quarterly and comprehends the period between 1948-67 and 2016. Employment rate series come from the U.S. Bureau of Labour Statistics as one minus the unemployment rate. The wage-share was computed using data from the Bureau of Economic Analysis taking the ratio between the compensation of employees and domestic income at production prices.[3] Finally, several scholars have investigated the existence of distributive cycles using the rate of capacity utilisation instead of employment (including Barbosa-Filho and Taylor, 2006). Hence, we also use the capacity utilisation index provided by the Board of Governors of the Federal Reserve System. Data for employment and income distribution was available from 1948 to 2016 while the capacity utilisation index is available only after 1967.

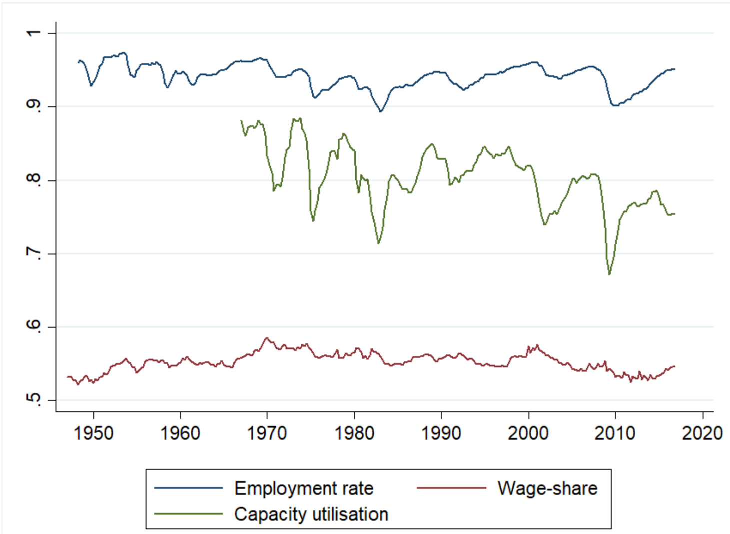
We test for the presence of structural breaks using the sequential Bai-Perron test. Several structural breaks were identified for employment (1958Q3, 1974Q4, 1987Q1, 2006Q4), the wage-share (2005Q1) and the rate of capacity utilisation (1974Q4, 1987Q3, 2001Q1). Therefore, we included dummy variables to capture the structural break effects. We assigned one

[3] There is some controversy when it comes of how to measure the share of wages and profits on income. Giovannoni (2014), for instance, shows that it is possible to differentiate between wage-share (share of wages on income), compensation-share (share of wages and benefits on income), and labour-share (share of wages and benefits adjusted for self-employment on income). In this paper, we use the compensation-share given its easy availability.

dummy variable for each indicator. They assume value 1 for years with break and 0 for years with no break.

Fig. 1 presents a general overview of our series. Employment rates fluctuate around 95% with a peak of 97.5% during the fifties and a valley of 90% in 1983 and after the 2007 financial crisis. Capacity utilisation, on the other hand, presents a declining trend going from 88% at the beginning of the series to 75% by the end of the period. Finally, the wage-share exhibits a relatively stable trajectory. It increases from 52% in the late forties to 57% just before the oil shocks and falls to 53% in 2010.

Figure 1: Employment rates, wage-share, and capacity utilisation



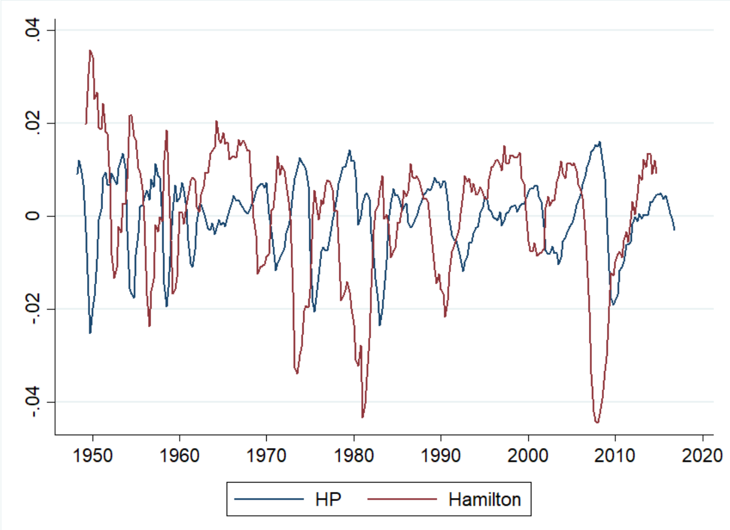
The reader might find useful a visual assessment of differences between the two methodologies used to separate trend and cycle. HP filter was applied using a 1600 smooth parameter. Hamilton procedure, on the other hand, basically corresponds to the error term of an Ordinary Least Squares (OLS) regression of the variable of interest in $t+8$ on the four most recent values as of date t. Figs. 2 to 4 show that the differences between series are remarkable, indicating that there is no reason to believe *a priori* that the correspondence between variables should be the same in each case. Notice that in several moments they go in opposite directions. That is, frequently, when the HP filter delivers a positive deviation from the long run trend, Hamilton's procedure shows a negative deviation.

3.2 Impulse response functions

Let us begin looking only to employment rates and the wage-share as in Goodwin (1967). We estimated a VAR model with two lags for HP-filtered data. Analogously, a VAR with three lags was estimated for detrended series using Hamilton's procedure. In both cases, we followed the Schwarz criteria for choosing the optimal number of lags. We preferred the Schwarz criterion over the popular Akaike criteria insofar as usually it assigns a lower number of lags, which in this case is desirable given the nature of the relationship we are addressing. Still, when serial correlation was found, we increased the number of lags until we removed it. Details on the estimation process are in the Empirical Appendix.

Fig. 5, on the left, presents the response of income distribution to a positive shock on the employment rate while, on the right, we have the response of employment to a positive

Figure 2: Employment rate – Comparing HP vs Hamilton filters



shock on the wage-share. The exercise was performed using HP-detrended time series and depicts the well-known profit-squeeze mechanism. An increase in employment rates leads to an increase in the wage-share while a higher wage-share delivers lower employment.

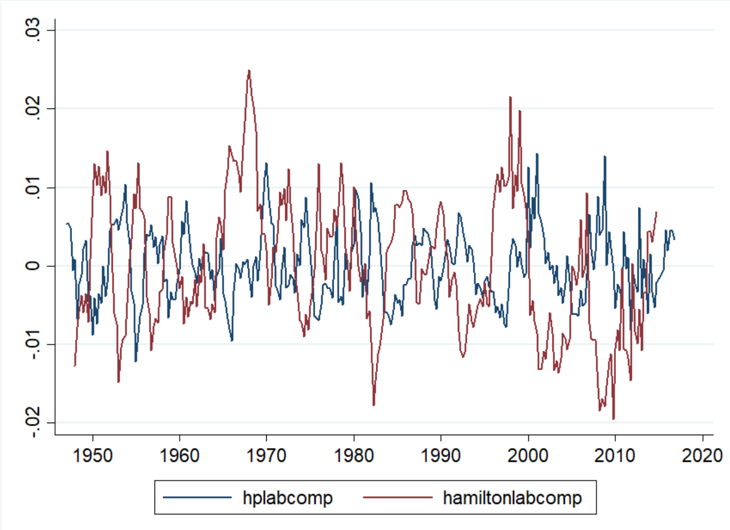
On the other hand, Figs. 6 reports the same exercise using the Hamilton filter. An increase in employment increases the bargaining power of workers so that real wages grow faster than labour productivity. This leads to the observed increase in the wage-share, which is equivalent to a reduction in the profit-share and could be interpreted as a reduction of investment profitability. On the other hand, an increase in the wage-share, that is supposed to capture a decrease in profitability, leads to a contraction in capital accumulation and ultimately to the observed negative response of employment. Notice, however, that using the Hamilton detrending method significantly increases the magnitude as well as the time length of the response. When using the HP filter, effects disappear after ten periods while in the second case they continue for at least another four quarters.

Several scholars have investigated the existence of distributive cycles using the rate of capacity utilisation instead of employment rates (e.g. Barbosa-Filho and Taylor, 2006; Kiefer and Rada, 2015). From a theoretical point of view, there are several problems of treating the goods market and the labour market as equivalent. Still, it might be useful to see if the mechanism holds when we confront utilisation rates and income distribution. A VAR model with four lags was estimated for HP and Hamilton filtered data. Fig. 7, on the left, presents the response of income distribution to a positive shock on capacity utilisation while, on the right, we have the reaction of utilisation rates to a shock on the wage-share. On the one hand, a shock on utilisation leads to higher wage-share. On the other hand, there seems to be no response of income distribution to a shock on utilisation rates.

Results do not significantly change when we repeat the exercise using the Hamilton procedure. For instance, we recover the positive correspondence going from utilisation to the wage-share. Effects are stronger and dure longer than in the previous case. Furthermore, a shock on the wage-share does not produce a response of our utilisation index. Fig. 8 depicts this case.

Our estimates indicate that when we look at the relationship between employment and income distribution, the profit-squeeze mechanism founds clear support independently on the

Figure 3: Wage-share – Comparing HP vs Hamilton filters



detrending method used. Moreover, the Hamilton procedure allows us to obtain higher effects in both directions. On the other hand, results do not hold when we substitute employment by capacity utilisation. Because higher utilisation is in general associated with higher employment, it seems that the Phillips curve mechanism works and explains why a positive shock on capacity utilisation increases the wage-share. However, income distribution does not respond to changes in u. A satisfactory explanation for this requires that we go back to the theoretical model and allow labour and goods markets to interact. This is done in the next section.

4 A 3D growth-cycle

Even though the rate of employment and the level of capacity utilisation are strongly correlated, labour and goods markets follow very particular dynamics, and we need to treat them separately. It is our purpose in this section to extend the original growth-cycle model to take into account this differentiation. Exercises of this type have been already performed extensively over the past decades. Still, this section brings some novelties. We show that (i) introducing an independent investment function that only relies on the accelerator effect, and (ii) allowing the growth rate of output to follow what we refer to as "Skott's rule", the system always admits a family of periodic solutions. This result holds independently of the functional forms and parameter values we chose. The empirical relevance of the obtained dynamic system will be discussed in the next section.

4.1 Endogenising capacity utilisation

Once capacity utilisation is allowed to vary, it follows from the Leontief efficiency condition that:

$$\frac{\dot{u}}{u} = \frac{\dot{Y}}{Y} - \frac{\dot{K}}{K} \tag{9}$$

i.e. the rate of change in capacity utilisation now depends on the difference between the rate of growth of output and capital accumulation. Two critical modifications must be introduced.

Figure 4: Capacity utilisation – Comparing HP vs Hamilton filters

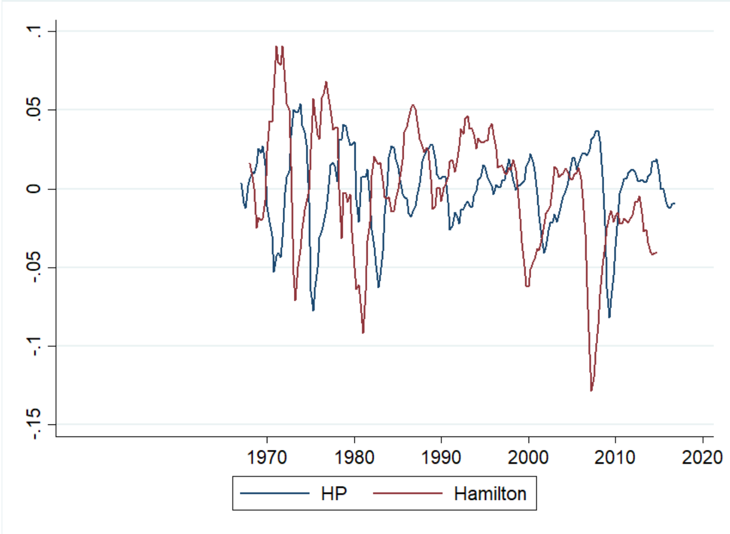
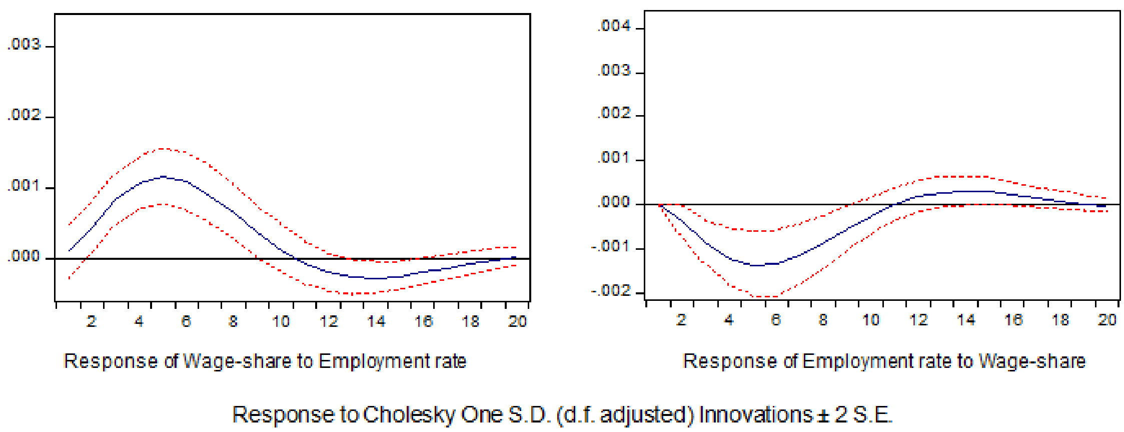


Figure 5: Impulse response – HP filter



First, capital accumulation cannot be determined by savings. This requires the adoption of an independent investment function in Keynesian lines. Secondly, a new behavioural relation is necessary for the growth rate of output.

For this paper, we adopt the following general specification for the ratio between investment to the capital stock:

$$\frac{I}{K} = h(u), \;\; h_u > 0 \tag{10}$$

so that $\dot{K}/K = I/K$. The basic idea is investors use capacity utilisation as a predictor the future state of demand. In other words, capital accumulation is a function only of the accelerator effect. Different specifications of a flexible accelerator can be found in the literature and of course in Goodwin himself. While one could also make the case that capital accumulation depends on a measure of profitability (such as the wage-share), there is not a consensus about its inclusion (see, for example, Skott, 1989b, 2012). Thus, we maintain the model as simple as possible and leave room only for the accelerator.

Figure 6: Impulse response – Hamilton filter

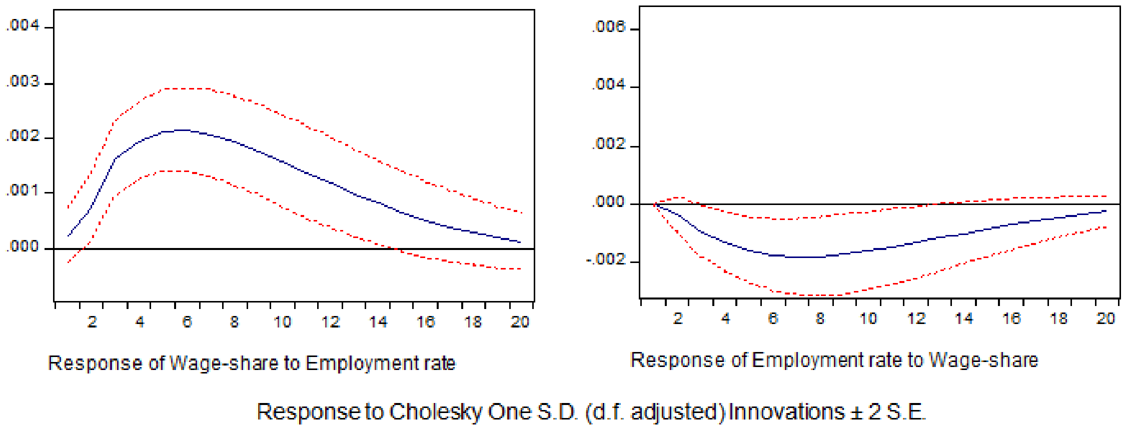
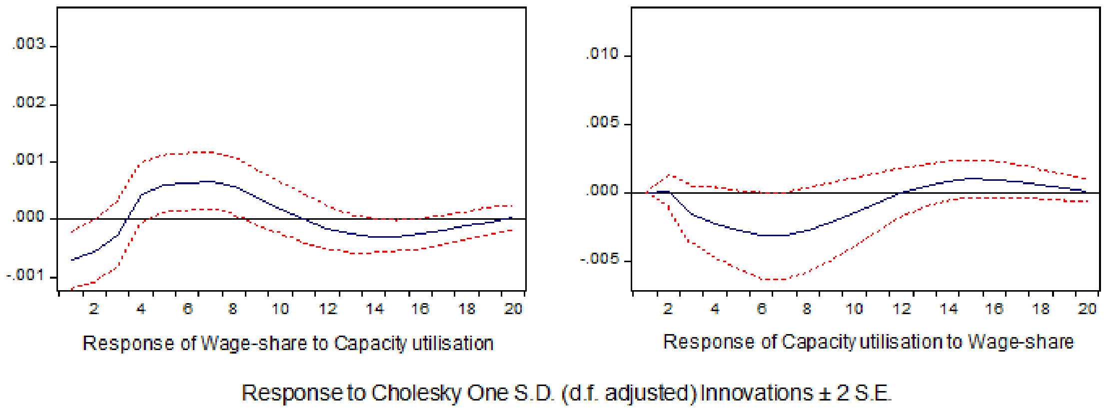


Figure 7: Impulse response – HP filter



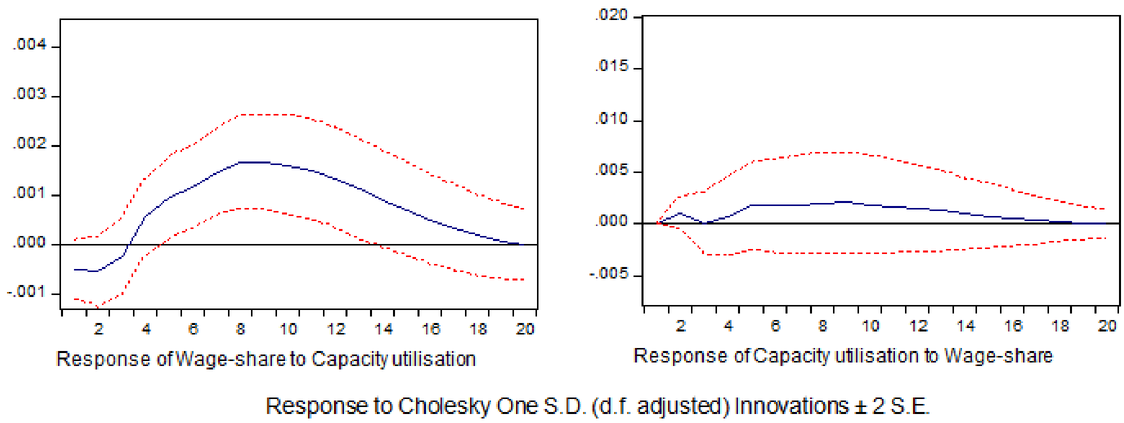
In what concerns $\dot{Y}/Y$, let us consider a mechanism similar to the one described by Skott (1989b):

$$\frac{\dot{Y}}{Y} = g(\varpi), \quad g_{\varpi} < 0 \tag{11}$$

The motivation of this last expression reproduces Skott's argument (see, for instance, Skott and Ryoo, 2008, p. 837). In a continuous-time setting, given the effects of lags and adjustment costs, the level of output is predetermined. Hence, firms choose the rate of growth of production at each moment, instead of its level, and this choice is made to balance the costs of changes against the benefits of moving toward a preferred level of output. The authors proceed arguing that such costs and benefits are determined by demand and cost signals from output and input markets.

We make the case that both can be seen to be somehow captured by the wage-share. In what concerns the demand signal, recall that by assumption the level of output is predetermined. Under the assumption of flexible prices and sticky nominal wages, a rise in demand leads to an increase in the price of output. Even though it is true that we are not explicitly modelling this increase, the real wage and consequently the share of wages in income respond

Figure 8: Impulse response – Hamilton filter



to such unanticipated movements in prices. The conclusion is that a positive demand shock generates a reduction in the wage-share to which firms respond to increasing $\dot{Y}/Y$.

Furthermore, as far as production decisions are concerned, the wage-share also captures the costs of changing output through its effects on the availability of labour with the desired qualifications. Recall that by definition $\varpi = w/q$, that is, the cost of labour weighted by labour productivity. A higher ϖ reduces the capacity of the firm to increase the growth rate of production for any given of demand.[4]

Substituting Eqs. (10) and (11) in (9), we obtain variations in capacity utilisation as a function of income distribution and the accelerator effect on investment. Analogously, making use of Eqs. (4) and (11), we have a dynamic expression for the employment rate. Maintaining the expression for variations in income distribution, the dynamic system becomes:

$$\begin{aligned} \frac{\dot{e}}{e} &= g(\varpi) - \alpha - \eta \\ \frac{\dot{\varpi}}{\varpi} &= f(e) - \alpha \\ \frac{\dot{u}}{u} &= g(\varpi) - h(u) \end{aligned} \tag{12}$$

An increase in employment rates continues to increase the wage-share through the real wage Phillips curve which in a second moment reduces employment and capacity utilisation through Skott's mechanism. A strong accelerator effect ironically exercises a stabilising force in the goods market for a given $g(\cdot)$. The potential persistence of these dynamics is addressed in the next subsection.

[4]Skott and collaborators have on different occasions make $g(\cdot)$ a function also of the employment rate. To maintain the exercise as simple as possible, we explicitly avoid this route though acknowledge and reproduce the spirit of their original motivation. It must be noted, however, that the inclusion of e changes the qualitative properties of our system making it locally stable. It is also worth to notice that making $h(\varpi, u)$ does not modify the dynamic properties of the model as long as $g_\varpi > h_\varpi$.

4.2 Equilibrium points and local stability analysis

In steady-state $\dot{e}/e = \dot{\varpi}/\varpi = \dot{u}/u = 0$. This gives us as equilibrium conditions:

$$\begin{aligned} g(\varpi) &= \alpha + \eta \\ f(e) &= \alpha \\ g(\varpi) &= h(u) \end{aligned} \tag{13}$$

These conditions are well-known requirements that show how the growth rate of output adjusts to the so-called "natural growth rate" and to capital accumulation, while a stable income distribution is only possible if real wages growth at the same pace as labour productivity. The interaction between labour and goods markets is intermediate by the wage-share. Hence, we can state and prove the following Proposition regarding the existence and uniqueness of an internal equilibrium.

Proposition 1 *The dynamic system (13) has a unique internal equilibrium point* (e^*, ϖ^*, u^*) *that satisfies:*

$$\begin{aligned} e^* &= f^{-1}(\alpha) \\ \varpi^* &= g^{-1}(\alpha + \eta) \\ u^* &= h^{-1}(\alpha + \eta) \end{aligned}$$

Proof. See Mathematical Appendix. ■

The economic intuition works as follows. The natural growth rate determines the long-run growth trend, $\alpha + \eta$. The whole system adjusts to this trend.[5] For instance, firms adjust their capacity utilisation or output-capital ratio taking into account the strength of the accelerator effect. A high h_u is related to a lower u^* because firms can easily adjust their capital stock to changes in aggregate demand and, therefore, there is no need to maintain high utilisation levels.

A similar mechanism determines the equilibrium rate of employment. Highly combative workers can obtain strong real wage increases. This means that small variations of employment by firms are potentially harmful to profitability because of their impact on the wage-share. In this way, a high f_e is related to a lower e^*. However, it is important to emphasise that the slope of function f captures how combative workers are and not necessary unionisation levels. Finally, for a given bargain power of workers, the wage-share adjusts to the natural growth trend in a process intermediated by the capacity of firms to increase production, g_{ϖ}.

In what regards the unique internal equilibrium point, we can now state and prove the following Proposition about its local stability.

Proposition 2 *In the neighbourhood of the internal equilibrium point* (e^*, ϖ^*, u^*)*, the dynamic system (13) always has a negative real root and a pair of pure imaginary roots, thus admitting a family of periodic solutions.*

Proof. See Mathematical Appendix. ■

[5] Different authors have discussed on theoretical and empirical grounds how to endogenise the natural growth rate. For a recent review of the literature, see Tavani and Zamparelli (2017).

The combination of a negative real root and a pair of pure imaginary roots is a necessary condition for obtaining persistent and bounded fluctuations, giving rise to a *converging vortex spiral* (see Reyn, 1964). As we will show in the next section, different initial conditions lead to spiral convergence to stable orbits of different amplitude. Still, one should keep in mind that we are not dealing with plane centres where each point belongs to a closed orbit from which other trajectories neither approach nor diverge. This last situation requires the trace and determinant of the Jacobian matrix to be equal to zero, which is not our case. Furthermore, our results do not depend on a critical value for any parameter of the model and are in line with Goodwin's (1967) aim of generating persistent endogenous cycles.

5 Empirical estimation and numerical simulations

We proceed by presenting an econometric exercise that is used in a second step to calibrate our theoretical model. Departing from what we did in the previous section, we will avoid working with filtered data. By means of numerical simulations and using our estimates, is our purpose to evaluate the empirical relevance of the model developed in the previous section.

5.1 Empirical estimation

Considerable attention has been paid over the past decades to assess the existence of relationships in level between variables. Different approaches have been adopted, such as the Engle-Granger two-step residual based procedure or the Johansen system-based reduce rank regressions. It must be noted, however, that those tests require series to be unequivocally non-stationary and integrated of the same order. This might be a problem when it is not known with certainty whether the underlying regressors are trend or first-difference stationary.

Hence, we make use of the Autoregressive Distributive Lag (ARDL) estimator developed by Pesaran and Shin (1998) and later extended by Pesaran et al. (2001). This methodology has several advantages over other cointegration methods as it allows the undertaking of analysis regardless of whether the variables are a mixture of stationary, $I(0)$, and integrated of order one, $I(1)$. Moreover, it also allows us to work with the original series without any detrending procedure. We performed the traditional Augmented Dickey-Fuller (ADF) and the Dickey-Fuller test with GLS detrending (DF-GLS) to verify that series are at most integrated of order one. Results are reported in the Empirical Appendix.

At this point, we need to choose functional forms for the three behavioural equations of the model, namely, $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$. We specify these functions as follows:

$$f(e) = -f_1 + f_2 e \tag{14}$$

$$g(\varpi) = g_1 \varpi \tag{15}$$

$$h(u) = -h_1 + h_2 u \tag{16}$$

where all functional forms we have chosen are linear to emphasise that the dynamics obtained are not due to ad-hoc non-linearities. The system is intrinsically non-linear as a result of the interaction between its basic structure and the adopted behavioural rules. Still, notice that for a sufficiently low employment rate or capacity utilisation, the growth rate of real wages and capital accumulation will be negative. This is because it is unreasonable to suppose that firms continue to invest when utilisation equals zero, or that workers can obtain real wage increases when there is no employment.

Table 1: Employment rate dynamic equation

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
Emp.(-1)	1.050490	0.010948	95.95669	0.0000
Emp.Wage.Share.(-1)	-0.091359	0.019724	-4.631923	0.0000
DUMMY80	$-7.01E-05$	0.000462	-0.151637	0.8796

R-squared	0.946199	Mean dependent var	0.941770
Adjusted R-squared	0.945802	S.D. dependent var	0.016319
S.E. of regression	0.003799	Akaike info criterion	-8.297242
Sum squared resid	0.003911	Schwarz criterion	-8.257682
Log likelihood	1139.722	Hannan-Quinn criter.	-8.281364
Bounds Coint. (F-statistic)	10.87055		

The dynamic system can be rewritten as:

$$\begin{aligned} \frac{\dot{e}}{e} &= -g_1\varpi - \alpha - \eta \\ \frac{\dot{\varpi}}{\varpi} &= -f_1 + f_2 e - \alpha \\ \frac{\dot{u}}{u} &= -g_1\varpi + h_1 - h_2 u \end{aligned} \tag{17}$$

which in discrete form is equivalent to:

$$e_t = \beta e_{t-1} - g_1 e_{t-1}\varpi_{t-1} \tag{18}$$

$$\varpi_t = \gamma\varpi_{t-1} + f_2\varpi_{t-1}e_{t-1} \tag{19}$$

$$u_t = -g_1 u_t \varpi_{t-1} + \rho u_{t-1} - h_2 u_{t-1}^2 \tag{20}$$

where $\beta = 1 - \alpha - \eta$, $\gamma = 1 - f_1 - \alpha$, and $\rho = 1 + h_1$.

When presenting the dynamic model, a continuous time approach was preferred motivated by Gandolfo (2009, pp. 568-573). For instance, although individual economic decisions are in general made in discrete time intervals, it is difficult to believe that they are coordinated in such a way as to be perfectly synchronised. Moreover, a specification in continuous time is particularly useful for the formulation of dynamic adjustment processes based on excess demand, and it is interesting to note that the first writers on the topic explicitly advocated the use of continuous time models. However, once we move on to empirics, data is available only in discrete form and, therefore, we are obliged to rewrite the model as such.

In Section 3, we identified several structural breaks in time series using the Bai-Perron test. Hence, in the present exercise, we have included a dummy variable that assumes value 1 for years after 1980 and 0 otherwise. The year 1980 was chosen as different studies have pointed out to important transformations in the US economy since then. Table 1 reports our estimates of Eq. (18).

All coefficients have the expected sign and are statistically different from zero. For instance, β is fairly close to one, while the interaction between income distribution and employment in $t-1$ reduces the rate of employment in t. Also notice that the F-statistic of the Bounds

Table 2: Wage-share dynamic equation

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
Wage.Share(-1)	0.912191	0.026246	34.75589	0.0000
Emp.Wage.Share(-1)	0.093143	0.027661	3.367256	0.0009
DUMMY80	0.000211	0.000497	0.425548	0.6708

R-squared	0.906250	Mean dependent var	0.552758
Adjusted R-squared	0.905558	S.D. dependent var	0.012382
S.E. of regression	0.003805	Akaike info criterion	−8.294096
Sum squared resid	0.003924	Schwarz criterion	−8.254536
Log likelihood	1139.291	Hannan-Quinn criter.	−8.278217
Bounds Coint. (F-statistic)	6.064843		

cointegration test rejects the null of no cointegration at 1% (see Pesaran et al., 2001, and Narayan, 2005, for critical values of the respective asymptotic distribution). Even though we are interested in the short-run coefficients (and not in the long-run relationship which we do not report here), cointegration guarantees that the omitted variable issue does not affect the reliability of our estimates. The reason for this is that an omitted variable will either be stationary – in which case the estimated coefficients are invariant to its inclusion – or it will be non-stationary – in which case we will not be able to obtain a stable cointegrating relationship if we leave it out.[6]

Table 2 reports our estimates of Eq. (19). Coefficient γ is as expected close to one and the interaction between employment and income distribution in $t-1$ has the expected and statistically significant positive impact on the wage-share in t. Furthermore, we fairly reject the null of no cointegration at 1%. Results are in line with those obtained in the previous section and give support to the profit-squeeze mechanism.

The final step consists in estimating the last dynamic equation of the model which corresponds to capacity utilisation. Table 3 reports our main findings. Coefficients of interest are statistically different from zero and have the expected signs. The interaction between the utilisation index with income distribution in $t-1$ hurts capacity utilisation in t. Our estimation of h_2 shows the importance of the accelerator effect on investment and capital accumulation. Precisely because investment strongly responds to u, there is an increase in accumulation that *ceteris paribus* reduces the level of utilisation.

Residuals of all ARDL regressions were checked for serial correlation to assess valid inference and not spurious relations. If residuals are correlated, the estimated coefficients would be biased and inconsistent. We conclude that our estimates are consistent. We report those last tests in the Empirical Appendix.

[6] Only if an omitted variable is strongly correlated with one of the variables in the cointegration analysis we can end up with spurious estimates. For a further discussion and references on the econometric properties of the time-series approach and its advantages/disadvantages in comparison to cross-country analysis, see Gobbin and Rayp (2008).

Table 3: Capacity utilisation dynamic equation

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
Util(-1)	1.208181	0.059971	20.14607	0.0000
Util.Wage.Share(-1)	−0.277316	0.103545	−2.678219	0.0080
Util^2(-1)	−0.061608	0.030414	−2.025654	0.0442
DUMMY80	−0.005726	0.002467	−2.321304	0.0213

R-squared	0.925150	Mean dependent var	0.803007
Adjusted R-squared	0.923999	S.D. dependent var	0.042086
S.E. of regression	0.011602	Akaike info criterion	−6.055324
Sum squared resid	0.026250	Schwarz criterion	−5.989127
Log likelihood	606.5047	Hannan-Quinn criter.	−6.028532
Bounds Coint. (F-statistic)	4.681756		

5.2 Numerical Simulations

With these results in hands, we reconvert the model to its continuous-time form to illustrate and provide an economic interpretation of the *converging vortex spiral*, whose existence was proved in the last section. Substituting our estimates in Eqs. (18) to (20) and rewriting the system as in (17), we obtain:

$$\begin{aligned}\frac{\dot{e}}{e} &= -0.091\varpi + 0.05\\ \frac{\dot{\varpi}}{\varpi} &= -0.087 + 0.093e\\ \frac{\dot{u}}{u} &= 0.2 - 0.27\varpi - 0.06u\end{aligned} \tag{21}$$

with the respective equilibrium point given by $(e^*, \varpi^*, u^*) = (0.935, 0.55, 0.85)$.

Taking as initial conditions $(e_0, \varpi_0, u_0) = (0.95, 0.6, 0.9)$ and $(e_0, \varpi_0, u_0) = (0.915, 0.57, 0.825)$, Fig. 9 shows trajectories converging to the correspondent stable orbits. They are supposed to capture two specific moments over the business cycle. In the first one, the rate of employment, wage-share, and utilisation are above their equilibrium values. On the other hand, the second case corresponds to a situation in which all variables are below equilibrium. Our system provides a representation of a weak type of path dependence and, therefore, of the statement that "history matters". The reason for this is that different initial conditions lead to cycles of different amplitude. Still, one should keep in mind that the obtained stable orbits are not plane centres. As shown by Reyn (1964), plane centres require the trace and determinant of the Jacobian matrix to be equal to zero, which is not our case.

After dropping the first interactions, we report in Fig. 10 the respective cyclical time series. The blue line is always more volatile than the red one basically because the initial conditions are more distant from the fixed point. Cycles are different in their amplitude but similar in their time length. In both cases, fluctuations are inside the $[0, 1]$ interval. This is particularly interesting (and important to emphasise) because the parameters used in the numerical simulations come directly from our estimations without any further manipulation.

Figure 9: Converging vortex spiral for $(e_0, \varpi_0, u_0) = (0.95, 0.6, 0.9)$ in blue and $(e_0, \varpi_0, u_0) = (0.915, 0.57, 0.825)$ in red

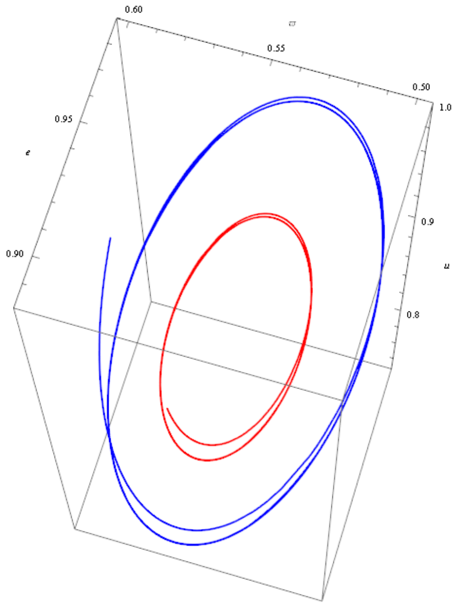
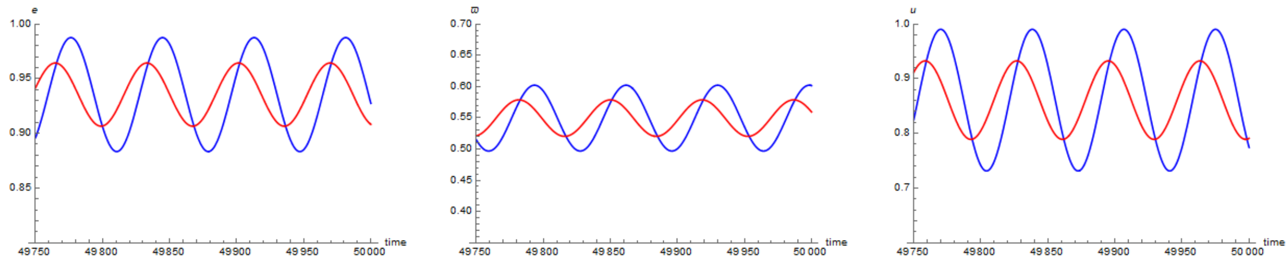


Figure 10: Time series for $(e_0, \varpi_0, u_0) = (0.95, 0.6, 0.9)$ in blue and $(e_0, \varpi_0, u_0) = (0.915, 0.57, 0.825)$ in red



Taking a closer look at the figure above, we can attempt to sketch a description of the dynamic interactions among the three variables along any given cycle. The basic profit-squeeze mechanism is preserved with a somehow new flavour. Expansion of employment leads to an increase in the bargaining power of workers increasing real wages above labour productivity gains. This results in a higher wage-share which implies rising production costs as well as a reduction in the capacity of firms to respond to positive demand shocks. At a certain point, firms reduce the growth rate of output and, consequently, employment rates and capacity utilisation fall. A reduction in e reduces the wage-share which in turn makes it possible recovery of $g(\cdot)$. Firms also adjust their capital accumulation taking into account the accelerator effect. As a result, we observe higher volatility in the goods market than in the labour market or income distribution.

6 Final considerations

In the past forty years, Goodwin's (1967) distributive cycle model has consolidated itself among alternative theories of growth and distribution as a "system for doing macrodynamics". More than one hundred contributions have extended its basic structure in almost all possible directions. A significant amount of research has also been devoted to investigating the empirical relevance of the profit-squeeze mechanism that underlines the theoretical model. We have aimed in this article to provide some new empirical and theoretical insights to an intensively studied topic.

VAR models have been used to study profit-squeeze cycles since at least Goldstein (1999) and Barbosa-Filho and Taylor (2006). However, it must be noted that most exercises rely on Hodrick-Prescott (HP) filtered time series. It is well-known by now that the HP filter produces series with spurious dynamic relations. Recently, Hamilton (2018) has strongly argued against its use providing a simple alternative that successfully separates trend and cycle with none of the HP drawbacks. Hence, in this paper, we have compared the results delivered by both methodologies using quarterly data for the United States between 1948-67 and 2016. Distributive cycles were found to be robust to different filters when confronting employment rates and the wage-share.

Furthermore, we also developed an extension of the original growth-cycle model that includes the employment rate, wage-share, and capacity utilisation as endogenous variables. We introduced an independent investment function and allowed output's growth rate to follow what we referred to as "Skott's rule". We showed analytically that our system *always* admits a family of periodic solutions. This result holds independently of the functional forms and the chosen parameter values. We also estimated the model econometrically using the Autoregressive Distributive Lag (ARDL) approach. Through numerical simulations and using our estimates, we were able to confirm that fluctuations are persistent and bounded.

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Empirical Appendix

We begin reporting in table A1 the respective Autocorrelation LM tests for each VAR model estimated in Section 2. In all cases, we cannot reject the null hypothesis of no serial correlation at lags 1 to l. Hence, the VAR models estimated are not spurious.

Table A1: VAR – Autocorrelation LM tests

Model	VAR(2) – e vs ϖ, HP		VAR(3) – e vs ϖ, Ham.		VAR(4) – u vs ϖ, HP		VAR(4) – u vs ϖ, Ham.	
Lags	Rao F-stat	Prob.	Rao F-stat	Prob.	Rao F-stat	Prob.	Rao F-stat	Prob.
1	0.881194	0.4749	0.275282	0.8939	0.384649	0.8196	1.247752	0.2905
2	0.863880	0.5469	0.585587	0.7900	0.326425	0.9557	0.704794	0.6873
3	0.887386	0.5598	0.600950	0.8419	0.501166	0.9137	1.382161	0.1725
4	-	-	0.548934	0.9203	0.436097	0.9724	1.176318	0.2852
5	-	-	-	-	0.481538	0.9726	1.177983	0.2712

The respective VAR coefficients as well as the inverse roots of the Autoregressive characteristic polynomial can be easily checked from the Eviews Workfile.

Table A2 reports a summary of ADF and DF-GLS unit root tests in first difference for the employment rate, wage-share, and capacity utilisation. Lag length was chosen using the Schwarz information criterion with a maxlag= 14. Outcomes indicate that series are at most integrated of order one.

Table A2: Unit root tests (1st difference)

Employment rate				
Method	Intercept (t-statistic)	Prob.	Trend/Intercept (t-statistic)	Prob.
ADF	-7.997830	<0.01	-8.000229	<0.01
DF-GLS	-2.742298	<0.01	-7.303117	<0.01
Wage-share (t-statistic)				
Method	Intercept (t-statistic)	Prob.	Trend/Intercept (t-statistic)	Prob.
ADF	-20.80148	<0.01	-20.81807	<0.01
DF-GLS	-20.68952	<0.01	-20.89264	<0.01
Capacity utilisation (t-statistic)				
Method	Intercept (t-statistic)	Prob.	Trend/Intercept (t-statistic)	Prob.
ADF	-7.446137	<0.01	-7.424846	<0.01
DF-GLS	-4.973135	<0.01	-6.480929	<0.01

To assess a valid inference and not spurious regressions, residuals of all three ARDL regressions were checked for serial correlation making use of standard ADF unit root tests. Once more, lag length was chosen using the Schwarz information criterion with a maxlag= 14. If residuals are correlated the estimated coefficients would be biased and inconsistent. Since errors were found to be stationary, we conclude that our estimates are consistent.

Table A3: Unit root tests (levels)

Employment rate dynamic eq., residuals				
Method	Intercept (t-statistic)	Prob.	Trend/Intercept (t-statistic)	Prob.
ADF	-8.206466	<0.01	-8.193350	<0.01
DF-GLS	-8.150246	<0.01	-8.189363	<0.01
Wage-share dynamic eq., residuals				
Method	Intercept (t-statistic)	Prob.	Trend/Intercept (t-statistic)	Prob.
ADF	-21.61119	<0.01	-21.57213	<0.01
DF-GLS	-2.117763	<0.05	-6.724009	<0.01
Cap. utilisation dynamic eq., residuals				
Method	Intercept (t-statistic)	Prob.	Trend/Intercept (t-statistic)	Prob.
ADF	-7.675994	<0.01	-7.733431	<0.01
DF-GLS	-4.971238	<0.01	-6.349136	<0.01

Mathematical Appendix

Proof of Proposition 1

The demonstration of this Proposition follows almost imediately from the equilibrium conditions stated in (13). The growth rate of real wages is determined by $f : \Re \rightarrow \Re$ which is monotonically increasing in e. Therefore, its inverse is also an increasing function and we obtain $e^* = f^{-1}(\alpha)$. The growth rate of output follows Skott's mechanism and is determined by $g : \Re \rightarrow \Re$ which is monotonically decreasing in ϖ. Hence, its inverse is also a decreasing function and we obtain $\varpi^* = g^{-1}(\alpha + \eta)$. Capital accumulation is monotonically increasing on the accelerator effect, u, with $h : \Re \rightarrow \Re$. It follows that $u^* = h^{-1}(g(g^{-1}(\alpha + \eta))) = h^{-1}(\alpha + \eta)$. Finally, in order to obtain values with economic meaning, we have to impose $0 < e^*,\ \varpi^*,\ u^* < 1$.

Proof of Proposition 2

In this demonstration, we first linearise the dynamic system around the internal equilibrium point, so as to obtain:

$$\begin{bmatrix} \dot{e} \\ \dot{\varpi} \\ \dot{u} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & J_{12} & 0 \\ J_{21} & 0 & 0 \\ 0 & J_{32} & J_{33} \end{bmatrix}}_{J^*} \begin{bmatrix} e - e^* \\ \varpi - \varpi^* \\ u - u^* \end{bmatrix}$$

where the elements of the Jacobian matrix, J^*, are such that:

$$\begin{aligned} J_{12} &= g_{\varpi} e^* < 0 \\ J_{21} &= f_e \varpi^* > 0 \\ J_{32} &= g_{\varpi} u^* < 0 \\ J_{33} &= -h_u u^* < 0 \end{aligned}$$

so that the characteristic equation can be written as:

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0$$

where the coefficients are given by:

$$\begin{aligned}
b_1 &= -trJ^* = -J_{33} > 0 \\
b_2 &= \begin{vmatrix} 0 & 0 \\ J_{32} & J_{33} \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & J_{33} \end{vmatrix} + \begin{vmatrix} 0 & J_{12} \\ J_{21} & 0 \end{vmatrix} > 0 \\
b_3 &= -\det J^* = -\begin{vmatrix} 0 & J_{12} & 0 \\ J_{21} & 0 & 0 \\ 0 & J_{32} & J_{33} \end{vmatrix} > 0
\end{aligned}$$

The necessary and sufficient condition for the local stability of (e^*, ϖ^*, u^*) is that all roots of the characteristic equation have negative real parts, which, from Routh-Hurwitz conditions, requires:

$$b_1,\ b_2,\ b_3 > 0 \text{ and } b_1 b_2 - b_3 > 0$$

The crucial condition for local stability becomes the last one. Through direct computation, we find that:

$$b_1 b_2 - b_3 = -h_u u^* g_{\varpi} e^* f_e \varpi^* + h_u u^* f_e \varpi^* g_{\varpi} e^* = 0$$

This means that we actually have a negative real root and a pair of pure imaginary roots thus admiting a family of periodic solutions.