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Alternative Approaches to Technological Change when Growth is BoPC

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# Alternative Approaches to Technological Change when Growth is BoPC*

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**Abstract**

Dávila-Fernández and Sordi (2018) have recently extended Goodwin's (1967) model to study the interaction between distributive cycles and international trade for economies in which growth is Balance-of-Payments Constrained. This paper examines the implications of adopting (i) Kaldor-Verdoorn's law, and (ii) classical-Marxian technical change to the main results of the model. The Kaldorian specification leaves the system with no internal equilibrium solution while the Marxian specification makes it stable. A Hopf bifurcation analysis shows that the combination of both formulations might give rise to persistent and bounded cyclical fluctuations. Given the lack in the literature of reliable estimates for the classical-Marxian case, we provide a panel-VAR estimation for a sample of 16 OECD countries between 1980-2012 that gives some support to its central argument. Our estimates were used to calibrate the models developed in the first part of the paper.

**Keywords:** Growth cycle, Goodwin, Thirlwall's law, Distributive cycles, Hopf bifurcation, Technical change.

**JEL:** E12; E32; O40

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# 1 Introduction

The analysis of the role of technical change in growth processes has been for a long time of central importance in economic theory. Different approaches have been proposed over the years to study distinct aspects of the phenomenon. Among alternative theories of growth and distribution, two particular viewpoints on the evolution of technology deserve special attention given their influence to Marxian and Post-Keynesian macrodynamic modelling. On the one hand, according to Kaldor-Verdoorn's law, labour productivity grows in line with output's growth rate or capital accumulation. On the other hand, classical-Marxian technical change states that factor productivity growth rates respond positively to factor cost shares.

Moreover, the seminal paper on the growth cycle by Goodwin (1967) has consolidated itself throughout the past decades as a powerful "system for doing macrodynamics". In the last fifty years, a significant number of contributions have tried to generalise its formulation in all possible directions. The introduction of technical change considerations has not been an exception, especially in what concerns the dependence of labour productivity growth on the share of labour on income, as initially discussed by Shah and Desai (1981), and further elaborated by van der Ploeg (1987), Foley (2003), Julius (2006) or Tavani and Zamparelli (2015; 2018), among others.

Recognising that the relationship between growth and income distribution has been a central issue for non-neoclassical theories of social conflict, Dávila-Fernández and Sordi (2018, hereafter DF&S), have extended Goodwin's (1967) model to study the interaction between distributive cycles and international trade for economies in which growth is Balance-of-Payments Constrained (BoPC), i.e. follows Thirlwall's (1979) law. The authors have demonstrated that under very general conditions and without relying on price-adjustment mechanisms, output's growth rate fluctuates around the external constraint while preserving the persistent endogenous oscillations that characterise the original growth cycle model.

However, these results strongly rely on a learning-by-doing mechanism in which changes in labour productivity are a function of the level of effective capacity utilisation. DF&S made the case that even though technological progress to some extent is capital embodied, machines must be operating in order to productivity gains to be effectively incorporated. Still, the implications of adopting different specifications of technical change have not been discussed. It is our purpose in this article to address some of those issues, especially regarding the local stability properties of the system.

We proceed in three steps. First, and following the Kaldorian literature, we introduce labour productivity gains as a function of capital accumulation or output's growth rate. In a second step, we adopt a classical-Marxian technical change approach and make both labour and capital productivity growth rates to depend positively on factor cost shares. Finally, a combination of these two specifications is studied. The Kaldorian case leaves the system with no internal equilibrium solution while the Marxian specification makes the system stable. Furthermore, a Hopf bifurcation analysis shows that the combination of both formulations might give rise to persistent and bounded cyclical paths.

When it comes to the empirics of technical change, there is robust literature on Kaldor-Verdoorn's law that gives support to its formulation (e.g. McCombie and De Ridder, 1983; 1984; Angeriz et al, 2008; 2009; Romero and McCombie, 2016a; Magacho and McCombie, 2017; Romero and Britto, 2017). The same cannot be said about the classical-Marxian construction. Therefore, it is also our purpose to address the relation between functional income distribution and the evolution of technology empirically.

We applied panel Vector Autoregression (pVAR) techniques to a sample of 16 OECD countries between 1980 and 2012. Orthogonalised Impulse Response Functions (OIRFs) indicate that one standard deviation impulse on the profit-share/wage-share ratio decreases labour productivity growth rates by 2% though it has a neglectable effect on capital productivity. Moreover, changes in income distribution are responsible for up to 40% of changes in labour productivity growth rates. To the best of our knowledge, we are the first to provide more consistent estimations that give some support to the classical-Marxian technical change argument.

Our estimates are used to calibrate the models developed in the first part of the paper. Numerical simulations show that the main dynamics of DF&S are robust to alternative specifications of technological change. Furthermore, the introduction of a forcing term motivated by Goodwin's discussion of "Schumpeter clock" gives rise to irregular fluctuations similar to those observed in real data.

This article is organised as follows. In the next section, we revisit DF&S main dynamic equations discussing the implications of adopting different specifications for technical change analytically. Section 3 brings our econometric exercise that provides some empirical support to the Marxian argument. Readers not interested in the theoretical discussion can go directly to this part. In section 4 we use those estimates to calibrate our theoretical model. Some final considerations follow.

# 2 Kaldorian and classical-Marxian technical change

Suppose an economy in which output is produced with capital and labour inputs. We characterised the production technology by the pair $(\rho, q)$ where $\rho$ and $q$ represent capital and labour productivities, respectively. Technical progress is expressed as a combination of changes in both variables. In this section, we briefly revisit the main structure of DF&S model and study the implications of adopting the Kaldorian and classical-Marxian formulations.[1]

Define $e$ as the rate of employment, $Y$ as the level of output, $N$ represents total labour force, $\varpi$ is the wage-share, $w$ corresponds to real wages, $u$ is the level of capacity utilisation, and $K$ is the capital stock. The employment rate is defined as $e = L/N$ where $L$ stands for labour employed in production. Capacity utilisation is given by $u = Y/Y^*$ with $Y^*$ as production at full capacity. Labour and capital productivity are defined as $q = Y/L$ and $\rho = Y^*/K$, respectively. The main structure of the dynamic system is given by:[2]

$$\frac{\dot{e}}{e} = \frac{\dot{Y}}{Y} - \frac{\dot{q}}{q} - \frac{\dot{N}}{N} \tag{1}$$

$$\frac{\dot{\varpi}}{\varpi} = \frac{\dot{w}}{w} - \frac{\dot{q}}{q} \tag{2}$$

$$\frac{\dot{u}}{u} = \frac{\dot{Y}}{Y} - \frac{\dot{K}}{K} - \frac{\dot{\rho}}{\rho} \tag{3}$$

[1] See Tavani and Zamparelli (2017) for a comprehensive and recent survey on endogenous technical change in alternative theories of growth and distribution.

[2] For any variable $x$, $\dot{x}$ indicates its time derivative ($dx/dt$).

**Assumption 1** The balance-of-payments is always in equilibrium so that output's growth rate is demand-side determined and given by Thirlwall's law, i.e.

$$\frac{\dot{Y}}{Y} = y_{bp}$$

Though it is true that there are short-run deviations from such a long-run trend, this assumption allows us to isolate and focus on the implications of adopting different specifications of technical change.[3] Another way of thinking of it is to implicitly assume that the adjustment of the rate of growth of demand to the external constraint takes places through changes in the rate of domestic absorption, more specifically, through expansionary or contractionary fiscal policy.

When the economy exceeds the BoPC rate of growth, i.e. $\dot{Y}/Y > y_{bp}$, and hence a current account deficit emerges, the government adopts a contractionary fiscal policy to correct the external deficit. This fall goes in hand with a perception that there will be a crisis shortly if the government fails to curb the growth of imports. Guarini and Porcile (2016), for instance, referred to instability in the exchange rate market and outflows of foreign capital that follow this perception. They argued that crowding in effects of government expenditures may also induce a similar fall of private expenditure. Inversely, for $\dot{Y}/Y < y_{bp}$, the government has space for more expansionary fiscal policy. In fact, from the expenditures identity, we have that the difference between savings and investment is always equal to the difference between exports and imports. Hence, a balance-of-payments that is always in equilibrium requires that domestic savings must be equal to the firm's investment plans. Since we are going to introduce an independent investment function, we need savings to be the adjustment variable.

Eqs. (1)-(3) are direct manipulations of accounting identities.[4] In steady state, $\dot{e}/e = \dot{\varpi}/\varpi = \dot{u}/u = 0$. For positive values of employment, wage-share, and utilisation, we obtain the following equilibrium conditions:

$$y_{bp} = \frac{\dot{q}}{q} + \frac{\dot{N}}{N} \tag{4}$$

$$\frac{\dot{w}}{w} = \frac{\dot{q}}{q} \tag{5}$$

$$y_{bp} = \frac{\dot{K}}{K} + \frac{\dot{\rho}}{\rho} \tag{6}$$

From Eq. (4), we have that output's growth rate must equal the natural growth rate in order to deliver a constant employment rate. It establishes a correspondence between

[3]Although the BoPC growth model addresses the investigation of the long-run, it also has profound implications for short-run dynamics. For a formal analysis of how deviations from long-run paths are generated and corrected, see Soukiazis et al. (2012; 2014), Garcimartin et al. (2016), and DF&S. The last one shows that as long as savings and investment growth at the same rate, output's growth rate follows Thirlwall's law.

[4]Suppose a Leontief production function such that $Y = \min\{\rho K u; qNe\}$. The efficiency condition states that $Y = \rho K u = qNe$. Notice that, in a sense, this is an accounting identity because $Y = (Y^*/K)\,K\,(Y/Y^*) = (Y/L)\,N\,(L/N)$. Taking logarithms and time derivates of $Y = \rho K u$ and $Y = qNe$, we obtain Eqs. (1) and (3). Finally, the wage-share is defined as the share of wages in output, $\varpi = wL/Y = w/q$. Taking logarithms and time derivates we arrive at Eq. (2).

the capacity of the economy to expand production and how this interacts with changes in the supply of labour. Eq. (5) consists of the equilibrium condition to distributive conflict. A stable income distribution requires real wages to grow at the same pace as labour productivity. Finally, and analogously to the first expression, a constant level of capacity utilisation is the result of output growing at the same rate of the sum between capital accumulation and productivity. It shows how the capacity of the economy to expand production interacts with changes in capital accumulation. We are now ready to study how this structure responds to different behavioural assumptions.

## 2.1 Labour productivity and Kaldor-Verdoorn's law

It has been observed that changes in factor productivity do not occur symmetrically through time. For a large group of capitalist countries and over long periods of time, capital productivity has remained constant or even declined while labour productivity shows a clear positive trend.

The Kaldorian tradition has indeed paid little attention to changes in capital productivity. In a well-known paper, Kaldor (1961, p. 178) himself stated as one of his "stylised facts" a constant capital-output ratio, i.e. constant capital productivity. In what concerns labour, Kaldor-Verdoon's law states that labour productivity growth rates are directly related either to capital accumulation or output growth. The basic idea goes back to Adam Smith's pin factory and highlights the importance of dynamic returns to scale or macroeconomic increasing returns that are involved in learning-by-doing processes.[5] Two alternative specifications are:

$$\frac{\dot{q}}{q} = G\left(\frac{\dot{K}}{K}\right), \; G_{\dot{K}/K} > 0 \tag{7}$$

or

$$\frac{\dot{q}}{q} = G\left(\frac{\dot{Y}}{Y}\right), \; G_{\dot{Y}/Y} > 0 \tag{8}$$

where both functions are monotonically increasing in their main arguments.

Five behavioural relations are needed in order to close the model. Set $\dot{q}/q = G\left(\dot{K}/K\right)$ as in Eq. (7) and $\dot{\rho}/\rho = 0$. Furthermore, make the labour force grow at an exogenous rate, $n$. Assuming a real wages Phillips curve, $\dot{w}/w = F(e)$ with $F'(\cdot) > 0$, and a conventional capital accumulation function, $\dot{K}/K = H(\varpi, u)$ with $H_\varpi < 0$ and $H_u > 0$, the dynamical system (1)-(3) becomes:

$$\begin{aligned} \dot{e} &= e[y_{bp} - G\left(H(\varpi, u)\right) - n] & (9)\\ \dot{\varpi} &= \varpi[F(e) - G\left(H(\varpi, u)\right)] & (10)\\ \dot{u} &= u\left[y_{bp} - H(\varpi, u)\right] & (11) \end{aligned}$$

For a given rate of growth of output, an increase in capital accumulation increases labour productivity reducing employment rates and the wage-share. A reduction in $\varpi$ rises the profitability of investment. This leads to an increase in the growth rate of the capital stock which in turn implies higher labour productivity. Even though the last equation indicates

[5] For a comprehensive review of Kaldor-Verdoorn's law see McCombie et al. (2002).

that $u$ stabilises itself, increases in capacity utilisation seem to trigger instability through an increase in capital accumulation.

In steady state, $\dot{e}/e = \dot{\varpi}/\varpi = \dot{u}/u = 0$. Equilibrium conditions for this first set of behavioural equations are:

$$y_{bp} = G(H(\varpi, u)) + n \tag{12}$$
$$F(e) = G(H(\varpi, u)) \tag{13}$$
$$y_{bp} = H(\varpi, u) \tag{14}$$

Therefore, we can state and prove the following Proposition regarding the existence of a non-trivial equilibrium solution.

**Proposition 1** *The probability that the dynamic system (9)-(11) has an internal non-trivial equilibrium solution is zero.*

***Proof.*** *Substituting Eq. (14) in (12) we obtain $y_{bp} = G(y_{bp}) + n$. Since $y_{bp}$ and $n$ are parameters in the model, only by chance this expression is actually true. In a continuous sample space, the probability of any elementary event, consisting of a single outcome, is zero.* ■

At this point, it is quite intuitive to realise that a similar problem arises when adopting the second specification of Kaldor-Verdoorn's law. Substituting Eq. (8) in the first dynamic equation, and keeping all other behavioural relations, we obtain:

$$\dot{e} = e\left[y_{bp} - G(y_{bp}) - n\right] \tag{15}$$

In steady-state $\dot{e}/e = 0$ and a non-trivial solution exists only if $y_{bp} = G(y_{bp}) + n$. Given that $y_{bp}$ and $n$ are exogenous parameters, it is easy to understand that the equality is unlikely to be satisfied.

Such result comes from the fact that we are fixing $\dot{Y}/Y = y_{bp}$, i.e. the growth rate of output required to maintain the equilibrium in the balance of payments. However, a simple relaxation of this assumption does not come without problems. Suppose output's growth rate does not follow the external constraint. Keeping $\dot{\rho}/\rho = 0$, from Eqs. (4) and (5) it is clear that, in order to obtain a constant rate of employment and capacity utilisation, we need $\dot{Y}/Y = \dot{q}/q + n$ and $\dot{Y}/Y = \dot{K}/K$. Define $\Phi = \dot{Y}/Y - G(\cdot)$. This means that as long as $\Phi$ has an inverse, $\dot{Y}/Y = G\left(\dot{Y}/Y\right) + n = \Phi^{-1}(n)$ and growth becomes supply-side determined. Such result goes against recent empirical evidence indicating that the natural rate of growth is not only endogenous but also determined by the external constraint (see, for example, Lanzafame, 2014).

Setterfield (2006) and Gabriel et al. (2016) among others have tried to overcome the problem using linear specifications of $G(\cdot)$. Suppose $G\left(\dot{Y}/Y\right) = \alpha_0 + \alpha_1\dot{Y}/Y$ where $\alpha_1$ is the so-called Verdoorn coefficient and is assumed to capture the presence of dynamic economies of scale. Hence, they endogenised $\alpha_1$ allowing an adjustment towards the external constraint. Nonetheless, this is still quite unsatisfactory. As shown by McCombie and Spreafico (2016), such interpretation of the linear coefficients is wrong because it implies $G(\cdot)$ to be a sub-product of a neoclassical production function instead of a behavioural relation.[6] They demonstrated that if a linear form is adopted "the intercept *cannot* and *should not* be interpreted as the separate contribution to economic growth of the rate of exogenous technical change" while "the Verdoorn coefficient also *should not* be interpreted as a measure of increasing returns to scale per se" (p. 1131, emphasis added).

Another alternative would be to make the growth rate of the labour force, $n$, an endogenous variable. For OECD countries, for example, this could be justified by immigration. In developing countries, one could make the case that as the excess of labour supply is absorbed by the modern sector, $n$ changes. However, we do not deal with such cases here since endogenising $n$ goes beyond the scope of this article.

## 2.2 Classical-Marxian technical change

Classical-Marxian technological change is based on the assumption that labour-saving or capital-saving innovations depend on the share of labour and capital costs in production. The inspiration for this idea is not purely classical, and the first modern reference is Hicks (1932) – with the induced (or biased) innovation hypothesis – while further contributions include neoclassical (e.g. Kennedy, 1964; Samuelson, 1965; and more recently Funk, 2002; Acemoglu, 2003) and non-neoclassical scholars (see Okishio, 1961; Duménil and Levy, 1995; Foley, 2003; Kemp-Benedict, 2018).

We continue redefining the labour productivity growth rate, such that:

$$\frac{\dot{q}}{q} = G(\varpi), \; G_{\varpi} > 0 \tag{16}$$

$$\frac{\dot{\rho}}{\rho} = J(\varpi), \; J_{\varpi} < 0 \tag{17}$$

Notice that there is a fundamental difference between this case and the first one discussed in the previous subsection. Back then, an increase in the wage-share reduced capital accumulation and as consequence labour productivity growth. Now, changes in income distribution have the opposite effect, at least in what concerns labour. An increase in the wage-share indicates that real wages are higher relative to labour productivity. Hence, firms respond to increasing the search for labour saving techniques.

In a similar scenario to the one adopted for the Kaldorian specifications, make the labour force grow at an exogenous rate $n$, assume a real wages Phillips curve, $\dot{w}/w = F(e)$ with $F'(\cdot) > 0$, and a conventional capital accumulation function, $\dot{K}/K = H(\varpi, u)$ with $H_{\varpi} < 0$ and $H_u > 0$. Finally, making use of Eqs. (16) and (17), we can rewrite the dynamic system (1)-(3) as:

$$\dot{e} = e\left[y_{bp} - G(\varpi) - n\right] \tag{18}$$

$$\dot{\varpi} = \varpi\left[F(e) - G(\varpi)\right] \tag{19}$$

$$\dot{u} = u\left[y_{bp} - H(\varpi, u) - J(\varpi)\right] \tag{20}$$

An increase in the wage-share reduces the rate of employment and capital productivity growth through technical change functions. It also brings down investment's profitability resulting in lower capital accumulation. Lower capital accumulation and productivity rates

[6] Needless to say that the problems of such production functions are well known. For a comprehensive discussion see Petri (2004) and Felipe and McCombie (2013).

increase the level of capacity utilisation. Such an increase may sound counterintuitive but comes from the fact that capacity of production is expanding slower relatively to aggregate demand. On the other hand, higher capacity utilisation increases capital accumulation through the accelerator effect, which reduces $u$, stabilising the system. Furthermore, a reduction in employment rates has a negative impact on real wages because workers are not able to obtain the same wage increases as in the past. This leads to a reduction in the wage-share and also stabilises the system.

In steady state, $\dot{e}/e = \dot{\varpi}/\varpi = \dot{u}/u = 0$. We obtain the following equilibrium conditions:

$$y_{bp} = G(\varpi) + n \tag{21}$$
$$F(e) = G(\varpi) \tag{22}$$
$$y_{bp} = H(\varpi, u) + J(\varpi) \tag{23}$$

Therefore, we can state and prove the following Proposition regarding the existence and uniqueness of a non-trivial equilibrium solution.

**Proposition 2** *The dynamic system (18)-(20) has a unique internal equilibrium point that satisfies*

$$\begin{aligned} e^* &= F^{-1}(y_{bp} - n) \\ \varpi^* &= G^{-1}(y_{bp} - n) \\ y_{bp} &= H\left(G^{-1}(y_{bp} - n), u^*\right) + J\left(G^{-1}(y_{bp} - n)\right) \end{aligned}$$

***Proof.*** *See Mathematical Appendix B1.* ■

The equilibrium solution values are very similar to those of DF&S with one striking difference. While in the original model the wage-share was responsible for adjusting capacity utilisation to the external constraint, now $u$ adjusts itself once $e$ and $\varpi$ are previously determined by the Phillips curve and labour technical change functions. Moreover, notice that in equilibrium the share of wages on income only depends on the shape of $G(\cdot)$. An increase in output's growth rate leads to higher employment and wage-share.

Intuitively, if the economy is growing faster, firms will hire more workers increasing the employment rate. This strengths the position of workers in the wage bargain process and ultimately increases the wage-share. If the increase in output's growth rate is permanent so will be the changes in $e$ and $\varpi$. Additionally, this also causes an increase in the level of capacity utilisation because machines are used more intensively.

It is also worth noting the effect of an increase in the sensitivity of labour productivity on income distribution. If small increases in the wage-share lead to high jumps in labour productivity, the $\varpi$ required in order to match the long-run labour productivity growth trend, $y_{bp} - n$, will be lower. Therefore, an increase in the slope of $G(\cdot)$ causes a reduction in the equilibrium wage-share. If it is easier for firms to find new production techniques when facing increases in labour costs, the bargaining power of workers is reduced and, therefore, they get a smaller piece of the cake.

Such a mechanism has some similarities with a concept put forward not long ago and in a different set up as "Power Biased Technical Change" (see Skott and Guy, 2007). New technologies, in particular, the so-called ICTs, have allowed firms to monitor workers more closely. Even though one could debate to which extend ICTs have increased labour productivity growth rates, a higher capacity of monitoring allows firms to respond quicker and faster to changes in labour costs and could be interpreted as an increase in the slope of $G(\cdot)$. The outcome in both cases involves a reduction in the income share that goes to those affected by the increase in surveillance.

Moreover, a lower wage-share increases capital profitability and, hence, capital accumulation. This means that, for a given $y_{bp}$, equilibrium capacity utilisation will be lower. Higher profitability makes corporations to rely less on the accelerator effect which in turn is reflected in a reduction of utilisation levels. Fig. 1 on the left shows the response of income distribution and capacity utilisation to an increase in $y_{bp}$ while, on the right, we depict their response to an increase in $G_{\varpi}$.

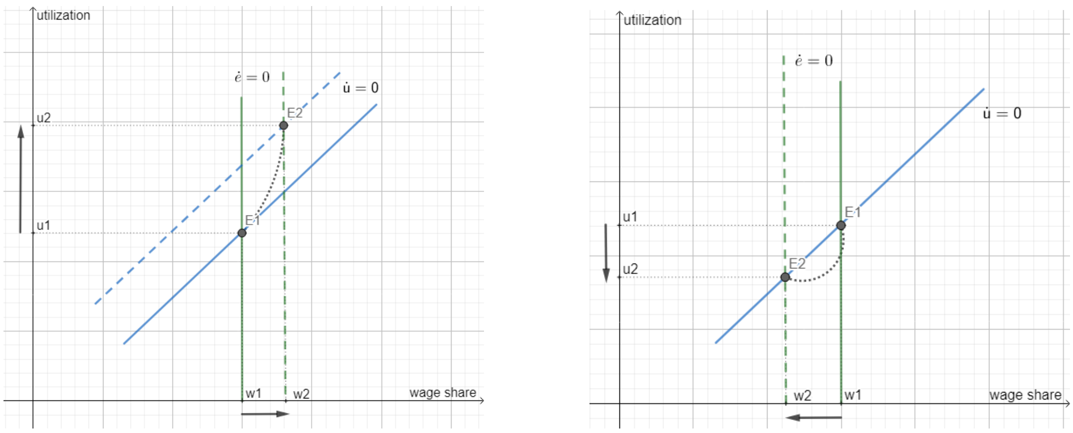



Figure 1: Response of equilibrium to changes in $y_{bp}$ (left) and $G_{\varpi}$ (right)

With regard to the unique internal equilibrium point, we can now state and prove the following Proposition concerning its local stability.

**Proposition 3** *The unique internal equilibrium point of the dynamic system (18)-(20) is locally stable.*

***Proof.*** *See Mathematical Appendix B2.* ■

Adopting a classical-Marxian specification for factor productivity growth changes dramatically the capacity of the model of generating cycles rooted in the functioning of the labour market and the dynamics of distributive conflict. That is, it breaks one of the main results of Goodwin (1967) and also of DF&S. We know that incorporating the induced innovation hypothesis can potentially lead to the disappearance of the growth cycle (see Tavani and Zamparelli, 2017). The reason for this is that increases in the employment rate are immediately corrected through the effect of income distribution on labour productivity. Still, several exercises have shown that under standard parameterisations, the direction of adjustment is not monotonic being characterised by persistent fluctuations of decreasing amplitude.

## 2.3 Combining both approaches

So far we have shown that incorporating Kaldor-Verdoorn's law to our framework leaves the system with no internal equilibrium solution while the Marxian specification makes the system locally stable. However, one may wonder what happens if both approaches were combined. The simplest way to do it is by modelling changes in labour productivity as:

$$\frac{\dot{q}}{q} = G\left(\frac{\dot{K}}{K}, \varpi\right), \; G_{\dot{K}/K} > 0 \text{ and } G_{\varpi} > 0 \tag{24}$$

or

$$\frac{\dot{q}}{q} = G\left(\frac{\dot{Y}}{Y}, \varpi\right), \; G_{\dot{Y}/Y} > 0 \text{ and } G_{\varpi} > 0 \tag{25}$$

where from $\dot{K}/K = H(\varpi, u)$, we have that $G_{\dot{K}/K} = G_H$, and it follows that $G_H H_{\varpi} < 0$ while $G_H H_u > 0$.

**Assumption 2** The sensitivity of labour productivity growth rates to changes in the wage-share is such that:

$$G_{\varpi} > |G_H H_{\varpi}|$$

Increases in the wage-share reduce investment profitability and therefore capital accumulation. On the one hand, there is a reduction in the growth rate of labour productivity through the Kaldor-Verdoorn's effect. On the other hand, the Marxian component implies that an increase in real wages relative to productivity forces capitalists to search for labour saving techniques, increasing the growth rate of labour productivity. Since our econometric exercise reports a positive net impact of cost shares on factor productivity shares, as we will show in the next section, this assumption sounds quite plausible.

Maintaining the previous behavioural relations and making use of Eq. (24), the dynamic system (1)-(3) becomes:

$$\dot{e} = e[y_{bp} - G\left(H(\varpi, u), \varpi\right) - n] \tag{26}$$
$$\dot{\varpi} = \varpi[F(e) - G\left(H(\varpi, u), \varpi\right)] \tag{27}$$
$$\dot{u} = u\left[y_{bp} - H(\varpi, u) - J\left(\varpi\right)\right] \tag{28}$$

An increase in the level of capacity utilisation has a positive effect on capital accumulation through the accelerator. This in turns leads to a reduction in employment rates, in the share of wages, and in $u$ itself. The first two effects are the result of an increase in labour productivity due to the Kaldorian part of the $G(\cdot)$ function. The latter comes from the fact that the capital stock is growing faster relative to aggregate demand. A reduction in the wage-share results in a reduction of labour productivity growth rates as well as an increase in the rate of growth of the capital stock and productivity. This, in turn, pushes employment, wage-share and utilisation up. At this point what seems to be a cycle restarts.

In steady state, $\dot{e}/e = \dot{\varpi}/\varpi = \dot{u}/u = 0$. The respective equilibrium conditions are:

$$y_{bp} = G\left(H(\varpi, u), \varpi\right) + n \tag{29}$$
$$F(e) = G\left(H(\varpi, u), \varpi\right) \tag{30}$$
$$y_{bp} = H(\varpi, u) + J\left(\varpi\right) \tag{31}$$

Contrary to the pure Kaldorian case, we can now state and prove the following Proposition regarding the existence and uniqueness of a non-trivial equilibrium solution.

**Proposition 4** *The dynamic system (26)-(28) has a unique internal equilibrium point that satisfies*

$$\begin{aligned} e^* &= F^{-1}(y_{bp} - n) \\ G(H(\varpi^*, u^*), \varpi^*) &= y_{bp} - n \\ H(\varpi^*, u^*) + J(\varpi^*) &= y_{bp} \end{aligned}$$

***Proof.*** *See Mathematical Appendix B3.* ■

The equilibrium value of the employment rate is the same as in the simple Marxian-biased case. This comes from the fact that in both models employment is solely determined in the labour market. Higher output growth rates or less combative workers are capable of increasing steady-state employment. The last two equations simultaneously determine the wage-share and capacity utilisation. Still, some interesting implications follow regarding comparative statics. When proving Proposition 4, we showed that Eqs. (29) and (31) can be rewritten as $u = \Psi(\varpi)$ and $u = \Theta(\varpi)$, respectively, with $\Psi'(\cdot) < 0$ and $\Theta'(\cdot) > 0$. We can now briefly discuss the effects of changes in the BoPC growth rate and in the shape of $G(\cdot)$.

A relaxation of the external constraint, meaning an increase in $y_{bp}$, moves $\Psi(\cdot)$ to the right and $\Theta(\cdot)$ to the left. This implies an increase of equilibrium capacity utilisation. The net effect on income distribution, however, is indetermined, contrasting with the classical-Marxian case where the net effect was positive. Fig. 2 represents this case.

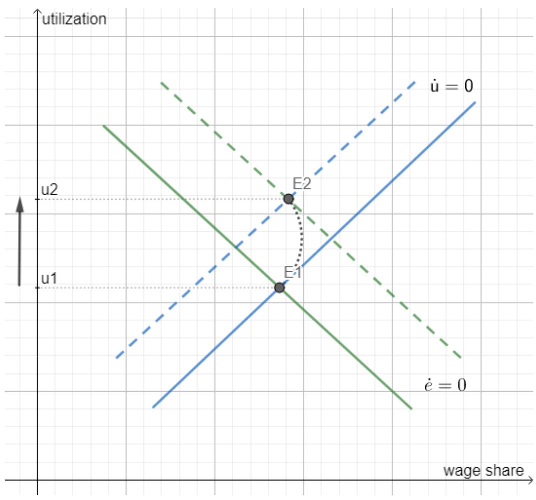



Figure 2: Response of equilibrium to changes in $y_{bp}$

An extensive literature on complexity has shown that there is a positive relationship between economic complexity, product diversification and growth (e.g. Hidalgo et al., 2007; Hausmann et al., 2014). One of the main findings in this literature is that more-sophisticated products are located in a densely connected core whereas less-sophisticated products occupy a less-connected periphery. Furthermore, recent contributions have also pointed out a robust negative correspondence between income inequality and economic complexity (Hartmann et al., 2017; Gala et al., 2017). Our model provides an explanatory mechanism for those findings.

On the one hand, we have that the BoPC growth rate reflects the non-price competitiveness of an economy (or region) which in turn is determined by the complexity and diversification of its productive structure (see Gouvea and Lima, 2010; 2013; Romero and McCombie, 2016b; Dávila-Fernández et al, 2018). On the other hand, observed trends show that the labour income share has typically fallen alongside an increase in income inequality. Countries that have managed to reduce inequality have also shown increases in the labour share (ILO, IMF, OECD and World Bank, 2015; ILO and KIEP, 2015).[7] In light of the results presented so far, as long as the sensitivity of capital accumulation to profitability is relatively weak, there is a positive relationship between $y_{bp}$ and $\varpi$ that explains the relations mentioned above. It is important to notice that in the simple classical-Marxian case such correspondence exists without any requirements for $H_{\varpi}$.

Assuming that investment is not very sensitive to changes in income distribution, an increase (reduction) in economic complexity increases (reduces) $y_{bp}$ and, therefore, employment rates. Higher (lower) employment leads to an increase (decrease) in the bargaining power of workers. In this way, they can get a bigger (smaller) piece of the pie increasing (decreasing) the wage-share. This leads to an increase (decrease) in labour productivity growth rates which in turn guarantees a stable employment rate at equilibrium. In other words, a reduction in economic complexity could explain the reduction in wage-shares and the slowdown of labour productivity growth observed in several OECD countries.

In what concerns the Kaldorian component of $G(\cdot)$, an increase in the sensitiveness of $\dot{q}/q$ to capital accumulation might produce a simultaneous increase or decrease in the level of capacity utilisation and wage-share. Net effects depend on the structural parameters of the economy. A higher Kaldor-Verdoorn effect implies a reduction of the slope and intercept of $\Psi(\cdot)$ while $\Theta(\cdot)$ remains unchanged. If the reduction of the intercept is smaller relatively to the change in the slope, both variables move upwards as in Fig. 3 on the left. Otherwise, we fall in the second situation, as in Fig. 3 on the right.

On the contrary, a higher classical-Marxian effect increases the slope of $\Psi(\cdot)$ with respect to income distribution without changing the intercept. The natural growth rate becomes very sensitive to changes in income distribution, and because wage-share and labour productivity are positively related, a smaller $\varpi$ is required to keep steady-state employment. This result follows the intuition described for the pure Marxian case. If firms can easily translate increases in labour costs to a change in production techniques, the bargaining power of workers is reduced and consequently the equilibrium wage-share. Furthermore, a lower wage-share means higher capital accumulation. For a given $y_{bp}$ this implies that a lower $u$ is required to bring utilisation levels to equilibrium. This is because firms now rely less on the accelerator for making their investment plans which allow for a reduction in $u$. We depict this case in Fig. 4.

With respect to the unique internal equilibrium point, we can now state and prove the

[7] In theory, the relationship between the share of wages on income and inequality is not clear-cut, depending largely on how labour and capital incomes are distributed as well as the magnitude of other sources of household incomes and the impact of taxes and social transfers. Recent evidence confirms that declines in the labour income share have a significant relationship with income inequality, especially when the decline in labour shares was concentrated at the lower end of the labour income distribution.

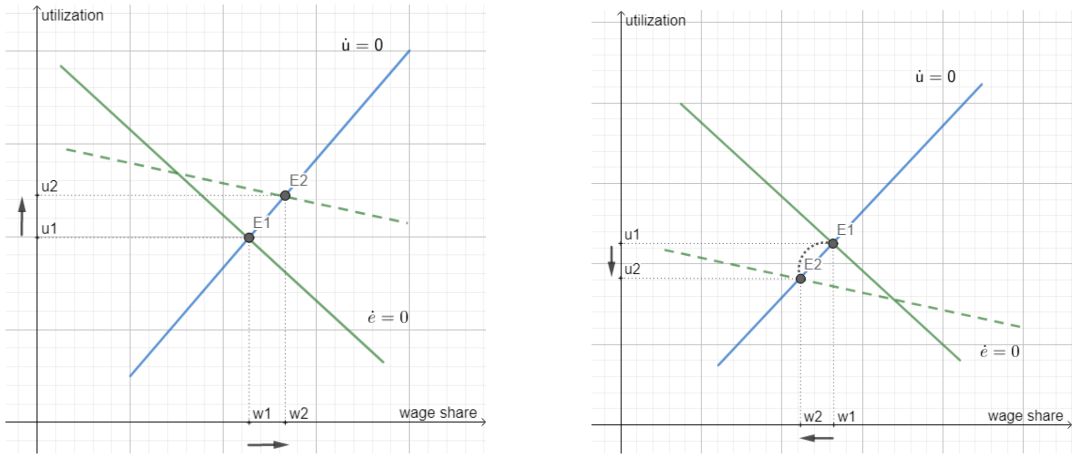



Figure 3: Different responses of equilibrium to changes in Kaldor-Verdoorn effect

following Proposition regarding its local stability.

**Proposition 5** *If the Kaldorian and classical-Marxian effects on factor productivity growth rates are such that*

$$\varpi^* \left(G_H H_\varpi + G_\varpi\right)^2 > \left|u^* G_H H_u \left[H_\varpi + J'(\varpi^*)\right]\right|$$

*the internal equilibrium* $(e^*, \varpi^*, u^*)$ *of the dynamic system (29)(31) is locally asymptotically stable.*

***Proof.*** *See Mathematical Appendix B4.* ■

This condition concerns the response of factor productivity to changes in income distribution. By assumption, an increase in the wage-share is followed by an increase in the growth rate of labour productivity, i.e. $G_H H_\varpi + G_\varpi > 0$. However, a higher $\varpi$ also triggers a reduction in capital accumulation and labour productivity growth which in turn causes an increase in capacity utilisation. Through Kaldor-Verdoorn's law, higher utilisation rates will further increase labour productivity leading to a reduction in the wage-share. The proposition above guarantees that this second effect is not so strong so that we have a smooth convergence to equilibrium. Such relation, of course, might not necessarily be satisfied in which case two Propositions follow.

**Proposition 6** *If the Kaldorian and classical-Marxian effects on factor productivity growth rates are such that*

$$\varpi^* \left(G_H H_\varpi + G_\varpi\right)^2 < \left|u^* G_H H_u \left[H_\varpi + J'(\varpi^*)\right]\right|$$

*and the sensitivity of real wages to changes in the employment rate satisfies*

$$F'(e^*) < -\frac{u^* \left[\varpi^* \left(G_H H_\varpi + G_\varpi\right) + u^* H_u\right] \left\{\left(G_H H_\varpi + G_\varpi\right) H_u - G_H H_u \left[H_\varpi + J'(\varpi^*)\right]\right\}}{e^* \varpi^* \left\{\varpi^* \left(G_H H_\varpi + G_\varpi\right)^2 + u^* G_H H_u \left[H_\varpi + J'(\varpi^*)\right]\right\}},$$

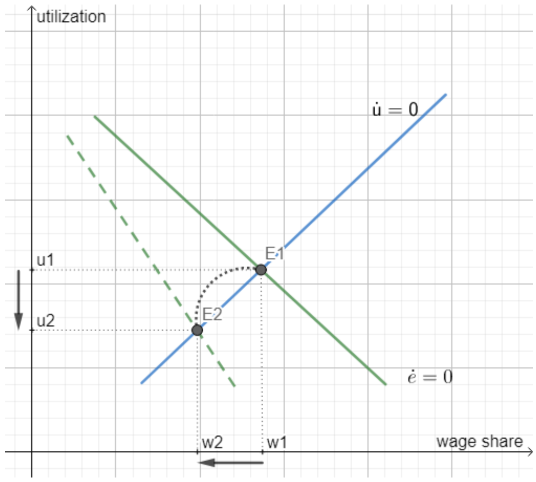



Figure 4: Response of equilibrium to changes in the Marx-biased effect

*then, the internal equilibrium $(e^*, \varpi^*, u^*)$ of the dynamic system (29)(31) is locally asymptotically stable.*

***Proof.*** *See Mathematical Appendix B5.* ■

Even when the inequality in Proposition 5 is violated, convergence to equilibrium is ensured if the worker's real wages do not strongly increase with small changes in employment rates. However, for higher values of $F'(e^*)$, it may happen that the last part of Proposition 6 also does not hold. Thus, the dynamic behaviour of the model may drastically change from the qualitative point of view, as the sensitivity of real wages to changes in $e$ increases, with all the other parameters remaining constant. Using $F'(e^*)$ as a bifurcation parameter, our purpose is now to apply the Hopf Bifurcation Theorem (HBT) for 3D systems to show that persistent cyclical behaviour of the variables can emerge as $F'(e^*)$ is increased (Gandolfo, 2009).

**Proposition 7** *If the Kaldorian and classical-Marxian effects on factor productivity growth rates are such that*

$$\varpi^* \left(G_H H_\varpi + G_\varpi\right)^2 < \left|u^* G_H H_u \left[H_\varpi + J'(\varpi^*)\right]\right|,$$

*then, for values of $F'(e^*)$ in the neighbourhood of the critical value*

$$F'(e^*)|_{HB} = -\frac{u^* \left[\varpi^* \left(G_H H_\varpi + G_\varpi\right) + u^* H_u\right] \left\{\left(G_H H_\varpi + G_\varpi\right) H_u - G_H H_u \left[H_\varpi + J'(\varpi^*)\right]\right\}}{e^* \varpi^* \left\{\varpi^* \left(G_H H_\varpi + G_\varpi\right)^2 + u^* G_H H_u \left[H_\varpi + J'(\varpi^*)\right]\right\}} \tag{32}$$

*and for which the real negative root of the characteristic equation satisfies*

$$\lambda_1 \neq u^* \left\{\frac{G_H H_u \left[H_\varpi + J'(\varpi^*)\right]}{G_H H_\varpi + G_\varpi} - H_u\right\}$$

*the dynamic system (29)(31) has a family of periodic solutions.*

***Proof.*** *See Mathematical Appendix B6.* ■

These results seem to be in line with Goodwin's (1967) aim of generating cycles rooted in the functioning of the labour market and the dynamics of distributive conflict. They correspond to an adaptation of DF&S four-dimensional model to a three-dimensional set up in which the behavioural equation for labour productivity was adjusted to the Kaldor-Marx story. Permanent periodic solutions might emerge as a result of an increase in the sensitivity of workers' wage demands to the employment rate.

If instead, the labour productivity growth rate follows Eq. (25), we go back to the pure Marxian case. This is because aggregate demand does not deviate from the external constraint, i.e. $\dot{Y}/Y = y_{bp}$. Therefore, the Kaldorian part of $G(\cdot)$ becomes a constant and the wage-share turns out to depend only on the shape of the Marxian component and on $y_{bp} - n$.

It is important to notice that, in this last case, we also recover the negative relationship between economic complexity and income inequality previously discussed. However, as in the simple classical-Marxian setup, the existence of such correspondence and the mechanism behind it does not require a sufficiently low response of capital accumulation to changes in income distribution. Furthermore, the system becomes once again locally stable presenting asymptotically convergence to the internal equilibrium solution.

A natural next step would be to perform numerical simulations in order to investigate the responsiveness of the model to different scenarios. Nonetheless, at this point of the analysis, any attempt of calibrating the system would be unsatisfactory. The reason for this is that there are not reliable estimates in the literature for the classical-Marxian effect of $\varpi$ on $\dot{q}/q$ and $\dot{\rho}/\rho$. Therefore, we proceed presenting some estimates of our own for functions $G(\cdot)$ and $J(\cdot)$.

# 3 Estimating classical-Marxian technical change

There is a robust literature on Kaldor-Verdoorn's law giving support to its formulation (e.g. McCombie and De Ridder, 1983; 1984; Angeriz et al., 2008; 2009; Romero and McCombie, 2016a; Magacho and McCombie, 2017; Romero and Britto, 2017). The same cannot be said about the classical-Marxian specification. It is our purpose in this section to provide more accurate estimates of the relation between functional income distribution and changes in factor productivity.

Several neoclassical contributions have investigated both theoretically and empirically the existence of induced (or biased) technical change using a CES production function in which each factor is paid accordingly to its marginal productivity. As usually is the case in that literature, the critical estimated parameter is the elasticity of substitution between capital and labour, though there seems to be little empirical consensus on its value and nature (see León-Ledesma et al., 2010, for a comprehensive review).

Among non-neoclassical economists, there have been some attempts to address the empirics behind factor productivity dynamics. Before continuing, however, an important clarification is necessary. In this article, we are interested in the relationship between factor productivity and factor cost shares also referred to as classical-Marxian technical change. Though somehow related, this concept is different from the also well known Marx-biased technical change (MBTC).

MBTC corresponds to the hypothesis that for a constant wage-share, labour productivity historically increases while capital productivity decreases, i.e. technical change is labour-saving and capital-using. It was first proposed by Foley and Michl (1999) and has been observed in specific periods of time for different countries and regions (e.g. Marquetti, 2003; Pichardo, 2007; Marquetti and Porsse, 2017). Moreover, authors such as Michl (2002), Sasaki (2008), and Basu (2010) have assessed empirically the predictions of a standard MBTC and neoclassical models making use of the so-called "viability condition".[8] Nonetheless, those contributions do not test the assumption that labour or capital-saving technical change increase under higher factor costs.

In this respect, Hein and Tarassow (2010) and Tridico and Pariboni (2017) are maybe the closest references to our exercise.[9] The former applies single equation techniques to a sample of 6 OECD countries between 1960 and 2007 while the latter uses panel data for 26 OECD countries from 1990 to 2013. Still, the evidence provided is limited for at least three reasons. First, their tests are only concerned with labour saving technical change. Secondly, the treatment used to remove cyclical effects is not convincing. Hein and Tarassow (2010), for instance, introduce up to three lags to control for cyclical variations. However, in their procedure, "insignificant variables were excluded and the equations re-estimated" (p. 742). This is quite strange because then, for some countries, only lag $t-1$ or $t-2$ were estimated (see, for example, Table 5, p. 748). It is difficult to understand the meaning of such estimations. Tridico and Pariboni (2017), on the other hand, do not provide any treatment for cyclicality. Finally, their regressions are likely to suffer from endogeneity by omitted variable and simultaneity.

### 3.1 Empirical methodology

Our empirical strategy consists in estimating functions $G(\cdot)$ and $J(\cdot)$ as in Eqs. (16) and (17). Under the hypothesis that firms make a local search relative to their current technology and implement discoveries following a profitability criteria, Kemp-Benedict (2017; 2018) has provided some general conditions that these functions need to satisfy. Hence, we adopt the functional forms:

$$\frac{\dot{q}}{q} = a - b\left(\frac{1-\varpi}{\varpi}\right) \tag{33}$$

$$\frac{\dot{\rho}}{\rho} = c + d\ln\left(\frac{1-\varpi}{\varpi}\right) \tag{34}$$

where $a$, $b$, $c$, and $d$ are parameters. Eqs. (33) and (34) provide the functional forms we estimate, allowing factor productivity to respond positively to changes in factor costs.

We make use of a panel-data Vector Autoregression methodology (pVAR). This technique combines the traditional VAR approach, which treats all variables in the system as endogenous, with the panel-data approach, which allows for unobserved individual heterogeneity. Time-series VAR models originated in the macroeconometrics literature as an alternative to multivariate simultaneous equation models. With the introduction of VAR in panel data settings, pVAR models have been used in multiple applications across fields.

[8] Foley and Michl (1999) derived the viability condition from a profit-maximising entrepreneur that chooses a new technique only if it can generate a higher expected rate of profit at the ongoing wage rate, compared with the old technique. See Basu (2018) for selective review on quantitative empirical research in Marxism political economy.

[9] One should also mention that in related literature, some scholars have found that increases in real wages are positively related to labour productivity growth (see, for example, Hartwig, 2014).

In this paper, we follow the procedure put forward by Love and Zicchino (2006) and Abrigo and Love (2016). A detailed description of the database, as well as pre-estimation treatments of the respective time-series, are provided in the Empirical Appendix.

## 3.2 Panel-VAR estimates

Ascertaining the order of integration of the variables under analysis is an essential precondition to establishing whether the use of panel-VAR tests is warranted. In this respect, we performed the Levin, Lin and Chu test that assumes a common unit root process, and the Im, Pesaran and Shin test, the ADF and PP test that assume individual unit root processes. Results are reported in the Empirical Appendix and indicate that series are stationary. Therefore, we can proceed and investigate the correspondence between factor productivities and cost shares using our pVAR methodology.

### 3.2.1 Labour productivity

Two pVAR models for labour productivity, $\dot{q}/q$, and income distribution, $(1-\varpi)/\varpi$, were estimated following Schwarz lag order selection criteria. To simplify notation, we write $\pi = 1 - \varpi$. We preferred Schwarz lag selection over the popular Akaike insofar as usually it assigns a lower number of lags which in this case is desirable given the limited size of our sample.[10] Model I uses labour productivity in PPPs while in model II we have constant national prices converted to 2011 US$ dollars. Table 1 reports our estimates. Results indicate that the growth rate of labour productivity responds negatively to an increase in the profit/wage-share ratio. That is, an increase in the wage-share increases $\dot{q}/q$ as expected from theory. Stability conditions of the estimated panels are checked in the Empirical Appendix.

Table 1: Income distribution vs Labour productivity growth, pVAR estimates

| Model | I | | II | |
|---|---|---|---|---|
| Explanatory | $\dot{q}/q$ | $\pi/\varpi$ | $\dot{q}/q$ | $\pi/\varpi$ |
| $(\dot{q}/q)_{t-1}$ | 0.107678 | 0.0683345 | 0.3916637*** | 0.3716756*** |
| $(\pi/\varpi)_{t-1}$ | -0.3507041*** | 0.7405059*** | -0.2891601*** | 0.6392449*** |
| No. Lags instruments | 5 | | 4 | |
| No. Obs. | 378 | | 378 | |
| No. Panels | 16 | | 16 | |
| Average t | 23.625 | | 23.625 | |

*, **, and *** stand by 10%, 5% and 1% of significance

Granger (1969) argued that causality in economics could be tested for by measuring the ability to predict the future values of a time series using prior values of another time

[10] Schwarz criterion is strongly consistent while Akaike is generally more efficient though not consistent. In other words, while the former will asymptotically deliver the correct model order, the latter will deliver on average a too large a model (Brooks, 2014). In our case, Akaike criteria recommended 2 lags for model I and 3 lags for model II. Still, estimates using 2 or 3 lags do not change significantly and are available under request.

series. Though the question of "true causality" is deeply philosophical, Granger causality tests are useful because they allow us to investigate if values of income distribution provide statistically significant information about future values of labour productivity growth rates. Table 2 reports our results for the two models estimated. In both cases we reject the null hypothesis and have that $(1-\varpi)/\varpi$ Granger causes $\dot{q}/q$.

Table 2: Granger causality

| Model | | I | | II | |
|---|---|---|---|---|---|
| Equation | Excluded | chi2 | Prob>chi2 | chi2 | Prob >chi2 |
| $\dot{q}/q$ | $\pi/\varpi$ | 8.073 | 0.004 | 29.596 | 0.000 |
| $\pi/\varpi$ | $\dot{q}/q$ | 0.615 | 0.433 | 7.781 | 0.005 |

$H_0$: no-causality in Granger sense

With our pVAR model in hands, we can also make use of variance decomposition analysis to assess the amount of information income distribution contributes to labour productivity in the autoregression. This allows us to determine how much of the forecast error variance of the latter can be explained by exogenous shocks in the former. As table 3 shows, with a 10-year forecast horizon, something around $20-40\%$ of the variation in $\dot{q}/q$ can be explained by income distribution.

Table 3: Variance decomposition

| Response variable | Impulse variable | Model I | Model II |
|---|---|---|---|
| $\dot{q}/q$ | $\pi/\varpi$ | 0.1909791 | 0.3848296 |
| $\dot{q}/q$ | $\dot{q}/q$ | 0.8090209 | 0.6151704 |
| $\pi/\varpi$ | $\pi/\varpi$ | 0.7638016 | 0.7894201 |
| $\pi/\varpi$ | $\dot{q}/q$ | 0.2361984 | 0.2105799 |
| Forecast horizon t | | 10 | 10 |

As a final step, we plot the response of $\dot{q}/q$ to a shock in $(1-\varpi)/\varpi$ as in traditional impulse response analysis. Fig. 5 reports OIRFs and only reinforces what we have found so far, that is, an increase in the share of wages on income seems to incentive firms at a macro-level to adopt labour saving techniques, increasing labour productivity growth rates. Most of these effects happen between 1 and 5 years after the shock.

### 3.2.2 Capital productivity

Two pVAR models for capital productivity, $\dot{\rho}/\rho$, and income distribution, $\ln(\pi/\varpi)$, were estimated. Model I uses data for real GDP and capital stock in PPPs while in model II both series are in national prices converted to 2011 US$ dollars. In the first case, Schwarz and Akaike lag selection criteria chose a two lag model. In the second case, Schwarz selected one lag while Akaike two lags. Following the procedure adopted in the last subsection, we preferred the former. Table 4 reports our estimates.

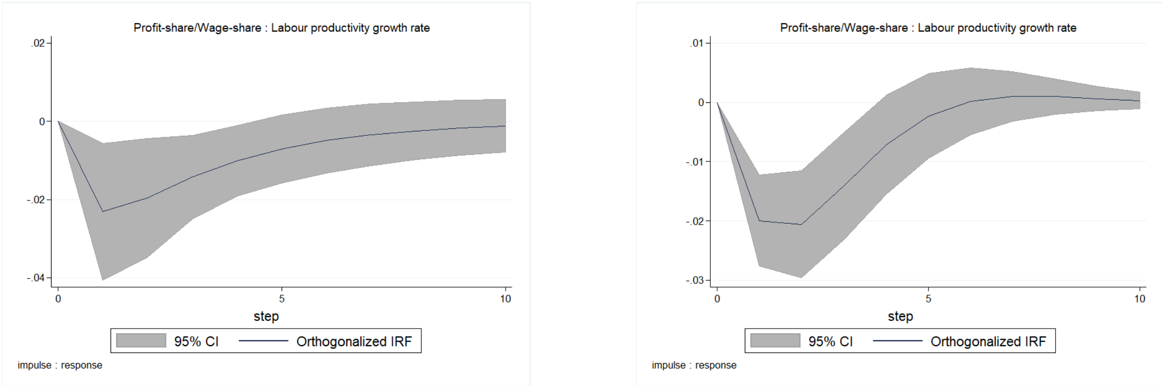



Figure 5: Impulse response functions, Model I (left) and II (right)

For both models, an increase of the profit share relative to the wage-share seems to be positively related to an increase in capital productivity growth rates. However, net effects here are not as apparent as in the labour productivity case. In the first of them, for instance, an increase in $\ln(\pi/\varpi)$ has an adverse effect in $t-1$ though a positive and slightly higher in $t-2$. In model II, the respective coefficient is positive but statistically not different from zero. Stability conditions of the estimated panels are checked in the Empirical Appendix.

Table 4: Income distribution and capital productivity growth, pVAR estimates

| Model | I | | II | |
|---|---|---|---|---|
| Explanatory | $\dot{\rho}/\rho$ | $\ln(\pi/\varpi)$ | $\dot{\rho}/\rho$ | $\ln(\pi/\varpi)$ |
| $(\dot{\rho}/\rho)_{t-1}$ | 0.2705985*** | -0.0835614 | 0.2837008*** | 0.2918696 |
| $(\dot{\rho}/\rho)_{t-2}$ | -0.1326931* | -0.0575909 | - | - |
| $\ln(\pi/\varpi)_{t-1}$ | -0.2341877** | 0.7806212*** | 0.0146131 | 0.7800752*** |
| $\ln \pi/\varpi)_{t-2}$ | 0.2436164** | -0.3626047*** | - | - |
| No. Lags instruments | 4 | | 5 | |
| No. Obs. | 329 | | 377 | |
| No. Panels | 16 | | 16 | |
| Average t | 20.563 | | 23.563 | |

*, **, and *** stand by 10%, 5% and 1% of significance

Granger causality tests confirm these first insights. Table 5 reports our results for the two models estimated. In the first case we have that $\ln(\pi/\varpi)$ unilaterally Granger causes $\dot{\rho}/\rho$ though this relationship disappears in the second model. This means that while in the first model past values of income distribution predict changes in capital productivity in the second model such correspondence does not hold anymore.

Table 5: Granger causality

| Model | | I | | II | |
|---|---|---|---|---|---|
| Equation | Excluded | chi2 | Prob>chi2 | chi2 | Prob>chi2 |
| $\dot{\rho}/\rho$ | $\ln(\pi/\varpi)$ | 9.853 | 0.007 | 0.622 | 0.430 |
| $\ln(\pi/\varpi)$ | $\dot{\rho}/\rho$ | 4.597 | 0.100 | 2.360 | 0.124 |

$H_0$: no-causality in Granger sense

Variance decomposition analysis emphasises the small relationship between income distribution and capital productivity growth rates. As Table 8 reports, within a 10-year forecast horizon, only $1-5\%$ of variation in $\dot{\rho}/\rho$ can be explained by $\ln(\pi/\varpi)$. Numbers do not change when we look at the response of income distribution to changes in capital productivity growth rates. For such magnitudes, it is quite safe to say that there is no correspondence at all among the variables. There is a clear contrast with the labour productivity case where changes in income distribution could explain up to 40% of variations in productivity.

Table 6: Variance decomposition

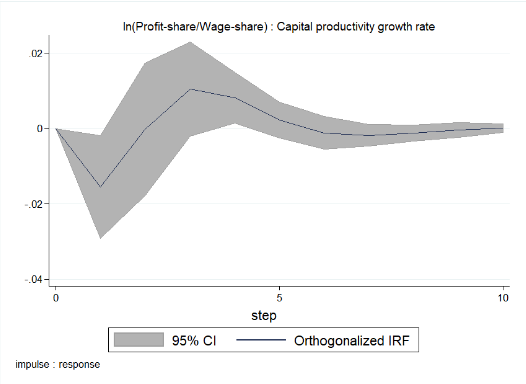
| Response variable | Impulse variable | Model I | Model II |
|---|---|---|---|
| $\dot{\rho}/\rho$ | $\ln(\pi/\varpi)$ | 0.0554572 | 0.0108349 |
| $\dot{\rho}/\rho$ | $\dot{\rho}/\rho$ | 0.9445428 | 0.9891652 |
| $\ln(\pi/\varpi)$ | $\ln(\pi/\varpi)$ | 0.9451099 | 0.9648765 |
| $\ln(\pi/\varpi)$ | $\dot{\rho}/\rho$ | 0.0548901 | 0.0351234 |
| Forecast horizon t | | 10 | 10 |

Fig. 6, through OIRFs, reinforces what we have found so far, that is, changes in the functional income distribution have little effect in firms' decisions to adopt capital saving techniques. In model I, it is possible to visualise the initial adverse effect of an increase in the profit share relative to the wage-share. Such negative impact was reversed immediately in the next three to four periods leading to a small positive net effect. On the other hand, model II depicts a non-statistically significant positive effect.

## 3.3 Discussion

The intuition behind functions $G(\cdot)$ and $J(\cdot)$ is that changes in functional income distribution reflect variations in factor cost shares. An increase in the wage-share means that real wages have increased relative to labour productivity. This implies a reduction in profitability and forces firms to increase their search for labour saving techniques, thus, increasing labour productivity growth rates. Analogously, an increase in profit shares is related to an increase in the cost of capital relative to capital productivity. Theory considers that firms should respond similarly and increase capital productivity growth rates.

Our results lead us to at least two possible conclusions. The first one comes from a straight reading of the estimates presented in this section. Variations in the wage (or profit) share only impact labour. The channel between the share of wages and labour productivity



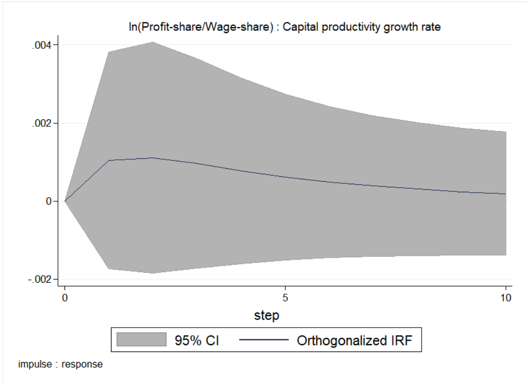




Figure 6: Impulse response functions, Model I (left) and II (right)

continues to be the same. Nevertheless, it seems that an increase in the profit share is not interpreted by the firm, at least at a macro level, as an increase in capital costs but only as labour becoming cheaper. Asymmetries in the ownership of the firm could justify such asymmetric behaviour. If those who own capital also own corporations, the latter will be more concerned with the factor of production it hires, that is labour. Therefore, asymmetries in firm's ownership are reflected in how they respond to changes in factor costs.

"In a perfectly competitive economy, it doesn't really matter who hires whom" (Samuelson, 1957, p. 894). This statement has been challenged for a long time by non-neoclassical economists and more recently by new-institutional scholars, who have shown that technologies are not neutral regarding the nature of property rights and corporate governance (see, for example, Pagano, 2013). The evidence presented in this article could be interpreted as reflecting such non-neutrality. Complementary, a higher wage-share drives low-productivity firms out of the market being the outcome of a selection process.

Alternatively, one could bring to attention that our estimates of capital productivity strongly depend on estimates of potential output. Potential output and the output gap are unobserved variables. Their estimation is deeply related to the estimation of production functions which involve controversial concepts such as the natural rate of unemployment and total factor productivity.[11] Potential output and output gap indicators are subject to a significant margin of uncertainty and might not be reliable. In this case, coefficients reported for capital productivity are not reliable as well and should be interpreted with caution.

# 4 Numerical Simulations

In section 2, we studied the analytical implications of adopting (i) Kaldor-Verdoorn's law, and (ii) classical-Marxian technical change to the main results of our model. The Kaldorian specification left the system with no internal equilibrium solution while the Marxian formulation made it stable. Nevertheless, a Hopf bifurcation analysis demonstrated that the

[11] For a review of the methodology employed by the IMF to estimate those variables, see De Masi (1997).

combination of both theories might give rise to persistent and bounded cyclical fluctuations when Verdoorn's effects are related to capital accumulation dynamics. Therefore, in this section, we present numerical simulations to investigate if the bifurcation is supercritical so that, under plausible settings, oscillations have economic meaning.

To this end, we must, first of all, choose functional forms for the main behavioural equations of the model, namely, $F(\cdot)$, $H(\cdot)$, $G(\cdot)$, and $J(\cdot)$. Given that our empirical exercise indicated that the effect of income distribution on the growth rate of capital productivity is neglectable, we set $J(\cdot) = 0$ and focused only on the first three relationships. Leaving aside, for a moment, any considerations about changes in labour productivity, the two remaining relations are expressed by:

$$
\begin{aligned}
F(e) &= \beta(e - \bar{e}) && (35)\\
H(\varpi, u) &= \gamma_1 - \gamma_2\varpi + \gamma_3 u && (36)
\end{aligned}
$$

where $\bar{e}$ is the rate of employment above which workers can obtain real wage increases. The functional form we have chosen in Eq. (35) captures the Marxian reserve army effect as discussed by DF&S. On the other hand, parameter $\beta$ corresponds to the sensitiveness of real wages to changes in employment rates. In Eq. (36), we adopt a linear specification of investment, where $\gamma_1$ captures the growth rate of the capital stock when wage income and capacity utilisation are both set to zero, while $\gamma_2$ and $\gamma_3$ stand for the sensitiveness of accumulation to income distribution and capacity utilisation, respectively.

We have discussed Kaldor-Verdoorn's effects on labour productivity under two different specifications. The first one makes $\dot{q}/q$ a function of capital accumulation while the second one takes into account output's growth rate. Local stability analysis demonstrated that those different formulations change the nature of the dynamic system from a qualitative point of view. When $\dot{q}/q$ depends on output's growth rate, we return to the pure Marxian case with simple convergence. Hence, in what follows, we investigate the case when labour productivity relies on $\dot{K}/K$.

In the empirical section, we used Eq. (33) to estimate the relationship between income distribution and labour productivity growth. This functional form was used following Kemp-Benedict's (2017; 2018) conditions for the existence of biased technical change. For our numerical simulations, however, a linear specification of $G(\cdot)$ was preferred. The reason for this is that we want to make it clear that the dynamics obtained do not depend on *ad-hoc* induced non-linearities. The system is intrinsically non-linear as a result of the interaction between its basic structure, given by equations (26)-(28), and the adopted behavioural rules, which are kept linear. Therefore, make:

$$
G\left(\frac{\dot{K}}{K}, \varpi\right) = a + b\varpi + \vartheta\left(\frac{\dot{K}}{K}\right) \tag{37}
$$

For this case, the steady-state internal equilibrium solution is such that satisfies:

$$
\begin{aligned}
e^* &= \bar{e} + \frac{y_{bp} - n}{\beta} && (38)\\
\varpi^* &= \frac{(1-\vartheta)y_{bp} - n - a}{b} && (39)\\
u^* &= \frac{y_{bp} - \gamma_1 + \gamma_2\varpi^*}{\gamma_3} && (40)
\end{aligned}
$$

In order to choose plausible parameter values, we have considered the evidence that is given in different empirical studies including DF&S. In particular, in what concerns the classical-Marxian effect, we dwell on our estimates of the impulse response functions.

$$
\begin{aligned}
y_{bp} &= 0.03,\ n = 0.01,\ \bar{e} = 0.85,\ a = -0.0275,\ b = 0.05 \\
\vartheta &= 0.5,\ \gamma_1 = -0.0125,\ \gamma_2 = 0.05,\ \gamma_3 = 0.1
\end{aligned}
$$

For this set of parameters, we have that the inequality in Proposition 5 is violated and, hence, there might be persistent and bounded fluctuations. Taking $F'(e^*) = \beta$ as bifurcation parameter, it turns out that $\beta_{HB} \approx 0.25055$. Therefore, for the simulations, we used a value slightly higher than this. Fig. 7 displays the solution path for initial values $(e_0,\ \varpi_0,\ u_0)$ equal to (0.95, 0.7, 0.75) which converge to a limit cycle around (0.93, 0.65, 0.75).

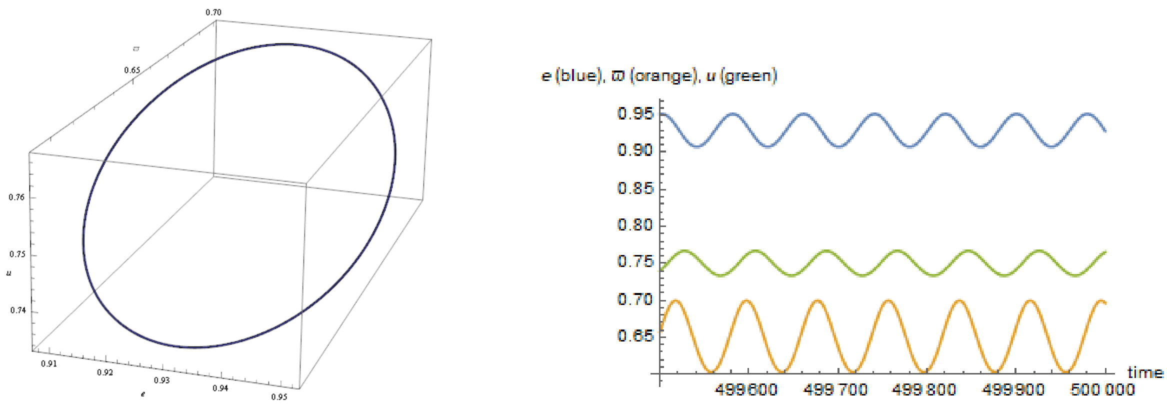



Figure 7: Limit cycle and time series

Still, it is important to understand the mechanism that is generating this outcome. Suppose that, for some exogenous reason, there is an increase in the employment rate. This will increase the bargaining power of workers and therefore the wage-share. Two effects follow. First, there is an increase in labour productivity which in turn brings down employment and the wage-share. Thus, we have a sequence similar to the one obtained by Goodwin, with:

$$
\begin{aligned}
e \uparrow &\Rightarrow \frac{\dot{w}}{w} \uparrow \Rightarrow \varpi \uparrow \Rightarrow \frac{\dot{q}}{q} \uparrow \Rightarrow e \downarrow \\
e \downarrow &\Rightarrow \frac{\dot{w}}{w} \downarrow \Rightarrow \varpi \downarrow \Rightarrow \frac{\dot{q}}{q} \downarrow \Rightarrow e \uparrow
\end{aligned}
$$

On the other hand, the increase in the wage-share that follows the increase in employment makes investment profitability to shrink and brings capital accumulation down. This leads to an increase in capacity utilisation. Through the accelerator effect, investment goes up and ultimately there is a reduction of u. Because of Kaldor-Verdoorn's law, one should expect an increase in labour productivity growth rate. Nonetheless, we know that by assumption this effect is small and does not overcome the Marxian effect. Still, the reintroduction of Kaldor-Verdoorn's law breaks the pure stability obtained from the Marxian mechanism, allowing the model to maintain its essential cyclical feature.

In his paper *On the Nonlinear Accelerator and the Persistence of Business Cycles*, Goodwin (1951) discussed the so-called "Schumpeter clock" relating the evolution of ideas to capital accumulation. New ideas require investment to occur regularly, but the latter goes by spurts. On the one hand, $I$ is limited by the capacity of the investment goods industry. On the other hand, machines once made, cannot be unmade, so that negative investment is constrained to attrition from time and innovations waves.

So far, our model has dealt with an induced component of investment. It is possible to evaluate the implications of introducing a cyclical autonomous component numerically. Making innovations a periodic function of time, as done by Sordi (1990), we can rewrite Eq. (36) as:

$$H(\varpi, u) \quad = \quad \gamma_1 - \gamma_2\varpi + \gamma_3 u + \gamma_A \cos(\tau t) \tag{41}$$

where $\gamma_A$ and $\tau$ are parameters.

In this way, we obtain a scenario in which a nonlinear system with a "natural" oscillation frequency interacts with an external periodic "force" resulting in a torus (see Fig. 8 on the left). A competition between two or more independent frequencies characterising the dynamics of the system is a well-known route to more complex behaviour. Fig. 8, on the right, depicts the solution path for the same initial values making $\gamma_A = 0.01$ and $\tau = 0.25$.

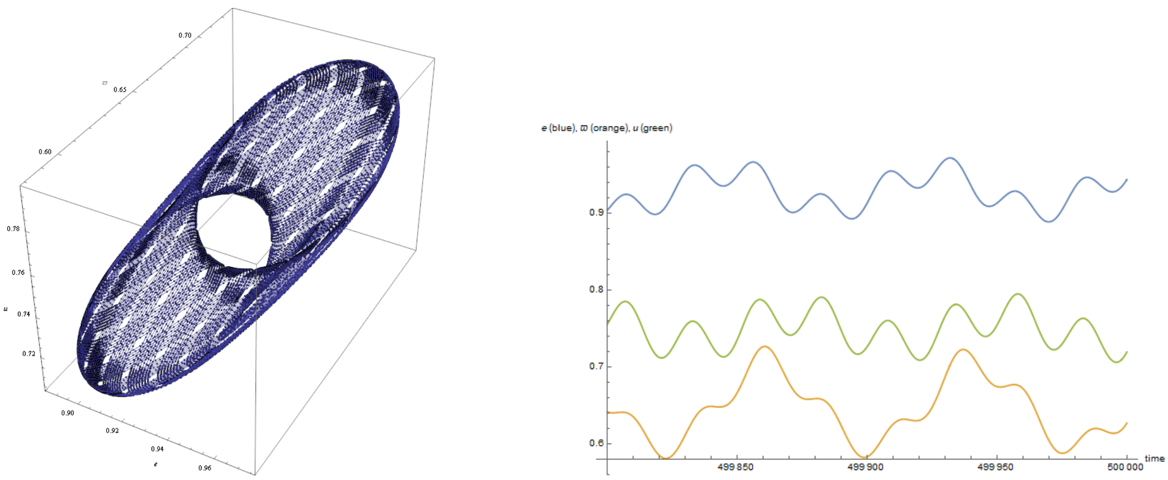



Figure 8: Torus and time series

The Newhouse-Ruelle-Takens theorem requires a three-dimensional torus for chaos possibility to arise in this context (Gandolfo, 2009). Providing that obtained quasi-periodic fluctuations result from a two-dimensional torus, there is no sensitive dependence on initial conditions. Still, the term quasi-periodic is used to describe the behaviour of the system given that it never exactly repeats itself (Lorenz, 1993).

One of the motives that lead Goodwin to the study of nonlinear dynamical systems was the advantage this structure offers to represent the interaction between cycle and trend. Hence, before concluding this article, we show how the growth cycle can be easily recovered by relaxing the assumption that output's growth rate is fixed and equal to Thirlwall's law.

Recalling the adjustment mechanism for GDP growth described in DF&S, we can write:

$$\frac{\dot{Y}}{Y} = y_{bp} + D(e - e^*, \varpi - \varpi^*, u - u^*) \tag{42}$$

where $D(e^*, \varpi^*, u^*) = 0$ and $D_e|_{e=e^*} = D_\varpi|_{\varpi=\varpi^*} = D_u|_{u=u^*} = 0$. It is easy to see that this modification does not change our local stability analysis. Since the system never actually reaches equilibrium, we have permanent and irregular fluctuations in the employment rate, wage-share, capacity utilisation, and output's growth rate.

# 5 Final Considerations

The analysis of the role of technical change in growth processes has been for a long time of central importance in economic theory. Different approaches have been proposed over the years to study distinct aspects of the phenomenon. On the other hand, in DF&S, we have extended Goodwin's (1967) model to study the interaction between distributive cycles and international trade for economies in which growth is BoPC. This article examined the implications of adopting (i) Kaldor-Verdoorn's law; and (ii) classical-Marxian technological change to the main results of the model.

We showed that using a Kaldorian approach to technical progress leaves the system with no internal equilibrium solution while the classical-Marxian formulation makes it stable. However, a Hopf bifurcation analysis demonstrated that the combination of both formulations might give rise to persistent and bounded cyclical fluctuations. Our numerical simulations confirmed that the Hopf bifurcation is supercritical and the limit cycle lies in a range of values with economic meaning. In other words, the central dynamics of DF&S are robust to alternative specifications of technological change. Moreover, the introduction of a forcing term motivated by Goodwin's discussion of "Schumpeter clock" gives rise to irregular fluctuations.

Furthermore, the models developed in this article provide a mechanism that helps to explain the positive correspondence found in the literature between economic complexity and income inequality. An increase in the diversification of the productive structure increases the BoPC growth rate and, therefore, employment rates. Higher employment leads to an increase in the bargaining power of workers allowing them to get a more significant piece of the pie, i.e. increasing the wage-share. This results in higher labour productivity which in turn guarantees a stable employment rate at equilibrium. Inversely, a reduction in economic complexity could explain the reduction in wage-shares and the slowdown of labour productivity growth observed in several OECD countries. New technologies, in particular, the so-called ICTs, have allowed firms to monitor workers more closely increasing the slope of $G(\cdot)$ and accentuating the aforementioned effect.

When it comes to the empirics of technological progress, studies testing the classical-Marxian formulation are limited. Several scholars have addressed the so-called MBTC hypothesis according to which, for a constant wage-share, labour productivity historically increases while capital productivity decreases, i.e. technical change is labour-saving and capital-using. Nevertheless, to the best of our knowledge, no reliable studies have tested the correspondence between factor productivity growth and cost shares. Therefore, we provided estimates of our own that give some support to the Marxian argument.

An increase in the means that real wages are higher relative to labour productivity. This implies a reduction in profitability and forces firms to increase their search for labour saving techniques, thus, increasing labour productivity growth rates. Analogously, an increase in profit shares is related to an increase in the cost of capital relative to capital productivity. Making use of a pVAR model for a sample of 16 OECD countries between 1980 and 2012, we found that one standard deviation impulse on the profit-share/wage-share ratio decreases labour productivity growth rates by 2% though it has a neglectable effect on capital productivity growth rate.

Our results lead us to two possible conclusions. The first one is that variations of income distribution only impact labour. Asymmetries in the ownership of the firm could justify such asymmetric behaviour. If those who own capital also own firms, the latter would be mainly concerned with the factor of production it hires, that is labour. Therefore, asymmetries in the firm's ownership are reflected in how the firm responds to changes in factor costs. Alternatively, one could bring to attention that our estimates of capital productivity strongly depend on estimates of potential output which are problematic given that involve controversial concepts such as the natural rate of unemployment or total factor productivity. Further research on those issues is required and encouraged.

# Empirical Appendix

Our data-set fundamentally comes from the Penn World Table 9.0 (PWT), which contains standardised macro series for a large number of economies from the 1950s onward. Output is measured both as real GDP at current PPPs and at constant 2011 national prices (in millions of 2011 US$). Total employment is given by the number of persons engaged in production (in millions). Hence, we obtain two indicators for labour productivity, computed as the ratio of those two measures and employment.

On the other hand, recall that capital productivity is defined as the ratio between potential output and the capital stock. The PWT lacks of estimates for $Y^*$ and, therefore, we adjust $Y$ making use of output gap series available from the World Economic Outlook Database (WEO). However, one should keep in mind that estimates of output gaps are subject to a significant margin of uncertainty. The WEO calculates the output gap as actual GDP less potential GDP as a percentage of potential GDP, i.e. $gap = (Y - Y^*)/Y^*$. This is equivalente to say that $Y^*/Y = 1/(1 + gap)$. Therefore, capital productivity is obtained as $Y^*/K = (Y/K)(Y^*/Y)$, where data for the capital stock comes from the PWT at both current PPPs and constant 2011 national prices (in millions of 2011 US$).

The reader may ask why not just approximate $\rho$ as $Y/K$. The reason is the following. Doing that, $\dot{\rho}/\rho$ by definition equals the difference between output and capital growth rates. The first one follows, in the long-run, Thirlwall's law. The second one depends on investment behaviour. But we know that the wage-share and investment are negative related through profitability. This means that an increase in the wage-share reduces capital accumulation resulting in an apparent increase in the growth rate of $\rho$, against the Marxian proposal. That is, making $\rho = Y/K$ is very likely to wrongly reject the classical-Marxian specification.

When addressing functional income distribution, aggregate labour share measures are influenced by the methods used to separate labour and capital income earned by entrepreneurs, sole proprietors, and unincorporated business. Thus, we use the novel database put forward by Karabarbounis and Neiman (2014) that focus on the labour share within the corporate sector and, thus, circumvent many of those measurement difficulties. By default, the profit share is equal to $1-\varpi$. Data for most OECD countries is available after the 1980s.

In order to keep our exercise as close as possible to DF&S, we use the same sample of 16 OECD countries (Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, New Zealand, Norway, South Korea, Sweden, United Kingdom, and United States). Our final dataset comprehends the period 1980 to 2012. Time span was determined due to data availability.

Over the past twenty years the Hodrick-Prescott (HP) filter has been used in macroeconomic analysis to separate trend from cycle when using macrodata. Although some of its drawbacks have been known for some time, the method continues to be widely adopted. Recently, Hamilton (2018) has strongly argued against its use showing this to be a serious mistake. He not only demonstrates that the HP filter introduces spurious dynamic relations that have no basis in the underlying data but provides an alternative that does the job without those distortions.

Hamilton's method consists in estimating an OLS regression of the type:

$$x_{t+h} = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \beta_3 x_{t-2} + \beta_4 x_{t-3} + v_{t+h}$$

where $x$ is a generic variable, $\beta$'s are the estimated coefficients, and $v$ is the error component. He proposes a 2-year horizon as a standard benchmark, in which case $h = 2$. Hence, residuals are:

$$\hat{v}_{t+2} = x_{t+2} - \hat{\beta}_0 - \hat{\beta}_1 x_t - \hat{\beta}_2 x_{t-1} - \hat{\beta}_3 x_{t-2} - \hat{\beta}_4 x_{t-3}$$

and correspond to time series cyclical component. Making $x - \hat{v}$ we obtain the trend. We apply this procedure to detrend our data for $q$, $\rho$, and $\varpi$. Once we got rid of the cyclical component, we compute labour and capital productivity growth rates. We proceed evaluating the stationarity of time-series.

We performed the Levin, Lin and Chu test that assumes a common unit root process, and the Im, Pesaran and Shin test, the ADF and PP tests that assume individual unit root processes. Number of lags was choosen following the Schwarz criteria. Results are reported in table A1. Series are found to be stationary.

Table A1: Panel Unit root tests

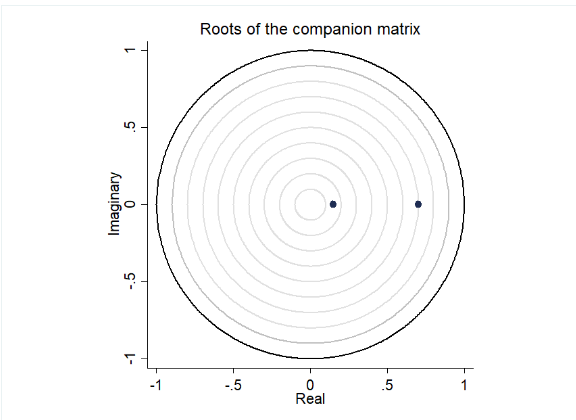
| $(1-\varpi)/\varpi$ | Intercept | Trend and Intercept |
|---|---|---|
| Method | Prob. | Prob. |
| Levin, Lin & Chu | 0.0000 | 0.0000 |
| Im, Pesaran and Shin | 0.0000 | 0.0000 |
| ADF | 0.0000 | 0.0000 |
| PP | 0.0000 | 0.1003 |

| $\ln((1-\varpi)/\varpi)$ | Intercept | Trend and Intercept |
|---|---|---|
| Method | Prob. | Prob. |
| Levin, Lin & Chu | 0.0000 | 0.0000 |
| Im, Pesaran and Shin | 0.0000 | 0.0000 |
| ADF | 0.0000 | 0.0000 |
| PP | 0.0000 | 0.0566 |

| Variable | PPPs, 2011 US$ | | cons. nat. prices, 2011 US$ | |
|---|---|---|---|---|
| $\dot{q}/q$ | Intercept | Trend and Intercept | Intercept | Trend and Intercept |
| Method | Prob. | Prob. | Prob. | Prob. |
| Levin, Lin & Chu | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Im, Pesaran and Shin | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| ADF | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| PP | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

| Variable | PPPs, 2011 US$ | | cons. nat. prices, 2011 US$ | |
|---|---|---|---|---|
| $\dot{\rho}/\rho$ | Intercept | Trend and Intercept | Intercept | Trend and Intercept |
| Method | Prob. | Prob. | Prob. | Prob. |
| Levin, Lin & Chu | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Im, Pesaran and Shin | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| ADF | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| PP | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Finally we check the stability condition of the estimated pVAR. We start ploting the stability diagram for models I and II relating labour productivity growth rates and income distribution as in figure A1. Real and imaginary roots of the companion matrix lie inside the unit circle confirming the model is stable.



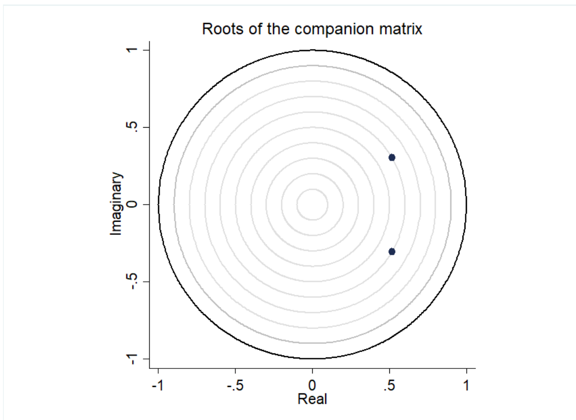



Figure A1: Stability condition, Model I (left) and Model II (right)

Figure A2 plot the unit circle for the relation between capital productivity and functional income distribution. Once more, models are found to be stable.

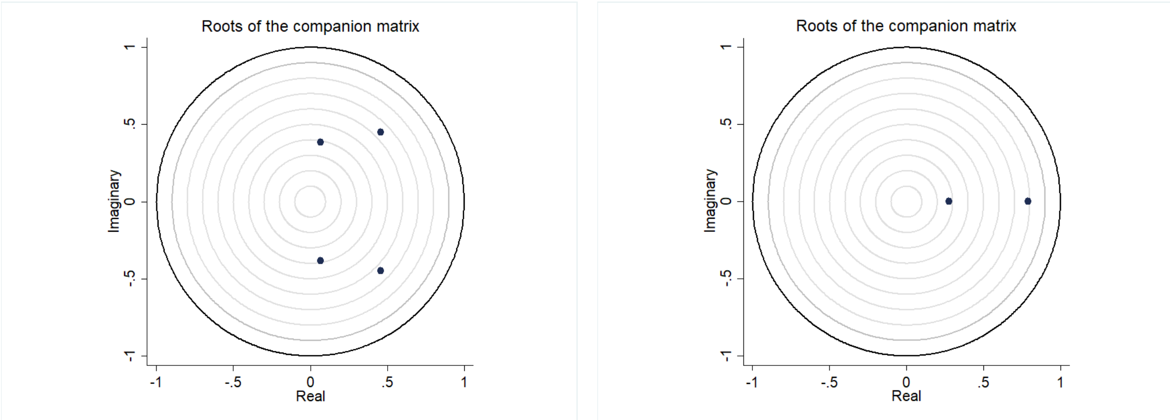



Figure A2: Stability condition, Model I (left) and Model II (right)

# Mathematical Appendix

## Proof of Proposition 2

To demonstrate proposition 2 we proceed in three steps. First, from equations (21) and (22) we obtain the rate of growth of real wages in terms of the external constrain, i.e. $F(e) = y_{bp} - n$, where $F : \Re \rightarrow \Re$ is monotonically increasing in $e$. Therefore, its inverse is also an increasing function and we obtain $e^* = F^{-1}(y_{bp} - n)$ as the unique equilibrium value of the rate of employment.

Looking at equation (21), it is straighforward that $G_{\varpi}(\varpi) = y_{bp} + n$, where, $G_{\varpi} : \Re \rightarrow \Re$ is a function monotonically increasing in $\varpi$. The inverse of $G_{\varpi}(\cdot)$ is also monotonically increasing so that $\varpi^* = G_{\varpi}^{-1}(y_{bp} - n)$ is the unique equilibrium value of the .

The equilibrium capacity utilisation is defined as the value of $u$ that brings utilisation and the balance-of-payments to equilibrium. Our investment function $H : \Re \rightarrow \Re$ is monotonically increasing in $u$ and decreasing in $\varpi$. Making use of the equilibrium value of the wage-share, we have that $y_{bp} = H\left[G^{-1}(y_{bp} - n), u\right] + J\left[G^{-1}(y_{bp} - n)\right]$. It follows that the unique equilibrium for capacity utilisation is determined and defined by $u^*$ that statisfies that condition.

Finally, in order to obtain values with economic meaning we have to impose $0 < F^{-1}(y_{bp} - n) < 1$, $0 < G^{-1}(y_{bp} - n) < 1$, and $0 < u^* < 1$.

## Proof of Proposition 3

In this Appendix we first derive the characteristic equation of the dynamic system (18)-(20) and prove Proposition 3. To do this, we linearise the dynamic system around the internal equilibrium point so as to obtain:

$$\begin{bmatrix} \dot{e} \\ \dot{\varpi} \\ \dot{u} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & j_{12} & 0 \\ j_{21} & j_{22} & 0 \\ 0 & j_{32} & j_{33} \end{bmatrix}}_{J^*} \begin{bmatrix} e - e^* \\ \varpi - \varpi^* \\ u - u^* \end{bmatrix}$$

where the elements of the Jacobian matrix $J^*$ are given by:

$$\begin{aligned} j_{11} &= 0 \\ j_{12} &= -G_\varpi e^* < 0 \\ j_{13} &= 0 \end{aligned}$$

$$\begin{aligned} j_{21} &= F'(e^*)\varpi^* > 0 \\ j_{22} &= -G_\varpi \varpi^* < 0 \\ j_{23} &== 0 \end{aligned}$$

$$\begin{aligned} j_{31} &= 0 \\ j_{32} &= -H_\varpi u^* > 0 \\ j_{33} &= -H_u u^* < 0 \end{aligned}$$

so that the characteristic equation can be written as

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0$$

where the coefficients are given by:

$$b_1 = -\operatorname{tr} J^* = -(j_{22} + j_{33}) > 0 \tag{43}$$

$$\begin{aligned} b_2 &= \begin{vmatrix} j_{22} & 0 \\ j_{32} & j_{33} \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & j_{33} \end{vmatrix} + \begin{vmatrix} 0 & j_{12} \\ j_{21} & j_{21} \end{vmatrix} \\ &= j_{22}j_{33} - j_{12}j_{21} > 0 \end{aligned} \tag{44}$$

$$b_3 = -\det J^* = j_{12}j_{21}j_{33} > 0 \tag{45}$$

The necessary and sufficient condition for the local stability of $(e^*, \varpi^*, u^*)$ is that all roots of the characteristic equation have negative real parts, which, from Routh-Hurwitz conditions, requires:

$$b_1 > 0,\ b_2 > 0,\ b_3 > 0 \text{ and } b_1b_2 - b_3 > 0.$$

Given (43)-(45) the crucial condition for local stability becomes the last one. Through direct computation we find that:

$$\begin{aligned} b_1b_2 - b_3 &= -(j_{22} + j_{33})(j_{22}j_{33} - j_{12}j_{21}) - j_{12}j_{21}j_{33} \\ &= (G_\varpi\varpi^* + H_u u^*)\left[G_\varpi\varpi^* H_u u^* + G_\varpi e^* F'(e^*)\varpi^*\right] - G_\varpi e^* F'(e^*)\varpi^* H_u u^* \\ &= G_\varpi\varpi^*\left[G_\varpi\varpi^* H_u u^* + G_\varpi e^* F'(e^*)\varpi^*\right] + H_u u^* G_\varpi\varpi^* H_u u^* > 0 \\ &= \underbrace{(G_\varpi\varpi^*)^2}_{>0}\underbrace{(H_u u^* + e^* F'(e^*)e^*)}_{>0} + \underbrace{(H_u u^*)^2}_{>0}\underbrace{G_\varpi\varpi^*}_{>0} > 0 \end{aligned}$$

Therefore, the system is locally stable.

## Proof of Proposition 4

To demonstrate proposition 4 we proceed in the following series of steps. First, from equations (29) and (30) we obtain the rate of growth of real wages in terms of the external constrain, i.e. $F(e) = y_{bp} - n$, where $F : \Re \rightarrow \Re$ is monotonically increasing in $e$. Therefore, its inverse is also an increasing function and we obtain $e^* = F^{-1}(y_{bp} - n)$ as the unique equilibrium value of the rate of employment.

Looking at equation (29), we have that $G\left[H(\varpi, u), \varpi\right] = y_{bp} + n$, where, $G : \Re \rightarrow \Re$ is a function monotonically increasing in its two arguments while $H : \Re \rightarrow \Re$ is decreasing in $\varpi$ and increasing in $u$. However, by assumption, $G_\varpi > |G_H H_\varpi|$ which implies $G(\cdot)$ is monotonically increasing in $\varpi$ and $u$. Hence, we can rewrite (29) as $u = \Psi(\varpi)$, where, $\Psi : \Re \rightarrow \Re$ is monotonically decreasing in $\varpi$. Analogously, from equation (31) a constantant level of capacity utilisation requires $H(\varpi, u) + J(\varpi) = y_{bp}$, with $J : \Re \rightarrow \Re$ is a decreasing function in $\varpi$. Since $H$ is also monotonically decreasing in the wage share we can rewrite (31) as $u = \Theta(\varpi)$, where, $\Theta : \Re \rightarrow \Re$ is monotonically increasing in $\varpi$.

Given that $\Psi$ is a decreasing function of $\varpi$ in $u$ and $\Theta$ is an increasing one, they intercept each other in one unique point that characterise the equilibrium solution of the system for these two variables. That is, though the employment rate is determined alone in the labour market, income distribution and capacity utilisation are simultaneously chosen. Finally, in order to obtain values with economic meaning we have to impose $0 < F^{-1}(y_{bp} - n) < 1$, $0 < \varpi^* < 1$, and $0 < u^* < 1$.

## Proof of Proposition 5

In this Appendix we first derive the characteristic equation of the dynamic system (26)-(28) and prove Proposition 5. To do this, we linearise the dynamic system around the internal equilibrium point so as to obtain:

$$\begin{bmatrix} \dot{e} \\ \dot{\varpi} \\ \dot{u} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ 0 & j_{32} & j_{33} \end{bmatrix}}_{J^*} \begin{bmatrix} e - e^* \\ \varpi - \varpi^* \\ u - u^* \end{bmatrix}$$

where the elements of the Jacobian matrix $J^*$ are given by:

$$\begin{aligned} j_{11} &= 0 \\ j_{12} &= -\left(G_H H_\varpi + G_\varpi\right) e^* < 0 \\ j_{13} &= -G_H H_u e^* < 0 \end{aligned}$$

$$\begin{aligned} j_{21} &= F'(e^*)\varpi^* > 0 \\ j_{22} &= -\left(G_H H_\varpi + G_\varpi\right) \varpi^* < 0 \\ j_{23} &= -G_H H_u \varpi^* < 0 \end{aligned}$$

$$\begin{aligned} j_{31} &= 0 \\ j_{32} &= -\left[H_\varpi + J'(\varpi^*)\right] u^* > 0 \\ j_{33} &= -H_u u^* < 0 \end{aligned}$$

so that the characteristic equation can be written as

$$\lambda^3 + b_1\lambda^2 + b_2\lambda + b_3 = 0$$

where the coefficients are given by:

$$b_1 = -\operatorname{tr} J^* = -(j_{22} + j_{33}) > 0 \tag{46}$$

$$\begin{aligned} b_2 &= \begin{vmatrix} j_{22} & j_{23} \\ j_{32} & j_{33} \end{vmatrix} + \begin{vmatrix} 0 & j_{13} \\ 0 & j_{33} \end{vmatrix} + \begin{vmatrix} 0 & j_{12} \\ j_{21} & j_{21} \end{vmatrix} \\ &= j_{22}j_{33} - j_{23}j_{32} - j_{12}j_{21} > 0 \end{aligned} \tag{47}$$

$$b_3 = -\det J^* = -j_{13}j_{21}j_{32} + j_{12}j_{21}j_{33} > 0 \tag{48}$$

The necessary and sufficient condition for the local stability of $(e^*, \varpi^*, u^*)$ is that all roots of the characteristic equation have negative real parts, which, from Routh-Hurwitz conditions, requires:

$$b_1 > 0,\ b_2 > 0,\ b_3 > 0 \text{ and } b_1b_2 - b_3 > 0.$$

Given (43)-(45) the crucial condition for local stability becomes the last one. Through direct computation we find that:

$$\begin{aligned} b_1b_2 - b_3 = {} & \left[\left(G_H H_\varpi + G_\varpi\right)\varpi^* + H_u u^*\right]\left[\left(G_H H_\varpi + G_\varpi\right)\varpi^* H_u u^* - G_H H_u \varpi^* \left[H_\varpi + J'(\varpi^*)\right] u^*\right. \\ & \left. + \left(G_H H_\varpi + G_\varpi\right) e^* F'(e^*)\varpi^*\right] + G_H H_u e^* F'(e^*)\varpi^* \left[H_\varpi + J'(\varpi^*)\right] u^* \\ & - \left(G_H H_\varpi + G_\varpi\right) e^* F'(e^*)\varpi^* H_u u^* \\ = {} & \left(G_H H_\varpi + G_\varpi\right)\varpi^* \left[\left(G_H H_\varpi + G_\varpi\right)\varpi^* H_u u^* - G_H H_u \varpi^* \left[H_\varpi + J'(\varpi^*)\right] u^*\right. \\ & \left. + \left(G_H H_\varpi + G_\varpi\right) e^* F'(e^*)\varpi^*\right] + H_u u^* \left[\left(G_H H_\varpi + G_\varpi\right)\varpi^* H_u u^* - G_H H_u \varpi^* \left[H_\varpi + J'(\varpi^*)\right] u^*\right] \\ & + G_H H_u e^* F'(e^*)\varpi^* \left[H_\varpi + J'(\varpi^*)\right] u^* \\ = {} & \varpi^* u^* \underbrace{\left[\left(G_H H_\varpi + G_\varpi\right)\varpi^* + H_u u^*\right]}\underbrace{\left[\left(G_H H_\varpi + G_\varpi\right) H_u - G_H H_u \left[H_\varpi + J'(\varpi^*)\right]\right]}_{>0} \\ & + \underbrace{e^* \varpi^* F'(e^*)}_{>0} \underbrace{\left[\varpi^* \left(G_H H_\varpi + G_\varpi\right)^2 + G_H H_u u^* \left[H_\varpi + J'(\varpi^*)\right]\right]}_{\gtrless 0} \end{aligned} \tag{49}$$

Therefore, $\varpi^* \left(G_H H_\varpi + G_\varpi\right)^2 > G_H H_u \left[H_\varpi + J'(\varpi^*)\right] u^*$ is a sufficient condition for $b_1b_2 - b_3 > 0$, and hence, guarantees local stability of the dynamical system.

## Proof of Proposition 6

When demonstrating Proposition 5, we showed that $\varpi^* \left(G_H H_\varpi + G_\varpi\right)^2 > G_H H_u \left[H_\varpi + J'(\varpi^*)\right] u^*$ is a sufficient condition for local stability. However, even if that inequality is no satisfied, local stability might still hold. Manipulating equation (49) we have that this is the case as long as:

$$F'(e^*) < -\frac{u^* \left[\varpi^* \left(G_H H_\varpi + G_\varpi\right) + u^* H_u\right] \left\{\left(G_H H_\varpi + G_\varpi\right) H_u - G_H H_u \left[H_\varpi + J'(\varpi^*)\right]\right\}}{e^* \varpi^* \left\{\varpi^* \left(G_H H_\varpi + G_\varpi\right)^2 + u^* G_H H_u \left[H_\varpi + J'(\varpi^*)\right]\right\}}$$

## Proof of Proposition 7

To prove Proposition 7 using the (existence part of) the Hopf Bifurcation Theorem and using $\partial F/\partial e$ as bifurcation parameter, we must: (HB1) show that the characteristic equation possesses a pair of complex conjugate eigenvalues $\theta\left[F'(e^*)\right] \pm i\omega\left[F'(e^*)\right]$ that become purely imaginary at the critical value $F'(e^*)_{HB}$ of the parameter – i.e. $\theta\left[F'(e^*)_{HB}\right]=0$ – and no other eigenvalues with zero real part exists at $\left[F'(e^*)\right]_{HB}$, and then (HB2) check that the derivative of the real part of the complex eigenvalues with respect to the bifurcation parameter is different from zero at the critical value.

(HB1) Given that the conditions $b_1>0$, $b_2>0$ and $b_3$ are all fulfilled, in order that the characteristic equation has one negative real root and a pair of complex roots with zero real part we must have:

$$b_1b_2-b_3=0$$

a condition which, given the expression for $b_1b_2-b_3$ derived in (49), is satisfied for

$$F'(e^*)|_{HB}=-\frac{u^*\left[\varpi^*\left(G_HH_\varpi+G_\varpi\right)+u^*H_u\right]\left\{\left(G_HH_\varpi+G_\varpi\right)H_u-G_HH_u\left[H_\varpi+J'(\varpi^*)\right]\right\}}{e^*\varpi^*\left\{\varpi^*\left(G_HH_\varpi+G_\varpi\right)^2+u^*G_HH_u\left[H_\varpi+J'(\varpi^*)\right]\right\}}$$

(HB2) By using the so-called sensitivity analysis, it is then possible to show that the second requirement of the Hopf Bifurcation Theorem is also met. Substituting the elements of the Jacobian matrix into the respective coefficients of the characteristic equation:

$$\begin{aligned}
b_1 &= \left(G_HH_\varpi+G_\varpi\right)\varpi^*+H_uu^*\\
b_2 &= \left(G_HH_\varpi+G_\varpi\right)\varpi^*H_uu^*-G_HH_u\varpi^*\left[H_\varpi+J'(\varpi^*)\right]u^*\\
&\quad+\left(G_HH_\varpi+G_\varpi\right)e^*F'(e^*)\varpi^*\\
b_3 &= -G_HH_ue^*F'(e^*)\varpi^*\left[H_\varpi+J'(\varpi^*)\right]u^*\\
&\quad+\left(G_HH_\varpi+G_\varpi\right)e^*F'(e^*)\varpi^*H_uu^*
\end{aligned}$$

so that

$$\begin{aligned}
\frac{\partial b_1}{\partial F'\left(e^*\right)} &= 0\\
\frac{\partial b_2}{\partial F'\left(e^*\right)} &= \left(G_HH_\varpi+G_\varpi\right)e^*\varpi^*>0\\
\frac{\partial b_3}{\partial F'\left(e^*\right)} &= e^*\varpi^*u^*\left\{\left(G_HH_\varpi+G_\varpi\right)H_u-G_HH_u\left[H_\varpi+J'(\varpi^*)\right]\right\}>0
\end{aligned}$$

When $F'(e^*)=F'(e^*)_{HB}$ as in (32), apart from $b_1>0$, $b_2>0$ and $b_3>0$ one also has $b_1b_2-b_3=0$. In this case, one root of the characteristic equation is real negative ($\lambda_1$), whereas the other two are a pair of complex roots with zero real part ($\lambda_{2,3}=\theta\pm i\omega$, with $\theta=0$). We thus have:

$$\begin{aligned}
b_1 &= -\left(\lambda_1+\lambda_2+\lambda_3\right)\\
&= -\left(\lambda_1+2\theta\right)\\
b_2 &= \lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_2\lambda_3\\
&= 2\lambda_1\theta+\theta^2+\omega^2\\
b_3 &= -\lambda_1\lambda_2\lambda_3\\
&= -\lambda_1\left(\theta^2+\omega^2\right)
\end{aligned}$$

such that:

$$\frac{\partial b_1}{\partial\left[F'(e^*)\right]} = -\frac{\partial \lambda_1}{\partial\left[F'(e^*)\right]} - 2\frac{\partial \theta}{\partial\left[F'(e^*)\right]} = 0$$
$$\frac{\partial b_2}{\partial\left[F'(e^*)\right]} = 2\theta\frac{\partial \lambda_1}{\partial\left[F'(e^*)\right]} + 2\left(\lambda_1+\theta\right)\frac{\partial \theta}{\partial\left[F'(e^*)\right]} + 2\omega\frac{\partial \omega}{\partial\left[F'(e^*)\right]} = P > 0$$
$$\frac{\partial b_3}{\partial\left[F'(e^*)\right]} = -\left(\theta^2+\omega^2\right)\frac{\partial \lambda_1}{\partial\left[F'(e^*)\right]} - 2\lambda_1\theta\frac{\partial \theta}{\partial\left[F'(e^*)\right]} - 2\lambda_1\omega\frac{\partial \omega}{\partial\left[F'(e^*)\right]} = R > 0$$

where $P = \left(G_H H_\varpi + G_\varpi\right) e^* \varpi^*$ and $R = e^* \varpi^* u^* \left\{\left(G_H H_\varpi + G_\varpi\right) H_u - G_H H_u \left[H_\varpi + J'(\varpi^*)\right]\right\}$.

For $\theta = 0$, the system to be solved becomes:

$$-\frac{\partial \lambda_1}{\partial\left[F'(e^*)\right]} - 2\frac{\partial \theta}{\partial\left[F'(e^*)\right]} = 0$$
$$2\lambda_1\frac{\partial \theta}{\partial\left[F'(e^*)\right]} + 2\omega\frac{\partial \omega}{\partial\left[F'(e^*)\right]} = P$$
$$-\omega^2\frac{\partial \lambda_1}{\partial\left[F'(e^*)\right]} - 2\lambda_1\omega\frac{\partial \omega}{\partial\left[F'(e^*)\right]} = R$$

or

$$\begin{bmatrix} -1 & -2 & 0 \\ 0 & 2\lambda_1 & 2\omega \\ -\omega^2 & 0 & -2\lambda_1\omega \end{bmatrix} \begin{bmatrix} \frac{\partial \lambda_1}{\partial\left[F'(e^*)\right]} \\ \frac{\partial \theta}{\partial\left[F'(e^*)\right]} \\ \frac{\partial \omega}{\partial\left[F'(e^*)\right]} \end{bmatrix} = \begin{bmatrix} 0 \\ P \\ R \end{bmatrix}$$

Thus:

$$\left.\frac{\partial \theta}{\partial\left[F'(e^*)\right]}\right|_{F'(e^*)=F'(e^*)_{HB}} = \frac{\begin{vmatrix} -1 & 0 & 0 \\ 0 & P & 2\omega \\ -\omega^2 & R & -2\lambda_1\omega \end{vmatrix}}{\begin{vmatrix} -1 & -2 & 0 \\ 0 & 2\lambda_1 & 2\omega \\ -\omega^2 & 0 & -2\lambda_1\omega \end{vmatrix}} = \frac{\left(P\lambda_1 + R\right)}{2\left(\lambda_1^2+\omega^2\right)}$$

and $\left.\frac{\partial \theta}{\partial\left[F'(e^*)\right]}\right|_{F'(e^*)=F'(e^*)_{HB}} \neq 0$ as long as $P\lambda_1 + R \neq 0$. Substituting the respective expressions of $P$ and $R$, that is equivalente to say that:

$$\lambda_1 \neq u^*\left\{\frac{G_H H_u\left[H_\varpi + J'(\varpi^*)\right]}{G_H H_\varpi + G_\varpi} - H_u\right\}$$

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