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From open economies to attitudes towards change
Growth and institutions in Latin America and Asia

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This article makes two contributions to the literature on growth and structural change. First, we estimate the multisectoral version of Thirlwall’s law and provide some empirical evidence on the stratification mechanism proposed in Dávila-Fernández et al. (2018). Second, we develop two models of structural change which assume that the capacity of adaptation of the economy is a function of attitudes towards change. Societies whose past experiences condition them to regard innovative change with antipathy are in sharp contrast to those whose heritage provide them with favourable attitudes. The models are used to discuss the experiences of Latin America and Asia since the 1960s. They highlight how a complex economy is likely to be associated with a better distribution of political and economic power. Our resulting nonlinear dynamic systems are shown to admit multiple equilibria. A Hopf-Bifurcation analysis establishes the possibility of persistent and bounded cyclical paths, allowing the investigation of further insights on the nature of structural and institutional change.

**Keywords:** Structural change; Institutions; Attitudes towards change; Hopf bifurcation; Path dependence.

**JEL:** E12; E32; O40
1 Introduction

What explains differences in growth rates between countries over time continues to be a question of central importance in economics. In search of an answer, modern theories of structural change have often emphasised the relationship between changes in the composition of production and expansions of the overall economic system. Growth has been found to be not only the result but also a source of structural change, in a relationship that involves the intertwining of technological and organisational domains (Andreoni and Scazzieri, 2014; Herrendorf et al., 2014).

Production processes are the principle loci of structural economic dynamics. Each production unit is endowed with a specific collection of capabilities. Their activation and transformation is the outcome of the interplay between cognitive dynamics and the productive structure (Pitelis and Teece, 2009; Jacobides and Winter, 2012). Furthermore, the existence of complementarities between different pieces of the puzzle tends to trigger circular and cumulative processes of development and underdevelopment. In this way, economic growth demands institutional and ideological adjustments while effective structural change requires coordinate revisions in many components of the economy (Pagano, 2011; Chang and Andreoni, 2019).

The interplay between institutions and structural change opens the door to policy intervention and more specifically to the elaboration of industrial policy. The main challenge consists in coherently addressing the institutional bottlenecks that emerge during the process of structural transformation (Andreoni and Chang, 2019). Without an alignment between institutional and structural cycles, the effectiveness of policy interventions is limited (Mazzucato, 2013; Andreoni et al. 2017). Hence, the capacity of an economy to continuously shift resources towards growth-enhancing activities depends on its ability of constantly adapting to changing conditions (Palma, 2009).

Institutions matter for economic growth and there is a certain consensus that the basic criterion for judgment is their efficiency in reducing transaction costs and enhancing market exchange (van Zanden, 2009). For instance, such principle can be found in North’s preference for the creation of “open access orders” rather than “natural states” (e.g. North et al., 2009). It is also part of Acemoglu and Robinson (2012) defence of “inclusive” rather than “extractive” economic and political arrangements. Frequently referred to as the “rules of the game”, institutions can be understood as a combination of formal rules, written laws, informal norms of behaviour, and shared beliefs about the world. Explaining economic change requires a theory of institutional change because it is the institutional framework that defines the deliberate incentive structure of a society (North, 2005).

On the other hand, alternative theories of growth and distribution have emphasised over the years the importance of demand constraints on economic performance. One of the most successful empirical regularities among them is the dynamic Harrod trade-multiplier, also known as Thirlwall’s (1979) law. It states that if in the long-run balance-of-payments equilibrium on current account is a requirement, then economic growth can be approximated by the ratio of the growth rate of exports to the income elasticity of demand for imports, i.e. growth is Balance-of-Payments Constrained (BoPC). Despite its vintage, the model continues to stand as a powerful explanation of international growth rate differences, and has been used to study processes of structural change in developed and developing economies (see, for example, Fagerberg, 1988; Araújo and Lima, 2007; Cimoli and Porcile, 2014).

The literature on BoPC growth recognises the importance of institutions as part of the explanation for differences in economic performance among countries. However, institutional
asymmetries are mainly viewed as different functional forms or parameter values in the behavioural equations. In this way, the evaluation of different institutional arrangements is limited to comparative static analysis (e.g. Cimoli, 1988 and, more recently, Ribeiro et al. 2016; a comparison between Latin America and Asia growth paths can be found in Cimoli et al. 2019).

In order to go a step further, in this article we formalise a mechanism that explains how people with different attitudes towards change influence each other and contribute to the consolidation of certain types of institutions. Societies whose past experiences condition them to regard innovative change with antipathy and suspicion are in sharp contrast with those whose heritage provide them with a favourable attitude to change (North, 2005; North et al., 2009). While a deeper assessment on which types of institutions are more effective or desirable goes beyond the scope of the paper, our main point is that societies that are more open to change will find better ways to adapt to change.

This article makes two contributions to the literature on growth and structural change. First, using panel cointegration and panel Vector Autoregressive (pVAR) techniques, we estimate the multisectoral version of Thirlwall’s law and show that increases in the rate of growth compatible with equilibrium in the balance-of-payments might have a negative impact on the ratio of foreign trade income elasticities. Second, we develop two models of structural change assuming that the capacity of adaptation of the economy is a function of attitudes or sentiments towards change. We discuss the contrasting experiences of Latin America and Asia since the 1960s to highlight how a complex economy is likely to be associated with a better distribution of political and economic power. Without denying the crucial role of industrial policy, our contribution emphasises the role of shared beliefs and mental constructions that are behind institutions and policy interventions.

The first model we present leads to a 2-dimensional nonlinear dynamic system in which the composition of sentiments changes endogenously and plays a crucial role in the determination of growth trajectories. We proceed by introducing some power and income distribution considerations. The resulting 4-dimensional nonlinear dynamic system is also shown to admit multiple equilibria. Without having to impose any special condition on the values of the parameters, a Hopf-Bifurcation analysis establishes the possibility of persistent and bounded cyclical paths, providing further insights on the nature of structural and institutional change. This result is important because can be understood as a formalisation of what Andreoni and coauthors defined as “structural cycles” (see, for instance, Andreoni et al. 2017).

Our exercise has a clear message. There are different shared beliefs and perceptions in society regarding how to approach change which depend on several political and cultural values that shape the views of citizens. To the extent that there is a significant degree of uncertainty regarding the implications of such change, it is important to understand how agents with different attitudes interact. The sum of individual sentiments generates what we refer to as collective opinion, while the latter determines the explicit and implicit rules that influence our own beliefs. Thus, the capacity of adaptation of the economy depends on the design of public policies and institutions which are ultimately the result of people’s attitudes and sentiments towards change itself:

\[ \text{Attitudes towards change} \iff \text{Institutions} \iff \text{Structural change} \]

An implication of our analysis is that, before discussing specific policy instruments, we have to take account of how a particular society is willing to respond when facing the uncertainty that follows structural change. A society that regards innovative change with antipathy and suspicion will design industrial policies that reflect this perceived reality which will
always be Pareto-inferior. Khan and Blankenburg (2009) argued that the (in)compatibility of technological and institutional strategies explains why the same policies produced such different outcomes in Latin America and Asia before and after the liberalisation process of the 1990s. In this article, we stress that the underlying mechanism might be mental constructions that give support to different attitudes towards change.

The remainder of the paper is organized as follows. In the next section, we present our empirical exercise showing the validity of Thirlwall’s law using panel cointegration techniques for a sample of 34 Latin American and Asian countries. A pVAR estimator allows us to identify a feedback mechanism from the BoPC growth rate to the ratio of foreign trade income elasticities. In Section 3, we present our first model of attitudes towards change. Section 4 brings an extension that allows for feedback effects from structural change to sentiments through power and income distribution. Numerical simulations are performed based on the analytical models. Some final considerations follow.

2 Some empirics on structural change and the external constraint

The idea that growth is BoPC has been an important component of much demand-led growth theory since at least Prebisch (1959). The crucial element that distinguishes this approach from other growth models is the role of demand defining the nature of the constraint (Razmi, 2016). Suppose the following traditional functions for exports and imports hold:

\[ X = X(Z), \quad X_Z > 0 \]
\[ M = M(Y), \quad M_Y > 0 \]

where \( X \) are exports, \( M \), imports, \( Z \), the rest of the world’s output, and \( Y \) is the domestic output. Since we are abstracting from any price consideration, the real exchange rate is assumed to be constant and equal to one. For simplicity, it is also assumed that all trade consists in the exchange of final goods.

Equilibrium in trade, which in this framework approximates equilibrium in the balance-of-payments, implies:

\[ X(Z) = M(Y) \]

Thus, we can easily show that: \(^1\)

\[ y_{bp} = \frac{\dot{X}/X}{\pi} = \frac{\rho z}{\pi} \quad (1) \]

where \( \pi = \frac{dM}{dY} \) is the income elasticity of imports, \( \varphi = \frac{dX}{dZ} \), the income elasticity of exports, \( \rho = \varphi/\pi \), the ratio between foreign trade income elasticities, \( y_{bp} \), the BoPC rate of growth, and \( z = \dot{Z}/Z \) is the growth rate of the rest of the world.

Following Gouvêa and Lima (2010, 2013), multisectoral tests of the law can be performed by estimating:

\[ \ln M_{it} = \pi_{it} \ln Y_{it} + u_{it} \quad (3) \]

\(^1\)For any generic variable \( x \), the time derivative is indicated by \( \dot{x} \), while \( \dot{x}/x \) corresponds to its rate of growth. On the other hand, the derivative of any generic function \( x(\tau) \) will be indicated as \( dx/d\tau = x_{\tau} \).
where \( i = 1, 2, ..., n \) refers to the number of sectors and \( u \) stands as an error term. The BoPC growth rate is obtained by dividing the weighted sum of the sectoral rate of growth of exports by the weighted sum of the estimated sectoral income elasticities of imports, as in Eq. (1).

Alternatively, one can also estimate a sectoral exports function of the type:

\[
\ln X_{it} = \varphi_{it} \ln Z_t + v_{it} \tag{4}
\]

where \( v \) stands as an error term. In this case, Thirlwall’s law is computed as the weighted sum of the estimated sectoral income elasticities of demand for exports over imports, as in Eq. (2).

Several methodologies have been used over the years to test the validity of the so-called dynamic Harrod trade-multiplier. Here we provide some empirical evidence of our own using panel cointegration techniques for a sample of 20 Latin American and 14 Asian countries between 1980 and 2014 for a 11-sector level of aggregation. The multisectoral version of Thirlwall’s law was obtained applying the Fully Modified Ordinary Least Squares (FMOLS) and Dynamic Ordinary Least Squares (DOLS) estimators. To the best of our knowledge, we are the first to use such a technique when testing the robustness of the law.\(^2\) Two different panels were estimated, one for Latin America and the other for Asia. A detailed description of the database, the pre-estimation treatments, and the estimated coefficients is provided in the Empirical Appendix.

Fig. 1 shows that actual and estimated growth rates are indeed very close for both FMOLS and DOLS models. Asian countries, in red, present systematically higher rates of growth than Latin American countries, in blue.

An extensive literature on complexity has shown that there is a positive relationship between economic complexity, product diversification and the growth rate of output (e.g.\(^2\)The main advantage of this methodology is that, in the presence of a cointegrating relationship, it controls for serial correlation and endogeneity. Omitted variables are less likely to affect the reliability of our estimates given that they will either be stationary – in which case the estimated coefficients are invariant to their inclusion – or it will be nonstationary – in which case we will not be able to obtain a stable cointegrating relationship if we leave them out.

\(^2\)
Hidalgo et al., 2007; Hausmann et al., 2014). On the other hand, scholars in the BoPC tradition have pointed out that the ratio between foreign trade income elasticities reflects the non-price competitiveness of the economy, which in its turn is determined by the diversification and complexity of its productive structure (see, Gouvêa and Lima, 2010, 2013; Romero and McCombie, 2016).

Fig. 2 shows the estimated ratio between the sectoral income elasticities of exports and imports for Latin America and Asia. Values of $\rho_i$ greater than one indicate that sector $i$ is capable to respond more than proportionally to increases in foreign demand. Our exercise shows that Latin America is very competitive in industries such as animal and vegetal oils and less dynamic in manufacturing and services. Asia, on the other hand, has strong non-price competitiveness in chemicals and machinery ($\rho_i \approx 2$) and a significant capacity to respond to increases in foreign demand in manufacturing and services ($\rho_i \approx 1.5$).

![Figure 2: FMOLS and DOLS estimates of non-price competitiveness for a sample of 34 Latin American and Asian countries between 1980-2014 for a 11-sector level of aggregation.](image)

Different mechanisms have been proposed to explain changes in non-price competitiveness and, consequently, in the BoPC growth rate. Based on the concept of technological competitiveness, some authors have discussed the role played by technological specialisation across countries (see, among others, Fagerberg, 1988; Meliciani, 2002; Cimoli and Porcile, 2014). Other contributions have given particular attention to changes in the sectoral composition of the economy (e.g. Araújo and Lima, 2007; Nishi, 2016). Finally, the possibility of feedbacks from growth to non-price competitiveness has been explored by Fiorillo (2001), McCombie and Roberts (2002), and Setterfield (2011).

We proceed by assessing the empirical relevance of this last mechanism. Using our estimates of the sectoral elasticities of exports and imports, we are able to compute $\rho$ and $y_{bp}$ for every country and year of our sample. The correspondence between this two variables is investigated using a panel-data Vector Autoregression methodology. A detailed description of the pre-estimation treatments and estimated coefficients is provided in the Empirical Appendix.

Fig. 3 shows the response of non-price competitiveness to a shock in the BoPC growth rate in the simplest bivariate case that does not differentiate between regions and only

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3The strength of this approach comes from the fact that it treats all variables in the system as endogenous while allowing for unobserved individual heterogeneity.
controls for the size of the economy as an exogenous variable in the pVAR. An increase in $y_{bp}$ is associated with a reduction in $\rho$. Such a result is in line with the stratification mechanism formalised by Dávila-Fernández et al. (2018) and we shall discuss it in detail in the next section.

![Figure 3: Impulse-response functions based on pVAR estimates.](image)

We continue by investigating the dynamics of each single region. Fig. 4(a) reports the respective Impulse Response Functions (IRF) for Latin America when we add an extra exogenous control for the size of the export sector. Given that the literature has emphasised the importance of productive diversification to non-price competitiveness, we continue by including two extra controls for economic complexity: the Economic Complexity Index (ECI), as an endogenous variable, and the Complexity Outlook Index (COI), as an exogenous one. Fig. 4(b) indicates an increase in the magnitude of the response.

The same procedure was repeated for our sample of Asian countries. Figs. 4(c,d) suggest that the response of $\rho$ to a shock in $y_{bp}$ is slightly stronger in comparison to Latin America. Overall, one standard deviation impulse on the BoPC rate of growth decreases the ratio of foreign trade income elasticities by $0.2 - 1\%$. Notice, however, that in both cases controlling for economic complexity increases the strength of the response. In line with the nature of structural change, the effects disappear only after about 10 to 20 years depending on the region.

### 3 A simple model of structural change

It is our purpose in this section to develop a model of structural change in which agents with different attitudes towards change endogenously interact and play a crucial role in the determination of economic trajectories. Our approach follows closely the literature on mutual mimetic contagion in speculative markets formalised by Lux (1995). For expositional reasons, we divide it into two blocks of equations, referring to (i) the stratification mechanism and (ii) attitudes towards change.

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4This model has been intensively used to assess macroeconomic and stock market interactions. Recent applications have also discussed the role of the mechanism for the dynamics of attitudes towards the environment (see, for example, Dávila-Fernández and Sordi, 2019).
Figure 4: Impulse response functions based on pVAR estimates for a sample of 20 Latin American countries, (a)-(b), and 14 Asian countries, (c)-(d). Panels (a) and (c) control for the size of the economy and the export sector. Panels (b) and (d) include extra controls for economic complexity.
3.1 The stratification mechanism

The relation between growth and structural change is highly non-linear. The so-called stratification mechanism is given by:

\[
\frac{\dot{\rho}}{\rho} = \varepsilon (\alpha - y_{bp}) \tag{5}
\]

where \( \alpha \) indicates the maximum rate of growth the economy is able to adjust to, and \( \varepsilon \) corresponds to the speed at which changes induced by growth are translated into changes in \( \rho \).

Economic growth presses for change. Therefore, the institutional and productive arrangements that sustain economic activity need to adapt to the new requirements of change (Abramovitz, 1986; Chang, 2007, 2011). However, the capacity to adapt of an economy depends on several specific constraints. Effective structural change may require coordination in many components. This is because complementarities between the different parts that form the system in and over time are one of the key manifestations of structural interdependency (Andreoni and Chang, 2019). Similar to complex species, economies are characterised by development constraints such that the fitness of each mutation is limited by the other characteristics of the system (Pagano, 2011).

For example, the capital stock of a country consists of a complex web of interlocking elements. Given that they are initially built to fit together, it is difficult to make replacements without the costly rebuilding of other components. Growth opens several opportunity windows through dynamic economies of scale and the division of labour. In fact, a certain minimum scale of production is a necessary condition for the introduction of more effective production techniques (Andreoni and Scazzieri, 2014). However, different production processes are associated with different configurations of tasks, capabilities, capacities and materials. As the size of the system expands, it is likely that new opportunities of specialisation and increasing returns arise. It is also likely that additional scarcity constraints are discovered because an increasing number of non-produced inputs will become essential (Scazzieri, 1993).

The path of economic diversification is strongly conditioned by the positioning of the existing productive structure in the product space (Hidalgo et al., 2007; Alshamsi et al., 2018). Furthermore, the capacity of adaptation is also deeply related to distributive issues and the struggle between reform and status quo. Leaving aside any reference to which types of reforms or institutions are “good” or “bad”, coordination uncertainty may arise because it is not clear who is going to be better-off afterwards (Fernandez and Rodrik, 1991; Rodrik, 1996). Consequently, change paradoxically also creates incentives to “keep things like they are”, making change costly.

In Dávila-Fernández et al. (2018) formalisation of the stratification mechanism, \( \alpha \) captures the maximum rate of growth the economy is able to adapt to and translate into a more diversified productive structure. Hence, one could think of it as a threshold for increasing returns. For a constant growth rate of the rest of the world, it follows that \( \dot{y}_{bp}/y_{bp} = \dot{\rho}/\rho \). Hence, given Eq. (5):

\[
\frac{\dot{y}_{bp}}{y_{bp}} = \varepsilon (\alpha - y_{bp}) \tag{6}
\]

In what follows, our purpose is to endogenise the capacity of adaptation of the economy emphasising the role of institutions in the determination of \( \alpha \).
3.2 Institutions and attitudes towards change

Human interaction is a basic attribute of everyday life and is structured by the prevalence of recognised collective practices, or institutions (Lawson, 2012). Following North (2005), we understand that the “reality” of a political-economic system is never know to anyone. Still, humans elaborate beliefs about the nature of that “reality”. They can be both positive, about the way the system works, and normative, about how it should work. In any case, a certain perceived reality determines the set of beliefs of a given society. This in turn works as a basic input to any attempt to structure the environment, reducing the uncertainty that is intrinsic to any kind of human interaction. The human environment becomes the human construct of rules, norms, and conventions that define the framework of social interchange (Knight and North, 1997).

The idea that we do not reproduce reality but, rather, construct systems of “habits of thought common to the generality of men” goes back at least to Veblen (1909, p. 239). It also finds echo in Hayek (1952), who indicated that our learning about the external world is itself the product of a kind of experience. Such specification of how individual beliefs interrelate with social context provides a set of mechanisms by which it is possible to connect our explanation of economic and structural change to social institutions. This field is still in its infancy, but it is our intention to emphasise in this representation two main elements. One of them is related to how individuals and society approach change. The second concerns the interactions of the single individual with the collectivity that surrounds her/him.

The development of different belief systems produces diverse abilities to confront the problems that novelty posits. Instead of assuming that agents are fully rational in the conventional sense, we are inclined to think that they are more likely to form habits in line with Simon’s (1976) notion of procedural rationality. Hence, suppose that the population equals the labour force, \(N\), and is divided between those whose attitudes are open to change, \(N^+\), and those who have a more hostile position, \(N^-\), so that:

\[ N = N^+ + N^- \]

while we indicate with \(n\) the difference between the two groups:

\[ n = N^+ - N^- \]

Defining:

\[ \Phi = \frac{n}{N} = \frac{N^+ - N^-}{N^+ + N^-} \tag{7} \]

we have that \(\Phi \in [-1, 1]\) is an index describing the average sentiment of the population towards change, or, in other words, the degree of willingness to embrace change. If all citizens of this society are change-friendly, then \(\Phi = 1\). At the other extreme, a complete opposition to what is new in terms of social change results in \(\Phi = -1\). For an equal division of the population between these two groups, we have \(\Phi = 0\).

As a simplifying hypothesis, assume that \(N\) does not change over time, i.e. \(\dot{N}/N = 0\). Taking time derivatives of Eq. (7), we obtain:

\[ \dot{\Phi} = \frac{\dot{n}}{N} = \frac{\dot{N}^+ - \dot{N}^-}{N^+ + N^-} \tag{8} \]
i.e., for a constant $N$, changes in the sentiment index fundamentally depend on the difference between variations in the two groups that form the population. Hence, we need to specify the behaviour of $\dot{N}^+$ and $\dot{N}^-$ taking into account that people might change their own views on the matter. To this end, we write:

\begin{align}
\dot{N}^+ &= N^-p^{+-} - N^+p^{-+} \quad (9) \\
\dot{N}^- &= N^+p^{-+} - N^-p^{+-} \quad (10)
\end{align}

where $p^{+-}$ is the probability of someone who is pro-openness to change her/his mind and $p^{-+}$ stands for the probability of the opposite case.

Clark (1997) argues that while every thought is formed in the brain, the flow of thoughts seem to depend on repeated interactions with external sources. Single individuals are part of groups and a common culture heritage provides means of reducing the divergent mental models among them (Zweynert, 2009). Moreover, cognitive activity does not happen without a context, where context is not a fixed set of surrounding conditions, but a highly dynamic process in which the cognition of the individual itself is a part of the whole (Hutchins, 1995). If cognitive process are context-dependent, the more we do or think habitually, the more can be carried out tacitly, without conscious reflection (Lawson, 2015). Analogously, the more people around us approach change in a certain way, the more likely is that we will adopt a similar attitude.

The probabilities $p^{+-}$ and $p^{-+}$ depend on the distribution of the population between the two groups. The higher the share of citizens in the economy that support or has a positive attitude towards change, the higher the probability of someone with the opposing attitudes changing her/his views. Similar reasoning in the other direction applies. In this way, our representation allows for an important source of institutional inertia, at least in terms of the approach to change. Therefore, consider:

\begin{align}
p^{+-} &= v^+(\Phi), \quad v_{\Phi}^+ > 0 \quad (11) \\
p^{-+} &= v^-(\Phi), \quad v_{\Phi}^- < 0 \quad (12)
\end{align}

where $v^+(\cdot)$ and $v^-(\cdot)$ stand as the probability functions of an agent coming to favour or to oppose a positive attitude towards change, respectively.

Substitute Eqs. (11) and (12) into (9) and (10) so that changes in $\dot{N}^+$ and $\dot{N}^-$ are a function of the sentiment index. Further inserting the resulting expressions in Eq. (8), and making use of the definitions of $N$ and $n$, we obtain the dynamic relation that governs sentiment dynamics towards openness:

\begin{equation}
\dot{\Phi} = (1 - \Phi) v^+(\Phi) - (1 + \Phi) v^-(\Phi) = \theta(\Phi) \quad (13)
\end{equation}

such that $\theta(\Phi) \gtrless 0$.

The literature on structural change recognises that economic growth demands institutional and ideological adjustments (see, for example, Andreoni and Chang, 2019; Chang and Andreoni, 2019). As discussed in detail by Narula (2004), adaptive capacities include the ability of integrating existing and exploitable resources into the production chain, foresighting and selecting the most appropriate technological trajectories, etc. Without entering into the merit of which institutions or types of institutions are more effective or desirable, our main point is that societies that are more open to change will find better ways to adapt to change. In this way, $\alpha$ is such that:

\begin{equation}
\alpha = \alpha(\Phi), \quad \alpha_{\Phi} > 0, \quad \alpha(0) > 0 \quad (14)
\end{equation}
Conformity can be costly in a world of uncertainty and, in the long-run, produces stagnation and decay as humans confront new challenges that require innovative institutional creation (North, 2005). A society that regards innovative change with antipathy or suspicion will design policies and build institutions that reflect this perceived reality which will always be Pareto-inferior. Empirical evidence for our claim is far from conclusive but we provide here some further intuition using our estimates of non-price competitiveness. We rely on two particular questions of the World Values Survey as proxies for attitudes towards change.

Fig. 5, on the left, depicts a positive correlation between the share of population that sees future technology as something good for society and the ratio of foreign trade elasticities, $\rho$. This holds true over the years for Latin American, in blue, and Asian, in red, countries. Analogously, on the right, there is a negative correlation between those who see future technology as bad for their lives and non-price competitiveness.

Figure 5: Attitudes or sentiments towards technology and non-price competitiveness for a sample of 17 Latin American and Asian countries between 1980-2014.

3.3 The dynamic system

The dynamic system of our simple model consists of two differential equations, one for the BoPC rate of growth and another for sentiments or attitudes towards change. Substituting Eq. (14) in (6), we obtain the dynamics of structural change in terms of variations in the rate of growth consistent with equilibrium in the balance-of-payments. On the other hand, sentiment dynamics are governed by Eq. (13). Hence, we have:

$$\dot{y}_{bp} = \varepsilon \left[ \alpha (\Phi) - y_{bp} \right] y_{bp}$$
$$\dot{\Phi} = \theta (\Phi)$$ (15)

Notice that $\theta (\cdot)$ is a highly non-linear function which leaves the door open to the existence of multiple non-trivial equilibria. This is particularly interesting for the literature on growth and institutional economics because it indicates the complexity of the problem in our hands and the possibility of path dependence. In order to provide a more concrete view of its structure and properties, we define functional forms for $p^{-\uparrow}$ and $p^{+\downarrow}$. Hence, suppose:

$$v^+ (\Phi) = \zeta \exp (\mu \Phi)$$
$$v^- (\Phi) = \zeta \exp (-\mu \Phi)$$ (16)
where $\zeta > 0$ captures the speed of change, and $\mu > 0$ is a measure of the “strength of infection” or “herd behaviour”. This last parameter is important for the existence of a unique or multiple equilibria values as we will show in the next section.

Substituting Eqs. (16) and (17) into (13), we can rewrite the dynamic system (15) as:

\[
\begin{align*}
\dot{y}_{bp} &= \varepsilon \left[ \alpha (\Phi) - y_{bp} \right] y_{bp} \\
\dot{\Phi} &= \zeta \left[ (1 - \Phi) \exp (\mu \Phi) - (1 + \Phi) \exp (-\mu \Phi) \right]
\end{align*}
\] (18)

where sentiments towards change influence $y_{bp}$ through the capacity of adaptation, but there is no feedback from structural change to $\Phi$.

### 3.4 Equilibrium points and local stability analysis

In steady-state, $\dot{y}_{bp} = \dot{\Phi} = 0$. This gives us the following equilibrium conditions:

\[
\begin{align*}
\left[ \alpha (\Phi) - y_{bp} \right] y_{bp} &= 0 \\
(1 - \Phi) \exp (\mu \Phi) &= (1 + \Phi) \exp (-\mu \Phi)
\end{align*}
\] (19)

Structural change is understood as variations in $y_{bp}$. There is no structural change under two conditions. Either the rate of growth required by the balance-of-payments equilibrium on current account is equal to the capacity of adaptation of the economy, $y_{bp} = \alpha (\Phi)$, or there is no growth in the first place, $y_{bp} = 0$. Finally, the sentiment index towards change can only stabilise when the probabilities of changing between groups equilibrate.

Given conditions (19), we can state and prove the following propositions regarding the existence of equilibria.

**Proposition 1** If the “herd behaviour” effect regarding sentiments towards change is weak enough, i.e. $\mu \leq 1$, the dynamic system (18) has two equilibrium solutions, $(y_{bp}^{E_1}, \Phi_{E_1})$ and $(y_{bp}^{E_2}, \Phi_{E_2})$, such that:

\[
\begin{align*}
y_{bp}^{E_1} &= \alpha (0) \\
\Phi_{E_1} &= 0
\end{align*}
\]

and

\[
\begin{align*}
y_{bp}^{E_2} &= \Phi_{E_2} = 0
\end{align*}
\]

**Proof.** See Mathematical Appendix B.1. □

A weak “herd behaviour” effect indicates that there is little interaction between citizens in what concerns their attitudes towards change. The individual’s position is not really influenced by the social context making the latter also relatively independent of personal attitudes. As a result, in equilibrium, sentiments are equally divided. Since there is no strong position about how to approach change, they become irrelevant to the determination of the capacity to adapt. It follows that the structure of the economy adjusts to this state. On the other hand, notice that the system even at this early stage already admits the case of a stagnant economy. If there is no growth, there is no structural change.

When there is sufficiently strong interaction, the nature of the system changes qualitatively exhibiting additional equilibria, as stated in the following proposition.
Proposition 2 If the “herd behaviour” effect regarding sentiments towards change is strong enough, i.e. \( \mu > 1 \), the dynamic system (18) has four additional equilibrium solutions:

\[
y_{E_i}^{bp} = \alpha \left( \Phi_{E_i} \right) \left( 1 - \Phi_{E_i} \right) \exp \left( \mu \Phi_{E_i} \right) = \left( 1 + \Phi_{E_i} \right) \exp \left( -\mu \Phi_{E_i} \right), \quad i = 3, 4
\]

and

\[
y_{E_j}^{bp} = 0 \quad \left( 1 - \Phi_{E_j} \right) \exp \left( \mu \Phi_{E_j} \right) = \left( 1 + \Phi_{E_j} \right) \exp \left( -\mu \Phi_{E_j} \right), \quad j = 5, 6
\]

where \( \Phi_{E_3} = \Phi_{E_5} < 0 \), and \( \Phi_{E_4} = \Phi_{E_6} > 0 \).

**Proof.** See Mathematical Appendix B.1. ■

In the case in which the individual’s position is strongly influenced by the social context, the economy may reach new equilibrium situations. If those who have a favourable attitude towards change become the main voice in society, they will contribute to the design of policies and institutions that reflect such approach. This in turn will influence in a determinant way the capacity of adaptation of the economy, leading to a higher rate of growth. On the other hand, it might happen that those that regard innovative change with suspicion become the majority. As a result, an opposite approach will be adopted by principle, leading to a lower equilibrium BoPC growth rate.

It is possible, however, that the economy falls in a state of stagnancy in which there is no growth regardless the social approach to change. That is, even when agents are willing to embrace change, there might be no structural change at all. Given the possibility of multiple equilibria, it becomes very important to understand the stability conditions of each one of these states. Concentrating on the dynamic equation of attitudes or sentiments, it is not difficult to see that if \( \mu \leq 1 \), then \( \theta_{\Phi} (0) < 0 \), whereas for \( \mu > 1 \), the following two different situations arise: (i) \( \theta_{\Phi} (0) > 0 \) and (ii) \( \theta_{\Phi} \left( \Phi_{E_i} \right) \) or \( \theta_{\Phi} \left( \Phi_{E_j} \right) < 0 \). Therefore, we are able to state and prove the following propositions regarding the local stability of equilibria.

Proposition 3 When the “herd behaviour” effect regarding sentiments towards change is weak enough, i.e. \( \mu \leq 1 \), the equilibrium point \( \left( y_{E_1}^{bp}, \Phi_{E_1} \right) \) is locally stable while \( \left( y_{E_2}^{bp}, \Phi_{E_2} \right) \) is a saddle point.

**Proof.** See Mathematical Appendix B.2. ■

In a society with low levels of interaction between citizens or in which individual’s positions are not really influenced by the social context, there is a balance between divergent opinions that results in a stable equilibrium. Structural change becomes relatively independent of \( \Phi \). Non-price competitiveness will adjust taking into account all other variables that influence \( \alpha \), with no particular effect coming from attitudes towards change.

We rely on numerical simulations to better appreciate the economic properties of the dynamic system. For this purpose, we need to define a functional form for \( \alpha (\cdot) \). We choose a linear specification to emphasise that the dynamics we obtained do not rely on specific non-linearities in the behavioural relations, with the exception of the very natural non-linearity in the switching process:

\[
\alpha (\Phi) = \alpha_0 + \alpha_1 \Phi
\]
Taking as parameter values:

\[ \alpha_0 = 0.03, \; \alpha_1 = 0.01; \; \varepsilon = 1.5, \; \zeta = 0.25 \]

Fig. 6 shows the direction field in the phase space \( (y_{bp}, \Phi) \) when \( \mu = 0.75 \).

The story is quite different if there is sufficient interaction between sentiments or attitudes. In this case, the first equilibrium point undergoes a pitchfork bifurcation becoming unstable and giving rise to two locally stable points.

**Proposition 4** When the “herd behaviour” effect regarding sentiments towards change is strong enough, i.e. \( \mu > 1 \), the additional equilibrium solutions \( (y_{bp}^{E1}, \Phi^{E1}) \) and \( (y_{bp}^{E2}, \Phi^{E2}) \) are locally stable while \( (y_{bp}^{E3}, \Phi^{E3}) \) and \( (y_{bp}^{E4}, \Phi^{E4}) \) are saddle points.

**Proof.** See Mathematical Appendix B.2. ■

This last proposition indicates that with enough interaction in society, some degree of polarisation will arise, either in one or in the other direction. We consider this is important because the engagement in an active institutional movement for structural change, independently of its political orientation, carries a set of collective and shared habits of thought. Fig. 7 presents the basins of attraction in the phase space \( (y_{bp}, \Phi) \) when we adopt \( \mu = 1.1 \). The stable manifolds of \( (y_{bp}^{E1}, \Phi^{E1}) \) and \( (y_{bp}^{E2}, \Phi^{E2}) \) separate the basin of attraction of \( (y_{bp}^{E3}, \Phi^{E3}) \) and \( (y_{bp}^{E4}, \Phi^{E4}) \). Initial conditions in the dark grey area converge to the “Asian” case in which the majority of agents supports change. On the other hand, initial conditions in the light grey area converge to the “Latin American” equilibrium.

Looking at the recent experience of Latin America and Asia, Khan and Blankenburg (2009) argued that the (in)compatibility of technological and institutional strategies explains why the same policies produced such different outcomes in the two regions, before and after the liberalisation process of the 1990s. The roots of such mismatch could be in the way both regions have approached change. One could argue that the real comparative advantage acquired by Asian countries during this period was to learn how to constantly adapt to change (Palma, 2009).
There is a clear contrast between the Import-substitution strategy adopted by the former and the Export-led growth model in the latter. We can go beyond that and assess why the liberalisation reforms of the 1990s also produced such different outcomes. Indeed, a liberalization process designed as a strategy to promote competitiveness, as in Asia, was likely to deliver different results comparing one that from the beginning was concerned with maintaining the status quo, as in Latin America.

4 An extension of the model

We now continue by extending the model to investigate the possibility of feedback effects from structural change to attitudes towards change. It is our intention to do so by considering the labour market and the role of income distribution in affecting the configuration of interest groups and their bargain power. We assume firms produce using a Leontief production technology. Income is divided between labour and capital such that income distribution is determined by distributive conflict in the labour market.

4.1 Supply conditions

Consider the following production function:

\[ Y = \min \left\{ \frac{K}{q}, qNe \right\} \]

where \( K \) stands for capital and \( \vartheta \) is the capital-output ratio. The level of employment is \( L \), such that \( q = Y/L \) corresponds to labour productivity, and \( e = L/N \) stands for the rate of employment.

For a constant capital-output ratio, the Leontief dynamic efficiency condition states that:

\[ \frac{\dot{K}}{K} = \frac{\dot{q}}{q} + \frac{\dot{N}}{N} + \frac{\dot{e}}{e} \]  

(20)
Recall that, as a simplifying hypothesis, we are assuming that the labour force is constant. The supply-side of the economy adjusts to the demand-side determined growth rate of output, $y_{bp}$, through $K$ and $e$. Hence, it follows that:

\[
\frac{\dot{K}}{K} = y_{bp} \tag{21}
\]

\[
\frac{\dot{e}}{e} = y_{bp} - \frac{\dot{q}}{q} \tag{22}
\]

In order to keep the system as simple as possible, we abstract from a more elaborated specification of capital accumulation, and assume firms automatically adjust their capital stock to the rate of growth of demand determined by the external constraint. On the other hand, employment rates allow for the adjustment between demand and labour productivity. For $y_{bp} > \dot{q}/q$ employment will increase, while for $y_{bp} < \dot{q}/q$ technical change is such that labour vacancies are continuously destroyed. In equilibrium, a constant rate of employment requires output to grow at the same pace as labour productivity. Eqs. (21) and (22) guarantee a constant capital-output ratio and that the so-called “natural rate of growth” follows the external constraint.

4.2 Distributive conditions

In an economy with two factors of production and no government, the income identity is:

\[Y = wL + rK\]

where $w$ is the real-wage and $r$ is the rate of return on capital. The wage-share is defined as $\varpi = wL/Y$. It immediately follows that:

\[\frac{\dot{\varpi}}{\varpi} = \frac{\dot{w}}{w} - \frac{\dot{q}}{q} \tag{23}\]

Changes in income distribution depend on the behaviour of real-wages and labour productivity.

4.3 Power relations and attitudes towards change

Radical economists have long argued that the relative power among individuals dictates the nature of institutions such that they become biased in favour of the more powerful elite (for a discussion, see Khalil, 2013). Existing institutions can affect the configuration of interest groups and their bargain power. Groups with a vested interest in the status quo may attempt to block subsequent institutional change (Kingston and Caballero, 2009). Hence, we allow for changes in the distribution of power to influence the probability of adopting different attitudes towards change. Eqs. (11) and (12) are modified, so that:

\[p^{++} = v^+ \left( \Phi, \frac{\dot{\Gamma}}{\Gamma} \right), \quad v_\Phi^+ > 0, \quad v_{\Gamma/\Gamma}^+ < 0 \tag{24}\]

\[p^{+-} = v^- \left( \Phi, \frac{\dot{\Gamma}}{\Gamma} \right), \quad v_\Phi^- < 0, \quad v_{\Gamma/\Gamma}^- > 0 \tag{25}\]
where $\Gamma$ corresponds to a measure of power distribution in society. The idea is that a concentration of power in the hands of few, captured by an increasing $\Gamma$, favours the status quo, reducing the probability of switching towards $N^+$.

Recent empirical evidence has indicated that money is a political resource and that when the rich get richer relative to the poor, they will also be more powerful relative to the poor (Goodin and Dryzeck, 1980). Indeed, studies on the so-called “relative power theory” have shown that higher income inequality is related to a reduction in protest participation (Solt, 2015), a reduction in political engagement and campaign participation (Solt, 2008; Ritter and Solt, 2019), an increase and acceptance of authoritarianism (Solt, 2012), an increase in nationalism (Solt, 2011), and an increase in religiosity (Solt et al., 2011). These last results support the view that higher inequality concentrates power and makes the individual more likely to accept current conditions as natural, facing change with suspicion.

Therefore, we suppose:

$$\frac{\dot{\Gamma}}{\Gamma} = \psi \frac{\dot{\sigma}}{\sigma}, \psi > 0 \quad (26)$$

where $\sigma$ stands as a proxy for income inequality such as the Gini index, for example. Parameter $\psi$, on the other hand, captures the correspondence between power and income distribution.

### 4.4 Remaining behavioural relations

Real-wages follow a Phillips curve of the type:

$$\frac{\dot{w}}{w} = F(e), \quad F_e > 0 \quad (27)$$

The motivation behind this expression is that when employment rates are high, the bargain power of workers increases and they are able to obtain a higher increase in real wages (see Goodwin, 1967 and, for empirical support, Grasselli and Maheshwari, 2018).

The analysis of the role of technical change in growth processes has been for a long time of central importance in economic theory. Among alternative theories of growth and distribution, two explanations deserve special attention. On the one hand, according to Kaldor-Verdoorn’s law, labour productivity grows in line with the growth rate of output. On the other hand, classical-Marxian technical change states that the rate of growth of labour productivity responds positively to changes in the labour (or wage) share. Hence, we assume:

$$\frac{\dot{q}}{q} = G(y_{bp}, \varpi), \quad G_{y_{bp}} > 0, \quad G_{\varpi} > 0 \quad (28)$$

There is a robust empirical literature on Kaldor-Verdoorn’s law (for recent assessments, see Romero and Britto, 2017; Magacho and McCombie, 2018) and some evidence for the classical-Marxian case (e.g. Dávila-Fernández, 2018).

Finally, we establish a direct link between functional and personal income distribution. In theory, this relationship is not clear-cut. It depends on how labour and capital incomes are distributed as well as on the tax structure and social transfers. Still, recent evidence confirms that declines in the labour income share have a significant impact on income inequality (e.g. ILO, IMF, OECD and World Bank, 2015; ILO and KIEP, 2015). Therefore, we suppose:

$$\frac{\dot{\sigma}}{\sigma} = -\gamma \frac{\dot{\varpi}}{\varpi}, \quad \gamma > 0 \quad (29)$$

such that an increase in the wage-share is related to a reduction in income inequality.
4.5 Dynamic system

Our new dynamic system consists of four differential equations in the BoPC rate of growth, employment rate, wage-share, and sentiments or attitudes towards change. The dynamic equation for structural change was already reported in (15). Substituting Eq. (28) into (22), we obtain the behaviour of employment rates. Making use of Eqs. (23), (27), and (28), changes in income distribution are a function of the difference between the real-wage Phillips curve and the Kaldor-Verdoorn and classical-Marxian mechanisms. The dynamic equation for sentiments is obtained by substituting Eqs. (24) and (25) into (13). We write the full system as follows:

\[
\begin{align*}
\dot{y}_{bp} &= \epsilon \left[ \alpha (\Phi) - y_{bp} \right] y_{bp} \\
\dot{e} &= \left[ y_{bp} - G \left( y_{bp}, \varpi \right) \right] e \\
\dot{\varpi} &= \left[ F \left( e \right) - G \left( y_{bp}, \varpi \right) \right] \varpi \\
\dot{\Phi} &= \theta \left( \frac{1}{\Gamma} \right)
\end{align*}
\]

where \( \theta_{\varpi/\varpi} > 0 \) because an increase in the wage-share reduces income and power inequality.

In order to provide a more concrete view of its structure and properties, we modify Eqs. (16) and (17) to incorporate the effect of changes in income distribution on the probability of individuals being more (or less) friendly to change. Hence, suppose that:

\[
\begin{align*}
p^{++} &= \zeta \exp \left( \mu \Phi - \mu_1 \frac{\hat{\varpi}}{\Gamma} \right) \\
p^{+-} &= \zeta \exp \left( -\mu \Phi + \mu_1 \frac{\hat{\varpi}}{\Gamma} \right)
\end{align*}
\]

in which case, making use of Eq. (26) and (29), the dynamic system reads:

\[
\begin{align*}
\dot{y}_{bp} &= \epsilon \left[ \alpha (\Phi) - y_{bp} \right] y_{bp} \\
\dot{e} &= \left[ y_{bp} - G \left( y_{bp}, \varpi \right) \right] e \\
\dot{\varpi} &= \left[ F \left( e \right) - G \left( y_{bp}, \varpi \right) \right] \varpi \\
\dot{\Phi} &= \zeta \left[ (1 - \Phi) \exp \left( \mu \Phi + \mu_1 \psi \frac{\hat{\varpi}}{\varpi} \right) - (1 + \Phi) \exp \left( -\mu \Phi - \mu_1 \psi \frac{\hat{\varpi}}{\varpi} \right) \right]
\end{align*}
\]

Sentiments towards change influence \( y_{bp} \) through the capacity of adaptation, but now there is a feedback from structural change to \( \Phi \). An increase in the rate of growth of the economy, for instance, increases employment which in turn strengthens the bargain power of workers. Stronger workers are able to obtain higher wages, reducing inequality. Such a reduction impacts the distribution of power in society, making people more willing to accept change.
4.6 Equilibrium points and local stability analysis

In steady-state $\dot{y}_{bp} = \dot{e} = \dot{\varpi} = \dot{\Phi} = 0$. This gives us the following equilibrium conditions:

\begin{align}
0 &= \varepsilon \left[ \alpha (\Phi) - y_{bp} \right] y_{bp} \\
0 &= \left[ y_{bp} - G (y_{bp}, \varpi) \right] e \\
0 &= \left[ F (e) - G (y_{bp}, \varpi) \right] \varpi \\
(1 - \Phi) \exp (\mu \Phi) &= (1 + \Phi) \exp (-\mu \Phi)
\end{align}

The rationality behind the first and last expressions is the same as discussed in the previous model. The novelties come from the labour market and income distribution equations. In equilibrium, a non-zero rate of employment requires that output grows at the same rate as labour productivity. Analogously, real-wages have to grow at the same pace as productivity in order to obtain a stable and positive wage-share.

Visual inspection of conditions (34) indicates that the system is likely to admit multiple equilibrium solutions. To maintain the exercise as simple as possible and to compare the results with those of the previous section, we proceed limiting our analysis to the cases in which equilibrium employment rates and the wage-share are different from zero. Therefore, we can state and prove the following propositions.

**Proposition 5** If the “herd behaviour” effect regarding sentiments towards change is weak enough, i.e. $\mu \leq 1$, the dynamic system (33) admits two equilibrium solutions, $(y_{bp}^{E_1}, \Phi^{E_1})$ and $(y_{bp}^{E_2}, \Phi^{E_2})$, that satisfy:

\begin{align}
y_{bp}^{E_1} &= \alpha (0) \\
G (\alpha (0), \varpi^{E_1}) &= \alpha (0) \\
e^{E_1} &= F^{-1} (\alpha (0)) \\
\Phi^{E_1} &= 0
\end{align}

and

\begin{align}
y_{bp}^{E_2} &= \Phi^{E_2} = 0 \\
G (0, \varpi^{E_2}) &= 0 \\
e^{E_2} &= F^{-1} (0)
\end{align}

such that the rate of employment and wage-share are different from zero.

**Proof.** See Mathematical Appendix B.3. $\blacksquare$

A weak “herd behaviour” effect indicates that agents are not really influenced by the social context, making the latter also relatively independent of personal attitudes. The labour market adjusts to the reality imposed by the combination of sentiments and structural change. In the case in which growth is different from zero, causality runs from attitudes towards change to non-price competitiveness, determining the BoPC growth rate through the stratification mechanism.

Firms adjust employment rates in order to satisfy demand. In this way, a higher rate of growth is associated with higher employment. This in turn will increase the bargain power of workers, leading to higher real-wages and consequently to a higher wage-share. On the
other hand, an increase in the share of output that goes to workers means that real-wages are growing faster than labour productivity. Firms respond searching for production techniques that are labour saving, increasing labour productivity. Income distribution stabilises when the bargain power of workers equilibrates technical change while Kaldor-Verdoorn’s law attenuates the impact of growth on employment rates and income distribution.

It might happen, nonetheless, that the economy is in a situation of stagnation with zero growth and no structural change. In this case, the system is somehow independent of attitudes towards change. The labour market and income distribution adjust to this situation which will be Pareto-inferior to the previous one.

The design of policy and the consolidation of institutions take time. Even when there are variations in the way a certain society approaches change, they are not immediately translated into a greater or lower $\alpha$. When employment rates are high, an increase in the bargain power of workers reduces income inequality, increasing the probability that individuals will question the status quo and adopt attitudes more favourable to change. Through the stratification mechanism, this leads to an increase in the growth-trend. If this process is too strong, we run the risk of entering into an unrealistic loop of explosive growth rates. Hence, we assume that the capacity of adaptation adjusts slowly to changes in sentiments and state the following crucial assumption:

**Assumption** The sensitivity of the capacity of adaptation to changes in sentiments is sufficiently small, such that:

$$\alpha \Phi < \frac{G_y y^E (\varepsilon y^E - \theta \Phi)}{\theta \frac{m}{m^E} (1 - G^y) y^E}$$

When there is sufficiently strong interaction between agents, the nature of the system changes qualitatively exhibiting additional equilibria, as stated in the following proposition.

**Proposition 6** If the “herd behaviour” effect regarding sentiments towards change is strong enough, i.e. $\mu > 1$, the dynamic system (33) has four additional equilibrium solutions such that the rate of employment and wage-share are different from zero:

- $y_{E_3} = \alpha \left( \Phi_{E_3} \right)$
- $G \left( \alpha \left( \Phi_{E_3} \right), \varepsilon_{E_3} \right) = \alpha \left( \Phi_{E_3} \right)$
- $e^{E_3} = F^{-1} \left( \alpha \left( \Phi_{E_3} \right) \right)$
- $(1 - \Phi_{E_3}) \exp (\mu \Phi_{E_3}) = (1 + \Phi_{E_3}) \exp (-\mu \Phi_{E_3})$, $i = 3, 4$

and

- $y_{E_5} = 0$
- $G \left( 0, \varepsilon_{E_5} \right) = 0$
- $e^{E_5} = F^{-1} (0)$
- $(1 - \Phi_{E_5}) \exp (\mu \Phi_{E_5}) = (1 + \Phi_{E_5}) \exp (-\mu \Phi_{E_5})$, $j = 5, 6$

where $\Phi_{E_3} = \Phi_{E_5} < 0$, and $\Phi_{E_4} = \Phi_{E_6} > 0$.

**Proof.** See Mathematical Appendix B.3.

Once we allow for a sufficiently strong interaction between agents, we are able to obtain some degree of polarisation that is reflected in the kind of policies and institutions designed
to respond to change. As a result, the capacity of adaptation of the economy will differ depending on the \( \Phi \) that stabilises sentiments. Once the BoPC growth rate is determined through the stratification mechanism, labour market and income distribution adjust to the conditions imposed by the external constraint.

Given the multiplicity of equilibria possibilities, it becomes very important to understand the stability conditions of each one of these states. We are able to state and prove the following propositions regarding the local stability of equilibria.

**Proposition 7** When the “herd behaviour” effect regarding sentiments towards change is weak enough, i.e. \( \mu \leq 1 \), the equilibrium point \( (y_{E1}^1, e^{E1}, \omega^{E1}, \Phi^{E1}) \) is locally stable as long as the sensitivity of sentiments to changes in income distribution, \( \theta_{\omega/\omega} \), is such that:

\[-(b_1B_1 + D_1) D_1 \theta_{\omega/\omega}^2 + (b_1B_1 C_1 - b_1A_1 D_1 + 2C_1 D_1) \theta_{\omega/\omega} + b_1A_1 C_1 - b_1^2b_4 - C_1^2 > 0\]

where \( b_1, b_2, b_3 \), and \( b_4 \) are the coefficients of the characteristic equation, and

\[A_1 = \varepsilon G_{\omega} y_{yp} e^{E1} - \varepsilon \theta_{\phi} y_{yp}^{E1} + G_{\omega} F_e e^{E1} \omega^{E1} - G_{\omega} \theta_{\phi} \omega^{E1}\]
\[B_1 = \varepsilon \alpha_{\phi} G_{\omega} y_{yp}^{E1} > 0\]
\[C_1 = -\theta_{\phi} G_{\omega} F_e e^{E1} \omega^{E1} - \varepsilon G_{\omega} \theta_{\phi} y_{yp}^{E1} \omega^{E1} + \varepsilon F_e G_{\omega} y_{yp}^{E1} e^{E1} \omega^{E1}\]
\[D_1 = \varepsilon F_e \alpha_{\phi} (1 - G_{\omega}) y_{yp}^{E1} e^{E1} > 0\]

The equilibrium point \( (y_{E2}^1, e^{E2}, \omega^{E2}, \Phi^{E2}) \) will be locally stable as long as the capacity of adaptation, \( \alpha (0) \), is sufficiently small such that:

\[-\varepsilon \alpha (0) + G_{\omega} \omega^{E2} - \theta_{\phi} > 0\]
\[-G_{\omega} \omega^{E2} \theta_{\phi} + G_{\omega} e^{E2} F_e \omega^{E2} + \varepsilon \alpha (0) \theta_{\phi} - \varepsilon \alpha (\Phi) G_{\omega} \omega^{E2} > 0\]
\[-\theta_{\phi} F_e \omega^{E2} G_{\omega} e^{E2} + \varepsilon \alpha (0) G_{\omega} \omega^{E2} \theta_{\phi} - F_e \omega^{E2} G_{\omega} e^{E2} \varepsilon \alpha (0) > 0\]
\[-\varepsilon (C_2 E_2 + F_2) \alpha (0)^3 + (A_2 C_2 E_2 + \varepsilon B_2 E_2 + \varepsilon C_2 D_2 + 2 \varepsilon A_2 F_2) \alpha (0)^2\]
\[-(A_2 B_2 E_2 + A_2 C_2 D_2 + \varepsilon B_2 D_2 + A_2^2 F_2) \alpha (0) + (A_2 B_2 - H_2) H_2 > 0\]

where

\[A_2 = G_{\omega} \omega^{E2} - \theta_{\phi}\]
\[B_2 = -G_{\omega} \omega^{E2} \theta_{\phi} + G_{\omega} e^{E2} F_e \omega^{E2}\]
\[C_2 = \varepsilon \theta_{\phi} - \varepsilon G_{\omega} \omega^{E2}\]
\[D_2 = -\theta_{\phi} F_e \omega^{E2} G_{\omega} e^{E2}\]
\[E_2 = \varepsilon G_{\omega} \omega^{E2} \theta_{\phi} - F_e \omega^{E2} G_{\omega} e^{E2} \varepsilon\]
\[H_2 = \theta_{\phi} F_e \omega^{E2} G_{\omega} e^{E2} \varepsilon\]

**Proof.** See Mathematical Appendix B.4. ■

Even though relatively long, the proposition above has a simple clear message. When there is weak interaction between agents, the population will be equally divided between agents pro- and against- change. The productive structure adjusts to this set of beliefs in terms of growth rates, employment, and income distribution. The latter influences the
distribution of power in society. If variations in income distribution have little impact on how people understand the status quo, then \((y_{bp}, e, \varpi, \Phi)\) is locally stable.

On the other hand, a situation in which \((y_{bp}, e, \varpi, \Phi)\) is locally stable is possible and stands as a “growth trap”. It is related to a low capacity of adaptation to change. When the institutional framework is incapable of delivering a minimum \(\alpha\), then a state of stagnation with zero growth is locally stable. Notice, however, that this is a very exceptional case. If any of the conditions listed in the second part of the proposition are violated, the economy is able to leave the trap and this equilibrium point becomes unstable.

There are no reasons to believe a priori that the inequality in the first part of Proposition 7 necessary holds. Indeed, if such condition is violated, the dynamic behaviour of the model may drastically change, from the qualitative point of view, as the sensitivity of sentiments to the wage-share increases. Using \(\theta_{\varpi/\varpi}\) as a bifurcation parameter, our purpose is to apply the Hopf Bifurcation Theorem (HBT) for 4-dimensional systems to show that persistent cyclical behaviour of the variables can emerge (Asada and Yoshida, 2003).

**Proposition 8** The equilibrium point \((y_{bp}, e, \varpi, \Phi)\) for values of \(\theta_{\varpi/\varpi}\) in the neighbourhood of the critical value \(\theta_{\varpi/\varpi_{HB}}\) such that

\[
-(b_1 B_1 + D_1) D_1 \theta_{\varpi/\varpi_{HB}}^2 + (b_1 B_1 C_1 - b_1 A_1 D_1 + 2 C_1 D_1) \theta_{\varpi/\varpi_{HB}} + b_1 A_1 C_1 - b_2^2 b_4 - C_1^2 = 0
\]

loses stability and the dynamic system admits a family of periodic solutions around it.

**Proof.** See Mathematical Appendix B.4. ■

This result is important because it could be interpreted as a representation of the “structural cycles” described by Andreoni et al. (2017) as “transformational phases of technology transition and organisational reconfiguration that business organisations experience when they shift towards higher-value product segments opportunities” (p. 888). An understanding of these cyclical dynamics matters because the effectiveness of policy interventions is limited by the alignment between structural-cycles and policy design-implementation.

Similarly to what was done in the previous section, we rely on numerical simulations to better appreciate the economic content of our results. We define the following functional forms for the real-wage Phillips curve and the rate of growth of labour productivity:

\[
F(e) = -f_0 + f_1 e
\]

\[
G(y_{bp}, \varpi) = -g_0 + g_1 y_{bp} + g_2 \varpi
\]

where

\[
f_0 = 0.15, \ f_1 = 0.2, \ g_0 = 0.01, \ g_1 = 0.5, \ g_2 = 0.05
\]

and, from the dynamic equation for sentiments:

\[
\mu_1 = 2.35, \ \gamma = 2
\]

Fig. 8 shows the emergence of the limit cycle around \((y_{bp}, e, \varpi, \Phi)\). When \(\psi\) is sufficiently low such that \(\theta_{\varpi/\varpi} < \theta_{\varpi/\varpi_{HB}}\), trajectories converge to an equal distribution of sentiments. Further increasing \(\psi\) leads to a situation in which \(\theta_{\varpi/\varpi} > \theta_{\varpi/\varpi_{HB}}\) and the economy experiences long-waves of progressive and regressive structural change. Fluctuations between rates of growth and sentiments are clockwise oriented. On the other hand, employment rates and the wage-share depict a counter-clockwise motion similar to the original.
Figure 8: Emergence of the limit cycle when $\mu = 0.75$ in the phase space ($y_{bp}$, $\Phi$) and ($e$, $\varpi$). Panels (a)-(b) show convergence to the equilibrium point $(y_{bp}^E, e^E, \varpi^E, \Phi^E)$ when $\phi = 2$. Panels (c)-(d) depict convergence to the limit cycle when $\phi = 5$. 
growth-cycle by Goodwin (1967). We will come back to this point to discuss in detail the rationality behind the endogenous fluctuations.

In a society with low levels of interaction between citizens or in which individual’s positions are not really influenced by the social context, there is a balance between divergent opinions. They might be stable or generate periodic fluctuations depending on the sensitivity of sentiments to income distribution. We proceed by assessing the implications of allowing for a sufficient interaction between sentiments.

Proposition 9 When the “herd behaviour” effect regarding sentiments towards change is strong enough, i.e. \( \mu > 1 \), the equilibrium points \((y_{bp}^E, e_{E_1}, \varpi_{E_1}, \Phi_{E_1}) \) and \((y_{bp}^E, e_{E_1}, \varpi_{E_1}, \Phi_{E_1}) \) are locally stable as long as the sensitivity of sentiments to changes in income distribution, \( \theta_{\varpi/\varpi} \), is such that:

\[
-(b_1B_i + D_i) D_i \theta_{\varpi/\varpi}^2 + (b_1B_iC_i - b_1A_iD_i + 2C_iD_i) \theta_{\varpi/\varpi} + b_1A_iC_i - b_1^2b_4 - C_i^2 > 0
\]

where

\[
\begin{align*}
A_i & = \varepsilon G_{\varpi, y_{bp}^E}w_{E_i} - \varepsilon \theta_{\Phi} y_{bp}^E_i + G_{\varpi} e_{E_i} w_{E_i} - G_{\varpi} \theta_{\Phi} w_{E_i}, \\
B_i & = \varepsilon \theta_{\Phi} y_{bp}^E_i > 0, \\
C_i & = -\varepsilon \theta_{\Phi} G_{\varpi} e_{E_i} w_{E_i} + \varepsilon G_{\varpi} \theta_{\Phi} y_{bp}^E_i w_{E_i} + \varepsilon F_{\varpi} G_{\varpi} e_{E_i} w_{E_i}, \\
D_i & = \varepsilon F_{\alpha} \alpha \Phi \left(1 - G_{y_{bp}^E} \right) y_{bp}^E_i e_{E_i} > 0.
\end{align*}
\]

The equilibrium points \((y_{bp}^E, e_{E_1}, \varpi_{E_1}, \Phi_{E_1}) \) and \((y_{bp}^E, e_{E_1}, \varpi_{E_1}, \Phi_{E_1}) \) will be locally stable as long as \( \alpha (\Phi_{E_1}) \) is sufficiently small such that:

\[
-\varepsilon \alpha (\Phi_{E_1}) + G_{\varpi} w_{E_1} \theta_{\Phi} > 0 \\
-G_{\varpi} w_{E_1} \theta_{\Phi} + G_{\varpi} e_{E_1} F_{\varpi} w_{E_1} + \varepsilon \alpha (\Phi_{E_1}) \theta_{\Phi} = \varepsilon \alpha (\Phi_{E_1}) G_{\varpi} w_{E_1} \theta_{\Phi} > 0 \\
-\theta_{\Phi} F_{\varpi} e_{E_1} G_{\varpi} w_{E_1} e_{E_1} + \varepsilon \alpha (\Phi_{E_1}) G_{\varpi} w_{E_1} \theta_{\Phi} - F_{\varpi} e_{E_1} G_{\varpi} w_{E_1} \varepsilon \alpha (\Phi_{E_1}) > 0 \\
-\theta_{\Phi} F_{\varpi} e_{E_1} G_{\varpi} e_{E_1} e_{E_1} \varepsilon \alpha (\Phi_{E_1}) > 0 \\
-\varepsilon (C_j E_j + F_j) \alpha (\Phi_{E_1})^3 + (A_j C_j E_j + \varepsilon B_j E_j + \varepsilon C_j D_j + 2\varepsilon A_j F_j) \alpha (\Phi_{E_1})^2 \\
- (A_j B_j E_j + A_j C_j D_j + \varepsilon B_j D_j + A_j^2 F_j) \alpha (\Phi_{E_1}) + (A_j B_j - H_j) H_j > 0
\]

where

\[
\begin{align*}
A_j & = G_{\varpi} w_{E_1} - \theta_{\Phi} \\
B_j & = -G_{\varpi} e_{E_1} \theta_{\Phi} + G_{\varpi} e_{E_1} F_{\varpi} w_{E_1}, \\
C_j & = \varepsilon \theta_{\Phi} - \varepsilon G_{\varpi} w_{E_1}, \\
D_j & = -\theta_{\Phi} F_{\varpi} e_{E_1} G_{\varpi} e_{E_1}, \\
E_j & = \varepsilon G_{\varpi} e_{E_1} \theta_{\Phi} - F_{\varpi} e_{E_1} G_{\varpi} e_{E_1} e_{E_1}, \\
H_j & = \theta_{\Phi} F_{\varpi} e_{E_1} G_{\varpi} e_{E_1} e_{E_1}.
\end{align*}
\]

All other equilibrium points are unstable.

Proof. See Mathematical Appendix B.4. □
This last proposition indicates that when there is enough interaction in society, some degree of polarisation will arise. Those equilibrium points in which there is an equal distribution between sentiments towards change become unstable. Moreover, the system admits a “growth trap” case where, independently on social beliefs, there is no structural change. Such state is stable only if the capacity of adaptation is very small such that a series of restrictive conditions are satisfied.

The most interesting case occurs when polarisation in collective beliefs produces very different sets of institutions leading to different capacities of adaptation. As long as variations in income distribution have little influence on the dynamics of Φ, both equilibrium points \((y_{bp}, \omega, E_3, \Phi)\) and \((y_{bp}, e, E_4, \Phi)\) will be stable. Hence, we recover our initial representation of the experiences of Latin America and Asia. The first region stands as an example of societies whose past experience condition it to regard change with antipathy and suspicion. The institutions adopted reflect this characteristic and produce low levels of structural change and growth. As a result, the labour market adjusts to this situation leading to lower rates of employment and high inequality. This in turn reinforces the status quo nature of the system.

On the other hand, Asia has historically adopted more favourable attitudes towards change, regarding it as something to be faced with certain optimism. At least this seems to be the case from the evidence reported in the previous section. This equilibrium is Pareto-superior in terms of rates of growth, employment and also income distribution. The model provides a mechanism that helps to explain the results discussed by Hartmann et al. (2019), highlighting how a complex economy is likely to be associated with a better distribution of political and economic power.

Applying the existence part of the HBT, we are able to show that the system admits persistent cyclical behaviour providing a representation of “structural cycles”. We proceed by stating and proving the following proposition.

**Proposition 10** The equilibrium points \((y_{bp}, \omega, E_3, \Phi)\) and \((y_{bp}, e, E_4, \Phi)\), for values of \(\omega/\omega^i\) in the neighbourhood of the critical value \(\omega/\omega^i_{HB}\) such that

\[
-b_1B_i + D_i \theta^2_{\omega/\omega^i_{HB}} + (b_1B_iC_i - b_1A_iD_i + 2C_iD_i) \theta_{\omega/\omega^i_{HB}} + b_1A_iC_i - b_1^2b_4 - C_i^2 = 0
\]

lose stability and the dynamic system admits a family of periodic solutions around them.

**Proof.** See Mathematical Appendix B.4. ■

Cyclical behaviour is rooted on income distribution and how it can potentially affect attitudes towards change. Fig. 9 shows the emergence of the limit cycle. When the sensitivity of attitudes towards change with respect to income distribution is sufficiently low, such that \(\omega/\omega < \omega/\omega^i_{HB}\), trajectories converge either to the “Asian” or to the “Latin America” equilibrium. The former is Pareto-superior to the latter in terms of growth, employment, and wage-share. Further increasing \(\omega\) leads to a situation in which \(\omega/\omega > \omega/\omega^i_{HB}\) and a limit cycle emerges around \((y_{bp}, \omega, E_3, \Phi)\), \((y_{bp}, e, E_3, \Phi)\) and \((y_{bp}, e, E_4, \Phi)\). In such state of affairs, the economy experiences long-waves of progressive and regressive structural change.

Given the nature of our model, the growth-cycle we obtained should be interpreted as long-waves of structural change with the economy going through phases of high and low public support to change. The cycle works as follows. An increase in positive attitudes towards change leads to an increase in the capacity of adaptation of the economy, resulting
Figure 9: Emergence of the limit cycle when $\mu = 1.1$ in the phase space $(y_{bp}, \Phi)$ and $(e, \varpi)$. Panels (a)-(b) and (c)-(d) show convergence to the equilibrium points $(y_{E_3}, x, \Phi E_3, \Phi E_3)$ and $(y_{E_4}, x, \Phi E_4, \Phi E_4)$ when $\phi = 0.5$ and 1, respectively. Panels (e)-(f) depict convergence to the limit cycle when $\phi = 2$. 
in higher growth. Firms respond hiring more workers. Higher employment rates mean greater bargain power for labour, increasing the wage-share.

A higher wage-share has two opposite effects in the system. On the one hand, there is a reduction in inequality leading to a better distribution of power. This makes society even more favourable to change, increasing the capacity of adaptation of the economy. On the other hand, as the economy grows, the existence of dynamic economies of scale increases the rate of growth of labour productivity through the Kaldor-Verdoorn mechanism. Furthermore, an increasing wage-share indicates that real-wages are growing faster than labour productivity, and firms respond searching for labour-saving production techniques.

Since the capacity of adaptation moves slower than labour productivity, the rate of employment decreases, reverting the trajectory of the economy. A reduction of employment rates reduces the bargain power of workers increasing inequality. At this point society starts to regard innovative change with antipathy leading to reverse structural change. As consequence, a reduction in the BoPC rate of growth will eventually lead to a reduction in the wage-share. There is a reduction in the rate of growth of labour productivity which allows employment to recover. At this point, the cycle restarts:

\[
\Phi \uparrow \Rightarrow y_{bp} \uparrow \Rightarrow e \uparrow \Rightarrow \varpi \uparrow \Rightarrow \frac{\dot{q}}{q} \uparrow \Rightarrow e \downarrow \Rightarrow \varpi \downarrow \Rightarrow \Phi \downarrow
\]

\[
\Phi \downarrow \Rightarrow y_{bp} \downarrow \Rightarrow e \downarrow \Rightarrow \varpi \downarrow \Rightarrow \frac{\dot{q}}{q} \downarrow \Rightarrow e \uparrow \Rightarrow \varpi \uparrow \Rightarrow \Phi \uparrow
\]

as shown in Fig. 10.

5 Final considerations

The dynamic Harrod trade-multiplier model, also referred to as Thirlwall’s law, has now reached its fortieth anniversary. Despite its vintage, the model continues to stand as a powerful explanation of international growth rate differences. Given the large support found in the literature, it is not an exaggeration to say that it stands as one of the most powerful empirical regularities among alternative theories of growth and distribution.
Extensions of the model have explored the multisectoral dimension of structural change, the effect of capital flows, technical change, capital accumulation, and income distribution. Institutions matter for economic growth and structuralist theorists in this tradition have investigated distinct patterns of international trade through the institutional asymmetries that characterise different functional mechanisms in the labour and goods market. It must be noted, however, that there has been little effort to formally investigate additional insights on the role of institutions and institutional dynamics into the model.

This article has made two contributions to the literature on growth and structural change. First, using panel cointegration techniques, we estimated the multisectoral version of Thirlwall’s law. Panel Vector Autoregressive estimations showed that increases in the rate of growth compatible with equilibrium in the balance-of-payments have a negative impact on the ratio of foreign trade income elasticities. Second, we endogenised the capacity of adaptation of the economy, which we assumed to be a function of attitudes or sentiments towards change. In this way, a mechanism is provided that highlights how a complex economy is likely to be associated with a better distribution of political and economic power.

Our modelling exercise has a clear message. The capacity of adaptation of the economy depends on the design of public policies and institutions which are ultimately the result of people’s attitudes towards change. This link was motivated empirically using data from the World Values Survey indicating that a positive attitude to technology is positively correlated with non-price competitiveness. An implication of our analysis is that before discussing any kind of “structural policy”, policy makers need to take into account how a particular society is willing to respond when facing the uncertainty that follows structural change. A society that regards innovative change with suspicion will design industrial policies that reflect this perceived reality.

Figure 11: Co-evolution of structural and attitudes towards change.

The roots of the (in)compatibility between technological and institutional strategies in Latin America and Asia might be found in the way both regions have approached change. There is an obvious contrast between the Import-substitution strategy adopted by the former and the Exports-led growth model in the latter. We can go beyond that and explore why the liberalisation reforms of the 1990s also produced such different outcomes. A liberalization
process designed to maintain a series of privileges, as in Latin America, was likely to deliver different results than one that from the beginning was compromised with a strategy of promoting competitiveness, as in Asia.

We developed two models of structural change. The first one leads to a 2-dimensional nonlinear dynamic system in which the composition of sentiments changes endogenously and plays a crucial role in the determination of growth trajectories. The system was shown to admit multiple equilibria that can be used to describe the Latin American and Asian cases. We continued by introducing the labour market and income distribution considerations. The resulting 4-dimensional nonlinear dynamic system was also shown to admit multiple equilibria. Fig. 11 summarises the main channels of the model.

A Hopf-Bifurcation analysis established the possibility of persistent and bounded cyclical paths, providing further insights on the nature of structural and institutional change and a formalisation of “structural cycles”. Numerical simulations were performed based on the analytical models. Structural change requires institutional and ideological adjustments, a transformation of the habits of thought common to the generality of men.

A Empirical appendix

Our dataset is annual and comprehends the period from 1980 to 2014 for 20 Latin American and 14 Asian countries (Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, Guatemala, Honduras, Haiti, Jamaica, Mexico, Nicaragua, Panama, Peru, Paraguay, El Salvador, Uruguay, Venezuela; Bangladesh, China, Indonesia, India, South Korea, Sri Lanka, Myanmar, Mongolia, Malaysia, Nepal, Pakistan, Singapore, Thailand). The time span was chosen given data availability.

Export and import series were obtained from the Atlas of Economic Complexity that uses raw data for goods, as reported to the United Nations Statistical Division (COMTRADE), and for services, from the International Monetary Fund (IMF). We chose a 11-sector level of aggregation because it allows us to address sectoral differences keeping the analysis as simple as possible. Gross Domestic Product (GDP) series and price deflators were obtained from the Penn World Table (PWT) 9.0. For each country, output of the rest of the world corresponds to the sum of GDP of all countries in the PWT minus domestic GDP. Finally, ECI and COI indexes also come from the Atlas of Economic Complexity. With the exception of these last two indicators, data was converted to logarithmic form.

We proceed by evaluating the stationarity of time-series. We perform the Im, Pesaran & Shin (IPS), the ADF and the PP tests with individual intercepts. These tests allow for individual unit root processes and are all characterized by the combining of individual unit root tests to derive a panel-specific result. The probability of rejecting the null-hypothesis of non-stationarity for imports in levels and first differences is reported in Table A1. We allow for automatic lag selection based on the SIC criteria. Our results indicate that series are integrated of order one, I(1).

We repeat the procedure for exports. Table A2 reports the probability of rejecting the null-hypothesis of non-stationarity. With the exception of animwgoil and othermanuf, IPS indicates that series are I(1). The ADF test rejects the presence of unit root also for chemi. On the other hand, the PP test suggests that mach might be stationary, a result in conflict with the previous two tests.

We conclude this first part of our exercise by reporting in Table A3 that domestic and the rest of the world GDP series are integrated of order one. We cannot reject the null
### Table A1: Unit root test – Sectorial imports (Prob.)

<table>
<thead>
<tr>
<th></th>
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<th>First differences</th>
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<tr>
<td></td>
<td>IPS</td>
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<tr>
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<tr>
<td>bev&amp;tob</td>
<td>0.9707</td>
<td>0.9700</td>
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<td>crudmat</td>
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</tr>
<tr>
<td>fuel</td>
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<td>1.0000</td>
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<tr>
<td>animvegoil</td>
<td>0.9999</td>
<td>0.9992</td>
</tr>
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<td>chemi</td>
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<td>1.0000</td>
</tr>
<tr>
<td>manuf</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>mach</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>othmanuf</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>oth</td>
<td>1.0000</td>
<td>1.0000</td>
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<tr>
<td>serv</td>
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</table>


H0: Series are non-stationary.

### Table A2: Unit root test – Sectorial exports (Prob.)

<table>
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<tr>
<th></th>
<th>Levels</th>
<th>First differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IPS</td>
<td>ADF</td>
</tr>
<tr>
<td>food</td>
<td>1.0000</td>
<td>1.0000</td>
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<td>bev&amp;tob</td>
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<td>animvegoil</td>
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<td>0.0000</td>
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<tr>
<td>chemi</td>
<td>0.4871</td>
<td>0.0203</td>
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<td>manuf</td>
<td>0.9947</td>
<td>0.9213</td>
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<tr>
<td>mach</td>
<td>0.3766</td>
<td>0.1490</td>
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<td>othmanuf</td>
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<tr>
<td>oth</td>
<td>0.9986</td>
<td>0.5846</td>
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<tr>
<td>serv</td>
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<td>1.0000</td>
</tr>
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</table>


H0: Series are non-stationary.
hypothesis of non-stationarity for all three tests.

## Table A3: Unit root test – Output (Prob.)

<table>
<thead>
<tr>
<th></th>
<th>Levels</th>
<th>First differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IPS</td>
<td>ADF</td>
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<tr>
<td>Y</td>
<td>1.0000</td>
<td>1.0000</td>
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<tr>
<td>Z</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
</tbody>
</table>


H0: Series are non-stationary.

Once we have determined that series are I(1), we can continue looking for cointegration. We make use of Pedroni’s test, based on the Engle Granger two-step cointegration procedure, allowing for individual intercepts. Tables A4 and A5 report the probability of rejecting the null-hypothesis of no-cointegration. Following the specification of Eqs. (3) and (4), we investigate the presence of cointegration between imports and domestic output, and between exports and foreign GDP for each one of the eleven sectors of our sample. The optimal number of lags was chosen following the SIC criteria.

## Table A4: Cointegration test – M and Y (Prob.)

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<th>Between</th>
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<td>rho</td>
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<td>crudmat</td>
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<td>0.0000</td>
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<tr>
<td>animvegoil</td>
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<td>0.0000</td>
</tr>
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<td>chemi</td>
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<td>0.0000</td>
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<tr>
<td>manuf</td>
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<td>mach</td>
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<td>0.0001</td>
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<td>othmanuf</td>
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<td>0.0000</td>
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<tr>
<td>serv</td>
<td>0.0013</td>
<td>0.0007</td>
</tr>
</tbody>
</table>


H0: Series are non-cointegrated.

Imports and domestic output were found to cointegrate for almost all sectors with the exception of bev&tob and othmanuf for which Pedroni’s test does not allow us to reject the null-hypothesis of no-cointegration. Hence, for these two sectors, we further perform Kao residual and Fisher cointegration tests. In both cases, we were able to find a long-run relationship between the two series. Thus, we conclude that sectoral imports and GDP are cointegrated. A similar problem occurred when testing the correspondence between exports and the rest of the world’s output. A cointegrating vector was found in all cases but in othmanuf. Further performing Kao and Fisher tests indicates that we can reject the null of no-cointegration also for this sector.

With these results in hands, we continue estimating Eq. (3) using panel cointegration techniques. We make use of the Fully Modified Ordinary Least Squares (FMOLS) and
Dynamic Ordinary Least Squares (DOLS) estimators which have been extended to a panel setting by Kao and Chiang (2001) and Pedroni (2001). Two different panels were estimated, one for Latin America and the other for Asia. Table A6 presents our results for the first region.

In Table A7, we report our sectoral estimates of the import’s function for Asia. On average they are lower than in Latin America. Fig. 1 was obtained by dividing the weighted sum of the sectoral rate of growth of exports by the weighted average sum of the sectoral income elasticities of imports, $\pi_i$.

We proceed estimating Eq. (4) to obtain the income elasticity of exports. Both models deliver similar estimates in terms of the magnitude and significance of the coefficients. Table A8 reports our results for Latin America.

Table A9 brings a similar exercise now performed to Asian countries. Fig. 2 shows the ratio between the income elasticity of exports and imports for each one of the 11 sectors of our sample, $\rho_i$. They can be interpreted as the sectoral non-price competitiveness by region because capture the response of domestic output to an increase in foreign demand.

We are ready to investigate the possibility of feedbacks from non-price competitiveness...
### Table A7: Income elasticity of imports

<table>
<thead>
<tr>
<th>Sector</th>
<th>FMOLS</th>
<th>DOLS</th>
<th>Sector</th>
<th>FMOLS</th>
<th>DOLS</th>
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<td>1.557268***</td>
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*, **, and *** stand by 10%, 5%, and 1% of significance

### Table A8: Income elasticity of exports

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*, **, and *** stand by 10%, 5%, and 1% of significance

### Table A9: Income elasticity of exports

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<td>2.386535***</td>
</tr>
<tr>
<td>φ&lt;sub&gt;bev&amp;tob&lt;/sub&gt;</td>
<td>2.913317***</td>
<td>2.652585***</td>
<td>φ&lt;sub&gt;mach&lt;/sub&gt;</td>
<td>3.555252***</td>
<td>3.447251***</td>
</tr>
<tr>
<td>φ&lt;sub&gt;crudmat&lt;/sub&gt;</td>
<td>1.630531***</td>
<td>1.797677***</td>
<td>φ&lt;sub&gt;othmanuf&lt;/sub&gt;</td>
<td>3.196750***</td>
<td>3.040570***</td>
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<tr>
<td>φ&lt;sub&gt;fuel&lt;/sub&gt;</td>
<td>3.685269***</td>
<td>3.550736***</td>
<td>φ&lt;sub&gt;oth&lt;/sub&gt;</td>
<td>4.248160***</td>
<td>4.427334***</td>
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<tr>
<td>φ&lt;sub&gt;animvegoil&lt;/sub&gt;</td>
<td>2.837457***</td>
<td>2.517164***</td>
<td>φ&lt;sub&gt;serv&lt;/sub&gt;</td>
<td>2.665054***</td>
<td>2.601053***</td>
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<tr>
<td>φ&lt;sub&gt;chemi&lt;/sub&gt;</td>
<td>3.411887***</td>
<td>3.603706***</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*, **, and *** stand by 10%, 5%, and 1% of significance
to the rate of growth consistent with equilibrium in the balance-of-payments. Since the sectoral composition of the economy changes every year, we are able to compute a $\rho$ and $y_{bp}$ for each country every year of the sample. We have two different measures for each variable coming from our FMOLS and DOLS estimates. We make use of the IPS, ADF, and PP unit root tests to verify if series are stationary. Given that we include the ECI as an endogenous variable in some of our pVAR regressions, we also tested it for the presence of unit root. Series were found to be I(0). Results are available under request.

The pVAR model combines the traditional VAR approach – which treats all variables as endogenous – with the panel-data approach – which allows for unobserved individual heterogeneity. In applying the VAR procedure to panel data, we need to impose that the underlying structure is the same for each cross-sectional unit. Such constraint in general does not hold in practice. To avoid this problem, Love and Zicchino (2006) used Helmert procedure of forward mean-differencing. In this paper, we follow their routine as discussed in Abrigo and Love (2016). Table A10 reports our main findings for the simplest case that does not differentiate between regions.

<table>
<thead>
<tr>
<th>Table A10: pVAR estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
</tr>
<tr>
<td>Using “FMOLS” estimates</td>
</tr>
<tr>
<td>$\rho_t$</td>
</tr>
<tr>
<td>$\rho_{t-1}$</td>
</tr>
<tr>
<td>$y_{bp_{t-1}}$</td>
</tr>
<tr>
<td>$\ln Y$</td>
</tr>
<tr>
<td>Granger bi-causality</td>
</tr>
<tr>
<td>Stability</td>
</tr>
</tbody>
</table>

*, **, and *** stand by 10%, 5%, and 1% of significance

An increase in the BoPC growth rate is related to a decrease in non-price competitiveness. Notice that we introduced an exogenous variable to control for the size of the economy, $\ln Y$. Bigger economies are also significantly related to lower growth rates, as predicted by standard theory. In Fig. 3, we plotted the respective IRFs. The models estimated are stable and Granger-causality goes in both directions.

We proceed by investigating the robustness of such result to different regions and to the inclusion of further controlling variables. Tables A11 and A12 report our pVAR estimates for Latin America. The first one is used to obtain the IRFs of Fig. 4(a) while the second gives us Fig. 4(b).

To conclude, Tables A13 and A14 report our pVAR estimates for Asian countries. The first one is used to obtain the IRFs of Fig. 4(c) while the second gives us Fig. 4(d). Confidence intervals in the IRFs were calculated using 200 Monte Carlo draws from the distribution of the fitted reduced-form panel VAR model.
### Table A11: pVAR estimates, Latin America

<table>
<thead>
<tr>
<th>III</th>
<th>IV</th>
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</thead>
<tbody>
<tr>
<td>Using “FMOLS” estimates</td>
<td>Using “DOLS” estimates</td>
</tr>
<tr>
<td>( \rho_t )</td>
<td>( y_{bp_t} )</td>
</tr>
<tr>
<td>( \rho_{t-1} )</td>
<td>0.7810022***</td>
</tr>
<tr>
<td>( y_{bp_{t-1}} )</td>
<td>-0.1913766*</td>
</tr>
<tr>
<td>( \ln Y )</td>
<td>-0.1024037***</td>
</tr>
<tr>
<td>( \ln X )</td>
<td>0.0384309***</td>
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</tbody>
</table>

Granger: \( y_{bp} \rightarrow \rho \)

*Stability Yes Yes

\*, **, and *** stand by 10%, 5%, and 1% of significance

### Table A12: pVAR estimates, Latin America

<table>
<thead>
<tr>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using “FMOLS” estimates</td>
<td>Using “DOLS” estimates</td>
</tr>
<tr>
<td>( \rho_t )</td>
<td>( y_{bp_t} )</td>
</tr>
<tr>
<td>( \rho_{t-1} )</td>
<td>0.7480102***</td>
</tr>
<tr>
<td>( y_{bp_{t-1}} )</td>
<td>-0.4346721***</td>
</tr>
<tr>
<td>( ccl_{t-1} )</td>
<td>0.000537</td>
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<tr>
<td>( \ln Y )</td>
<td>-0.134825***</td>
</tr>
<tr>
<td>( \ln X )</td>
<td>0.0514403***</td>
</tr>
<tr>
<td>( coi )</td>
<td>0.0401915***</td>
</tr>
</tbody>
</table>

Granger: \( y_{bp} \rightarrow \rho \)

*Stability Yes Yes

\*, **, and *** stand by 10%, 5%, and 1% of significance

### Table A13: pVAR estimates, Asia

<table>
<thead>
<tr>
<th>VII</th>
<th>VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Using “FMOLS” estimates</td>
<td>Using “DOLS” estimates</td>
</tr>
<tr>
<td>( \rho_t )</td>
<td>( y_{bp_t} )</td>
</tr>
<tr>
<td>( \rho_{t-1} )</td>
<td>0.9113049***</td>
</tr>
<tr>
<td>( y_{bp_{t-1}} )</td>
<td>-0.3386177***</td>
</tr>
<tr>
<td>( \ln Y )</td>
<td>-0.128768***</td>
</tr>
<tr>
<td>( \ln X )</td>
<td>0.0631626***</td>
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</tbody>
</table>

Granger: \( y_{bp} \rightarrow \rho \)

*Stability Yes Yes

\*, **, and *** stand by 10%, 5%, and 1% of significance
Table A14: pVAR estimates, Asia

<table>
<thead>
<tr>
<th>IX</th>
<th>Using “FMOLS” estimates</th>
<th>Using “DOLS” estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \rho )</td>
<td>( y_{bp} )</td>
</tr>
<tr>
<td><strong>( \rho_{t-1} )</strong></td>
<td>( 0.8055225^{***} )</td>
<td>( 0.0309571 )</td>
</tr>
<tr>
<td>( y_{bp_{t-1}} )</td>
<td>( -0.4620842^{***} )</td>
<td>( 0.201788^{**} )</td>
</tr>
<tr>
<td>( eci_{t-1} )</td>
<td>( -0.0505208^{**} )</td>
<td>( -0.0019713 )</td>
</tr>
<tr>
<td><strong>ln ( Y )</strong></td>
<td>( 0.041501 )</td>
<td>( -0.0214781 )</td>
</tr>
<tr>
<td><strong>ln ( X )</strong></td>
<td>( -0.0130621 )</td>
<td>( 0.0081763 )</td>
</tr>
<tr>
<td><strong>coi</strong></td>
<td>( -0.0068192 )</td>
<td>( 0.0086321 )</td>
</tr>
</tbody>
</table>

Granger: \( y_{bp} \rightarrow \rho \)  
Stability: Yes, Yes

*,, **, and *** stand by 10%, 5%, and 1% of significance

B Mathematical appendix

B.1 Proofs of Propositions 1 and 2

The determination of equilibrium follows a sequence that runs from attitudes towards change to structural change. The properties of the dynamic equation:

\[
\dot{\Phi} = \zeta \left[ (1 - \Phi) \exp (\mu \Phi) - (1 + \Phi) \exp (-\mu \Phi) \right]
\]

have been extensively discussed in the literature. Lux (1995) demonstrated that this equation has a unique equilibrium given by \( \Phi = 0 \) as long as \( \mu \leq 1 \). In the case \( \mu > 1 \), two additional equilibrium points are admitted, one positive and negative. Since in our model \( \Phi \) is determined independently of the rest of the economy, Lux’s demonstration is valid also in our case. Fig. B1 shows a graphically representation. From condition (19), we easily see that once the equilibrium value of \( \Phi \) is determined, the BoPC growth rate is equal to \( y_{bp} = \alpha(\Phi) \) or zero.

Therefore, for \( \mu \leq 1 \), we have two equilibrium points given by \((y_{bp}^{E_1}, \Phi^{E_1}) = (\alpha(0), 0)\) and \((y_{bp}^{E_2}, \Phi^{E_2}) = (0, 0)\). On the other hand, when \( \mu > 1 \), the system has four additional
equilibrium solutions that satisfy:

$$y_{Ei}^{bp} = \alpha (\Phi_{Ei})$$

$$(1 - \Phi_{Ei}) \exp (\mu \Phi_{Ei}) = (1 + \Phi_{Ei}) \exp (-\mu \Phi_{Ei}),$$

and

$$y_{Ej}^{bp} = 0$$

$$(1 - \Phi_{Ej}) \exp (\mu \Phi_{Ej}) = (1 + \Phi_{Ej}) \exp (-\mu \Phi_{Ej}),$$

where $i = [3; 4]$ and $j = [5; 6]$ stand for each additional equilibrium solution, $\Phi_{E3} = \Phi_{E5} < 0$, and $\Phi_{E4} = \Phi_{E6} > 0$.

### B.2 Proofs of Propositions 3 and 4

We linearise the dynamic system around the internal equilibrium point so as to obtain:

$$\begin{bmatrix} y_{bp} \\ \Phi \end{bmatrix}' = \begin{bmatrix} -\varepsilon y_{bp} + \varepsilon [\alpha (\Phi) - y_{bp}] & \varepsilon \alpha \Phi y_{bp} \\ 0 & \theta_{\Phi} \end{bmatrix} \begin{bmatrix} y_{bp} - y_{Ei}^{bp} \\ \Phi - \Phi_{Ei} \end{bmatrix}$$

where $J$ stands for the Jacobian matrix.

When $\mu \leq 1$, we have that $\theta_{\Phi} < 0$ for all $\Phi$. For the first equilibrium point, $(y_{E1}^{bp}, \Phi_{E1})$, the correspondent Jacobian matrix reads:

$$J_1 = \begin{bmatrix} -\varepsilon \alpha (0) & \varepsilon \alpha \Phi y_{bp} \\ 0 & \theta_{\Phi} \end{bmatrix}$$

with

$$\text{tr } J_1 = -\varepsilon \alpha (0) + \theta_{\Phi} < 0$$

$$\det J_1 = -\varepsilon \alpha (0) \theta_{\Phi} > 0$$

hence, $(y_{bp}^{E1}, \Phi_{E1})$ is locally stable.

For the second equilibrium point, $(y_{bp}^{E2}, \Phi_{E2})$, we have that:

$$J_2 = \begin{bmatrix} \varepsilon \alpha (0) & 0 \\ 0 & \theta_{\Phi} \end{bmatrix}$$

such that

$$\text{tr } J_2 = \varepsilon \alpha (0) + \theta_{\Phi} \begin{cases} > 0 & \text{if } \theta_{\Phi} > 0 \\ < 0 & \text{if } \theta_{\Phi} < 0 \end{cases}$$

$$\det J_2 = \varepsilon \alpha (0) \theta_{\Phi} < 0$$

which means it is a saddle point.

When $\mu > 1$, the dynamic system has four additional equilibrium solutions. Furthermore, two different situations arise: (i) $\theta_{\Phi} (0) > 0$ and (ii) $\theta_{\Phi} (\Phi_{Ei}^{E})$ or $\theta_{\Phi} (\Phi_{Ej}^{E}) < 0$, where $i = [3; 4]$ and $j = [5; 6]$ stand for each additional equilibrium solution. Notice that $\Phi_{E3} = \Phi_{E5} < 0$ and $\Phi_{E4} = \Phi_{E6} > 0$. Matrix $J_1$ continues to be the same but $\theta_{\Phi} > 0$. This means that $\det J_1 < 0$ and the first equilibrium, $(y_{bp}^{E1}, \Phi_{E1})$, becomes a saddle point. On the other hand, matrix $J_2$
also does not change but since $\theta > 0$, the implication is that $\text{tr } J_2 > 0$ and $\text{det } J > 0$. It follows that the second equilibrium is an improper or spiral node.

Looking at the third and fourth equilibrium points, $(y_{bp}^E, \Phi^E_3)$ and $(y_{bp}^E, \Phi^E_4)$, the Jacobian matrix is:

$$J_i = \begin{bmatrix} -\varepsilon \alpha (\Phi^E_i) & \varepsilon \alpha \Phi^E_i (\Phi^E_i) \\ 0 & \theta \end{bmatrix}$$

such that

$$\text{tr } J_i = -\varepsilon \alpha (\Phi^E_i) + \theta \Phi < 0$$
$$\text{det } J_i = -\varepsilon \alpha (\Phi^E_i) \theta > 0$$

We conclude the two equilibrium points are locally stable.

Regarding the fifth and sixth equilibrium points, $(y_{bp}^E, \Phi^E_5)$ and $(y_{bp}^E, \Phi^E_6)$, the correspondent Jacobian matrix reads:

$$J_j = \begin{bmatrix} \varepsilon \alpha (\Phi^E_j) & 0 \\ 0 & \theta \end{bmatrix}$$

where

$$\text{tr } J_j = \varepsilon \alpha (\Phi^E_j) + \theta \Phi > 0$$
$$\text{det } J_j = \varepsilon \alpha (\Phi^E_j) \theta < 0$$

and we can conclude that they are saddle points.

### B.3 Proofs of Propositions 5 and 6

The proof of these propositions follows the steps of our previous demonstration of proposition 1 and 2. Given the properties of the exponential transition probabilities and the stratification mechanism, we obtain two equilibrium values when $\mu \leq 1$ and six equilibrium values when $\mu > 1$. Since we disregard the cases in which employment rates and income distribution are zero, it follows from the second and third equations of (34) that $G(y_{bp}, \varpi) = y_{bp}$ and $F(e) = y_{bp}$. Therefore, the two equilibrium points when $\mu \leq 1$ satisfy:

$$y_{bp}^{E_1} = \alpha (0)$$
$$G(\alpha (0), \varpi^{E_1}) = \alpha (0)$$
$$e^{E_1} = F^{-1}(\alpha (0))$$
$$\Phi^{E_1} = 0$$

and

$$y_{bp}^{E_2} = \Phi^{E_2} = 0$$
$$G(0, \varpi^{E_2}) = 0$$
$$e^{E_2} = F^{-1}(0)$$

On the other hand, when $\mu > 1$, we have four additional equilibrium points that satisfy:

$$y_{bp}^{E_i} = \alpha (\Phi^{E_i})$$
$$G(\alpha (\Phi^{E_i}), \varpi^{E_i}) = \alpha (\Phi^{E_i})$$
$$e^{E_i} = F^{-1}(\alpha (\Phi^{E_i}))$$
$$(1 - \Phi^{E_i}) \exp (\mu \Phi^{E_i}) = (1 + \Phi^{E_i}) \exp (-\mu \Phi^{E_i})$$,
so that the characteristic equation can be written as

\[ e^{E_j} = F^{-1}(0) \]

\[ G(0, E_j) = 0 \]

where \( i = [3; 4] \) and \( j = [5; 6] \) stand for each additional equilibrium solution, \( \Phi_{E3} = \Phi_{E5} < 0 \), and \( \Phi_{E4} = \Phi_{E6} > 0 \).

### B.4 Proofs of Propositions 7 to 10

In order to prove propositions 7 to 10 we differentiate between two cases: (i) the normal case and (ii) the "growth trap" case. The latter basically corresponds to those equilibrium points in which the growth rate of output is equal to zero.

#### B.4.1 The normal case

The Jacobian matrix in the first case reads

\[
J = \begin{pmatrix}
-\varepsilon y_{bp}^E & 0 & 0 & \varepsilon \alpha \Phi y_{bp}^E \\
(1 - G_{ybp}) e^{E} & 0 & -G_{\varepsilon \varepsilon} e^{E} & 0 \\
-G_{ybp} \varepsilon & -G_{\varepsilon \varepsilon} E & F_{e \varepsilon \varepsilon} E & -G_{\varepsilon \varepsilon} E \\
-\theta_{\varepsilon / \varepsilon} G_{ybp} & \theta_{\varepsilon / \varepsilon} F_e & -\theta_{\varepsilon / \varepsilon} G_{\varepsilon} & \theta_{\Phi}
\end{pmatrix}
\]

so that the characteristic equation can be written as

\[
\lambda^4 + b_1 \lambda^3 + b_2 \lambda^2 + b_3 \lambda + b_4 = 0
\]

where

\[
b_1 = -\text{tr}J = \varepsilon y_{bp}^E + G_{\varepsilon \varepsilon} E - \theta_\Phi \begin{cases}
\geq 0 & \text{when } \theta_\Phi > 0 \\
> 0 & \text{when } \theta_\Phi < 0
\end{cases}
\]

\[
b_2 = \begin{vmatrix}
-\varepsilon y_{bp}^E & 0 & \varepsilon \alpha \Phi y_{bp}^E \\
(1 - G_{ybp}) e^{E} & 0 & -G_{\varepsilon \varepsilon} e^{E} \\
-G_{ybp} \varepsilon & -G_{\varepsilon \varepsilon} E & F_{e \varepsilon \varepsilon} E \\
-\theta_{\varepsilon / \varepsilon} G_{ybp} & \theta_{\varepsilon / \varepsilon} F_e & -\theta_{\varepsilon / \varepsilon} G_{\varepsilon}
\end{vmatrix} + \begin{vmatrix}
-\varepsilon y_{bp}^E & 0 & 0 \\
(1 - G_{ybp}) e^{E} & 0 & -G_{\varepsilon \varepsilon} E \\
-G_{ybp} \varepsilon & -G_{\varepsilon \varepsilon} E & F_{e \varepsilon \varepsilon} E \\
-\theta_{\varepsilon / \varepsilon} G_{ybp} & \theta_{\varepsilon / \varepsilon} F_e & -\theta_{\varepsilon / \varepsilon} G_{\varepsilon}
\end{vmatrix}
\]

\[
b_3 = \begin{vmatrix}
0 & -G_{\varepsilon \varepsilon} e^{E} & 0 \\
F_{e \varepsilon \varepsilon} E & -G_{\varepsilon \varepsilon} E & 0 \\
\theta_{\varepsilon / \varepsilon} F_e & -\theta_{\varepsilon / \varepsilon} G_{ybp} & \theta_\Phi
\end{vmatrix} + \begin{vmatrix}
-\varepsilon y_{bp}^E & 0 & 0 \\
(1 - G_{ybp}) e^{E} & 0 & -G_{\varepsilon \varepsilon} E \\
-G_{ybp} \varepsilon & -G_{\varepsilon \varepsilon} E & F_{e \varepsilon \varepsilon} E \\
-\theta_{\varepsilon / \varepsilon} G_{ybp} & \theta_{\varepsilon / \varepsilon} F_e & -\theta_{\varepsilon / \varepsilon} G_{\varepsilon}
\end{vmatrix}
\]

\[
b_4 = \begin{vmatrix}
0 & -G_{\varepsilon \varepsilon} e^{E} & 0 \\
F_{e \varepsilon \varepsilon} E & -G_{\varepsilon \varepsilon} E & 0 \\
\theta_{\varepsilon / \varepsilon} F_e & -\theta_{\varepsilon / \varepsilon} G_{ybp} & \theta_\Phi
\end{vmatrix} + \begin{vmatrix}
0 & -G_{\varepsilon \varepsilon} e^{E} & 0 \\
0 & -G_{\varepsilon \varepsilon} E & 0 \\
0 & -G_{\varepsilon \varepsilon} E & 0
\end{vmatrix}
\]
\[ b_1 = \text{det } J = \begin{vmatrix} G_{y_p} e^E & 0 & 0 & \varepsilon \alpha \Phi y_p^E \\ F_{e} e^E & G_{y_p} - G_{\infty} e^E & 0 & 0 \\ -\theta_{\infty/\infty} G_{y_p} e^E & 0 & -G_{\infty} e^E & 0 \\ -\theta_{\infty/\infty} G_{y_p} e^E & 0 & 0 & -G_{\infty} e^E \end{vmatrix} \]

\[ = -\varepsilon \alpha \Phi y_p^E \begin{vmatrix} 1 - G_{y_p} e^E & 0 & 0 & 0 \\ -G_{y_p} e^E & 0 & -G_{\infty} e^E & 0 \\ -\theta_{\infty/\infty} G_{y_p} e^E & 0 & 0 & -G_{\infty} e^E \\ -\theta_{\infty/\infty} G_{y_p} e^E & 0 & 0 & -G_{\infty} e^E \end{vmatrix} \]

\[ = -\varepsilon \alpha \Phi y_p^E \begin{vmatrix} 1 - G_{y_p} e^E & 0 & 0 & 0 \\ -G_{y_p} e^E & 0 & -G_{\infty} e^E & 0 \\ -\theta_{\infty/\infty} G_{y_p} e^E & 0 & 0 & -G_{\infty} e^E \\ -\theta_{\infty/\infty} G_{y_p} e^E & 0 & 0 & -G_{\infty} e^E \end{vmatrix} \]

When \( \theta_\Phi < 0 \) we have that \( b_1, b_2, b_3, \) and \( b_4 > 0 \). The necessary and sufficient condition for the local stability is that all roots of the characteristic equation have negative real parts, which, from Routh–Hurwitz conditions, requires:

\[ b_1, b_2, b_3, b_4 > 0 \text{ and } b_1 b_2 b_3 - b_1^2 b_4 - b_3^2 > 0 \]

The crucial condition for local stability becomes the last one. Through direct computation we find that:

\[ b_1 b_2 b_3 - b_1^2 b_4 - b_3^2 = (\varepsilon y_p^E + G_{\infty} e^E - \theta_\Phi) (\varepsilon G_{y_p} y_p^E e^E - \varepsilon \theta_\Phi y_p^E + \varepsilon \alpha \Phi \theta_{\infty/\infty} G_{y_p} y_p^E + G_{\infty} F_e e^E e^E - G_{\infty} \theta_\Phi e) \times \]

\[ - \theta_{\infty/\infty} G_{y_p} e^E - \varepsilon \theta_\Phi y_p^E e^E - \varepsilon \theta_{\infty/\infty} F_e \alpha \Phi (1 - G_{y_p}) y_p^E e^E + \varepsilon F_e G_{\infty} y_p^E e^E e^E \]

\[ + (\varepsilon y_p^E + G_{\infty} e^E - \theta_\Phi)^2 \theta_\Phi F_e G_{\infty} y_p^E e^E e^E \]

\[ - (G_{\infty} y_p^E e^E - \varepsilon \theta_\Phi y_p^E e^E - \varepsilon \theta_{\infty/\infty} F_e \alpha \Phi (1 - G_{y_p}) y_p^E e^E + \varepsilon F_e G_{\infty} y_p^E e^E e^E)^2 \]

Rewrite

\[ b_2 = \varepsilon G_{y_p} y_p^E e^E - \varepsilon \theta_\Phi y_p^E + \varepsilon \alpha \Phi \theta_{\infty/\infty} G_{y_p} y_p^E + G_{\infty} F_e e^E e^E - G_{\infty} \theta_\Phi e^E \]

\[ = \varepsilon G_{y_p} y_p^E e^E - \varepsilon \theta_\Phi y_p^E + G_{\infty} F_e e^E e^E - G_{\infty} \theta_\Phi e^E + \varepsilon \alpha \Phi G_{y_p} y_p^E \theta_{\infty/\infty} \]

\[ = A + B \theta_{\infty/\infty} \]

\[ b_3 = -\varepsilon \theta_\Phi G_{\infty} F_e e^E e^E - \varepsilon \theta_{\infty/\infty} F_e \alpha \Phi (1 - G_{y_p}) y_p^E e^E + \varepsilon F_e G_{\infty} y_p^E e^E e^E \]

\[ = -\varepsilon \theta_\Phi G_{\infty} F_e e^E e^E - \varepsilon \theta_{\infty/\infty} F_e \alpha \Phi (1 - G_{y_p}) y_p^E e^E + \varepsilon F_e G_{\infty} y_p^E e^E e^E \]

\[ = C - D \theta_{\infty/\infty} \]
where
\[
A = \varepsilon G_yy_{bp}\varpi^E - \varepsilon \theta y_{bp}^E + G_w F_e e^E \varpi^E - G_w \theta \varpi^E \\
B = \varepsilon \alpha \Phi G_{yp} y_{bp}^E > 0 \\
C = -\theta \Phi G_w F_e e^E \varpi^E - \varepsilon G_w \theta y_{bp} \varpi^E + \varepsilon F_e G_y y_{bp} e^E \varpi^E \\
D = \varepsilon F_e \alpha \Phi (1 - G_{yp}) y_{bp}^E e^E > 0
\]
then we obtain
\[
b_1 b_2 b_3 - b_1^2 b_4 - b_3^2 \\
= b_1 (A + B \theta_{\varpi/\varpi})(C - D \theta_{\varpi/\varpi}) - b_1^2 b_4 - (C - D \theta_{\varpi/\varpi})^2 \\
= b_1 [(A + B \theta_{\varpi/\varpi}) C - (A + B \theta_{\varpi/\varpi}) D \theta_{\varpi/\varpi}] - b_1^2 b_4 - (C - D \theta_{\varpi/\varpi})^2 \\
= b_1 AC + b_1 BC \theta_{\varpi/\varpi} - b_1 AD \theta_{\varpi/\varpi} - b_1 BD \theta_{\varpi/\varpi}^2 - b_1^2 b_4 - C^2 - D^2 \theta_{\varpi/\varpi}^2 + 2CD \theta_{\varpi/\varpi} \\
= -(b_1 B + D) D \theta_{\varpi/\varpi}^2 + (b_1 BC - b_1 AD + 2CD) \theta_{\varpi/\varpi} + b_1 AC - b_1^2 b_4 - C^2
\]
This means that as long as
\[-(b_1 B + D) D \theta_{\varpi/\varpi}^2 + (b_1 BC - b_1 AD + 2CD) \theta_{\varpi/\varpi} + b_1 AC - b_1^2 b_4 - C^2 > 0
\]
the equilibrium points \((y_{bp}^E, e^E, \varpi^E, \Phi^E)\), when \(\mu \leq 1\); \((y_{bp}^E, e^E, \varpi^E, \Phi^E)\) and \((y_{bp}^E, e^E, \varpi^E, \Phi^E)\), when \(\mu > 1\) are locally stable.

In what concerns \((y_{bp}^E, e^E, \varpi^E, \Phi^E)\) and \((y_{bp}^E, e^E, \varpi^E, \Phi^E)\), notice that \(- (b_1 B + D) D < 0\) and \(Ab_1 C - b_1^2 b_4 - C^2 > 0\), which implies, by continuity, that there is a \(\theta_{\varpi/\varpi} = \mu_B\) such that \(b_1 b_2 b_3 - b_1^2 b_4 - b_3^2 = 0\). Therefore, we can use \(\theta_{\varpi/\varpi}\) as our bifurcation parameter. On the other hand, when \(\theta_{\Phi} > 0\) then \(b_4 < 0\). This implies that, for \(\mu > 1\), the equilibrium point \((y_{bp}^E, e^E, \varpi^E, \Phi^E)\) is unstable.

### B.4.2 The “growth trap” case

The Jacobian matrix reads
\[
J = \begin{bmatrix}
\varepsilon \alpha (\Phi^E) & 0 & 0 & 0 \\
(1 - G_{yp}) e^E & 0 & -G_w e^E & 0 \\
-G_{yp} \varpi^E & F_e \varpi^E & -G_w \varpi^E & 0 \\
-\theta_{\varpi/\varpi} G_{yp} & \theta_{\varpi/\varpi} F_e & -\theta_{\varpi/\varpi} G_w & \theta \Phi
\end{bmatrix}
\]
where
\[
b_1 = -\text{tr} J = -\varepsilon \alpha (\Phi^E) + G_w \varpi^E - \theta_{\Phi} \\
b_2 = \begin{vmatrix}
-G_w \varpi^E & 0 & G_w e^E & F_e \varpi^E + \varepsilon \alpha (\Phi^E) \theta_{\Phi} - \varepsilon \alpha (\Phi^E) G_w \varpi^E \\
-\theta_{\varpi/\varpi} G_w & \theta_{\varpi/\varpi} F_e & -\theta_{\varpi/\varpi} G_{yp} & \theta_{\varpi/\varpi} G_w \\
0 & -G_w e^E & 0 & -G_w \varpi^E \\
0 & G_w \varpi^E & 0 & -G_w \varpi^E
\end{vmatrix}
\]
\[
b_3 = \begin{vmatrix}
0 & -G_w e^E & 0 & \varepsilon \alpha (\Phi^E) \theta_{\Phi} - \varepsilon \alpha (\Phi^E) G_w \varpi^E \\
G_w \varpi^E & -G_w e^E & 0 & 0 \\
0 & G_w \varpi^E & 0 & 0 \\
0 & 0 & 0 & 0
\end{vmatrix}
\]
\[
= -\theta \Phi F_e \varpi^E G_w e^E + \varepsilon \alpha (\Phi^E) G_w \varpi^E \theta_{\Phi} - F_e \varpi^E G_w e^E \varepsilon \alpha (\Phi^E)
\]
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\[
\begin{align*}
 b_4 &= \det \mathbf{J} = \theta_\Phi \\
 &= -\theta_\Phi F_e e^E G_\omega e^E \varepsilon_\omega (\Phi^E) \\

eq -\theta_\Phi F_e e^E G_\omega e^E \varepsilon_\omega (\Phi^E)
\end{align*}
\]

and

\[
\begin{align*}
 b_1 b_2 b_3 - b_1^2 b_4 - b_3^2 &= -\varepsilon (C E + F) \alpha (\Phi^E)^3 + (A C E + \varepsilon B E + \varepsilon C D + 2 \varepsilon A F) \alpha (\Phi^E)^2 \\
&- (A B E + A C D + \varepsilon B D + A^2 F) \alpha (\Phi^E) + (A B - H) H
\end{align*}
\]

The Routh–Hurwitz conditions, requires:

\[
\begin{align*}
 b_1, b_2, b_3, b_4 &> 0 \text{ and } b_1 b_2 b_3 - b_1^2 b_4 - b_3^2 > 0
\end{align*}
\]

which means that the equilibrium points \((y_{bp}, e^{E_2}, \omega^{E_2}, \Phi^{E_2})\), when \(\mu \leq 1\); \((y_{bp}, e^{E_3}, \omega^{E_3}, \Phi^{E_3})\) and \((y_{bp}, e^{E_6}, \omega^{E_6}, \Phi^{E_6})\), when \(\mu > 1\) are locally stable as long as the capacity of adaptation, \(\alpha (\Phi^E)\), is sufficiently small such that:

\[
\begin{align*}
 b_1 &= -\varepsilon \alpha (\Phi^E) + G_\omega e^E - \theta_\Phi > 0 \\
 b_2 &= -G_\omega e^E \theta_\Phi + G_\omega e^E F_e e^E + \varepsilon \alpha (\Phi^E) \theta_\Phi - \varepsilon \alpha (\Phi^E) G_\omega e^E > 0 \\
 b_3 &= -\theta_\Phi F_e e^E G_\omega e^E + \varepsilon \alpha (\Phi^E) G_\omega e^E \theta_\Phi - F_e e^E G_\omega e^E \varepsilon \alpha (\Phi^E) > 0 \\
 b_4 &= -\theta_\Phi F_e e^E G_\omega e^E \varepsilon \alpha (\Phi^E) > 0 \\
 b_1 b_2 b_3 - b_1^2 b_4 - b_3^2 &= -\varepsilon (C E + F) \alpha (\Phi^E)^3 + (A C E + \varepsilon B E + \varepsilon C D + 2 \varepsilon A F) \alpha (\Phi^E)^2 \\
&- (A B E + A C D + \varepsilon B D + A^2 F) \alpha (\Phi^E) + (A B - H) H > 0
\end{align*}
\]
where

\[ A = G_{\pi E} - \theta_N \]
\[ B = -G_{\pi E} \theta_N + G_{\pi E} F_{e E} \]
\[ C = \varepsilon \theta_N - \varepsilon G_{\pi E} \]
\[ D = -\theta_N F_{e E} G_{\pi E} \]
\[ E = \varepsilon G_{\pi E} \theta_N - F_{e E} G_{\pi E} \varepsilon \]
\[ H = \theta_N F_{e E} G_{\pi E} \varepsilon \]

On the other hand, when \( \theta_N > 0 \) we have \( b_4 < 0 \). This implies that, for \( \mu > 1 \), the equilibrium point \((y^{E_2}_{b_{\theta}}, e^{E_2}, \omega^{E_2}, \Phi^{E_2})\) is unstable.

References


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