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(Ir)rational explorers in the financial jungle: modelling Minsky with heterogeneous agents*

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Abstract

Over the past decades, several scholars have formalised Minsky's profound insight that increasing financial fragility accompanies periods of economic stability. It must be noted, however, that a deep assessment of the role of expectations formation with heterogeneous agents has been provided only by those contributions focusing on stock-market price dynamics. Macroeconomic models dealing with debt dynamics, on the other hand, have not yet presented such an account. It is our purpose to fill this gap in the literature by formalising switches between different heuristics in a model where solvency aspects matter. Our system is capable of generating time-series that reproduce important empirical stylised facts such as fat-tails and asymmetric skewness. In the absence of a stochastic component, the model still leads to sensitivity to initial conditions. Moreover, while the destabilising role of extrapolative behaviour is part of conventional wisdom, we show under which conditions fundamentalists, the existence of resource constraints, and the time horizon of the economic unit may also lead to instability.

Keywords: Financial instability; real-financial interactions; heterogeneous expectations; complex dynamics; Minsky

JEL: G01, C61, D84

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1 Introduction

The deep global recession triggered by the subprime mortgage crisis has renewed the interest for the study of the role of financial actors and institutions in generating economic fluctuations. Ten years or so after the collapse of Lehman Brothers, Hyman Minsky's profound insight that increasing financial fragility accompanies periods of economic stability is more alive than ever.

An initially robust financial system can be endogenously turned into a fragile structure as prolonged periods of tranquillity induce agents to take riskier financial practices. While Minsky (1975, 1982) and the so-called Financial Instability Hypothesis (FIH) have just recently regained momentum among mainstream economists and the financial press, several scholars have formalised over the past decades different parts of his theory. Seminal contributions include Taylor and O'Connell (1985), Foley (1987) and Delli Gatti et al. (1993), the latter demonstrating the possibility of the emergence of persistent, bounded, and irregular fluctuations.

Keeping debt at the centre of the analysis, more recent studies have explored the interaction between real and financial sectors using a stock-flow consistent approach (e.g. Ryoo, 2010; Nikolaidi, 2014; Daffermos, 2018). Other scholars have introduced a financial sector into Goodwin's growth-cycle model, obtaining a macrodynamic system in which growth and fluctuations are indissolubly fused (see, for example, Keen, 1995; Grasselli and Costa Lima, 2012; Sordi and Vercelli, 2014). This strand of the literature has provided a successful differentiation between firms and household debt in a growth-cycle framework with equally important developments in terms of economic policy (Asada et al., 2011; Costa Lima et al., 2014; Giraud and Grasselli, 2019).

On the other hand, the literature on bounded rational heterogeneous agents has successfully shown that trading activity of interacting speculators can account for a large part of the dynamics of financial markets (see Day and Huang, 1990; Chiarella, 1992). Different sources of behavioural heterogeneity have been identified, ranging from optimistic and pessimistic expectations to disposition effects, overconfidence, etc. (for a review on recent developments, see Lux, 2009; Hommes, 2013; Dieci and He, 2018). They provide the basis for a rich representation of the interconnections between real and financial markets with a clear interest in more complex dynamics.

One of the first steps in this direction was given by Westerhoff (2012). In his model, nonlinear interactions between aggregated demand, chartists, and fundamentalists result in complex "bull & bear" dynamics. Even though this literature not always explicitly mentions Minsky, there is a clear favourable attitude towards the FIH. Endogenous switches between different heuristics have been formalised by Naimzada and Pireddu (2015), Cavalli et al. (2018), and Flaschel et al. (2018), among others. Most of the time, they rely on Lux (1995) or Brock and Hommes (1997) description of agents behaviour under bounded rationality.

The contributions revisited so far have adopted a *macro*economic perspective of the problem. A seminal *micro*economic formalisation of Minsky was first presented by Chiarella and Di Guilmi (2011). Their article started a fruitful research line that has not neglected the incorporation of stock-flow considerations (e.g. Caiani et al., 2016; Di Guilmi and Carvalho, 2017; Salle and Seppecher, 2018). Similar efforts include Farmer (2013) and Bhattacharya et al. (2015) that have combined some Minskyan insights and the basic Dynamic Stochastic General Equilibrium (DSGE) model.

Given the richness of approaches assessing the intrinsic instability of the capitalist financial system, any attempt to present a comprehensive survey inevitably runs the risk of not making justice to some authors. Still, following Nikolaidi and Stockhammer (2017), it is possible to differentiate between those contributions concerned with the dynamics of *debt* and those interested in *stock-market prices*. Focusing on macrodynamic studies, we notice that those in the second group have presented a deep assessment of the role of expectations formation with heterogeneous agents, while Minsky debt models have not yet provided such an account. The main purpose of this article is to fill such a gap by formalising switches between different heuristics in a model where solvency matters.

The hypothesis regarding the process of formation and revision of expectations was central to both Keynes and Minsky reasoning. Recalling a metaphor used by Descartes in his *Discourse on the Method*, Sordi and Vercelli (2012, hereafter SV) compared the behaviour of economic agents to the one of an explorer that has to cross a forest of unknown size. It is rational for her/him to proceed in a straight direction to minimise the risk of getting lost. However, s/he has to take into account that food reserves are limited so that such strategy will only be pursued up to a well-defined threshold. At that point, a rational explorer will go back following the same path in reverse because s/he does not know how far the border is. This simple metaphor captures something crucial that typically happens in financial behaviour. Differentiating between liquidity and solvency dimensions, the authors showed that a simple mechanism of heterogeneous expectations generates persistent, bounded, and irregular fluctuations. Still, they did not provide an explicit account of the relationship between real and financial variables, nor discussed the possibility of shifts of the margin of safety regarding inter-temporal solvency conditions.

It is our intention to extend the analysis of SV to consider these elements. Our formalisation illustrates how decisions of economic units (banks, firms, and households) are heavily affected by their current and expected financial conditions. We study how the interaction between the real and the financial sides of the economy may lead to financial instability. The parameter capturing the response of the margin of safety to output is the main source of endogenous fluctuations. Irregular oscillations arise only for sufficiently weak resource constraints, low response of expectations to discrepancies in the liquidity index, and a low time horizon of the relevant unit. While the destabilising role of extrapolative behaviour is part of conventional wisdom, we discuss under which conditions regressive expectations can also lead to instability.

Furthermore, given the empirical evidence highlighting that the within-country time-series distribution of output is persistently fatter than the Gaussian one (e.g. Fagiolo et al., 2008; Ascari et al., 2015; Naimzada and Pireddu, 2015), we demonstrate that our model is capable of reproducing such stylised fact. We show that even if the system is hit by purely Gaussian uncorrelated shocks, fat-tail distributed time-series arise via the endogenous transmission mechanisms embodied in the model. To obtain high kurtosis and skewed distributions the model has to be nonlinear. We demonstrate that even in the absence of a stochastic component, there is still sensitivity to initial conditions.

The main disadvantage of our approach comes from aggregation. Still, our position is that aggregate systems have a place in economic theory and are useful as such to the extent that they illuminate empirical regularities in a simple but rigorous way. Employing both analytical and numerical tools, we detect the mechanisms and channels through which stability leads to instability in the two interacting markets.

In terms of policy implications, three results not so obvious (or common) in the literature on the FIH are worth stressing. First, the time horizon of the economic unit seems to be more important than the interest rate as a source of endogenous instability. Second, fundamentalists are not always good for the economy. If their speed of reaction is too strong, there is an increase in volatility resulting from the lags involved in the interaction between real and financial variables. Last but not least, fiscal policy might be a useful instrument for reducing instability if it manages to either avoid dramatic reductions or increases in aggregate demand. Such approach could reduce the complexity of the instruments to be designed and should be further investigated in future research.

The remaining of the article is organised as follows. Section 2 presents a simple liquidity-solvency model allowing for the interaction between real and financial sides of the economy. Section 3 introduces heterogeneous expectations differentiating between chartism and fundamentalism behaviour. In Section 4, the system is submitted to normally distributed stochastic shocks. Numerical simulations permit us to go deeper into the mechanisms that generate endogenous fluctuations. Section 5 brings some final considerations.

2 Financial instability and extrapolative expectations

Bounded rational agents stick to behavioural rules that are the more rigid the higher the degree of uncertainty. Paradoxically, in this context, "uncertainty becomes the basic source of predictable behaviour" (Heiner, 1983, p. 570). In light of recent advances in complex dynamics and in behavioural economics, it is our purpose to study the interaction between the real and financial sides of the economy making explicit reference to expectations formation under the FIH.

2.1 Modelling liquidity and solvency interactions

In a modern economy, as argued by SV, economic units make decisions focusing on the interaction between current and inter-temporal financial constraints. Both dimensions have a crucial role in shaping their behaviour. Current financial conditions may be approximated by a liquidity index (f) which is given by the difference between financial inflows and outflows. On the other hand, the solvency of an economic unit (f^*) is defined as the sum of the expected liquidity over a given time horizon (T), discounted by the nominal interest rate (r):

$$f^*_{t+1} = \sum_{s=0}^{T} \frac{E_t\left[f_{t+1+s}\right]}{(1+r)^s} \tag{1}$$

where $E_t\left[\cdot\right]$ represents the conditional expectation operator. This expression may be seen as a generalisation of the concept of capital marginal efficiency introduced by Keynes in the *General Theory* (ibid., p. 553).

Minsky (1982) stressed that a situation of financial stability may be destabilising. Empirical evidence showing that a sequence of high returns triggers agents to overestimate the probability of high returns in the future, while bad returns yield lower forecasts of future returns, has been recently provided by Böck and Zörner (2019). During periods of tranquillity, the expectations of the economic unit are systematically validated by the market, leading to a progressive reduction of the perception of risk associated with forecasting mistakes.

The simplest way to take account of this is to assume that units extrapolate liquidity trends, such that:

$$E_t\left[f_{t+1}\right] = f_t + \rho\left(f_t - \bar{f}\right) \tag{2}$$

with $\rho > 0$ indicating the speed of reaction of expectations to discrepancies between the liquidity index and its reference level in periods of tranquillity ($\bar{f}$). This last variable was defined as liquidity expectations under robust financial conditions. SV (p. 549) demonstrated that:

$$\bar{f} = af^{*} \tag{3}$$

where

$$a = \frac{r\,(1+r)^{T}}{(1+r)^{T+1}-1} \leq 1$$

By combining Eqs. (1)-(3), we obtain the dynamic equation for changes in solvency:

$$f^{*}_{t+1} \;=\; \beta f_t + (1-a\beta)\, f^{*}_t \tag{4}$$

where

$$\beta = (1+\rho)\left[\frac{(1+r)^{T+1}-(1+\rho)^{T+1}}{(1+r)^{T}\,(r-\rho)}\right] > 0 \tag{5}$$

We proceed by determining the motion of current financial conditions. In this regard, the metaphor of the (ir)rational explorer in the (financial) jungle turns out to be particularly illuminating (ibid., p. 546). For a limited amount of food and water supplies, someone crossing a forest of unknown size and dimension will proceed in a straight direction to minimise the risk of getting lost only until a well-defined threshold. After reaching this safety point, the explorer needs to follow the same path in reverse because s/he does not know how far the border is.

In the same way, given that higher risk implies a higher rate of return, firms and households are willing to increase their financial exposure as long as their financial structure is within a desired solvency safety margin (μ). However, after the safety threshold is reached, their focus changes to the struggle of securing their own financial survival. In mathematical terms:

$$f_{t+1} \;=\; f_t - \alpha\,(f^{*}_t - \mu_t) \tag{6}$$

with $\alpha > 0$ representing the speed of reaction to discrepancies between the solvency index and the safety margin. Moreover, the authors assume:

$$\mu_t = \mu_0 > 0,\ \forall t$$

The two-dimension dynamic system formed by Eqs. (4) and (6) provides a useful structure to study the role of expectations within the core of the FIH. It must be noted, however, that there is no explicit account of the relationship between real and financial variables. Moreover, the margin of safety is supposed to be constant and exogenously determined.

2.2 Introducing the real sector

In order to go one step forward, we extend the model by introducing a nonlinear mechanism for income variations that takes into account the existence of resource constraints. In a closed economy without government, aggregate demand (Z) is divided between consumption (C) and investment (I). They are both assumed to be a function of the solvency index and of the level of output:

$$\begin{aligned} Z_t &= C\,(f^{*}_t, Y_t) + I\,(f^{*}_t, Y_t) \\ &= Z_0 - \psi f^{*}_t + \phi Y_t \end{aligned} \tag{7}$$

where $Z_0 > 0$ corresponds to autonomous expenditure, while $0 < \psi$ and $0 < \phi < 1$ are marginal propensity parameters.

Private expenditure increases with national income since consumption depends on current income and investment decisions include an accelerator component. On the other hand, an increase in solvency is associate with a lower level of demand. By definition, f^* depends on the sum of expected financial inflows over outflows. A high solvency index indicates that economic units are spending less than what they could given their inter-temporal financial constraint. "Hyper-solvent" units clearly have space for increasing their expenditures and refusing to do so has negative implications in the current level of economic activity.

This last relationship is supposed to capture indirectly the wealth effect on consumption and the so-called cash-flow effect on investment (see, for example, Fazzari et al., 2008 and Westerhoff, 2012). The former states that households' spending accompanies changes in perceived wealth, while the latter suggests cash-flow raises the amount of investment that firms can undertake without incurring the costs associated with debt or new share issues. By depicting the correspondence between aggregate demand and the solvency constraint, we are able to tackle both. For instance, as long as f^* is greater than the margin of safety, economic units respond by increasing their financial exposure, i.e. reducing their liquidity. A reduction in f brings about an increase in expenditures and consequently, as f^* falls, leads to higher demand.

Excess demand (E) corresponds to the difference between expenditures and output:

$$E_t = Z_t - Y_t \tag{8}$$

while firms adjusts output with a one-period production lag:

$$Y_{t+1} = Y_t + g\left(E_t\right) \tag{9}$$

The properties of the function $g\left(\cdot\right)$ describe how changes in production depend on E. As in standard macroeconomic models, we require $dg/dE > 0$ and $g\left(0\right) = 0$ such that in equilibrium there is no excess demand.

In addition, one should take into account the fact that production is subject to resource constraints. Output cannot increase too strongly in the upswing phase of the business cycle because at some point the economy would face input shortages and rationing of orders. This includes increases in labour costs as we approach low unemployment rates. There are also limits to how much output can fall. The existence of an autonomous consumption component, for example, partially explains the existence of such floor. One could also notice that machines, once made, cannot be unmade, so that capital destruction is limited to attrition from wear, time, and innovations.

Allen's (1967, pp. 374-383) well-known macroeconomic manual proposed a sigmoid shape smooth function that introduces such constraints into the nonlinear accelerator of investment in a rather elegant way (see also Sordi, 1987). This formulation has recently become quite popular among those studying real and financial market interactions (e.g. Naimzada and Pireddu, 2015; Cavalli et al., 2018). Following this literature, we write:

$$\begin{aligned} g\left(E_t\right) &= b_2\left(\frac{b_1 + b_2}{b_1 \exp\left[-E_t\right] + b_2} - 1\right) \\ &= g(f_t^*, Y_t) \qquad g_{f^*} < 0,\ g_Y < 0 \end{aligned} \tag{10}$$

where b_1 and b_2 are positive parameters determining output's floor and ceiling, respectively.

Moreover, we allow the desired solvency safety margin μ to change depending on the phase of the business cycle. In an environment with strong asymmetric information, traders necessarily have to rely on what can be observed in the market to take decisions concerning their actions. Keeping in mind Minsky's principle that stability is destabilising, we divide the solvency safety threshold into two components, one capturing structural conditions of the economic unit (μ_0) and another capturing those subject to business cycle fluctuations:

$$\mu_t = \mu_0 - \bar{\mu} g\left(f_t^*, Y_t\right) \tag{11}$$

such that a high $\bar{\mu}$ reflects an economy in which the financial side strongly responds to changes in output. During good times, households and firms increase their financial exposure because their confidence in their own financial structure is affected by the generalised optimism that usually follows periods of economic prosperity. A path of improving news leads an agent to focus on good future outcomes and neglect bad ones. The opposite situation happens during bad times. Deteriorating news lead agents to neglect good ones causing excessive pessimism (for empirical support, see Bordalo et al., 2018; Böck and Zörner, 2019).

Making use of Eqs. (7)-(10) and substituting Eq. (11) into (6), we allow for the interaction between real and financial sides of the economy. The three-dimensional dynamic system reads:

$$\begin{aligned}
f_{t+1} &= f_t - \alpha\left[f_t^* - \mu_0 + \bar{\mu} b_2\left(\frac{b_1+b_2}{b_1 \exp\left[-Z_0 + \psi f_t^* + (1-\phi)Y_t\right] + b_2} - 1\right)\right] \\
f_{t+1}^* &= \max\left\{\beta f_t + (1-a\beta)\, f_t^*, 0\right\} \\
Y_{t+1} &= Y_t + b_2\left(\frac{b_1+b_2}{b_1 \exp\left[-Z_0 + \psi f_t^* + (1-\phi)Y_t\right] + b_2} - 1\right)
\end{aligned} \tag{12}$$

where we impose $f_t^* \geq 0 \; \forall t$ to take into account that highly financially distressed units are virtually insolvent and either go to bankruptcy or need to be bailed-out.

Hence, we can state and prove the following Proposition regarding the existence and uniqueness of an internal equilibrium.

Proposition 1 *The dynamic system (12) has a unique equilibrium point* $(f^E,\ f^{*E},\ Y^E)$ *that satisfies*

$$\begin{aligned}
f^E &= a\mu_0 \\
f^{*E} &= \mu_0 \\
Y^E &= \frac{Z_0 - \psi\mu_0}{1-\phi}
\end{aligned} \tag{13}$$

where an economic meaningful equilibrium output requires that $Z_0 > \psi\mu_0$.

Proof. See Mathematical Appendix. ■

In steady-state, liquidity and solvency indexes are the same as those obtained by SV. The main innovation comes from the introduction of the real sector. Higher autonomous demand and a lower sensitivity of expenditures to the level of output lead to higher equilibrium values of Y through traditional Keynesian reasoning. Output also depends on the part of the so-called margin of safety that in its turn depends on structural conditions of the economy. Higher values of μ_0 are related to lower levels of production, reflecting the empirical regularity that higher risk implies higher returns.

Regarding the stability properties of the fixed point, we can state and prove the following Proposition:

Proposition 2 *The equilibrium point $(f^E,\ f^{*E},\ Y^E)$ is locally asymptotically stable in the region of the parameter space defined as:*

$$(4 - 2a\beta + \alpha\beta)(2 + \bar{g}_Y) + 2\alpha\beta\bar{\mu}\bar{g}_{f^*} > 0 \tag{14}$$

and

$$\begin{aligned} &-\alpha^2\beta^2\bar{g}_{f^*}^2\bar{\mu}^2 + \left(a\alpha\beta^2 - 2\alpha^2\beta^2 - 2\alpha^2\beta^2\bar{g}_Y - \alpha\beta\bar{g}_Y + 2a\alpha\beta^2\bar{g}_Y\right)\bar{g}_{f^*}\bar{\mu} \\ &+ (a-\alpha)\left(\alpha\beta^2 - a\beta^2\bar{g}_Y + 2\alpha\beta^2\bar{g}_Y + \bar{g}_Y^2 - a\beta^2\bar{g}_Y^2 + \alpha\beta^2\bar{g}_Y^2\right) > 0 \end{aligned} \tag{15}$$

where $\bar{g}_Y$ and $\bar{g}_{f^*}$ are the partial derivatives of $g(\cdot)$ at the equilibrium point. Moreover, if a change in one of the parameters determines the violation of the first condition while the second is satisfied, a Flip bifurcation occurs. On the contrary, if the second condition is violated while the first is satisfied, a Neimark-Sacker bifurcation occurs.

Proof. See Mathematical Appendix. ∎

These are necessary conditions for the existence of the associated bifurcations (see Lines et al., 2019). Still, their economic intuition is not straightforward. Before proceeding, it is worth spending some time studying at least one particular case. Suppose that the interest rate and the time horizon of the unit are such that the ratio between liquidity and solvency indexes at equilibrium (a) equalises the response of liquidity to solvency deviations from the margin of safety (α). We can state and prove the following Proposition:

Proposition 3 *When $a = \alpha$, the equilibrium point $(f^E,\ f^{*E},\ Y^E)$ is locally asymptotically stable provided that:*

$$(4 - \alpha\beta +)(2 + \bar{g}_Y) + 2\alpha\beta\bar{\mu}\bar{g}_{f^*} > 0 \tag{16}$$

and

$$\alpha\beta(1 + \bar{g}_{f^*}\bar{\mu}) + \alpha\beta + \bar{g}_Y > 0 \tag{17}$$

If a change in one of the parameters determines the violation of condition (16) while (17) is satisfied, a Flip bifurcation occurs. The opposite case is associated with a Neimark-Sacker bifurcation.

Proof. See Mathematical Appendix. ∎

Given that $\bar{g}_{f^*} < 0$, conditions (16) and (17) indicate that the response of liquidity to real outcomes might be critical for the dynamic properties of the system. Indeed, high values of $\bar{\mu}$, α, and β, increase the likelihood that the dynamic behaviour of the model may drastically change from a qualitative point of view.

For the general case, we rely on numerical evidence to show that such bifurcations happen and to present an economic interpretation of the dynamics we obtain. Parameter values were chosen to provide results economically meaningful. Although this parameter selection has an illustrative purpose only, similar qualitative results are observed for wider ranges. Our reference values are:

$$\alpha = 0.5,\ \rho = 0.3,\ r = 0.05,\ T = 1,\ Z_0 = 20.15$$
$$\mu_0 = 0.1,\ \bar{\mu} = 1,\ \phi = 0.7995,\ \psi = 1,\ b_1 = b_2 = 2.5$$

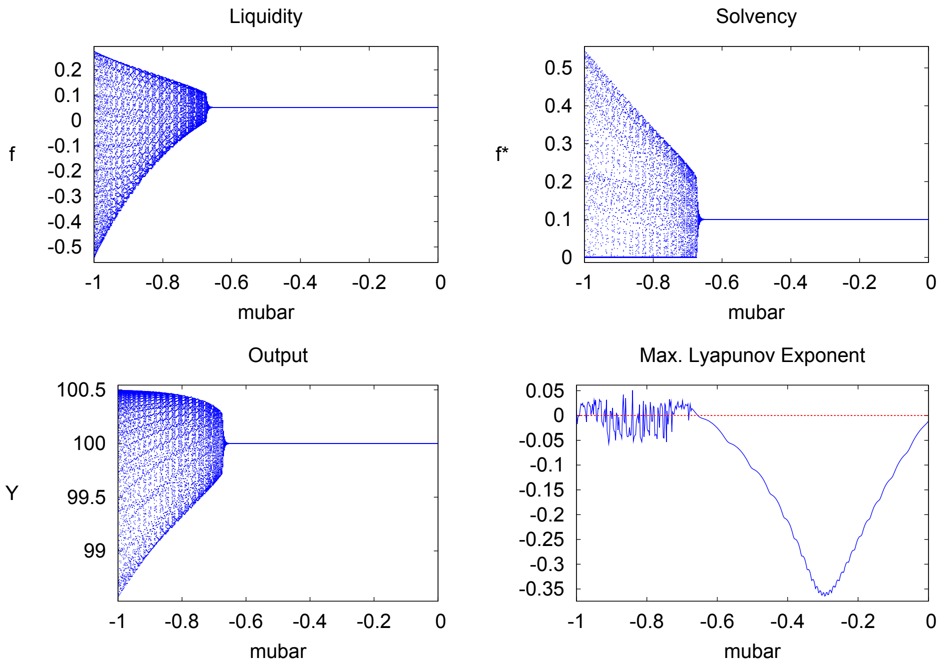


Figure 1: Bifurcation diagrams with respect to $\bar{\mu}$ for liquidity, solvency, output, and the correspondent Lyapunov exponent.

Following our previous discussion, a crucial parameter capturing the interaction between markets is $\bar{\mu}$. Fig. 1 reports the respective 1D bifurcation diagram. As we increase the interconnection between real and financial sides, the unique equilibrium point loses stability and a *Neimark-Sacker* bifurcation takes place. However, an equilibrium set may still exist because an attracting invariant closed curve co-exists with the unstable fixed point. The resulting orbits are sequences of points whose motion around the curve is either periodic or quasi-periodic. Indeed, the Maximum Lyapunov Exponent (MLE) alternates between values slightly above and below zero.[1] Our story follows closely Minsky's insight that during the boom phase of the business cycle, economic units become less careful in their financial decisions increasing their risk exposure. This leads to higher volatility in both markets.

We proceed by investigating the interplay of such effect with variations in the interest rate, the financial time horizon, and the response of liquidity to solvency deviations from the margin of safety. Fig. 2(a)-(c) presents different 2D bifurcation diagrams providing a qualitative representation of the stability region in the parameter space. Regions in red correspond to a combination that leads to convergence to the fixed point. Coloured in grey, we have regions of non-convergence, representing either chaotic areas or high frequency periodic cycles.

Overall, the system seems to depict a small response to changes in the interest rate. As long as $\bar{\mu}$ is low enough, varying r does not change qualitatively the behaviour of the model even below the so-called zero-lower bound. Dieci et al. (2018) found similar results in their study of the interaction between stock, bond and housing markets. Given that we do not

[1] A positive MLE is usually taken as an indication that the system is chaotic. It can be estimated numerically either by evaluating the Jacobian matrix at any point along a generated orbit or by using a finite differencing method along the orbit. In this article, we follow the second approach embodied in the algorithm of Diks et al. (2008).

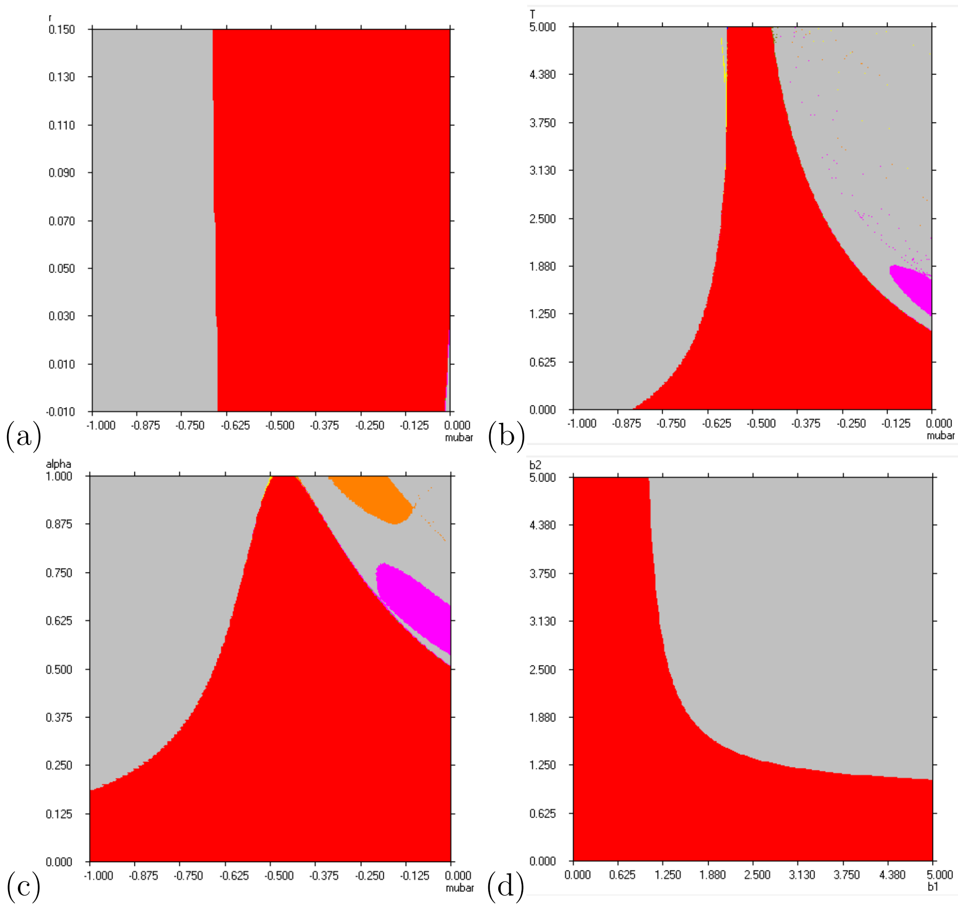


Figure 2: 2D bifurcation diagrams in the $(\bar{\mu}, r)$, $(\bar{\mu}, T)$, $(\bar{\mu}, \alpha)$, and (b_1, b_2) space.

allow the interest rate to directly influence aggregate demand, it only plays a marginal role through β. This last variable makes an important bridge between solvency and liquidity conditions. However, it relies more on expectations and on the time horizon of the economic unit, explaining the apparent insensitivity to r.

Also a $\bar{\mu}$ sufficiently close to zero combined with low values of T and α stabilises the system. This is basically because the transmission channels from output to liquidity and from liquidity to solvency are blocked. On the other hand, a sufficiently high time horizon of the unit and a strong response of liquidity to solvency are enough to bring about endogenous fluctuations, even under low values of $\bar{\mu}$. In this case, instability comes strictly from the financial market. Households and firms' solvency conditions depend on expectations about long-term financial flows. However, for a high T, there is an increasing projection of current states into the future because agents were supposed to be trend-extrapolators. As a result, instability rises. We also identified regions with cycles of periodicity five (in pink) and six (in orange). There is no straightforward economic meaning for them besides the fact that, for those parameters, the system generates endogenous cycles.

Finally, it is possible to assess the relevance of resource constraints, as in Eq. (10), for the dynamics of our model. Fig. 2(d) shows that either a sufficiently low b_1 or b_2 is enough for the equilibrium point to be stable. The asymmetric nature of this last effect is not obvious. It indicates that policies that avoid dramatic reductions in aggregate demand alone might have an important role in stabilising both the financial and real sides of the economy. Avoiding high peaks in output has a similar effect by limiting overconfidence in financial markets.

3 Heterogeneous agents and financial instability

Bounded rational agents rely on simple behavioural heuristics but their aggregation at the micro level "may generate sophisticated structure at the macro level" (Hommes, 2006, p. 1109). It must be noted, however, that the model developed in Section 2 assumed that agents are homogeneous. The interplay between different groups of agents is a necessary step to explain crucial stylised facts that characterise economic and financial time-series, such as high kurtosis, skewed distributions and crashes. To introduce some degree of heterogeneity, we extend the dynamic system (12) by differentiating between extrapolative ($E_t^e[\cdot]$) and regressive ($E_t^r[\cdot]$) expectations formation:

$$E_t^e\left[f_{t+1}\right] = f_t + \rho^e\left(f_t - \bar{f}\right) \tag{18}$$

$$E_t^r\left[f_{t+1}\right] = f_t - \rho^r\left(f_t - \bar{f}\right) \tag{19}$$

where ρ^e, $\rho^r > 0$ stand as the response of trend-extrapolators (or *chartists*) and those with regressive expectations (or *fundamentalists*), respectively, to deviation of liquidity from the tranquillity reference value.

Making use of Eqs. (18) and (19), aggregate expectations are given by the weighted sum between the two groups:

$$\begin{aligned} E_t\left[f_{t+1}\right] &= w_t E_t^e\left[f_{t+1}\right] + (1-w_t) E_t^r\left[f_{t+1}\right] \\ &= \left(1+\rho_t^{er}\right)\left(f_t - \bar{f}\right) + \bar{f} \end{aligned} \tag{20}$$

with

$$\rho_t^{er} = w_t \rho^e + (1-w_t)\left(-\rho^r\right) \tag{21}$$

and $w \in [0,1]$ standing for the share of chartists in the population.

Following Brock and Hommes (1997) and Naimzada and Pireddu (2015), the composition of the population is given by a discrete choice model:

$$w_t = \frac{\exp\left[\gamma x_t^e\right]}{\exp\left[\gamma x_t^e\right] + \exp\left[\gamma x_t^r\right]} \tag{22}$$

where x^e and x^r represent the "mood-state" of chartists and fundamentalists.[2]

We assume that agents can choose between extrapolating the trend or returning to the fundamentals. When the economy is facing a "boom" or periods of tranquillity, a unit is more likely to decide to extrapolate the trend. On the other hand, during a recession, agents become more risk averse and willing to converge to $\bar{f}$. Hence, suppose x^e and x^r are such that:

$$x_t^e = g\left(f_t^*, Y_t\right) \tag{23}$$

$$x_t^r = -g\left(f_t^*, Y_t\right) \tag{24}$$

For $g(\cdot) > 0$, we have that $w \in (0.5, 1]$ and chartists prevail, while for $g(\cdot) < 0$, it follows that $w \in [0, 0.5)$ and fundamentalists become the majority group. In steady state, $g(f^{*E}, Y^E) = 0$ and there is an equal distribution between the two heuristics.

Once we take into account heterogeneity, Eq. (5) can be rewritten as:

$$\beta_t = \frac{\left(1+\rho_t^{er}\right)\left[\left(1+r\right)^{T+1} - \left(1+\rho_t^{er}\right)^{T+1}\right]}{\left(1+r\right)^T\left(r-\rho_t^{er}\right)} \tag{25}$$

where, combining Eqs. (19)-(22), we have:

$$\rho_t^{er} = \frac{\rho^e \exp\left[\gamma g\left(f_t^*, Y_t\right)\right]^2 - \rho^r}{1 + \exp\left[\gamma g\left(f_t^*, Y_t\right)\right]^2}$$

so that β is a function of both f^* and Y.

It follows that the modified dynamic system thus becomes:

$$\begin{aligned} f_{t+1} &= f_t - \alpha\left[f_t^* - \mu_0 + \bar{\mu} b_2\left(\frac{b_1 + b_2}{b_1 \exp\left[-Z_0 + \psi f_t^* + (1-\phi)Y_t\right] + b_2} - 1\right)\right] \\ f_{t+1}^* &= \max\left\{\beta(f_t^*, Y_t) f_t + \left[1 - a\beta(f_t^*, Y_t)\right] f_t^*, 0\right\} \\ Y_{t+1} &= Y_t + b_2\left(\frac{b_1 + b_2}{b_1 \exp\left[-Z_0 + \psi f_t^* + (1-\phi)Y_t\right] + b_2} - 1\right) \end{aligned} \tag{26}$$

Hence, we can state and prove the following Proposition regarding the existence and uniqueness of an internal equilibrium as well as the local stability properties of the fixed point.

Proposition 4 *The dynamic system (26) has a unique equilibrium point (f^E, f^{*E}, Y^E) that satisfies (13). It is locally asymptotically stable provided that conditions (14) and (15) hold. If a change in one of the parameters determines the violation of the first condition, a Flip bifurcation occurs. On the other hand, a violation of the second condition is associated with a Neimark-Sacker bifurcation.*

[2]Notice that the description of endogenous switches between attitudes by the so-called Brock-Hommes discrete choice approach is closely related to the exponential functions in the transition probabilities of Lux (1995).

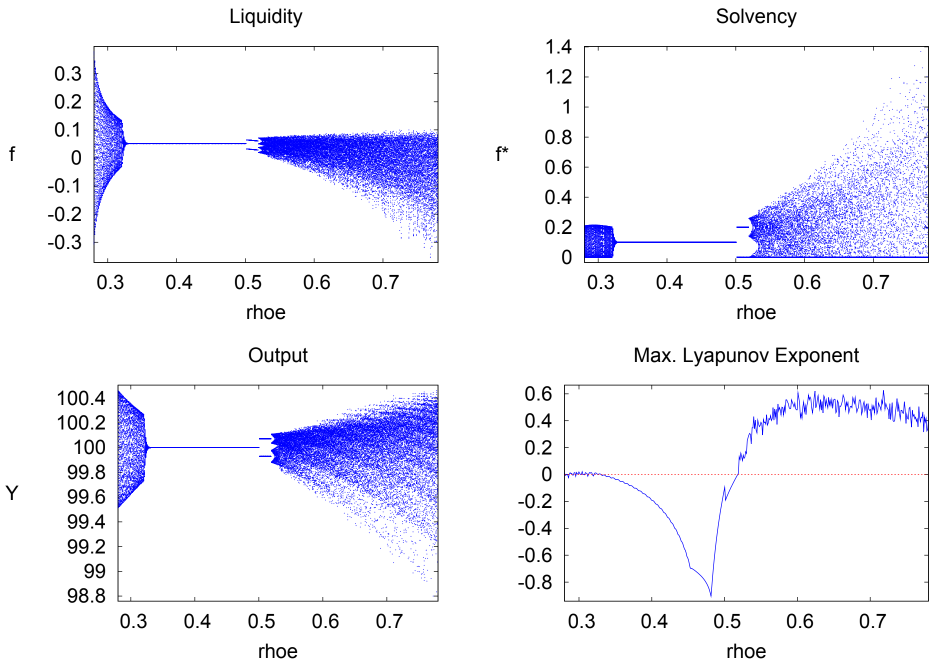


Figure 3: Bifurcation diagrams with respect to ρ^e for liquidity, solvency, output, and the correspondent Lyapunov exponent when $\bar{\mu} = 0.25$.

Proof. See Mathematical Appendix. ■

Once more, we rely on numerical simulations to verify if bifurcations occur and to further investigate the dynamic properties of the system. Our reference values are the same presented in Section 2, except for the adoption of the following extra calibration parameters:

$$\rho^e = 0.75, \;\; \rho^r = 0.5, \;\; \gamma = 2$$

Several studies on real-financial market interactions have suggested that chartists play a destabilising role in the economy. In our model, the crucial parameter is ρ^e, which indicates the reactivity of trend-extrapolators' expectations to liquidity deviations. Fig. 3 shows the respective 1D bifurcation diagram. For a sufficiently small response of expectations, a *Neimark-Sacker* bifurcation occurs and the stable fixed point loses its stability. A MLE oscillating between values slightly above and below zero indicates switches between periodic and aperiodic dynamics.

Very large values of ρ^e, on the other hand, lead to a degenerate *Flip* bifurcation. A stable period-2 cycle arises as the new limit set. Notice, however, that this is not the case of a classical period doubling, such as those observed in smooth maps. By continuously varying the bifurcation parameter, the period-2 cycle loses its stability through a second bifurcation, and so on, resulting in a cascade of bifurcations. Such a scenario typically leads to a parameter interval characterised by the emergence of chaotic trajectories, which is confirmed by a strictly positive MLE.

The reason for this behaviour is that when the reactivity of agents that extrapolate the trend is low, aggregate expectations strongly rely on regressive strategies. Given the lags involved in the interplay between real and financial variables, a too strong convergence to the fundamentals actually increases the volatility of the system. This is particularly true

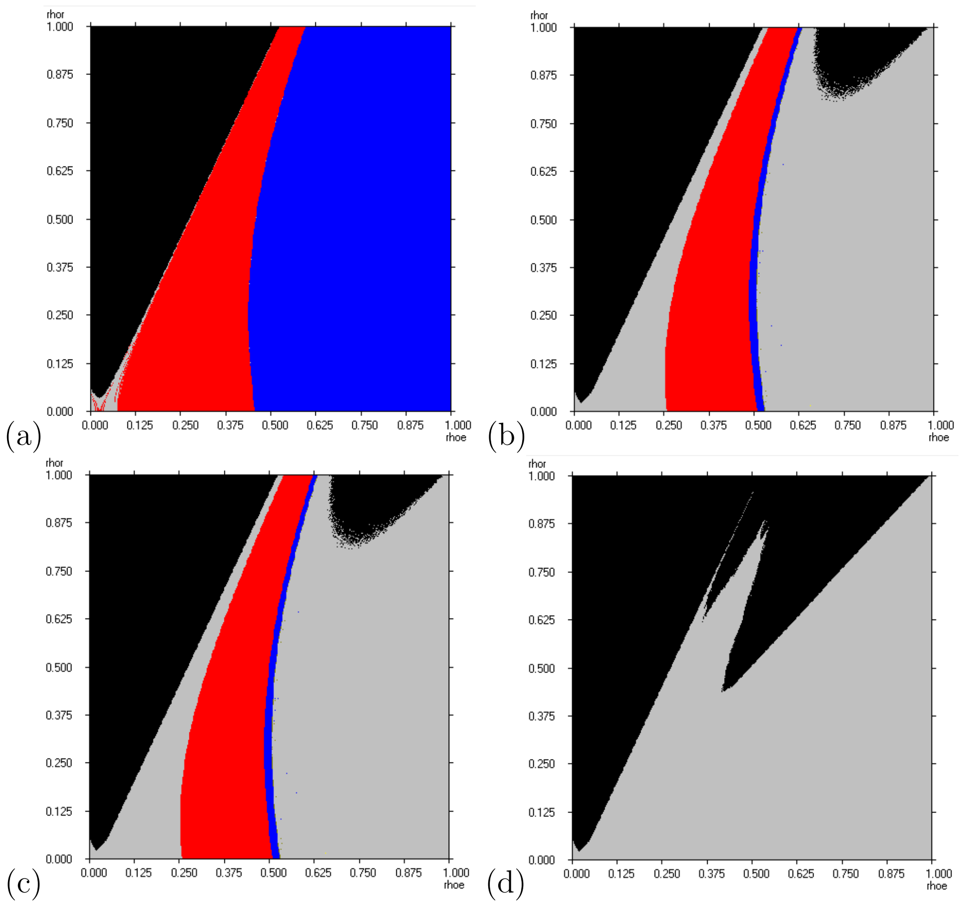


Figure 4: 2D bifurcation diagrams in the (ρ^e, ρ^r) space when (a) $\bar{\mu} = 0.25$ and $\alpha = 0.05$; (b) $\bar{\mu} = 0.25$ and $\alpha = 0.5$; (c) $\bar{\mu} = 1$ and $\alpha = 0.05$; (d) $\bar{\mu} = 1$ and $\alpha = 0.5$.

when $\rho^e < \rho^r$ so that in steady-state $\rho^{er} < 0$. At the other extreme, when ρ^e becomes sufficiently large, changes in aggregate expectations increasingly rely on chartists. In this case, the equilibrium loses stability due to an overreaction of expectations that amplifies the deviation between f and $\bar{f}$. For intermediary values of ρ^e, a region of stability emerges.

We now investigate simultaneous changes in chartists and fundamentalists' response for different values of the margin of solvency ($\bar{\mu}$) and of the speed of reaction (α). Looking at Fig. 4(a), we can observe what happens in a scenario characterised by a low interconnection between markets and by a low response of liquidity to solvency, i.e. $\bar{\mu} = 0.25$ and $\alpha = 0.05$. As in the previous section, regions in red correspond to a combination of parameters that leads to convergence to the fixed point while those coloured in grey stand for regions of non-convergence, representing areas of either chaotic or high frequency periodic cycles. The novelties are the blue and black regions: the first one points to the presence of a cycle of period two, while the second indicates divergence. When ρ^e is low, a chaotic behaviour may be observed. As it increases, however, the system first converges to equilibrium and later on a cycle of periodicity two arises.

By increasing the value of α, we move to Fig. 4(b). The blue area becomes very thin while regions characterised by non-convergence and chaos prevail. Panel (c) shows what happens when α is low but the interaction between the real and the financial side is stronger ($\bar{\mu} = 1$). It immediately stands out that this circumstance is equal to the previous one. We conclude that a strong interconnection between markets or between solvency and liquidity are

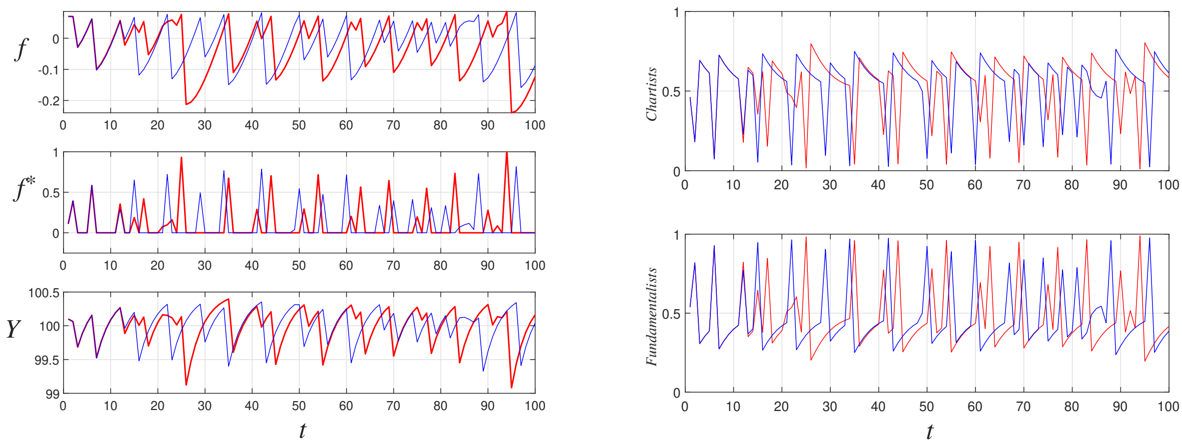


Figure 5: Sensitivity to initial conditions for (f_0, f_0^*, Y_0)=$(0.07, 0.11, 100.1)$, in red, and for (f_0, f_0^*, Y_0)=$(0.07, 0.1101, 100.1)$, in blue, when $\bar{\mu} = 0.25$, $\alpha = 0.5$, $\rho^e = 0.75$, and $\rho^r = 0.75$.

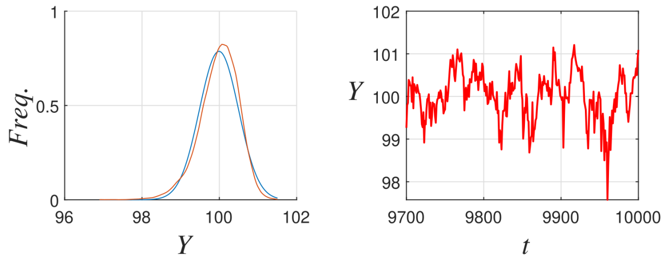
equivalent for the stability of the system. Finally, when both $\bar{\mu}$ and α are large, the system experiences high frequency cycles and a chaotic behaviour, as we can see in Fig. 4(d).

The view that the economy can be influenced permanently by the extent to which it has changed in the past is highly intuitive. Commonly referred to in social sciences as path dependency, it highlights that economic outcomes are historically contingent. Fig. 5 plots two different trajectories of our time-series for almost identical initial conditions. On the right, we can see the share of chartists and fundamentalists in the economy. After ten interactions, they start to diverge and follow a chaotic pattern, as already confirmed by our Lyapunov exponents. Such a result highlights the uniqueness of economic trajectories and that initial conditions matter.

Boom and bust dynamics strongly depend on the inter-temporal financial constraint. During good times, as Y goes up, there is a reduction of risk perception that reduces μ and increases the share of chartists in the economy. In this context, economic units operate in such a way that $f^* = 0$ and the solvency constraint is binding. As this process goes on, the liquidity position of the firm or household improves followed by an increase in ρ^{er} and, consequently, in β. Therefore, at a certain point, f^* becomes positive, which means aggregate demand and liquidity fall, frustrating expectations. Agents run to the fundamentals drastically reducing ρ^{er} and β. When the solvency constraint binds again, the economy starts to recover and the cycle restarts.

4 Introducing stochastic shocks

The last step of our analysis consists in understanding if our model is able to replicate, under a suitable parametric set-up, some qualitative features of real and financial markets. However, to deepen its descriptive power, we need some more sophisticated analysis on the features of the time-series. To do so, we introduce a stochastic component to each dynamic equation that is supposed to follow a random walk and captures unexpected or unpredictable events that affect an economy. Such factors are unexplained by our model but still may influence economic trajectories. Thus, we have the following map:



	Mean	Std.	Kurtosis	Skewness
f	0.0056	0.0186	4.2999	-0.5034
f*	0.1039	0.2079	14.1938	2.8412
Y	99.9910	0.5070	4.2316	-0.7169

Figure 6: Mean, Std., kurtosis, and skewness of the time-series. Average measures and statistics of 2000 Monte Carlo simulations (length 10000 iterations each).

$$
\begin{aligned}
f_{t+1} &= f_t - \alpha \left[f_t^* - \mu_0 + \bar{\mu} b_2 \left(\frac{b_1 + b_2}{b_1 \exp\left[-Z_0 + \psi f_t^* + (1-\phi)Y_t\right] + b_2} - 1 \right) \right] + \xi_{f,t} \\
f_{t+1}^* &= \max\left\{ \beta(f_t^*, Y_t) f_t + \left[1 - a\beta(f_t^*, Y_t)\right] f_t^* + \xi_{f^*,t}, 0 \right\} \\
Y_{t+1} &= Y_t + b_2 \left(\frac{b_1 + b_2}{b_1 \exp\left[-Z_0 + \psi f_t^* + (1-\phi)Y_t\right] + b_2} - 1 \right) + \xi_{Y,t}
\end{aligned} \tag{27}
$$

where

$$\xi_{f,t} \backsim N(\mu_f, \sigma_f^2) \qquad \xi_{f^*,t} \backsim N(\mu_{f^*}, \sigma_{f^*}^2) \qquad \xi_{Y,t} \backsim N(\mu_Y, \sigma_Y^2)$$

Stochastic shocks are i.i.d. and independent from each other. With a trial-and-error calibration approach, the mean and variances are set as:

$$
\begin{aligned}
&\mu_f = \mu_{f^*} = \mu_Y = 0 \\
&\sigma_f^2 = \sigma_{f^*}^2 = 0.01 \quad \sigma_Y^2 = 0.2
\end{aligned}
$$

We performed 2000 runs of Monte Carlo simulations, each one made up of 1000 iterations. It is well-known that GDP and asset prices often do not follow a normal distribution but suffer from the presence of fat-tails and asymmetric skewness, being highly volatile. For this reason, we are interested in showing that, even if the system is hit by purely Gaussian uncorrelated shocks, fat-tail distributed time-series arise via the endogenous transmission mechanisms embodied in the model. Fig. 6 displays this result for our relevant variables. The top-left panel shows the distribution of our simulated Y (in red) versus the theoretical normal one (in blue). Our model is able to reproduce both the presence of fat-tails and of left-skewness.[3]

It is important to understand the mechanism behind these last results. Fat-tails require that the underlying model is sufficiently nonlinear in order to amplify the stochastic component in such a way that extreme events are more likely to happen. Clearly, our system satisfies

[3]Magnitudes obtained are close to those reported in several empirical studies. For instance, Fagiolo et al. (2008) using monthly and quarterly data for a sample of OECD countries found that the rate of growth of output exhibited asymmetric skewness and a kurtosis varying between 3.5 and 9.5. For the United States, they obtained a skewness of -0.08 and a kurtosis of 4.2 (see also Ascari et al., 2015). Naimzada and Pireddu (2015), on the other hand, reported that the output-gap in the US has a skewness of 3.6.

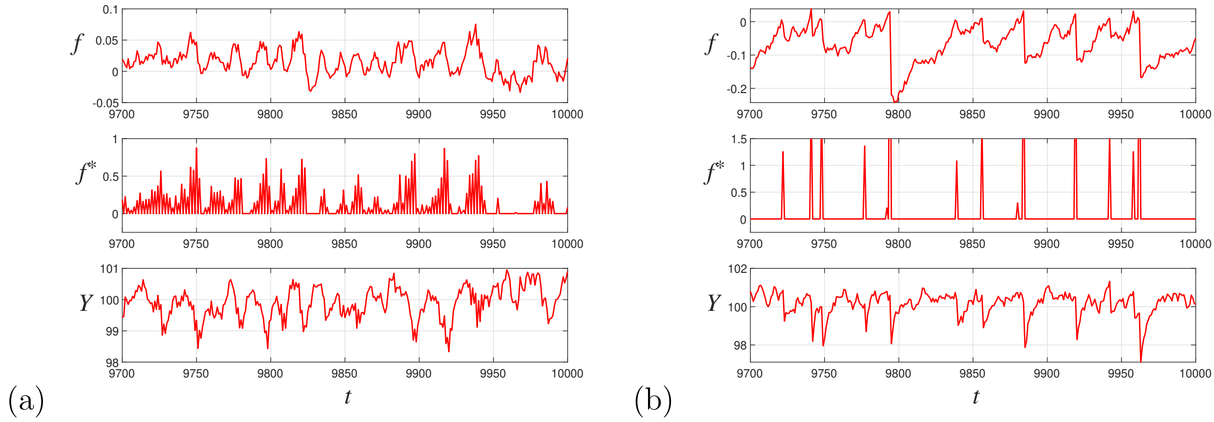


Figure 7: Stochastic trajectories for (a) $T = 0.5$ and (b) $T = 3$. Average measures and statistics of 2000 Monte Carlo simulations (length 10000 iterations each).

this condition. On the other hand, left-skewness in GDP series results from the interactions between the inter-temporal financial constraint and Y. Because financially distressed units are virtually insolvent and either go to bankruptcy or need to be bailed-out, at a macro level, it follows that f^* is strongly right-skewed. Given Eq. (7), this translates into left-skewed output.

Our analysis in the previous sections has revealed the following important insight: the time horizon of the economic unit is an important source of instability in a capitalist economy. To show that this is also the case once we take into account the interplay between different groups of agents, Fig. 7 reports the time-series of liquidity, solvency, and output for $T = 0.5$ and $T = 3$. As we increase T, two interesting results appear. First, there is a reduction in volatility because regressive expectations are anchored in the fundamentals. Second, economic units operate in such a way that the solvency constraint binds most of the time. Consequently, f^* becomes increasingly right-skewed leading to left-skewed output. In other words, GDP fluctuates less but crashes are also more likely to occur.

A question that remains to be answered concerns which specific shock is responsible for the main dynamics we obtained. To provide an answer, we followed a three-step procedure. First, we repeated our Monte Carlo simulations adding a noise component only to the first difference equation. As a second step, a stochastic shock was introduced only to the second equation. Finally, a noise term was added to output. Table 1 shows that output series depict high kurtosis and left-skewness only under liquidity shocks. Introducing ξ_{f^*} or ξ_Y actually leads to low kurtosis and neglectful effects in terms of asymmetric skewness.

Combining two stochastic components confirms our previous insight that unexpected events regarding the liquidity index are the main driving force behind economic crashes. As reported in Table 2, ξ_{f^*} and ξ_Y alone result in GDP series normally distributed. However, the introduction of the pairs ξ_f and ξ_{f^*} or ξ_f and ξ_Y is enough to recover trajectories with properties similar to those of Fig. 6.

Random disturbances ξ_f may be deemed to affect either the response of liquidity to solvency deviations from the margin of safety (α) or the margin of safety itself (μ). Notice, however, that an error term in α should interact with f^* and Y. By construction, this is not our case. It follows that the stochastic component must be acting through μ and might be understood as new information that changes the risk perception of households and firms.

Table 1: Mean, Std., kurtosis and skewness for individual stochastic shocks.

Shock	Values	Mean	Std.	Kurtosis	Skewness
ξ_f	f	0.0065	0.0177	3.1273	0.0137
-	f^*	0.892	0.1939	10.328	2.4565
-	Y	100.0102	0.3328	4.6079	-1.0821
Shock	Values	Mean	Std.	Kurtosis	Skewness
-	f	0.0121	0.0036	1.1769	-0.0090
ξ_{f^*}	f^*	0.1001	0.1017	1.0718	0.0529
-	Y	99.9999	0.0754	1.1635	-0.0344
Shock	Values	Mean	Std.	Kurtosis	Skewness
-	f	0.0116	0.004	1.9811	0.0916
-	f^*	0.1005	0.107	1.8138	0.3866
ξ_Y	Y	99.9959	0.3461	2.933	-0.1223

Table 2: Mean, Std., kurtosis and skewness for different combinations of stochastic shocks.

Shock	Values	Mean	Std.	Kurtosis	Skewness
ξ_f	f	0.0053	0.0184	4.3705	-0.3373
-	f^*	0.0913	0.196	17.95	2.9862
ξ_Y	Y	100.0156	0.4651	4.163	-0.5371
Shock	Values	Mean	Std.	Kurtosis	Skewness
-	f	0.0117	0.004	1.9592	0.0829
ξ_{f^*}	f^*	0.1005	0.1079	1.7341	0.381
ξ_Y	Y	99.99	0.344	3.01	-0.1767
Shock	Values	Mean	Std.	Kurtosis	Skewness
ξ_f	f	0.0064	0.0182	3.2854	-0.0698
ξ_{f^*}	f^*	0.1	0.1954	9.8445	2.4687
-	Y	100.006	0.3357	4.6644	-1.0631

Hence, we can rewrite Eq. (11) as:

$$\mu_t = \mu_0 + \bar{\mu}g(f^*, Y) + \frac{\xi_f}{\alpha} \tag{28}$$

Such a specification has a clear cut economic interpretation. In our model and numerical simulations, this variable is one of the main sources of endogenous fluctuations and establishes a crucial link between real and financial markets through f. This seems to be confirmed by the interaction between stochastic and deterministic forces. We conclude that shocks in the margin of safety of the (ir)rational explorer are amplified by the right-skewed solvency constraint leading to output fluctuations compatible with the possibility of crashes.

5 Final considerations

Minsky insisted that there is an inherent and fundamental instability in our sort of economy involving financial relations and the behaviour of agents during euphoric periods. He argued that processes that generate financial fragility are endogenous to the system and go in hand with periods of economic stability.

While the so-called FIH have just recently regained momentum with the deep global recession, a large number of scholars has formalised over the past decades different parts of this theory. It must be noted, however, that only those contributions focusing in stock-market price dynamics have provided an assessment of the role of expectation formation with heterogeneous agents. Macroeconomic models dealing with debt dynamics have failed short in presenting such an account. Therefore, this article filled a gap in the literature by formalising switches between different heuristics in a model where solvency aspects matter.

Building on Sordi and Vercelli (2012), we studied the interaction between current and inter-temporal financial constraints with the real economy, differentiating between chartist and fundamentalist agents. Though the destabilising role of extrapolative behaviour is part of conventional wisdom, we showed under which conditions regressive expectations and the existence of resource constraints can also lead to instability. Our system was capable of generating time-series that reproduce important empirical stylised facts such as negative skewness and fat-tails. We demonstrated that a random disturbance on liquidity conditions might be a crucial element driving such results. In the absence of a stochastic component, the model still was shown to present sensitivity to initial conditions.

In terms of policy implications, three results are worth stressing because they are not so obvious in the literature on the FIH. First, the time horizon of the economic unit seems to be more important than the interest rate as a source of endogenous instability. They both intermediate the relationship between liquidity and solvency, but the latter played only a minor role. This means policy makers need to understand what is behind changes in T and how regulation can be designed to avoid instability through this variable.

Second, fundamentalists are not always good for the economy. If their speed of reaction is too strong, there is an increase in volatility given the lags involved in the interaction between real and financial variables. On the other hand, a balanced combination of extrapolative and regressive expectations is stabilising. Hence, is not a matter of controlling the size of one or another group, but to guarantee a regulatory framework that allows smooth adjustments in expectations.

Last but not least, fiscal policy might be a useful instrument for reducing instability if it manages to avoid dramatic reductions or increases in aggregate demand. Such contracyclical

Keynesian policy is not a novelty in itself. However, our exercise indicates that either a sufficiently high floor or a low ceiling is enough for the equilibrium point to be stable. There is no need to tackle both at the same time. This means that policy makers could concentrate in guaranteeing just one of them. Such approach should certainly reduce the complexity of the instruments to be designed and should be further investigated in future research.

Minsky's insights help us to understand the key financial developments of recent decades. This article sheds some light on the complex relationship between real and financial markets putting another "brick" in the FIH "wall".

A Mathematical Appendix

A.1 Proof of Proposition 1

In steady-state $f_{t+1}=f_t$, $f^*_{t+1}=f^*_t$, and $Y_{t+1}=Y_t$. Hence, the dynamic system (12) reads:

$$\begin{aligned} 0 &= \alpha\left(f^*_t - \mu_0\right) \\ f_t &= a f^*_t \\ 0 &= g\left(f^*_t, Y_t\right) \end{aligned}$$

It follows that there is a unique equilibrium point $(f^E,\ f^{*E},\ Y^E)$ defined and given by:

$$\begin{aligned} f^E &= a\mu_0 \\ f^{*E} &= \mu_0 \\ Y^E &= \frac{Z_0 - \psi\mu_0}{1-\phi} \end{aligned}$$

A.2 Proof of Propositions 2 and 3

The Jacobian matrix that corresponds to our dynamic system (12) is such that:

$$J = \begin{bmatrix} 1 & -\alpha\left(1+\bar{\mu}\bar{g}_{f^*}\right) & -\alpha\bar{\mu}\bar{g}_Y \\ \beta & 1-a\beta & 0 \\ 0 & \bar{g}_{f^*} & 1+\bar{g}_Y \end{bmatrix}$$

where

$$\begin{aligned} g_{f^*}|_{(f^E,f^{*E},Y^E)} = \bar{g}_{f^*} &= \frac{-b_2\left(b_1+b_2\right)\psi b_1 \exp\left(-Z_0+\psi\bar{f}^*+(1-\phi)\bar{Y}\right)}{\left[b_1\exp\left(-Z_0+\psi\bar{f}^*+(1-\phi)\bar{Y}\right)+b_2\right]^2} \\ &= -\frac{\psi b_1 b_2}{b_1+b_2} < 0 \end{aligned}$$

$$\begin{aligned} g_Y|_{(f^E,f^{*E},Y^E)} = \bar{g}_Y &= \frac{-b_2\left(b_1+b_2\right)(1-\phi) b_1 \exp\left(-Z_0+\psi\bar{f}^*+(1-\phi)\bar{Y}\right)}{\left[b_1\exp\left(-Z_0+\psi\bar{f}^*+(1-\phi)\bar{Y}\right)+b_2\right]^2} \\ &= -\frac{(1-\phi) b_1 b_2}{b_1+b_2} < 0 \end{aligned}$$

are the partial derivatives of $g(\cdot)$ with respect to f^* and Y at the equilibrium point.

The elements of the Jacobian are such that:

$$
\begin{aligned}
j_{11} &= \frac{\partial f_{t+1}}{\partial f_t} = 1 > 0, \;\; j_{12} = \frac{\partial f_{t+1}}{\partial f_t^*} = -\alpha \left(1 + \bar{\mu} \bar{g}_{f^*}\right) \gtreqless 0, \;\; j_{13} = \frac{\partial f_{t+1}}{\partial Y_t} = -\alpha \bar{\mu} \bar{g}_Y > 0 \\
j_{21} &= \frac{\partial f_{t+1}^*}{\partial f_t} = \beta > 0, \;\; j_{22} = \frac{\partial f_{t+1}^*}{\partial f_t^*} = 1 - a\beta \gtreqless 0, \;\; j_{23} = \frac{\partial f_{t+1}^*}{\partial Y_t} = 0 \\
j_{31} &= \frac{\partial Y_{t+1}}{\partial f_t} = 0, \;\; j_{32} = \frac{\partial Y_{t+1}}{\partial f_t^*} = \bar{g}_{f^*} < 0, \;\; j_{33} = \frac{\partial Y_{t+1}}{\partial Y_t} = 1 + \bar{g}_Y \gtreqless 0
\end{aligned}
$$

so that the characteristic equation can be written as:

$$\lambda^3 + C_1 \lambda^2 + C_2 \lambda + C_3 = 0$$

where

$$
\begin{aligned}
C_1 &= -\mathrm{tr} J = -1 - 1 + a\beta - 1 - \bar{g}_Y \\
&= -3 + a\beta - \bar{g}_Y \\
C_2 &= \begin{vmatrix} 1 - a\beta & 0 \\ \bar{g}_{f^*} & 1 + \bar{g}_Y \end{vmatrix} + \begin{vmatrix} 1 & -\alpha \bar{\mu} \bar{g}_Y \\ 0 & 1 + \bar{g}_Y \end{vmatrix} + \begin{vmatrix} 1 & -\alpha \left(1 + \bar{\mu} \bar{g}_{f^*}\right) \\ \beta & 1 - a\beta \end{vmatrix} \\
&= \left(1 - a\beta\right)\left(1 + \bar{g}_Y\right) + 1 + \bar{g}_Y + 1 - a\beta + \alpha\beta \left(1 + \bar{\mu} \bar{g}_{f^*}\right) \\
&= 2 + \left(1 - a\beta\right)\left(1 + \bar{g}_Y\right) + \bar{g}_Y - a\beta + \alpha\beta \left(1 + \bar{\mu} \bar{g}_{f^*}\right) \\
C_3 &= -\det J = -\begin{vmatrix} 1 & -\alpha \left(1 + \bar{\mu} \bar{g}_{f^*}\right) & -\alpha \bar{\mu} \bar{g}_Y \\ \beta & 1 - a\beta & 0 \\ 0 & \bar{g}_{f^*} & 1 + \bar{g}_Y \end{vmatrix} \\
&= -\begin{vmatrix} 1 - a\beta & 0 \\ \bar{g}_{f^*} & 1 + \bar{g}_Y \end{vmatrix} + \beta \begin{vmatrix} -\alpha \left(1 + \bar{\mu} \bar{g}_{f^*}\right) & -\alpha \bar{\mu} \bar{g}_Y \\ \bar{g}_{f^*} & 1 + \bar{g}_Y \end{vmatrix} \\
&= -\left(1 - a\beta\right)\left(1 + \bar{g}_Y\right) + \beta \left[-\alpha \left(1 + \bar{\mu} \bar{g}_{f^*}\right)\left(1 + \bar{g}_Y\right) + \alpha \bar{\mu} \bar{g}_Y \bar{g}_{f^*}\right] \\
&= -\left(1 - a\beta\right)\left(1 + \bar{g}_Y\right) - \alpha\beta \left(1 + \bar{\mu} \bar{g}_{f^*}\right)\left(1 + \bar{g}_Y\right) + \alpha\beta \bar{\mu} \bar{g}_Y \bar{g}_{f^*} \\
&= -\left(1 - a\beta\right)\left(1 + \bar{g}_Y\right) - \alpha\beta \left(1 + \bar{g}_Y\right) - \alpha\beta \bar{\mu} \bar{g}_{f^*} \left(1 + \bar{g}_Y\right) + \alpha\beta \bar{\mu} \bar{g}_Y \bar{g}_{f^*} \\
&= -\left(1 - a\beta + \alpha\beta\right)\left(1 + \bar{g}_Y\right) - \alpha\beta \bar{\mu} \bar{g}_{f^*} - \alpha\beta \bar{\mu} \bar{g}_{f^*} \bar{g}_Y + \alpha\beta \bar{\mu} \bar{g}_Y \bar{g}_{f^*} \\
&= -\left(1 - a\beta + \alpha\beta\right)\left(1 + \bar{g}_Y\right) - \alpha\beta \bar{\mu} \bar{g}_{f^*}
\end{aligned}
$$

The necessary and sufficient conditions for the local stability of a given equilibrium point require that all eigenvalues of the Jacobian matrix, determined as roots of the characteristic equation, are less than unity in modulus:

$$1 + C_1 + C_2 + C_3 > 0 \quad \text{(I)}$$

$$1 - C_1 + C_2 - C_3 > 0 \quad \text{(II)}$$

$$1 - C_2 + C_1 C_3 - C_3^2 > 0 \quad \text{(III)}$$

$$3 - C_2 > 0 \quad \text{(IV)}$$

Through direct computation we find that:

$$\begin{aligned}
&1 + C_1 + C_2 + C_3 \qquad \text{(I)}\\
&= 1 - 3 + a\beta - \bar{g}_Y + 2 + (1 - a\beta)(1 + \bar{g}_Y) + \bar{g}_Y - a\beta + \alpha\beta(1 + \bar{\mu}\bar{g}_{f^*})\\
&- (1 - a\beta + \alpha\beta)(1 + \bar{g}_Y) - \alpha\beta\bar{\mu}\bar{g}_{f^*}\\
&= (1 - a\beta)(1 + \bar{g}_Y) + \alpha\beta(1 + \bar{\mu}\bar{g}_{f^*}) - (1 - a\beta + \alpha\beta)(1 + \bar{g}_Y) - \alpha\beta\bar{\mu}\bar{g}_{f^*}\\
&= (1 - a\beta - 1 + a\beta - \alpha\beta)(1 + \bar{g}_Y) + \alpha\beta + \alpha\beta\bar{\mu}\bar{g}_{f^*} - \alpha\beta\bar{\mu}\bar{g}_{f^*}\\
&= -\alpha\beta(1 + \bar{g}_Y) + \alpha\beta\\
&= -\alpha\beta\bar{g}_Y = \frac{\alpha\beta(1 - \phi)b_1 b_2}{b_1 + b_2} > 0, \text{ which is always satisfied.}
\end{aligned}$$

$$\begin{aligned}
&1 - C_1 + C_2 - C_3 \qquad \text{(II)}\\
&= 1 + 3 - a\beta + \bar{g}_Y + 2 + (1 - a\beta)(1 + \bar{g}_Y) + \bar{g}_Y - a\beta + \alpha\beta(1 + \bar{\mu}\bar{g}_{f^*})\\
&+ (1 - a\beta + \alpha\beta)(1 + \bar{g}_Y) + \alpha\beta\bar{\mu}\bar{g}_{f^*}\\
&= 6 - 2a\beta + 2\bar{g}_Y + (1 - a\beta)(1 + \bar{g}_Y) + \alpha\beta(1 + \bar{\mu}\bar{g}_{f^*}) + (1 - a\beta + \alpha\beta)(1 + \bar{g}_Y) + \alpha\beta\bar{\mu}\bar{g}_{f^*}\\
&= 6 - 2a\beta + 2\bar{g}_Y + 2 - 2a\beta + \alpha\beta + 2\bar{g}_Y - 2a\beta\bar{g}_Y + \alpha\beta\bar{g}_Y + \alpha\beta + 2\alpha\beta\bar{\mu}\bar{g}_{f^*}\\
&= 8 - 4a\beta + 4\bar{g}_Y + 2\alpha\beta - 2a\beta\bar{g}_Y + \alpha\beta\bar{g}_Y + 2\alpha\beta\bar{\mu}\bar{g}_{f^*}\\
&= 2(2 - a\beta)(2 + \bar{g}_Y) + \alpha\beta(2 + \bar{g}_Y + 2\bar{\mu}\bar{g}_{f^*})\\
&= (4 - 2a\beta + \alpha\beta)(2 + \bar{g}_Y) + 2\alpha\beta\bar{\mu}\bar{g}_{f^*} \gtrless 0
\end{aligned}$$

$$\begin{aligned}
&1 - C_2 + C_1 C_3 - C_3^2 \qquad \text{(III)}\\
&= 1 - 2 - (1 - a\beta)(1 + \bar{g}_Y) - \bar{g}_Y + a\beta - \alpha\beta(1 + \bar{\mu}\bar{g}_{f^*})\\
&- (-3 + a\beta - \bar{g}_Y)\left[(1 - a\beta + \alpha\beta)(1 + \bar{g}_Y) + \alpha\beta\bar{\mu}\bar{g}_{f^*}\right] - \left[(1 - a\beta + \alpha\beta)(1 + \bar{g}_Y) + \alpha\beta\bar{\mu}\bar{g}_{f^*}\right]^2\\
&= -(1 + \bar{g}_Y) - (1 - a\beta)(1 + \bar{g}_Y) + a\beta - \alpha\beta(1 + \bar{\mu}\bar{g}_{f^*})\\
&- \left[(1 - a\beta + \alpha\beta)(1 + \bar{g}_Y) + \alpha\beta\bar{\mu}\bar{g}_{f^*}\right]\left[-3 + a\beta - \bar{g}_Y + (1 - a\beta + \alpha\beta)(1 + \bar{g}_Y) + \alpha\beta\bar{\mu}\bar{g}_{f^*}\right]\\
&= -(2 - a\beta)(1 + \bar{g}_Y) + a\beta - \alpha\beta(1 + \bar{\mu}\bar{g}_{f^*})\\
&- \left[(1 - a\beta + \alpha\beta)(1 + \bar{g}_Y) + \alpha\beta\bar{\mu}\bar{g}_{f^*}\right]\left[-3 + a\beta - \bar{g}_Y + 1 + \bar{g}_Y - a\beta(1 + \bar{g}_Y) + \alpha\beta(1 + \bar{g}_Y) + \alpha\beta\bar{\mu}\bar{g}_{f^*}\right]\\
&= \big(-2 + a\beta + 2 - 2a\beta + 2\alpha\beta + a\beta\bar{g}_Y - a^2\beta^2\bar{g}_Y + a\alpha\beta^2\bar{g}_Y - \alpha\beta + a\alpha\beta^2 - \alpha^2\beta^2\\
&-\alpha\beta\bar{g}_Y + a\alpha\beta^2\bar{g}_Y - \alpha^2\beta^2\bar{g}_Y - \alpha^2\beta^2\bar{\mu}\bar{g}_{f^*} - \alpha\beta\bar{\mu}\bar{g}_{f^*} + a\alpha\beta^2\bar{\mu}\bar{g}_{f^*} - \alpha^2\beta^2\bar{\mu}\bar{g}_{f^*}\big)(1 + \bar{g}_Y)\\
&+ a\beta - \alpha\beta + \alpha\beta\bar{\mu}\bar{g}_{f^*} + a\alpha\beta^2\bar{\mu}\bar{g}_Y\bar{g}_{f^*} - \alpha^2\beta^2\bar{\mu}^2\bar{g}_{f^*}^2\\
&= (-a\beta + a\beta) + (\alpha\beta - \alpha\beta) + a\alpha\beta^2 - \alpha^2\beta^2 + (a\beta\bar{g}_Y - a\beta\bar{g}_Y) + (\alpha\beta\bar{g}_Y - \alpha\beta\bar{g}_Y)\\
&- a^2\beta^2\bar{g}_Y + \left(2a\alpha\beta^2\bar{g}_Y + a\alpha\beta^2\bar{g}_Y\right)\\
&- \left(\alpha^2\beta^2\bar{g}_Y + \alpha^2\beta^2\bar{g}_Y\right) - 2\alpha^2\beta^2\bar{\mu}\bar{g}_{f^*} + (\alpha\beta\bar{\mu}\bar{g}_{f^*} - \alpha\beta\bar{\mu}\bar{g}_{f^*}) + a\alpha\beta^2\bar{\mu}\bar{g}_{f^*}\\
&+ a\beta\bar{g}_Y^2 - \alpha\beta\bar{g}_Y^2 - a^2\beta^2\bar{g}_Y^2 + 2a\alpha\beta^2\bar{g}_Y^2 - \alpha^2\beta^2\bar{g}_Y^2 - \alpha^2\beta^2\bar{\mu}^2\bar{g}_{f^*}^2\\
&- 2\alpha^2\beta^2\bar{\mu}\bar{g}_Y\bar{g}_{f^*} - \alpha\beta\bar{\mu}\bar{g}_Y\bar{g}_{f^*} + \left(a\alpha\beta^2\bar{\mu}\bar{g}_Y\bar{g}_{f^*} + a\alpha\beta^2\bar{\mu}\bar{g}_Y\bar{g}_{f^*}\right)\\
&= (a - \alpha)\alpha\beta^2 - (a - \alpha)a\beta^2\bar{g}_Y + 2(a - \alpha)\alpha\beta^2\bar{g}_Y + (a - \alpha)\bar{g}_Y^2\\
&- 2\alpha^2\beta^2\bar{\mu}\bar{g}_{f^*} + a\alpha\beta^2\bar{\mu}\bar{g}_{f^*} - (a - \alpha)a\beta^2\bar{g}_Y^2\\
&+ (a - \alpha)\alpha\beta^2\bar{g}_Y^2 - \alpha^2\beta^2\bar{\mu}^2\bar{g}_{f^*}^2 - 2\alpha^2\beta^2\bar{\mu}\bar{g}_Y\bar{g}_{f^*} - \alpha\beta\bar{\mu}\bar{g}_Y\bar{g}_{f^*} + 2a\alpha\beta^2\bar{\mu}\bar{g}_Y\bar{g}_{f^*}\\
&= -\alpha^2\beta^2\bar{g}_{f^*}^2\bar{\mu}^2 + \left(a\alpha\beta^2 - 2\alpha^2\beta^2 - 2\alpha^2\beta^2\bar{g}_Y - \alpha\beta\bar{g}_Y + 2a\alpha\beta^2\bar{g}_Y\right)\bar{g}_{f^*}\bar{\mu}\\
&+ (a - \alpha)\left(\alpha\beta^2 - a\beta^2\bar{g}_Y + 2\alpha\beta^2\bar{g}_Y + \bar{g}_Y^2 - a\beta^2\bar{g}_Y^2 + \alpha\beta^2\bar{g}_Y^2\right) \gtrless 0
\end{aligned}$$

and finally

$$\begin{aligned}
&3 - C_2 \qquad \text{(IV)}\\
&= 3 - 2 - (1 - a\beta)(1 + \bar{g}_Y) - \bar{g}_Y + a\beta - \alpha\beta(1 + \bar{\mu}\bar{g}_{f^*})\\
&= 1 - 1 + a\beta - \bar{g}_Y + a\beta\bar{g}_Y - \bar{g}_Y + a\beta - \alpha\beta - \alpha\beta\bar{\mu}\bar{g}_{f^*}\\
&= 2a\beta - 2\bar{g}_Y + a\beta\bar{g}_Y - \alpha\beta - \alpha\beta\bar{g}_{f^*}\bar{\mu} \gtreqless 0
\end{aligned}$$

Therefore, the system is locally stable as long as:

$$(4 - 2a\beta + \alpha\beta)(2 + \bar{g}_Y) + 2\alpha\beta\bar{\mu}\bar{g}_{f^*} > 0$$

and

$$\begin{aligned}
&- \alpha^2\beta^2\bar{g}_{f^*}^2\bar{\mu}^2 + \left(a\alpha\beta^2 - 2\alpha^2\beta^2 - 2\alpha^2\beta^2\bar{g}_Y - \alpha\beta\bar{g}_Y + 2a\alpha\beta^2\bar{g}_Y\right)\bar{g}_{f^*}\bar{\mu}\\
&+ (a - \alpha)\left(\alpha\beta^2 - a\beta^2\bar{g}_Y + 2\alpha\beta^2\bar{g}_Y + \bar{g}_Y^2 - a\beta^2\bar{g}_Y^2 + \alpha\beta^2\bar{g}_Y^2\right) > 0
\end{aligned}$$

A separate violation of condition $1 - C_1 + C_2 - C_3$, while the other conditions hold, is associated with a *Flip* bifurcation (see Lines et al., 2019). Therefore, at:

$$(4 - 2a\beta + \alpha\beta)(2 + \bar{g}_Y) + 2\alpha\beta\bar{\mu}\bar{g}_{f^*} = 0$$

the fixed point loses stability and a stable period-2 cycle may arise as the new limit set.

On the other hand, a separate violation of condition $1-C_2+C_1C_3-C_3^2$ leads the modulus of a pair of complex conjugate eigenvalues to cross the unit circle. As the bifurcation parameter is varied beyond the critical value, the stable fixed point of the system loses its stability:

$$\begin{aligned}
&- \alpha^2\beta^2\bar{g}_{f^*}^2\bar{\mu}^2 + \left(a\alpha\beta^2 - 2\alpha^2\beta^2 - 2\alpha^2\beta^2\bar{g}_Y - \alpha\beta\bar{g}_Y + 2a\alpha\beta^2\bar{g}_Y\right)\bar{g}_{f^*}\bar{\mu}\\
&+ (a - \alpha)\left(\alpha\beta^2 - a\beta^2\bar{g}_Y + 2\alpha\beta^2\bar{g}_Y + \bar{g}_Y^2 - a\beta^2\bar{g}_Y^2 + \alpha\beta^2\bar{g}_Y^2\right) = 0
\end{aligned}$$

in which case we have a *Neimark-Sacker* bifurcation.

Setting $a = \alpha$, it immediately follows that the equilibrium point is locally stable iff:

$$(4 - \alpha\beta +)(2 + \bar{g}_Y) + 2\alpha\beta\bar{\mu}\bar{g}_{f^*} > 0$$

and

$$\alpha\beta(1 + \bar{g}_{f^*}\bar{\mu}) + \alpha\beta + \bar{g}_Y > 0$$

Consequently, if a change in one of the parameters determines the violation of the first condition while the second is satisfied, a Flip bifurcation occurs. The opposite case is associated with a Neimark-Sacker bifurcation.

A.3 Proof of Proposition 4

To prove Proposition 3 it is enough to notice that in steady-state $g(\cdot) = 0$. It follows that the Jacobian matrices of the dynamic systems (12) and (26) are equivalent. Hence, the local stability analysis repeats the steps performed when proving Proposition 2.

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