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Notes on Piketty's model

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## Abstract

Thomas Piketty's *Capital in the Twenty-First Century* is primarily an empirical investigation into the history of the distribution of income and wealth in developed countries. Piketty, however, goes beyond this approach, presenting a theory of the long-run tendency of wealth inequality and rooting his work deeply in economic theory. In this paper we review and develop the theoretical model of Piketty's book. We can divide the model into two parts: firstly, the "fundamental laws of capitalism" and the change in the functional distribution of income are analysed. Secondly, the evolution of personal wealth distribution is examined. Alongside the development of the model, the paper points out two shortcomings. We show the contradiction of the original model in explaining the increase of the capital/income ratio with the change in the functional distribution of income. Moreover, we highlight the inconsistency between the definition of capital and the model proposed. The paper concludes by outlining alternative approaches to the problem, calling for a major rethinking about the causes of rising wealth inequality.

**Keywords:** Piketty; economic inequality; fundamental laws of capitalism; functional distribution of income; economic growth

**JEL codes:** D31, E25, P10.

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# 1 Introduction

Since the first industrial revolution, income distribution has been at the centre of attention in economic studies. Classical economists, including Thomas Malthus (1798), David Ricardo (1817), and Karl Marx (1867), analysed and interpreted the economic changes over time by focusing on the problem of distribution. Their line of research was interrupted in the middle of the 20th century when the distribution of income became a minor issue in economics. In that period, "it came to be believed that a rising tide lifts all boats" (Stiglitz, 2015): the trickle-down hypothesis along with its theoretical justification, the marginal productivity theory, explained how regressive economic policies would bring increasing wealth to all sections of the population with a remuneration equal to the individual's contributions to society.

With the publication of Thomas Piketty's book *Capital in the Twenty-First Century* (2014), the subject attracted considerable attention in academic circles. The book tells the story of capital and labour income distribution in developed countries from the end of the 18th century to recent years, a study made possible by 15 years of research and data collection on more than 20 countries with the collaboration of over 30 researchers.<sup>1</sup> Analysing the data from the World Top Incomes Database, the largest database available today on the study of inequalities, Piketty shows how economic inequality in the developed countries since the 1980s is returning to the same level of the Belle Époque.<sup>2</sup> In the US the bottom 50% income share has collapsed from about 20% in 1980 to 12% in 2014. In the meantime, the income share of the top 1% increased from about 12% in the early 1980s to 20% in 2014 (Piketty et al., 2018). In Europe, the top decile income has been rising since the 1980s, and it is now close to 35% of total income (Piketty and Saez, 2014). Overall, the Gini coefficient, the most widely used income distribution index, has substantially increased in the last 30 years. Across the OECD, the coefficient has increased in 17 of 22 countries, by roughly 22% in Germany, 13% in Canada, 13% in the UK and 8% in Italy (OECD, 2011). These trends were interrupted in the first half of the twentieth century not by the structural dynamics forces announced by Simon Kuznets (1953), but by many other factors.<sup>3</sup> Among these we can mention the physical destruction of capital during the World Wars, the collapse of investments with the consequent fall of asset prices during the great depression and finally the implementation of economic and fiscal policies from the 1950s to the 1970s, aimed at reducing inequality and strengthening the welfare state.

That said, even if the core of Piketty's book has an empirical and historical nature, he goes beyond this approach advancing a theory of the long-term tendencies of capital inequality in developed countries and rooting his work deeply in economic theory (Piketty, 2014b).<sup>4</sup> In this context, the present paper develops and examines from a theoretical point of view the central macroeconomic claims made by Piketty.<sup>5</sup> We can divide the model into two parts. The first part is a supply-side long-run growth model à la Solow analysing the effect of a steady state increase of capital/income ratio on the functional distribution of income (Piketty and Zucman, 2014). In this part, we examine the two "fundamental laws of capitalism" to understand the dynamics that drive the accumulation of capital: by combining the laws, it is possible

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<sup>1</sup>The available database on the historical evolution of the world distribution of income and wealth can be found on-line at <https://wid.world/>.

<sup>2</sup>The French expression Belle Époque refers to the economic prosperity which characterised Europe from the end of the Franco-Prussian War (1871) to the start of World War I (1914).

<sup>3</sup>According to Kuznet's theory, "the inverted-U curve", during the capitalist development, income inequality automatically experience an increasing trend followed by a downturn process.

<sup>4</sup>Following the Piketty's model, capital and wealth are used synonymously.

<sup>5</sup>In this paper, we deal only with the theoretical aspect of Piketty's argument. We do not discuss Piketty's policy and economic recommendations discussed in the second part of the book. With respect to these, see for example Auerbach and Hassett (2015).

to study how the capital share varies on the functional distribution of income. In the second part, the increasing concentration of capital is explained in the context of real capital accumulation through saving (Piketty and Saez, 2014; Piketty and Zucman, 2015). More precisely, in a dynamics model of capital accumulation with random multiplicative shocks on saving, the tendency of the return on capital to exceed the growth rate of the economy is considered the real driving force towards the concentration of capital in the top 1% and 0.1% of wealth distribution.

Together with the development of Piketty's model, the paper aims to contribute to the literature pointing out two flaws: firstly, with respect to the first part of the model, we show the contradiction of using the second fundamental law of capitalism in a steady state condition to explain both the rising of capital/income ratio and the increasing capital share on the functional distribution of income. Secondly, we highlight the inconsistency of the definition of capital, the explanatory variable in the model, with the theoretical apparatus proposed by Piketty.

Before proceeding, we make two preliminary remarks. First, the history of economic inequality does not follow deterministic laws guided by purely economic mechanisms. It is a complex history that evolves driven not only by economic factors but also, and not secondarily, by political, social and historical forces. Institutional and political factors have always played a central role in the determination of the functional distribution of income. Hence, institutions, through economic and fiscal policies, will continue to play an essential role in reducing the adverse economic and social consequences of high and rising inequality. At the same time, we need a methodical approach for the study of the distribution of wealth, and the scientific approach can help us to understand which economic mechanisms are actually involved. We are entirely convinced, along with Piketty, that without a systematic analysis we will not be in a position to identify the real mechanisms underlying the increase in inequality, thereby running the risk of ideological thinking having the better of rational thought. Following this line of thought, we outline and analyse the theoretical model proposed by Piketty.

Secondly, *Capital* was widely discussed by economists and in the general press (see e.g. King, 2016). At the same time, we believe that the theoretical apparatus on which Piketty relies has been misunderstood in recent reviews and is often not well captured in the discussion that has surrounded the book. More specifically, we want to stress how the model must be divided into two parts, which, while to some extent overlapping, are quite distinct.<sup>6</sup> Moreover, we underline how the difference between the return on capital and the growth rate of the economy can act as an increasing concentration of capital only in a model of real capital accumulation and not as a simple inequality.<sup>7</sup> For this reason, we believe that developing and analysing Piketty's theoretical apparatus can serve to frame the model in the literature and, at the same time, to pinpoint the reasons for increasing wealth inequality from Piketty's perspective.

The remainder of the paper is organised as follows. Section 2 provides a review of the literature on Piketty. Section 3 introduces the "fundamental laws of capitalism" and the economic consequences of their union on the functional distribution of income. Section 4 analyses the effect in terms of wealth concentration of a rate of return on capital greater than the growth rate of the economy. Section 5 discusses the problem of considering capital in its two main components: the real and the financial. We

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<sup>6</sup>When referring to Piketty, Yew-Kwang (2015) commits an error in explaining the increasing share of capital in total income as a consequence of a higher rate of return to capital than the rate of growth of the economy. The two concepts are separate. In the same way, Patnaik (2014) sees movements in the wealth-income ratio and wealth inequalities both determined by the excess of the rate of return on capital over the rate of growth of the economy. However, the difference  $r > g$ , which enters in the second part of the model, only affects wealth inequality and not the increasing capital/income ratio, which in turn is explained in the first part of the model.

<sup>7</sup>It seems that this point was not captured for example by Mankiw (2015), Ray (2015) and Homburg (2015).

conclude by proposing alternative approaches to the problem.

## 2 A review of the literature on Piketty

Thomas Piketty's *Capital in the Twenty-First Century* (2014a) has redirected economists, especially of the mainstream theory, to an area of work that had been neglected in the years before his work (Krugman, 2014a). The major early reviews have in common the support for the examination of tax records and the collection of relatively vast historical data which document long-standing trends of wealth inequality and capital/income ratio in the most advanced economies (Milanovic, 2014; Ray, 2015). At the same time, the book generated an extensive debate in both mainstream and heterodox economics as well as outside the field of economics (Tribe, 2015) with many and various favourable (Krugman, 2014b; Milanovic, 2014; Solow, 2014) or critical reviews (Galbraith, 2014a, 2014b; Rowthorn, 2014; Aspromourgos, 2015). We will concentrate on the latter. Given the extensive literature on the subject, these critiques will be grouped in four different areas.

The first deals with the argument that inequality in the distribution of wealth does not matter. Along with the tradition of trickle down economy and the Pareto principle in welfare economics, wealth inequality does not matter if the investment decisions of the upper class result in higher standards of living for all, including the lowest class (Mankiw, 2015; Jones, 2014; Holcombe, 2014; Gissurarson, 2014). Along with this idea, Auerbach and Hassett (2015) point out that Piketty's policy proposal for a progressive global wealth tax is considered politically unrealistic and inefficient because of its impact on the incentive to save, thereby reducing capital accumulation with adverse effects on both the rich and poor classes. In the same vein, Calleo (2014) highlights the political consequences of Piketty's proposals, which would affect the middle class much more than the top 0.1% or 1%.

The second area of critiques complains that Piketty has ignored the fundamental role played by political institutions and historical events in the determination of wealth inequality (Kuttner, 2014; Deroncourt, 2017; Acemoglu and Robinson, 2015a). For example, Acemoglu and Robinson (2015a) critique Piketty's approach to capitalism and his general laws for the absence of institutions and political factors along with their endogenous evolution in the formation of inequality. In their vision, inequality dynamics is determined by a broad set of political factors that are largely excluded from the model.<sup>8</sup> Similarly, Patnaik (2014), Fainstein (2014) and Mongiovi (2015) have questioned the absence of class conflict in explaining the origins and the evolution of the high degree of wealth inequality.

Thirdly, the book has been criticised for ignoring the role of the distribution of wealth in developing countries in an explanation of the increasing wealth inequality in the developed ones (Milanovic, 2014; Ndhlovu, 2015; Lakner, 2017). While economic inequality has been narrowing at the global level and especially in the emerging- market, inequality has been rising within developed countries. For that reason, a comparison between the wealth dynamics of developed and developing countries may represent an important extension of the analysis.

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<sup>8</sup>On this point, we disagree with Acemoglu and Robinson (2015a). It must be noted that Piketty asserted that the fundamental economic laws are not the only factor, or even the primary one, driving inequality in wealth; exogenous institutional changes and political shocks have also been extremely important. Piketty (2014a) states that "[the history of the distribution of wealth] is shaped by the way economic, social, and political actors view what is just and what is not, as well as by the relative power of those actors and the collective choices that result. It is the joint product of all relevant actors combined [...] How this history plays out depends on how societies view inequalities and what kinds of policies and institutions they adopt to measure and transform them." In our opinion, it is essential to distinguish the theoretical model from the overall vision of the book. In this regard, see for example the end of Section 4.

Fourthly, there are critiques strictly related to the theoretical apparatus on which Piketty relies. We can identify three distinct lines of criticism. The first deals with the "second fundamental law of capitalism", that is the equivalence of capital/income ratio to the saving/growth ratio in steady state. By referring to a fall in the rate of growth with constant saving, the law explains the increasing share of capital. The condition has been the object of various critiques. Ray (2015) observes how the growth rate cannot be used as an explanation since growth is an outcome, rather than a prime cause. Even using the growth rate as an explanatory variable, in its change, it would not be independent of the saving rate (Homburg, 2015). In this sense, the saving rate does not appear to be entirely independent of the growth rate but it seems to be positively correlated with it (Krusell and Smith Jr., 2015). Therefore, the low economic growth would not be reflected in an increase in the capital/income ratio but, with a contemporaneous decrease in saving rate, we can obtain different predictions about capital/income movement.

The second line of critique concerns the "great contradiction of capitalism", namely the tendency of the rate of return on capital to exceed the rate of economic growth. These critiques undermine the relationship between the difference  $r - g > 0$  and the increasing concentration of capital (Cowell, 2014; Lopez-Bernardo et al., 2016; Galbraith, 2014b). First of all, for Ray (2015) and Homburg (2015), the real source of domination is not  $r > g$  but the assumption that capital owners save all their income in order to accumulate capital. If this hypothesis were relaxed, the growth rate of capital would become independent of the rate of return, changing over time in a higher, lesser, or in the same way as the economic output. At the same time, for Acemoglu and Robinson (2015a), the difference cannot be taken as a primitive condition to make future forecasts, as both the rate of return on capital and the growth rate will adjust to changes in the political and technological context. Moreover, this divergence would not imply the imagined concentration of capital but, considering important additional factors in the analysis, the final results would change significantly; among these, we can list consumption, taxation, generational change and the high degree of social mobility (Mankiw, 2015; Acemoglu and Robinson, 2015b). Considering these forces, the rate of return on capital should exceed the global growth rate of the economy by at least seven percentage points to generate an increasing concentration of wealth; we are far from this scenario. What is more, if  $r > g$  is not considered a problem,  $r < g$  could be. When the rate of growth is higher than the rate of return on capital, the economy has accumulated an excessive amount of capital, generating an inefficient dynamics situation.

Finally, the third line of critique comes from the interchangeable use of capital and wealth where all the forms of property are included without any distinction between them. This generates conceptual and empirical problems. Firstly, differently from capital, wealth is primarily housing and land, so it is not employed directly in the production process (Solow, 2014; Patnaik 2014; Rowthorn, 2014; Aspromourgos, 2015). Secondly, a more precise definition should consider the difference between reproducible and non-reproducible assets and their effects on the capital/income ratio. In fact, once housing prices and land values are removed from the definition of capital, the phenomenon of capital accumulation goes away from the empirical evidence (Bonnet et al., 2014; Homburg, 2015).

In conclusion, we can say that most of these critiques have directed their attention to the conclusions of the model, rather than how these conclusions are reached; with our work we take one step forward. The present study contributes to the debate by highlighting two new major structural theoretical shortcomings of Piketty's model.

### 3 The fundamental laws of capitalism

The first part of Piketty’s model explains the contemporaneous increase of capital stock and capital share in developed countries. The most useful way to measure capital is to divide that stock by the annual flow of income. For example, if a country’s total capital stock is the equivalent of five years of national income, the capital/income ratio is equal to 500%.

Let us start by examining the ongoing trends of the capital/income ratio in Europe and the US. As we can see from Figures 1 and 2, after a downturn during the two World Wars until the 1970/1980s, the capital/income ratio has started to increase. In the US, the value of the ratio almost reached those of the late 19th century with a value equal to 400% today. In European countries, the capital/income ratio generally varies between 400% and 600%. In Germany, France and the UK, the total capital stock is about the equivalent of 4, 5 and 6 years of national income respectively.<sup>9</sup>

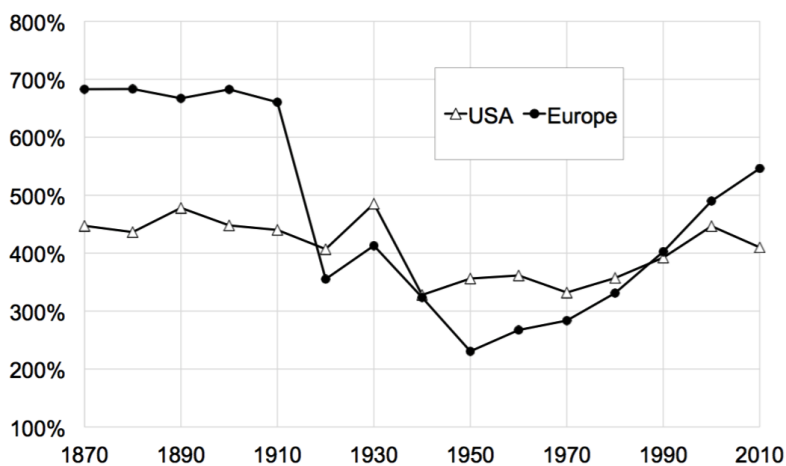


Figure 1: Capital/income ratios in Europe and in the US, 1870-2010. Piketty and Zucman (2013), available at <http://piketty.pse.ens.fr/en/capitalisback>.

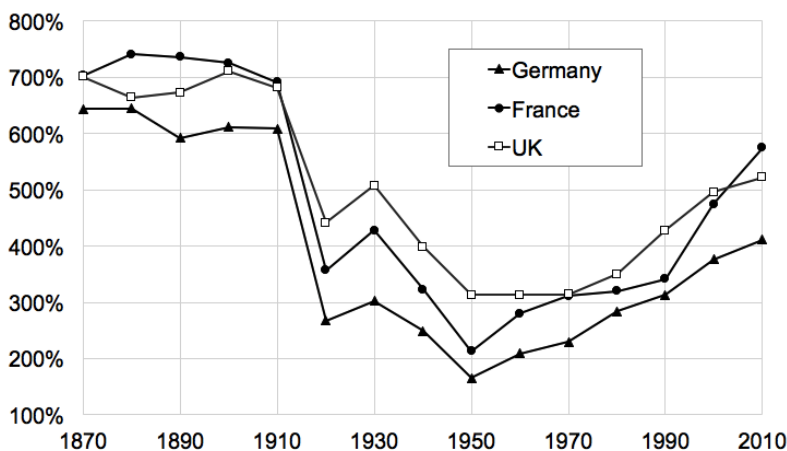


Figure 2: Capital/income ratios in Germany, France and the UK, 1870-2010. Piketty and Zucman (2013), available at <http://piketty.pse.ens.fr/en/capitalisback>.

<sup>9</sup>Supplementary data to this article can be found on-line at <http://piketty.pse.ens.fr/en/capital21c2>.

Together with this scenario, we can observe from Figure 3 an increase of capital share in the functional distribution of income between 1975 and 2010. The capital share increased in most rich countries absorbing between 15% and 25% of national income in 1975, and between 25% and 30% in 2000–2010.

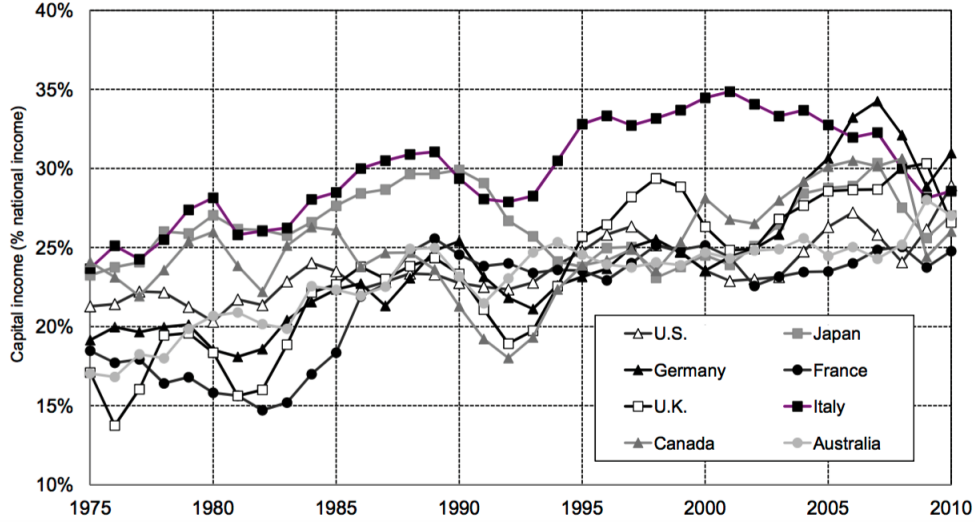


Figure 3: Capital share in the functional distribution of income, 1975–2010. Piketty and Zucman (2013), available at <http://piketty.pse.ens.fr/en/capitalisback>.

Let us see how this economic context is presented in the first part of Piketty's model.

Let  $K$  be the capital stock and  $Y$  the national income. If we indicate with  $r$  the average annual rate of return on capital, the share of capital income  $\alpha$  is defined as

$$\alpha = \frac{rK}{Y}$$

If we denote with  $\beta$  the capital/income ratio, we get what Piketty calls the "first fundamental law of capitalism"

$$\alpha = r\beta \tag{1}$$

This law is just a definition and links the three most important variables for the analysis of a capitalist economy: the capital share in the functional distribution of income, the rate of return on capital and the capital/income ratio (Piketty, 2014a); the law defines the distribution of income in terms of shares going to capital and labour. To understand the forces that in the long run push up the capital/income ratio, the "second fundamental law of capital" has to be introduced (Piketty, 2014b).

Piketty considers an exogenous growth model with homogeneous capital and consumer goods and where capital gains are excluded. Wealth comes from a process of accumulation through saving  $S$

$$K_{t+1} = K_t + S_t \quad S_t = s_t Y_t \tag{2}$$

where  $Y_t$  is the national income at time  $t$  and  $s_t$  is the saving rate net of depreciation of capital at time  $t$ . National income  $Y$  grows at the annual global exogenous growth rate  $g$



$$Y_{t+1} = (1 + g_t) Y_t \quad (3)$$

Dividing each term of Eq. (2) by Eq. (3), we get

$$\frac{K_{t+1}}{Y_{t+1}} = \frac{K_t}{Y_t} \frac{1}{(1 + g_t)} + \frac{S_t}{Y_t} \frac{1}{(1 + g_t)} \quad (4)$$

Knowing that  $s_t = S_t/Y_t$  and  $\beta_t = K_t/Y_t$ , from Eq. (4) we obtain

$$\beta_{t+1} = \beta_t \left( 1 + \frac{s_t}{\beta_t} \right) / (1 + g_t) \quad (5)$$

Piketty supposes that in the long run the growth rate and the saving rate stabilise at certain steady state level, respectively  $g_t = g$  and  $s_t = s$ . Solving Eq. (5) with respect to  $\beta$  in stationary level,  $\beta_{t+1} = \beta_t = \beta$

$$\beta = \beta \left( 1 + \frac{s}{\beta} \right) / (1 + g) \quad (6)$$

Piketty obtains the "second fundamental law of capitalism"

$$\beta = \frac{s}{g} \quad (7)$$

This formula links in the long-run the capital-income ratio  $\beta$  with the saving rate  $s$  and the economy's growth rate  $g$ , allowing us to understand the evolution of capital in advanced societies. In fact, Piketty explains the increase of capital/income ratio in a steady state context of a constant rate of saving with a decreasing rate of growth. As we can observe in Figure 4, the growth rate of per capita output has been declining both in Western Europe and North America since the second half of the 20th century.

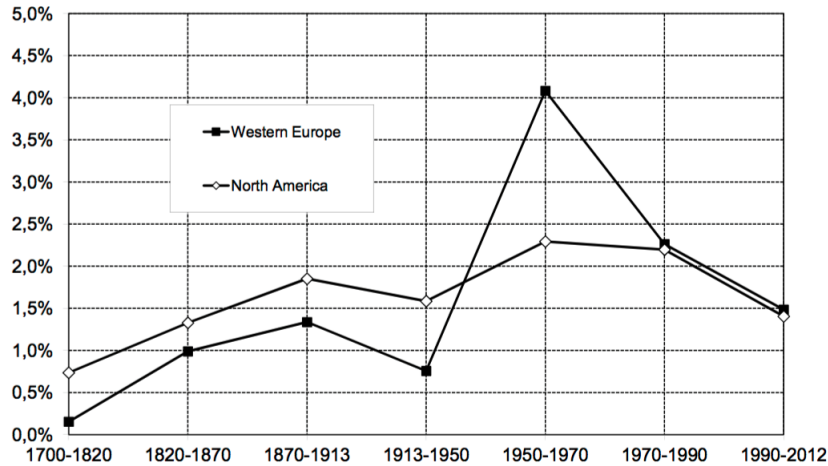


Figure 4: The growth rate of per capita output since the industrial revolution in Western Europe and North America. Available at <http://piketty.pse.ens.fr/en/capital21c2>

Proceeding with the theoretical model, the increasing capital/income ratio in steady-state value, Eq. (7), is inserted in the first fundamental law of capitalism, Eq. (1), to formalise how  $\alpha$ , the capital share, changes over time

$$\alpha = r \left( \frac{s}{g} \right) \quad (8)$$

From equation (8) we see that in steady state the variation of  $\alpha$  depends on the relative variation of  $r$  with respect to  $\beta$ . According to the neoclassical theory, to which Piketty's model belongs, in a context of perfect competition, the return on capital is determined by its marginal productivity. As the capital stock increases, the marginal productivity of the production factor tends to decrease. Piketty tries to understand how much the return on capital decreases when the corresponding capital/income ratio increases. This relies on the level of technology used to combine labour and capital in the production process of final goods. In a theoretical context, considering a two-factor production function with capital and labour, this link depends on the elasticity of substitution between labour and capital ( $\varepsilon_{K,L}$ ). Drawing on historical data, Piketty suggests that the elasticity of substitution between capital and labour appears to be greater than one, quantifying a value between 1.3 and 1.6 (Piketty, 2014a, p. 221). With an elasticity of substitution greater than one, the increase in capital/income ratio is compensated by a less than proportional decrease in  $r$ , thus resulting in a simultaneous increase in capital share  $\alpha$ .<sup>10</sup> Several empirical studies, however, indicate both a short-run and a long-run elasticity of substitution significantly less than one (Rowthorn, 2014; Semieniuk, 2017; Zamparelli, 2016).

### 3.1 Observation on the fundamental laws of capitalism

At this time of the analysis, we challenge the first part of the theoretical model proposed by Piketty. We highlight the contradiction of using the second fundamental law of capitalism to explain the increasing capital/income ratio and the simultaneous increase of capital share. From Eq. (5), the dynamics of capital/income ratio can be rewritten in the following way

$$\beta_{t+1} = \frac{\beta_t}{1 + g_t} + \frac{s_t}{1 + g_t}$$

from which

$$1 + g_t = \frac{\beta_t}{\beta_{t+1}} + \frac{s_t}{\beta_{t+1}}$$

i.e.

$$g_t = \frac{s_t}{\beta_{t+1}} - \frac{\Delta\beta_t}{\beta_{t+1}} \quad \text{where} \quad \Delta\beta_t = \beta_{t+1} - \beta_t \quad (9)$$

So it is possible to obtain the second fundamental law of capitalism presented in Eq. (7) when

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<sup>10</sup>In Piketty's conventional neoclassical framework, the standard Cobb–Douglas production function, whose elasticity is precisely equal to one, cannot be used. It predicts complete stability between labour and capital with capital share independent of the capital/income ratio. For a better description of the phenomenon, it is necessary to introduce the CES (Constant Elasticity of Substitution) production function with an elasticity greater than one. With a low elasticity of substitution, neoclassical theory predicts a falling, not an increasing, capital share.

$$g_t = \frac{s_t}{\beta_t} \quad \text{or} \quad \Delta\beta_t = 0 \quad \forall t \quad (10)$$

These conditions consist in the invariance of the capital/income ratio or, equivalently, in the growth rate at the level of "warranted rate of growth" as in the Harrod-Domar-Solow tradition (Solow, 1956). In this sense, the second law does not necessarily require the steady state level, although it does not exclude it: the steady state is a particular case of the second law. As long as  $\beta$  remains constant,  $g_t$  and  $s_t$  may vary over time compatible with (10).

However, with an increase in capital/income ratio,  $\Delta\beta_t > 0$ , Eq. (10) and the second fundamental law of capitalism cannot be satisfied, because necessarily  $g_t < (s_t/\beta_t)$ , and therefore the second law cannot explain anything. In fact, the capital/income ratio increases or decreases if  $g_t < (s_t/\beta_t)$  or  $g_t > (s_t/\beta_t)$  respectively. Moreover, Eq. (8) is valid only in the case of constant  $\beta$ , so on the condition that the second law holds. If the original model seeks to explain the contemporaneous increase of capital share with the capital/income ratio, the second fundamental law of capitalism cannot be used.

The increase of  $\beta$  should be explained from equation Eq. (5), with  $s$  constant and a simultaneous decrease of  $g$  in a dynamic process. Equally, the rise of the capital share should be illustrated with Eq. (1) and not inserting Eq. (7), which cannot be used, in Eq. (1).

In other words, Piketty explains that the second fundamental law applies only over the long run and only if the saving rate and the growth rate remain constant in order for an economy to reach in the long term a state of equilibrium. He uses a steady state condition to explain a dynamic process, analysing the dynamics as a parametric variation inside the steady state, and not as a movement from or toward the steady state. In our opinion, one reason why Piketty is considering a steady state condition is that he is considering a neoclassical supply-side growth model à la Solow, where the equilibrium is unique and stable both locally and globally. However, the steady state law used to explain the observed increase in capital/income ratio with the change of functional distribution is in contradiction with itself from Eq. (10), so that  $\beta = s/g$  cannot be used. The increasing process should be explained without Eq. (7) but with a dynamic analysis outside the steady state.

This section concludes the discussion of the first part of Piketty's model. We now pass to the second part, introducing what Piketty calls "the central contradiction of capitalism".

## 4 The $r > g$ inequality and the increasing concentration of wealth

Now we enter the second part of Piketty's model. Since the 1980s, together with a rising trend of capital/income ratio and capital share, from Figures 5, 6 and 7 we can observe an increasing concentration of wealth for the top decile (10%) and the top percentile (1%) in the distribution of wealth.

In Great Britain, between 1980 and 2010, the richest 1% saw their share of wealth pass from 22,7% to 28% and the richest 10% from 62% to 70,5%. Nowadays in Europe, the wealthiest 1% hold 24,4% of the continent's wealth, while top 10% nearly 64%. In the US the situation is more extreme: the top 1% of Americans hold 33,8% of the country's wealth and top 10% nearly 72% with an upward trend over the same 30 years.

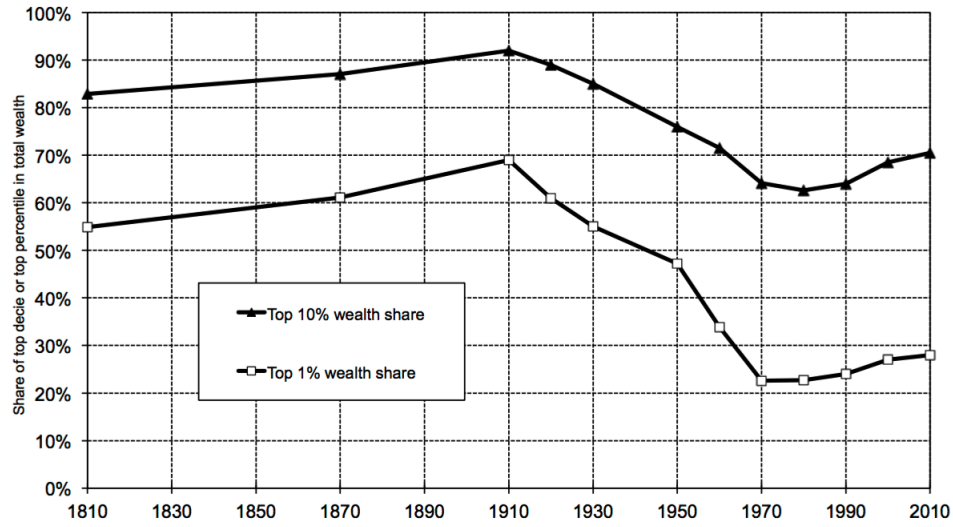


Figure 5: Wealth inequality in Britain, 1810-2010. Available at <http://piketty.pse.ens.fr/en/capital21c2>

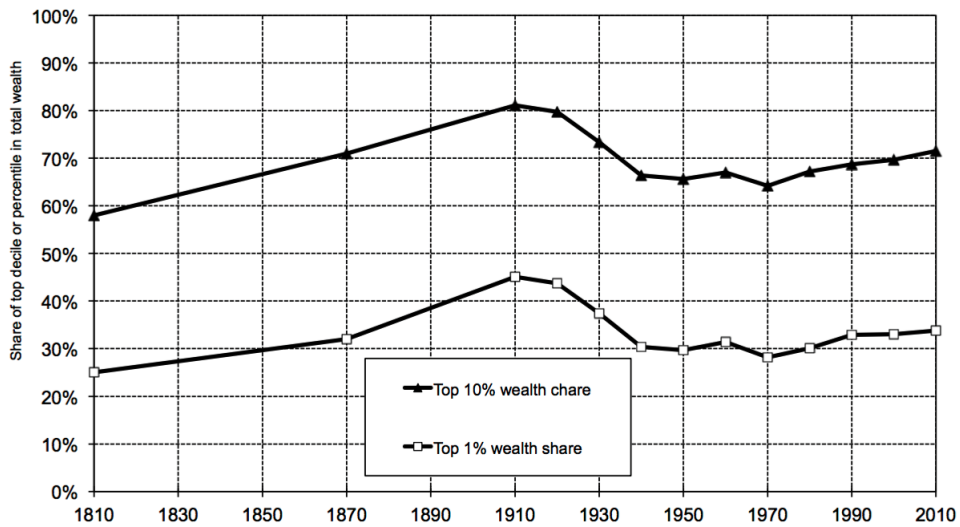


Figure 6: Wealth inequality in the US, 1810-2010. Available at <http://piketty.pse.ens.fr/en/capital21c2>

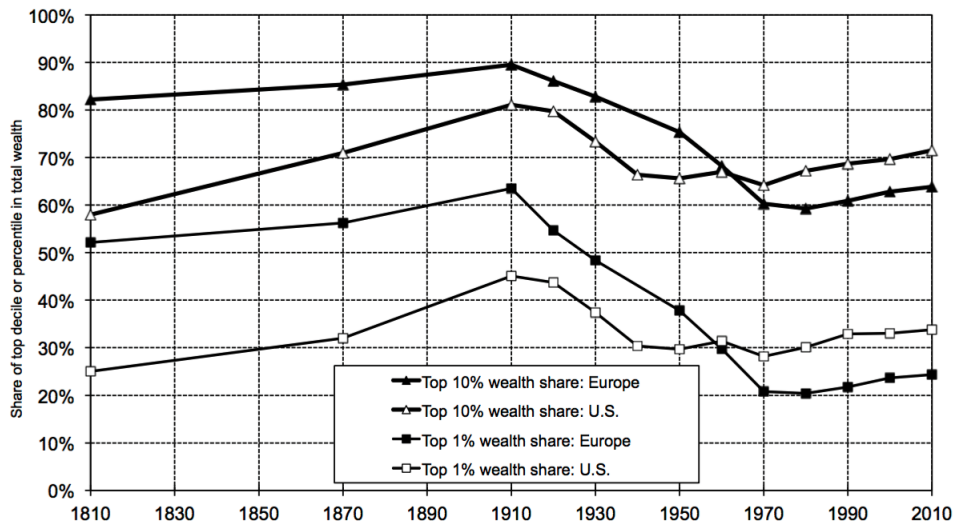


Figure 7: Wealth inequality in Europe and the US, 1810-2010. Available at <http://piketty.pse.ens.fr/en/capital21c2>

To explain the increasing wealth concentration, Piketty uses what he calls "the central contradiction of capitalism", namely the inequality  $r > g$ , where again  $r$  is the annual rate of return on capital and  $g$  is the annual growth rate of the economy. As can be seen in Figure 8, the rate of return to capital (pre-tax) has always been higher than the world growth rate, with a reduction of the gap during the 20th century. Figure 9 shows that the rate of return to capital (after tax and capital losses) fell below the growth rate during the first part of the 20th century and remained below until the 1980s. In general, in our modern economies,  $g$  is likely to decrease, while  $r$ , due to continuing globalisation and financial deregulation, might increase again in the 21st century.

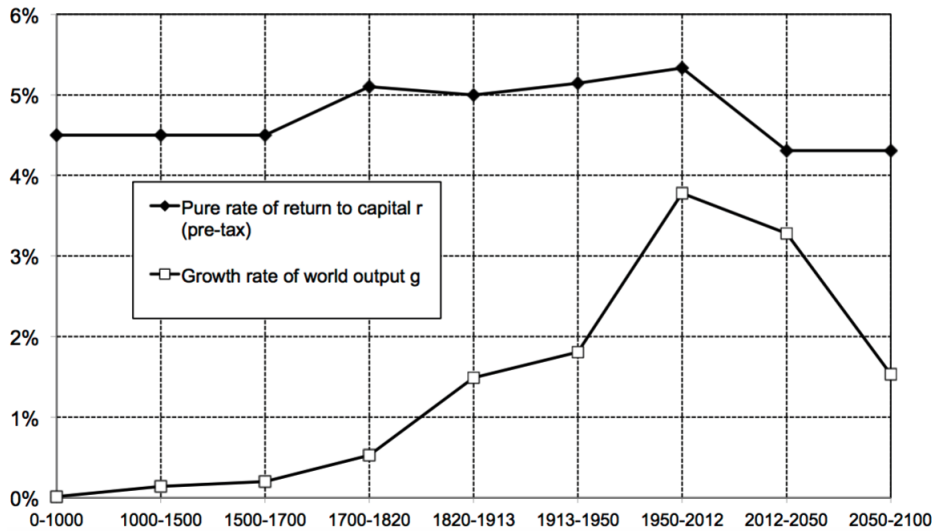


Figure 8: Pure rate of return on capital and growth rate at the world level, from antiquity until 2100. Available at <http://piketty.pse.ens.fr/en/capital21c2>

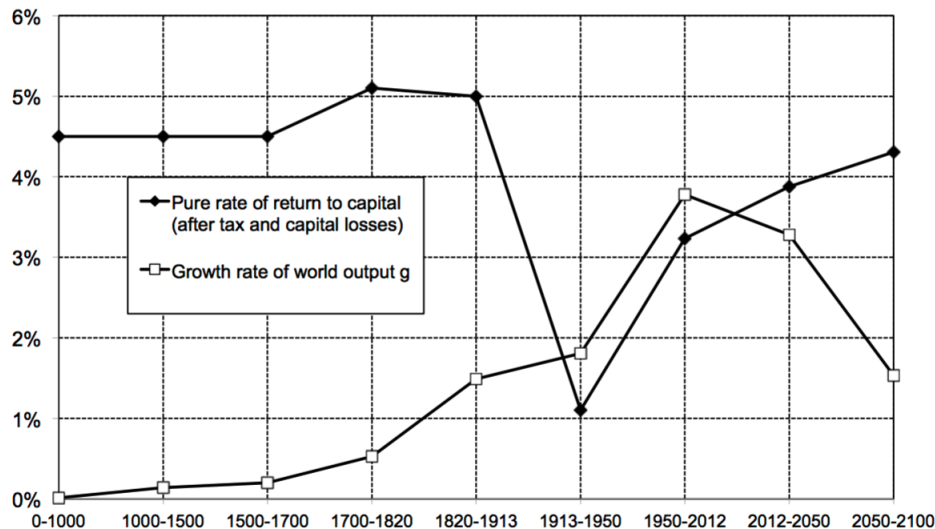


Figure 9: After tax pure rate of return on capital and growth rate at the world level, from antiquity until 2100. Available at <http://piketty.pse.ens.fr/en/capital21c2>

When  $r > g$ , a capitalist economy leads automatically to an increase in wealth concentration in the top decile of the distribution. In Piketty's words "when the rate of return on capital exceeds the rate of growth of output and income, as it did in the nineteenth century and seems quite likely to do again in the twenty-first, capitalism automatically generates arbitrary and unsustainable inequalities that radically undermine the meritocratic values on which democratic societies are based" (Piketty, 2014a).

Three main forces could prevent this scenario. The first is direct institutional action implemented to regulate the incidence of capital with social and fiscal policies, as happened during the three decades after the Second World War. This solution appears to have failed in recent years, characterised by substantial financial deregulation and intensive tax competition to attract capital (Piketty, 2015a).

The second is an economic force, represented by the decline of return on capital. However, this force could take a long time to manifest itself and, as already mentioned in Section 3, it can have limited results (Piketty and Zucman, 2014).

The third force is represented by a series of demographic and economic shocks that may prevent the process of capital accumulation and concentration. However, it is precisely in this context that the central contradiction acts. Only in a context of capital accumulation through saving the increase of the divergence between the rate of return and the rate of growth leads to an increase in the concentration of capital. More precisely, for a given economic shock during the process of capital accumulation, the distribution of capital converges, for the highest wealth values, towards a distribution whose upper tail follows the Pareto distribution. The Pareto coefficient, which measures the degree of concentration of capital, is directly determined by the difference between  $r$  and  $g$ : the larger the divergence between  $r$  and  $g$ , the higher the wealth concentration in the top decile and percentile of the distribution (Piketty, 2015b).

This point is very often misinterpreted in literature when referring to the original model. Hence the need to show how the relative distribution of capital for the highest wealth values tends toward a Pareto distribution and the difference  $r - g$  directly determines the Pareto coefficient. The convergence can be shown with a model of dynamic capital accumulation with binomial random multiplicative shocks on saving rate (Piketty and Zucman, 2015).

## 4.1 The dynamics model of capital accumulation

Consider a discrete time model  $t = 0, 1, 2, 3, \dots \in \mathbb{N}$  where each period has a duration of one year. With a unitary stationary population composed of a continuum of agents  $N_t = [0, 1]$ , there is no difference in considering the aggregate level or the average level of wealth  $W_t = w_t$  and national income  $Y_t = y_t$ . Suppose that the effective labour input in efficiency units  $L_t$  grows at an exogenous annual growth rate  $g$ . Domestic output is given by the production function  $Y_{dt} = F(K_t, L_t)$ , where  $K_t$  is the capital.

All individuals  $i \in [0, 1]$  have the same annual rate of return on capital  $r_{ti} = r_t$  and receive the same income from work  $y_{lti} = y_{lt}$ . Each type of economic agent chooses  $c_{ti}$  and  $w_{t+1i}$  so to maximise his utility function

$$V(c_{ti}, w_{ti}) = c_{ti}^{1-s_{ti}} w_{ti}^{s_{ti}}$$

under the budget constraint

$$c_{ti} + w_{t+1i} \leq y_{Lt} + (1 + r_t) w_{ti}$$

where  $s_{ti}$  is the saving rate. During the life of individual, random shocks composed of a sequence of independent and identically distributed random variables with mean  $s = E(s_{ti})$  cause idiosyncratic changes on the saving rate  $s_{ti}$ .

In a dynamic optimization problem the possible consumption  $c_{ti}$  is a fraction  $1 - s_{ti}$  of the total resources at the time  $t$ , given by the sum of the wage plus wealth  $y_{Lt} + (1 + r_t) w_{ti}$

$$c_{ti} = (1 - s_{ti}) [y_{Lt} + (1 + r_t) w_{ti}] \quad (11)$$

Substituting Eq. (11) in the budget constraint, we get

$$(1 - s_{ti}) [y_{Lt} + (1 + r_t) w_{ti}] + w_{t+1i} = y_{Lt} + (1 + r_t) w_{ti}$$

from which:

$$w_{t+1i} = s_{ti} [y_{Lt} + (1 + r_t) w_{ti}] \quad (12)$$

At aggregate level the national income is given by the sum of income from work plus income from capital

$$y_t = y_{Lt} + r_t w_t$$

so that, Eq. (12) becomes

$$w_{t+1} = s [y_{Lt} + (1 + r_t) w_t] = s [y_t + w_t] \quad (13)$$

Dividing Eq. (13) by  $y_{t+1} \approx (1 + g)y_t$ <sup>11</sup> we obtain

$$\frac{w_{t+1}}{y_{t+1}} = \frac{s}{y_t(1+g)} [y_{Lt} + (1 + r_t) w_t] = \frac{s}{y_t(1+g)} [y_t + w_t]$$

---

<sup>11</sup>The approximation symbol is used because the generic production function does not ensure the unique dependence of output from the rate of growth.

Now, indicating with  $\beta_t = w_t/y_t$  the capital/income ratio, with  $\alpha_t = r_t \times \beta$  the capital share of income and with  $(1 - \alpha_t) = y_{Lt}/y_t$  the labour share of income, we get

$$\beta_{t+1} = s \left( \frac{1 - \alpha_t}{1 + g} \right) + s \left( \frac{1 + r_t}{1 + g} \right) \beta_t = \frac{s}{1 + g} (1 + \beta_t) \quad (14)$$

with

$$\omega = s \left( \frac{1 + r}{1 + g} \right)$$

In an open economy the rate of return on capital is given,  $r_t = r$ . From Eq. (14) the capital/income ratio converges to a finite limit value if and only if

$$\omega = s \left( \frac{1 + r}{1 + g} \right) < 1$$

If  $\omega$  is greater than 1, the capital/income ratio would go to infinity. However in the long run, as the capital stock grows, the rate of return tends to fall, leading  $\omega$  to be less than one and the capital/income ratio to converge to a finite value.

In a closed economy, the rate of return on capital is equal to the marginal productivity of capital and is negatively correlated with the capital stock. As  $\beta$  increases,  $\omega$  comes to be less than one and  $\beta$  converges to a finite value.

Putting  $\beta_{t+1} = \beta_t = \beta$  and substituting in Eq. (14) we obtain the capital/income ratio in steady state level

$$\beta = \frac{s}{1 + g - s}$$

From now on, the normalised relative wealth is indicated by  $z_{ti} = w_{ti}/w_t$ . We divide both sides of Eq. (12) by  $w_{t+1} \approx (1 + g) w_t$  to obtain the individual-level transition equation for wealth

$$\frac{w_{t+1i}}{w_{t+1}} = \frac{s_{ti} [y_{Lt} + (1 + r_t) w_{ti}]}{(1 + g) w_t}$$

Multiplying and dividing by  $s$  and taking into account Eq. (14), we obtain

$$z_{t+1i} = \frac{s_{ti}}{s} \left[ \frac{y_{Lt}}{(1 + g) w_t} s + \omega z_{ti} \right]$$

In the long run, with  $y_{Lt} = (1 - \alpha) y_t$  where  $\alpha = r\beta = rs/(1 + g - s)$ , we get

$$z_{t+1i} = \frac{s_{ti}}{s} \left[ \frac{(1 - \alpha) y_t}{(1 + g) w_t} s + \omega z_{ti} \right]$$

from which

$$z_{t+1i} = \frac{s_{ti}}{s} [(1 - \omega) + \omega z_{ti}] \quad (15)$$

At this point, it is possible to show how the distribution of relative wealth  $\psi_t(z)$  converges to a distribution with a Pareto shape. With the presence of random shocks on saving, the relative wealth



distribution function for the highest value of  $z$ , i. e.

$$1 - \psi_t(z) = \text{prob}(z_{ti} \geq z)$$

converges to a Pareto distribution  $1 - \phi(z)$  and the Pareto coefficient depends on the  $\omega$  and therefore on  $r$  and  $g$ .

The long-run distribution is modelled with the assumption that a series of binomial multiplicative random shocks occurs to the saving parameter:  $s_{ti} = s_0 = 0$  with probability  $(1 - p)$  and  $s_{ti} = s_1 > 0$  with probability  $p$  (with  $s = ps_1$  and  $\omega < 1 < \omega/p$ ).

At time  $t = 0$  with probability  $(1 - p)$ ,  $s_{ti} = s_0 = 0$  and replacing it in Eq. (15) we get  $z_1 = 0$ . At time  $t = 1$  with probability  $(1 - p)p$ ,  $s_{ti} = s_1$ ,  $s = ps_1$  and knowing that  $z_1 = 0$ , we substitute this information in Eq. (15) obtaining

$$z_2 = \frac{1}{p}(1 - \omega) \quad (16)$$

At time  $t = 3$  with probability  $(1 - p)pp = (1 - p)p^2$ ,  $s_{ti} = s_1$ ,  $s = ps_1$  and knowing that  $z_2 = \frac{1}{p}(1 - \omega)$  with the same procedure we obtain

$$z_3 = \frac{(1 - \omega)}{p} \left(1 + \frac{\omega}{p}\right) \quad (17)$$

Multiplying the right side by  $\omega - p/\omega - p$  we get

$$z_3 = \frac{1 - \omega}{\omega - p} \left[ \left(\frac{\omega}{p}\right)^2 - 1 \right] \quad (18)$$

At  $t = 3$  with probability  $(1 - p)ppp = (1 - p)p^3$ ,  $s_{ti} = s_1$ ,  $s = ps_1$  and from Eq. (17), with the same procedure we substitute this information in Eq. (15) obtaining

$$z_4 = \frac{1}{p} \left\{ (1 - \omega) + \omega \left[ \frac{(1 - \omega)}{p} \left(1 + \frac{\omega}{p}\right) \right] \right\}$$

i.e.:

$$z_4 = \frac{1 - \omega}{p} \left( \frac{p^2 + \omega p + \omega^2}{p^2} \right)$$

Multiplying the right side by  $\omega - p/\omega - p$  we get

$$z_4 = \frac{1 - \omega}{\omega - p} \left[ \frac{(p^2 + \omega p + \omega^2)(\omega - p)}{p^3} \right]$$

from which

$$z_4 = \frac{1 - \omega}{\omega - p} \left[ \left(\frac{\omega}{p}\right)^3 - 1 \right] \quad (19)$$

Comparing Eq.(18) with Eq. (19), iterating the process and with probability  $(1 - p)p^k$ , we get

$$z_{k+1} = \frac{1 - \omega}{\omega - p} \left[ \left( \frac{\omega}{p} \right)^k - 1 \right]$$

For  $k \rightarrow \infty$ , we get

$$z = \frac{1 - \omega}{\omega - p} \left( \frac{\omega}{p} \right)^k \quad (20)$$

The cumulative distribution is given by  $1 - \phi(z) = \text{prob}(z \geq z_k)$  and to obtain it we have to sum up the probabilistic values for  $z \geq z_k$

$$1 - \phi(z) = \text{prob}(z \geq z_k) = \sum_{j \geq k} (1 - p)p^j$$

We obtain

$$1 - \phi(z) = (1 - p)p^k (1 + p + p^2 + p^3 + \dots)$$

from which

$$1 - \phi(z) = p^k (1 + p + p^2 + p^3 + \dots - p - p^2 - p^3 \dots) = p^k$$

Hence

$$1 - \phi(z) = \text{prob}(z \geq z_k) = \sum_{j \geq k} (1 - p)p^j = p^k \quad (21)$$

Now we can observe how the relative distribution of wealth, for the highest value of  $z$ , tends toward a distribution whose upper tail follows the Pareto distribution with a coefficient that is directly determined by  $r - g$ . Let us see this in detail.

From the law of cumulative distribution, Eq. (21), we obtain

$$\log [1 - \phi(z)] = \log p^k \quad (22)$$

We observe that

$$\log p^k = \log \left( \frac{1}{p} \right) k \frac{\log(p/\omega)}{\log(\omega/p)}$$

with

$$\frac{\log(p/\omega)}{\log(\omega/p)} = -1$$

so that

$$\log p^k = \frac{\log(1/p)}{\log(\omega/p)} \left[ -\log \left( \frac{\omega}{p} \right) \right]^k$$

Summing e subtracting the  $\log [(1 - \omega) / (\omega - p)]$  in the right side of the equation, we get

$$\log p^k = \frac{\log(1/p)}{\log(\omega/p)} \left\{ \log\left(\frac{1-\omega}{\omega-p}\right) - \log\left[\frac{1-\omega}{\omega-p} \left(\frac{\omega}{p}\right)^k\right] \right\} \quad (23)$$

Now we define  $a$  and  $\lambda$

$$a = \frac{\log(1/p)}{\log(\omega/p)} > 1$$

$$\lambda = \frac{1-\omega}{\omega-p}$$

Using Eq. (20), Eq. (23) becomes

$$\log p^k = \log\left(\frac{\lambda}{z}\right)^a \quad (24)$$

In conclusion, from Eq. (22) and Eq. (24), we obtain

$$\log[1 - \phi(z)] = \log p^k = \log\left(\frac{\lambda}{z}\right)^a$$

or:

$$1 - \phi(z) = \left(\frac{\lambda}{z}\right)^a \quad (25)$$

If we observe an increase of  $\omega = s(1+r)/(1+g)$ , that is an increase in the divergence between  $r$  and  $g$ ,  $\lambda = (1-\omega)/(\omega-p)$  can be considered a constant,  $a = \log\left(\frac{1}{p}\right)/\log\left(\frac{\omega}{p}\right) > 1$  decreases and so  $1 - \phi(z)$ , the stationary distribution of wealth, for the highest value of  $z$  becomes much more concentrated and wealth concentration increases. In fact, a smaller Pareto coefficient implies a thicker upper tail of the distribution and therefore a greater capital concentration.

More in general, whether we are considering binomial nature of the shocks (as in this example) or multinomial, whether they affect savings or other variables, it can be shown that capital accumulation models with random multiplicative or additive shocks lead to a distribution whose upper tail follows the Pareto distribution (Piketty and Zucman, 2015).

At this point, it is worth highlighting the points of contact and divergence between Vilfredo Pareto and Thomas Piketty. Pareto, in his work *Corso di Economia Politica* (1898), formulated what we now call the "Pareto Law". This law describes how, as income increases, the number of taxpayers tends to decrease. It is always valid in time and space and makes wealth inequality stable over time. Piketty does not criticise the validity of Pareto distribution but the uniqueness of the coefficient that characterises the distribution itself. According to Piketty, there is not one single Pareto coefficient, but there are several coefficients that vary according to the historical, social and economic context. With this different vision, wealth inequality is no longer a stable concept but, driven by changes in capital policies, variable over time.

## 5 Real capital vs financial capital

Over time, from the Industrial Revolution to the present, capital has changed its form from being a landed property capital to capital with an industrial and financial nature. Following Piketty (2014a), capital is defined as "the sum total of non-human assets that can be owned and exchanged on some market [...] [It] includes all forms of real property (including residential real estate) as well as financial and professional capital (plants, infrastructure, machinery, patents, and so on)", (Piketty, 2014a, p. 46). With this definition, capital is defined in its two main components, the real and the financial one, unified in a single variable.

We want to stress the specific contradiction between the definition of capital, the explanatory variable, and the model proposed. This way of defining capital creates a gap between the definition and the model. Both real and financial components are considered in the definition; simultaneously, a neoclassical theoretical model with only the real component is considered. In fact, the dynamics of capital's share and concentration is explained through the use of theoretical tools - aggregate neoclassical economic production function and marginal productivity of capital - that include only physical productive capital. In other words, Piketty treats capital in the terms of a standard neoclassical growth model, so that the financial component plays no role in the dynamics of distribution and concentration of wealth. In fact, in economics, capital is a production input factor that enters into the production process of machinery and buildings.

The inconsistency between the definition and the model proposed would be overcome by excluding the financial component from the definition of capital. In this sense, the rising income share of wealth-owners would be seen as an over-accumulation of capital through investment as is shown in the dynamics model of capital accumulation. Nevertheless, contrary to this explanation, the increasing of the capital/income ratio with the consequent concentration of capital is not caused by the real productive component but mainly by a valuation effect reflecting a disproportionate increase in the market value of financial assets (Stirati, 2016). In other words, although in the past, industrial capital was salient factor in the dynamics of the evolution of wealth and its distribution, nowadays economic inequality increases mainly because of rising stock prices and asset valuations (Galbraith, 2014a; Rowthorn, 2014). Moreover, the hypothesis of a real accumulation of capital turns out to be fragile, especially considering the important contribution of the service sector in the composition of national income in western countries. At the same time, in the service sector, productivity does not depend on the input of physical capital but conversely on what we call human capital, which is not considered in the definition (Weil, 2015).

Therefore, observing the high degree of financialisation of the developed economies, it seems necessary to introduce a model that deals not only with the real component, as in Piketty, but also with the financial one. This strategy would allow us to overcome the gap between the definition of capital and the original model that we have highlighted.

## 6 Conclusions

Piketty has the merit of bringing the debate about income and wealth inequality to the centre of economic studies, by questioning the solid ideas underlying the "goodness of capitalism". He presents remarkable empirical documentation of the state of inequalities and suggests a theoretical interpretation of the causes that produce the accumulation and distribution of capital in advanced capitalist economies (Piketty, 2014a, 2014b, 2015a, 2015b; Piketty and Zucman, 2015). In this sense, his work represents a valuable empirical contribution to the state of knowledge (Krugman, 2014b; Milanovic 2014; Solow, 2014). At the same time,

the model presented has opened great discussions, and still today, it is the subject of an extensive debate (King, 2016).

This article presented two major criticisms of Piketty's model. We have shown the contradiction of using the second fundamental law of capitalism to explain the rise of the capital/income ratio and the contemporaneous increase of capital share in the functional distribution of income. The contradiction is the result of the use of a law in a stationary condition that is unable to explain the increasing dynamics observed from the data. Secondly, we have highlighted the problem of adopting a neo-classical growth model when both the real and the financial components are included in the definition of capital. It is not possible to use the model proposed when the financial component is included in the definition. Moreover, nowadays capital share of national income and its concentration seem not determined by the rules of marginal productivity and real capital accumulation through saving, which are the main subject of the neoclassical theory of economic growth. Besides the forces Piketty places at centre stage, there are numerous other influences on the distribution of wealth.

By adding to or relaxing hypotheses in Piketty's model, new results could be obtained. With a modification of the original model, future analyses and investigations can be directed towards new paths to conduct a sensitivity analysis of the system. In this way, it would be possible to reconsider the original model to study the leading causes of the increase in capital/income ratio and the consequent concentration of wealth.

First of all, regarding the fundamental laws of capitalism, it is possible to extend the analysis by considering not only a static context but also a dynamic one, thereby introducing further economic forces able to restrain, or in opposition to increase, the process of wealth divergence.

Secondly, we need to go beyond the Piketty's analysis. Considering the various forms of capital and observing the high degree of financialisation of modern economies, it seems necessary to introduce a model capable of dealing not only with the real component but also with the financial one. Nowadays, capital is no longer a simple real factor of production, but the financial component plays a key role, especially in the determination of the functional distribution of income. A financial model of economic inequality would seem an appropriate place to start.

In conclusion, this paper pointed out the relevance of Piketty's contribution to the analysis of the dynamics of wealth inequality. At the same time, we have highlighted two significant flaws of the theoretical model presented. Inspired by these, future lines of research will be required to provide a better understanding of the phenomenon of persistent and rising inequality.

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