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Alessia Cafferata
Marwil J. Dávila-Fernández
Serena Sordi

Seeing what can(not) be seen: confirmation bias, employment dynamics and climate change

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Alessia Cafferata, Marwil J. Dávila-Fernández, Serena Sordi*

University of Siena

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Abstract

Psychologists among other behavioural scientists refer to the tendency of favouring, interpreting, and searching for information that supports one's prior beliefs as *confirmation bias*. Using Twitter data, we illustrate how this might affect environmental attitudes by contrasting #ClimateChangeIsReal and #ClimateChangeHoax engagement. Given the relevance of the topic to the field, we develop an agent-based model to investigate how employment conditions affect attitudes towards climate policies under such a cognitive bias. It is shown that persistent endogenous fluctuations might emerge via a super-critical Neimark-Sacker bifurcation. Furthermore, depending on the individual's response to the collective opinion, we might have coexistence of periodic attractors as a representation of path dependence. In terms of policy implications, we highlight that the adoption of a successful green-agenda depends on the ability of policy-makers to take advantage of favourable employment rates while appealing to different framing strategies.

Keywords: Climate change, confirmation bias, sentiment dynamics, group effect, adaptive learning.

JEL: D91, E71, O44, Q56.

* *Corresponding author*
Email address: serena.sordi@unisi.it

1 Introduction

Minds are a terrible thing to change. Behavioural scholars have identified the tendency to select information that seems to confirm prior opinions as one of the most problematic aspects of human reasoning. While initial beliefs become less relevant as evidence accumulates – a process known as "washing-out the priors" – individuals affected by *confirmation bias* tend to favour information that supports their personal opinions. Although there is an evolutionary justification for the existence of such a mechanism, it consists in a failure of the Bayesian updating of beliefs that has significant (negative) effects on the proper functioning of political systems (Lockwood, 2016), financial markets (Rabin and Schrag, 1999; Pouget et al., 2017), and society in general (for a comprehensive assessment, see Nickerson, 1998).

To the extent that climate change is also a social phenomenon (Hackmann et al., 2014; Pearson et al., 2016), a relevant research question consists in understanding the role of such a bias when investigating the respective trends in international public opinion. As shown by Capstick et al. (2015), despite the scientific evidence on climate change, the general public seems to be divided between those who believe and those who are not convinced there is an environmental challenge (see also Bliuc et al., 2015). Polarisation of opinions is dangerous in this case because successful policy implementation requires a certain degree of acceptance capable of delivering sustainable long-run results.

The costs associated with anthropogenic climate change increase the demand for insurance (Botzen and van den Bergh, 2012). This means that the implementation of a green-agenda is strongly related to the performance of the labour market. There is some evidence indicating that recessions negatively affect attitudes towards climate policies in the United States (US) and in Europe (Scruggs and Benegal, 2012; Shum 2012). When confronted with more "urgent" needs, people are less likely to engage in environmental action. In this paper, it is our purpose to go one step further and investigate the interaction between individuals' prior beliefs and employment dynamics. We develop a macro agent-based model to study how employment conditions affect environmental attitudes under *confirmation bias*. To the best of our knowledge, we are the first to attempt to provide a formal assessment of this relationship.

Our contribution relates to an emerging literature on macroeconomic ecological-sentiment analysis (e.g. Antoci et al., 2012; Dávila-Fernández and Sordi 2020; Rengs et al., 2020). In this way, we do not assume that agents are rational in the conventional sense. Instead, we adopt the notion of procedural rationality, which allows us to investigate the interaction between different heuristics as part of decision-making processes. Following Gowdy (2008), we recognise that assessing climate change requires a proper reference to social context. However, differently from previous efforts, we highlight the role and implications of the *confirmation bias*, a clear innovation in the field. It is shown that persistent endogenous fluctuations might emerge via a super-critical Neimark-Sacker bifurcation. Furthermore, depending on individuals' response to collective opinion, we might have coexistence of periodic attractors and, therefore, a representation of path dependence.

From an empirical point of view, social media have proved to be a unique space in which sentiments are confronted and updated. While traditional media do not allow for a deeper discussion between citizens, social networks are forums where diverse individuals can share their thoughts and opinions. Twitter, for example, has become a valuable resource for analysing trends and major events. Yu et al. (2013) have shown that it has a stronger relationship with stock-market performance than conventional media, though they emphasise that the impact of different types of social media varies significantly. Another example is Rill et al.

(2014), who designed a system to detect emerging political trends on Twitter, anticipating more mainstream information channels during the parliamentary election of 2013 in Germany. They observed that topics even emerged earlier in Twitter than in Google trends.

When it comes to climate change, Cody et al. (2015) analysed tweets containing the word "climate" collected between September 2008 and July 2014. The authors were able to determine how collective sentiment varies in response to climate change news, events and natural disasters, concluding that Twitter is a valuable resource for the spread of climate-change awareness. In this paper, we further justify our main research question by analysing the engagement on two different trend topics, namely, #ClimateChangeIsReal and #ClimateChangeHoax during a series of pro-environment demonstrations supported by Greta Thunberg in February 2020. We show that people refer to news and sources that support their beliefs about climate change in a biased way. There are important gender differences with a higher proportion of men among deniers. Furthermore, they also appear to be more aggressive, with a share of negative sentiments five times greater than the other group.

In terms of policy implications, our analysis indicates that the adoption of a successful green-agenda depends on the ability of policy makers to take advantage of favourable employment rates. This requires the understanding and use of different framing strategies. Given that *confirmation bias* is a problematic aspect of human reasoning but also seems to be an intrinsic part of being human, we should learn how to take advantage of it. Minds are a terribly difficult but not an impossible thing to change. Such a recommendation goes hand in hand with evidence indicating that bias effects might be dodged by emphasising the gains of climate change mitigation (e.g. Spence and Pidgeon, 2010; Feldman, 2018). Moreover, we highlight that more important than employment levels, policy makers should act when employment is increasing. Even though the trajectories we obtain are related to long-term processes of structural and institutional change, endogenous fluctuations might be used as a path towards a more desirable green-equilibrium.

The remainder of the paper is organised as follows. In the next Section, we present an environmental sentiment-analysis based on Twitter data. Section 3 develops a simple discrete-time agent-based model that allows for some degree of *confirmation bias* in the formation of attitudes towards climate policies. In Section 4, we introduce a "group effect" allowing agents to be decisively influenced by social context. We conclude with some final considerations.

2 Twitter as a social experiment

Social media have notably increased in importance over the last ten years. Their emergence has created a new communication system that allows users to connect, comment and share news and media content at speeds just unthinkable a few years ago. In the political arena, social media have become a crucial communication tool for the diffusion of social campaigns. As Chadwick (2011) suggested, studies of individual events and collective processes show that we are moving from a traditional "news cycle" – dominated by journalists and professional sources – to a more complex "information cycle" – that integrates ordinary people into the ongoing construction and contestation of news.

2.1 Twitter and climate change

Climate change is a heavily politicised topic, splitting public opinion. In the United States, for instance, it has become a divisive issue since the late 1990s, when a vigorous conservative campaign against climate science and climate change supporters led leaders of the Republican Party to adopt a highly skeptical view on global warming (Dunlap and McCright, 2008). In April 2013, 63% of US-Americans reported they believed climate change is happening but only the 49% believed that it is caused by humans. The percentage falls to 38% when asked if people around the world are currently being damaged by the consequences of climate change (Leiserowitz el al., 2014). In this Section, we provide some insights into how people address the environmental debate on social media. Moreover, with a sentiment analysis approach, we are particularly interested in the amount of information exchanged that may be imputable to one's prior assumptions.

We refer to the social media site Twitter, which allows 280 characters to its 152 million users to communicate whatever they like within a "tweet". Such expressions reflect what individuals are thinking or feeling about a multitude of arguments. The majority of topics trending on Twitter are headlines or persistent news, making Twitter a valuable source for studying climate change opinions (Kwak et al., 2010; Reyes-Menendez et al., 2018). Using simple text data techniques, we have considered a classification and interpretation of emotions that categorises them as positive, negative and neutral.

Twitter explorations of this kind have been used for heterogeneous purposes, from the analysis of social and linguistic phenomena (Lin et al., 2013) to use as a data source to create an earthquake reporting system in Japan (Sakaki et al., 2010). Given the sharp dispute on global warming, climate change has also become an object of sentiment analysis. For example, An et al. (2014) have analysed subjective *vs* objective and positive *vs* negative tweets mentioning climate change. In a similar vein, Williams et al. (2015) have mapped various climate hashtags among pro- and denier-communities on Twitter. Kirilenko et al. (2015) examined whether people living in the US connect local temperature to climate change and whether mass media influence the process. Other scholars have introduced an approach that automatically classifies tweets sentiments by using classifier ensembles and lexicons (da Silva et al., 2014), while there have been some attempts to analyse the average happiness of all tweets containing the word "climate" on different timescales (Cody et al., 2015).

Despite the almost global consensus of the scientific community, climate change remains far from having unambiguous support. Specifically, as far as the US is concerned, political partisanship may impact the way citizens (affected by *confirmation bias*) face the problem. On the one hand, Republicans or conservatives may give little weight to scientific evidence for the existence of climate change while they are more likely to select news that minimise the problem. On the other hand, Democrats may attribute more importance to every signal confirming global warming. In this paper, we focus on two different trends related to attitudes towards climate change to deepen these aspects.

2.2 #ClimateChangeIsReal *vs* #ClimateChangeHoax

We analyse two different trends on Twitter, both related to the topic of climate change, namely, #ClimateChangeIsReal and #ClimateChangeHoax. Our results are based on daily data between 21 and 28 of February, 2020.[1] These two keywords provide an interesting representation of the two opposing attitudes towards the environment before the COVID-19 outbreak. The time span corresponds to an interesting period during a series of pro-environment

[1] Data have been processed with the software R. The Application Programming Interface (API) key was conceded to us by Twitter on 11 February 2020. Replication codes are available under request. We would like to emphasise that even though our data come from a global platform, the fact that we rely on two hashtags in English might imply that our numbers are biased towards Anglo-Saxon countries.

demonstrations supported by Greta Thunberg. While protests started in October 2019, they culminated in February of the next year during the meeting between the green-activist with the Peace Nobel winner Malala Yousafzai. At the beginning of March 2020, Greta was invited by the Climate Commission of the European Parliament on the Climate Law to give a speech.

Table 1 describes the composition of the two hashtag samples. For every hashtag, we focused on the number of posts, number of users, their gender, the engagement rate, the reach metric, and the impression metric. We analysed 155 posts on #ClimateChangeIsReal and 153 posts on #ClimateChangeHoax. The first sample involves 145 users while the second 143. Posts against climate change have a significantly higher level of engagement. Such a measure indicates the total number of times a user interacted with a Tweet in terms of clicks, retweets, replies, follows, likes, links, cards, hashtags, embedded media, username, profile photo, or Tweet expansion. Despite a higher level of engagement, climate-change deniers reached a lower number of users. This means fewer Twitter users have actually seen the hashtag.

Table 1: Sentiment analysis on two different Twitter trends

Two different samples considered		
Variable	#ClimateChangeIsReal	#ClimateChangeHoax
Posts	155	153
Users	145	143
Engagement	903	1298
Reach	780862	710646
Impressions	788582	723643
Gender	55% Male, 45% Female	76% Male, 24% Female

Furthermore, climate-change supporters' tweets also have a higher impact. We measure impact by looking at the so-called impressions. Impact means that somebody has seen a tweet. Thus, an impression is generated when someone reads or sees a tweet. Regardless of a lower engagement, posts associated with the hashtag #ClimateChangeIsReal seem to involve a wider audience. Looking at gender composition, we have an almost equal distribution between the two sexes in the first sample, while 76% of individuals denying climate change are men. These results go in the same direction of Denton (2002), who has explained that women play a key role in natural resource management and are therefore more virtuous although vulnerable to the environment (for a review, see Arora-Jonsson, 2011).

Fig. 1 shows the most linked websites for climate change supporters and climate-change deniers. Here *confirmation bias* seems to appear. Leaving out from our analysis *bit.ly* which is an URL-shorter, we notice how sources are polarised. On the left, the reader can see which are the most important pro-climate websites. The most linked is *Youtube*, followed by *The Guardian*, a liberal-democrat newspaper and website. The other web-pages belong to environmentally friendly sources. Climate change supporters seem to refer to news that may confirm their point of view. The same holds true for those who do not believe in climate change, as we can observe in the right panel of Fig. 1. *Youtube* is not the most linked website anymore, giving way to *foxnews.com*, *politico.com* and *telegraph.com*. While *Politico* is a bipartisan journal and website, with the same percentage of Democrat and Republican readers in the US, the *Telegraph* and *Fox News* are known for belonging to conservatives.

People who do not believe in climate change refer to news that may support their beliefs.

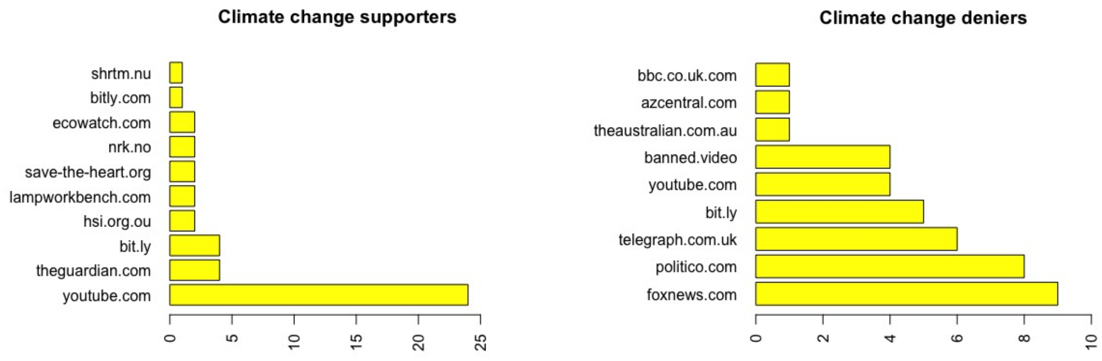


Figure 1: Most linked websites for climate change supporters and deniers.

Fig. 2 presents the distribution of sentiments within the two samples. The green area of the pie charts indicates positive sentiments, the blue area stands for neutral while the red area expresses negative sentiments. Twitter provides a "mention" metric on each tweet that is either positive, negative, or neutral. A positive tweet is a tweet that expresses constructive, optimistic, or confident sentiments. Analogously, a negative tweet contains words related to equivalent sentiments, such as anger, frustration, or delusion. When no emotions are implied, they are classified as neutral. Notice that words are analysed out of context, implying some minor misclassification by the software. While we do not give any score to different sentiments, this presentation already provides some interesting insights into the main attributes of the two samples. For pro-environmental users, the rate of negative sentiments is only 6%, while the majority of tweets are neutral. On the other hand, negative sentiments account for 34% of tweets from climate deniers.

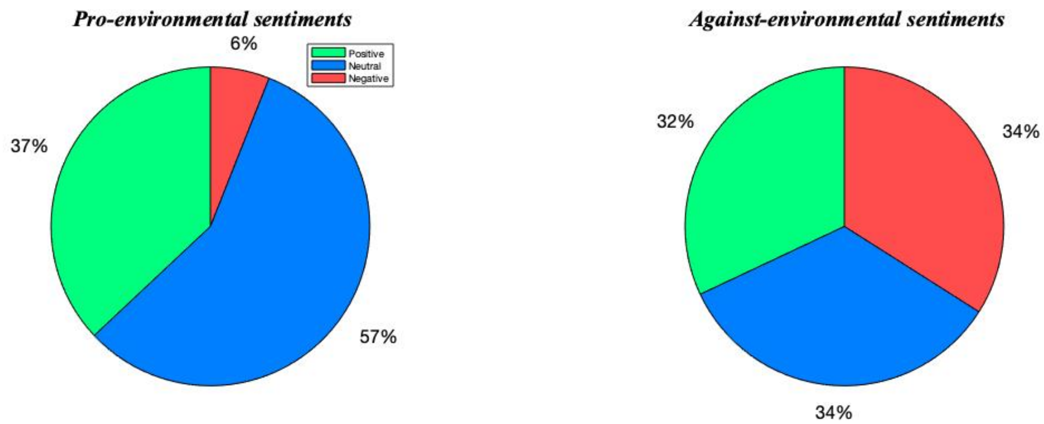


Figure 2: Sentiment of the two samples.

From a dynamic point of view, the existence of *confirmation bias* introduces an inertial component at the individual level. This is because agents do not change their views immediately after confronted with new evidence. On the contrary, as argued so far, people are biased in their searching processes. It is our purpose in what follows to provide a formal narrative that takes into account the presence of such a cognitive bias in the interplay between the (macro)economy and the environment.

3 A model of *green* confirmation bias

Climate change is not perceived in the same way by different people and polarisation in public opinion obstructs the implementation of green public policies. Thus, it is important to understand how people with opposite attitudes towards climate issues interact without ignoring basic elements that are part of human cognition.

Research on cultural psychology indicates that people's opinions about climate risk are not always based upon scientific evidence. Interpretations of scientific information on unknown phenomena are motivated by competing cultural philosophies or worldviews (Kahan et al., 2011, 2012). Reasoning at the individual level is context-dependent, leading to inevitable group behaviour effects. This results in agents engaging in *motivated reasoning*, also implying that they might have different mental goals (Druckman and McGrath, 2019)

On the other hand, individuals may aim to arrive at certain or predetermined conclusions, confirming a prior belief related to politics, religion or factors related to a certain life-style. This mechanism is characterised by several biases that lead to a mistaken interpretation of available information, causing a divergence of sentiments. The interaction between people who interpret evidence conditioned by existing beliefs or a pre-determined conclusion, i.e. affected by *confirmation bias*, results in complex attitudes towards climate policies at the individual and collective levels.

3.1 Attitudes towards *green* policies

Based on Lux (1995), scholars such as Dávila-Fernández et al. (2020) have provided some useful groundwork for setting up an elementary and rigorous sentiment analysis in ecological thinking.[2] Following a similar structure, suppose labour force is equal to population, N, the latter being constant and divided between people who are climate change supporters, N^+, and those who are climate-change deniers, N^-:

$$N = N_t^+ + N_t^- \tag{1}$$

The difference between these two groups is indicated with n:

$$n_t = N_t^+ - N_t^- \tag{2}$$

We introduce an index to define the average attitude of society towards environmental regulation:

$$\Phi_t = \frac{n_t}{N} \tag{3}$$

where $\Phi \in [-1, 1]$. If, in a certain period, all citizens are climate supporters, then $\Phi = 1$. On the other hand, if they are all climate change deniers, $\Phi = -1$. When agents are equally split between the two groups, $\Phi = 0$.

[2] For a deeper discussion on possible alternative specifications, see Franke and Westerhoff (2017). The two main mechanisms in this literature are the discrete choice and the transition probability approach. In both of them, agents face a binary decision and commit conditional to a set of probabilities. The model developed in this paper is in discrete-time, but analogous results can be derived in a continuous set-up.

Fiedler (2000) has documented a series of judgement biases when it comes to ecological thinking. They range from base-rate neglect to illusory correlation, being strongly related to the existence of a *confirmation bias*. Consequently, the way a narrative is framed influences how individuals acquire information (Jones and Sudgen, 2001) as well as cognitively organise concepts (Jones and Song, 2014). To keep our exercise as simple as possible, we assume that in every period individuals may update their beliefs in a biased way. Environmentally friendly agents are likely to adjust their expectations attributing different importance to facts that support their initial guess rather than news that goes in the opposite direction. On the other hand, those who do not consider global warming a priority may give more weight to information that minimises the problem.

Define α as the share of agents that are willing to change their view in a given period. The strength of the *confirmation bias*, $1-\alpha$, is such that the fraction of agents supporting and opposing environmental policies is given by:

$$\begin{aligned}\frac{N_t^+}{N} &= \alpha p_{t-1}^+ + (1-\alpha)\,\frac{N_{t-1}^+}{N}\\[2ex] \frac{N_t^-}{N} &= \alpha p_{t-1}^- + (1-\alpha)\,\frac{N_{t-1}^-}{N}\end{aligned} \tag{4}$$

where p^+ and p^- are the probability functions of having a positive or a negative attitude respectively. The closer α is to zero, the higher the bias. When $\alpha = 0$, agents just repeat their beliefs of the previous period, being resistant to the possibility of changing attitudes.

Changes in the number of environmentally friendly individuals are conditional upon the probability of adopting a pro- or anti-environmental position. In Eq. (4), subtracting the second expression from the first, we obtain the aggregate attitudes index as a function of the confirmation bias:

$$\Phi_t = \alpha\left(p_{t-1}^+ - p_{t-1}^-\right) + (1-\alpha)\,\Phi_{t-1} \tag{5}$$

This structure resembles Hommes et al. (2005) discrete choice approach with asynchronous updating. The reader might also find similarities with the so-called logit dynamics in evolutionary game theory. Notice that when there is no bias, i.e. $\alpha = 1$, Φ only depends on p^+ and p^-.

As argued at the beginning of this paper, the presence of *confirmation bias* introduces an inertial component at the individual level so that agents would change their minds less often than in the absence of such mechanism. Still, minds are a terribly difficult but not an impossible thing to change. In order to focus on the effects of the labour market on people's attitudes towards the environment, suppose:

$$\begin{aligned}p_{t-1}^+ &= \frac{\exp\left(\beta\left(\frac{e_t-e_{t-1}}{e_{t-1}}\right)\right)}{\exp\left(\beta\left(\frac{e_t-e_{t-1}}{e_{t-1}}\right)\right)+\exp\left(-\beta\left(\frac{e_t-e_{t-1}}{e_{t-1}}\right)\right)}\\[2ex] p_{t-1}^- &= \frac{\exp\left(-\gamma\left(\frac{e_t-e_{t-1}}{e_{t-1}}\right)\right)}{\exp\left(-\gamma\left(\frac{e_t-e_{t-1}}{e_{t-1}}\right)\right)+\exp\left(\gamma\left(\frac{e_t-e_{t-1}}{e_{t-1}}\right)\right)}\end{aligned} \tag{6}$$

where $\beta, \gamma > 0$ capture the response of the probability functions to changes in the employment rate. The discrete-choice approach refers to these parameters as the intensity of choice. When they are both close to zero, the two probabilities are nearly equal. On the other hand, when they are very large and tend to infinity, the two probabilities tend to 1 and 0, respectively. In this way, p^+ increases with employment while p^- presents the opposite behaviour.

3.2 Environmental regulation

The literature on the relationship between private costs and social benefits is quite controversial. This is not different when it comes to tackling climate change, with a prevailing belief that there is a trade-off between the two dimensions. However, at the beginning of the Nineties, Porter (1991) and Porter and van der Linde (1995) suggested that environmental regulation might potentially increase firms' competitiveness by changing the prevailing structure of incentives. If pollution is seen as resulting from an incomplete or inefficient utilisation of resources, strict environmental regulation could actually encourage innovation and induce efficiency.

In recent decades, the so-called Porter hypothesis (PH) has been discussed in at least three different versions. Some scholars have argued that any kind of regulation can induce innovation. Others have indicated that only certain types of innovation are fostered by regulation. A last group of contributions refers to a narrow PH in which only certain types of regulation stimulate innovation. Fabrizi et al. (2018) have provided a recent survey of the field and show that there is some empirical evidence supporting only the last two versions. In this paper, we adopt a friendly position towards PH, accepting its validity.

Define Z as a measure of environmental efficiency, for example, the amount of output per unit of Green House Gas (GHG) emissions. The degree of stringency of regulation is supposed to be captured by λ. However, efficiency conditions today are linked to the cumulative effects of past decisions. This means that Z not only depends on λ in period t, but also on the regulation adopted over a certain time-horizon, η. In mathematical terms:

$$Z_t = \prod_{\eta=1}^{t} (1 + f\lambda_{\eta-1}) \tag{7}$$

where $f > 0$ captures the efficiency of the regulation.

In reality, aspects related to the design of policy and regulations play a crucial role in the determination of efficiency conditions. While for the moment we abstract from a more detailed description of the channels involved in implementing and enforcing green policies, the main idea is that a more stringent regulation is positively associated with higher environmental efficiency. In terms of growth rates, it follows that:

$$\frac{Z_t - Z_{t-1}}{Z_{t-1}} = f\lambda_{t-1} \tag{8}$$

which is expressed as a function of λ.

As a simplifying hypothesis, suppose policy makers' attitudes towards the environment perfectly match those of citizens. In this way, environmental regulation mirrors aggregate sentiments:

$$\lambda_t = \Phi_t \tag{9}$$

This assumption does not come without an important cost given that such a utopian democracy does not really exist. For instance, several people are simply disenfranchised or do not care about the environment or politics. Future research should take into account these aspects and provide a more accurate description of power distribution in society. In any case, the approach adopted here is convenient from an operational point of view while preserving the main message of our contribution. Policy makers might not perfectly match society's expectations, but there must be some degree of adherence between them.

The outcome of an election somehow reflects the will of the majority of citizens and determines, among other things, the new policy guidelines on environmental risks. This, in turn, impacts the level of regulation stringency. Recall that $\Phi \in [-1, 1]$. Hence, when most people support environmental policies, $\Phi > 0$, policy makers adopt a green agenda with $\lambda > 0$. On the other hand, when citizens oppose environmental-friendly policies, $\Phi < 0$, regulation is such that $\lambda < 0$. Notice that an equal distribution between the two groups, $\Phi = 0$, does not produce any regulation in the first place, $\lambda = 0$. This is because none of the two groups is strong enough to prevail.

3.3 The production technology

The last block of equations consists in the determination of the supply side of the economy. Consider a production technology that combines energy, E, and labour. Following a long tradition in ecological economics, factors of production are complements rather than substitutes:

$$Y_t = \min\{E_t/\vartheta; q_t N e_t\} \tag{10}$$

where ϑ is the energy-output ratio, q stands for labour productivity and e is the employment rate, defined as $e = L/N$, with L indicating the level of employment. Labour productivity is defined as $q = Y/L$.

Given the Leontief dynamic efficiency condition, we have as a good approximation that output depends on the demand for energy by firms while employment rates adjust to production:[3]

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{E_t - E_{t-1}}{E_{t-1}} - \frac{\vartheta_t - \vartheta_{t-1}}{\vartheta_{t-1}} \tag{11}$$

$$\frac{e_t - e_{t-1}}{e_{t-1}} = \frac{Y_t - Y_{t-1}}{Y_{t-1}} - \frac{q_t - q_{t-1}}{q_{t-1}} \tag{12}$$

The first expression indicates that the use of energy goes *pari passu* with the rate of growth of output. On the other hand, the employment rate adjusts to the difference between $(Y_t - Y_{t-1})/Y_{t-1}$ and the rate of growth of labour productivity. When the former is growing faster than the latter, there is disequilibrium in the labour market and the employment rate increases. Therefore, to recover the so-called natural rate of growth the labour market must be in equilibrium.

Scholars in the field recognise that the amount of energy required in production is sensitive to the environmental efficiency of current technologies. Thus, an increase in the rate of growth of Z is associated with a lower energy-output ratio. Cleaner technologies are characterised by a more efficient use of resources:

$$\frac{\vartheta_t - \vartheta_{t-1}}{\vartheta_{t-1}} = b\left(\frac{Z_t - Z_{t-1}}{Z_{t-1}}\right) \tag{13}$$

[3] In continuous time, Eqs. (11) and (12) directly follow from taking log-derivatives of the Leontief efficiency condition. However, in discrete time, the rate of growth of the product of two variables is equal to the sum of the rates of growth of each variable plus their product. Still, because of the very small magnitude of this last term, we disregard it for the sake of simplicity.

where $b > 0$ stands for the respective elasticity. If there are no changes in environmental efficiency, ϑ will remain constant.

Suppose firms' energy needs grow at an exogenous rate, a. Inserting Eq. (13) into Eq. (11), the rate of growth of output can be disaggregated between an autonomous and an efficiency component:

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}} = a + b\left(\frac{Z_t - Z_{t-1}}{Z_{t-1}}\right) \tag{14}$$

By substituting Eq. (9) into Eq. (8) and the resulting expression into Eq. (14), we obtain the rate of growth of output in terms of aggregate attitudes towards the environment:

$$\frac{Y_t - Y_{t-1}}{Y_{t-1}} = a + bf\Phi_{t-1} \tag{15}$$

A well-designed environmental regulation is associated with a higher f, leading to the adoption of more environmentally friendly production arrangements. Moreover, b reflects the capacity of the productive structure to translate this increase in efficiency into greater competitiveness. We refer to bf as the intensity of PH. Adopting a friendly position towards it, when the majority of the population supports climate policies one should expect a higher rate of growth.

Finally, labour productivity is a central issue in economics and has been studied using both more conventional as well as alternative approaches (for a review see Tavani and Zamparelli, 2017). Keeping the exercise as simple as possible, in this representation, we want to highlight the role of two main mechanisms: dynamic economies of scale and induced technical change. Both can be directly linked to what is happening in the labour market, which is quite convenient for our narrative.

On the one hand, it is widely acknowledged by the profession that to a large extent technical progress is embodied in new machinery and equipment (Arrow, 1962; Romero and Britto, 2017). More recently, with the rise of intangible assets, one could think of innovation that comes in the form of new software and algorithms. In any case, labour must be in use for productivity gains to be effectively incorporated. Otherwise, there is just a "beautiful" creation with no impact on production. On the other hand, some empirical evidence indicates that factor productivity growth rates respond to factor cost-shares (Acemoglu, 2003; Dávila-Fernández, 2020). The idea is that, when employment rates are high, there is an increase in the bargaining power of workers that might be able to obtain higher wages relative to labour productivity. Firms respond by increasing their search for labour saving production techniques, thus increasing q. Formally, we have:

$$\frac{q_t - q_{t-1}}{q_{t-1}} = -c + ge_{t-1} \tag{16}$$

where $c > 0$ is an arbitrary constant and $g > 0$ captures the response of productivity to the employment rate.

3.4 Dynamic system

Substituting Eqs. (15) and (16) into Eq. (12), we obtain the dynamic relationship governing changes in the employment rate. On the other hand, substituting Eq. (6) into Eq. (5), we have attitudes towards climate policies depending on the employment rate and on the *confirmation bias*. Our 2-dimensional map is given by:

$$\begin{aligned}
e_t &= (1 + a + bf\Phi_{t-1} + c - ge_{t-1})\, e_{t-1} \\
\Phi_t &= \Phi_{t-1} + \alpha \left[\frac{\exp\left(\beta\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right)}{\exp\left(\beta\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right) + \exp\left(-\beta\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right)} \right. \\
&\quad \left. - \frac{\exp\left(-\gamma\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right)}{\exp\left(-\gamma\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right) + \exp\left(\gamma\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right)} - \Phi_{t-1} \right]
\end{aligned} \tag{17}$$

In steady-state, $e_t = e_{t-1}$ and $\Phi_t = \Phi_{t-1}$. The equilibrium conditions are such that:

$$\begin{aligned}
a &= -c + ge \\
\Phi &= 0
\end{aligned}$$

A stable rate of employment depends on the economy growing at the same pace as labour productivity. Moreover, environmental attitudes come to a state of rest when there is an equalisation between the probability of supporting and opposing climate policies. It is possible to say in advance that this will only occur when agents are split in equal proportions.

We can now state and prove the following Proposition regarding the existence of a unique non-trivial equilibrium point.

Proposition 1 *The dynamic system (17) has a unique non-trivial equilibrium solution,* $P_1 = (e_1^*, \Phi_1^*)$*, defined and given by:*

$$\begin{aligned}
e_1^* &= \frac{c + a}{g} \\
\Phi_1^* &= 0
\end{aligned} \tag{18}$$

Proof. See Mathematical Appendix A1. ■

Recall that the sentiments index only responds to changes in the rate of employment. This implies that, in equilibrium, the population is equally distributed between ecological positions. Aggregate attitudes are, thus, independent of any sort of confirmatory bias. Given the lack of social consensus regarding the environmental challenge, society is incapable of producing meaningful environmental policies. As a result, things remain the way they are, at least from a macroeconomic point of view.

On the other hand, the rate of employment only depends on energy used and on labour-productivity-related parameters. For instance, an increase in the autonomous component of energy requirements, a, is associated with an increase in the long-run rate of growth. In order to match demand, firms hire more workers, increasing the employment rate. Still, notice that if firms strongly increase their search for labour saving production techniques as a reaction to high employment rates, one should expect a lower e_1^*. Such results are independent of ecological variables because $\Phi_1^* = 0$, thus implying that, in equilibrium, $\lambda = 0$.

Regarding the local stability of P_1, we can state and prove the following Proposition:

Proposition 2 *The equilibrium point P_1 is locally asymptotically stable in the region of the parameter space defined as:*

$$ge_1^* + (1 - ge_1^*)\,\alpha - \frac{\alpha b f}{2}\,(\beta + \gamma) \quad > \quad 0$$

A violation of this condition is associated with the occurrence of a super-critical Neimark-Sacker bifurcation.

Proof. See Mathematical Appendix A2. ■

While we rule out the possibility of a *Flip* or a *Fold* bifurcation, the system admits a super-critical *Neimark-Sacker* bifurcation.[4] This comes with an important economic result, namely, the emergence of endogenous and persistent cycles in public opinion related to current macroeconomic conditions. They should be interpreted as long-run dynamics linked to processes of institutional and structural change, not to be confused with short-term business cycles. Using the response of sentiments to changes in employment, β, as a bifurcation parameter, we have that beyond a critical value:

$$\beta_{NS} = \left[\frac{ge_1^* + (1 - ge_1^*)\alpha}{\alpha b f}\right] 2 - \gamma \tag{19}$$

where

$$\begin{aligned} \lim_{\alpha \to 0} \beta_{NS} &= +\infty \\ \lim_{\alpha \to 1} \beta_{NS} &= \frac{2}{bf} - \gamma \end{aligned}$$

the dynamic system admits a family of periodic solutions.

The expression above establishes a direct connection between the existence of a periodic attractor and the degree of confirmatory bias. In fact, as α approaches zero, that is, as agents are fully biased, β_{NS} goes to infinity, basically implying the stability of the internal equilibrium point. However, as the *confirmation bias* falls, β_{NS} converges to a constant, which in turn is inversely related to the strength of PH. In our model, f captures the response of environmental efficiency to regulation while b stands as the elasticity of the energy-output ratio. The higher is their product, the more likely it is that persistent and periodic fluctuations will arise.

Proposition 2 indicates that we are dealing with a super-critical bifurcation, which in turn guarantees that the emerging orbit is stable. Still, to provide a more concrete view of its properties and the economic intuition behind our narrative of cycles in public opinion, it is useful to turn our attention to numerical analysis.

[4] In a dynamic system, when the modulus of a pair of complex conjugate eigenvalues is equal to one, a Neimark-Sacker bifurcation occurs. Its similarities with the Hopf bifurcation are obvious, but we would like to emphasise an important difference. While the invariant limit cycle in the latter case consists of a single orbit, in the former case it is an invariant set on which there exist many different orbits (see Medio and Lines, 2003, p. 159). The emerging periodic orbit might be stable or unstable. In the first situation, we say the bifurcation is super-critical, while in the second we refer to a sub-critical bifurcation. In this paper, following Kuznetsov (2004, pp. 157-187), we analytically demonstrate that cycles are stable.

3.5 Numerical simulations

We choose parameter values such as to obtain results that are economically meaningful. This selection, nonetheless, has only an illustrative purpose. Similar qualitative dynamics can be obtained for a wider range of values. We rely on well-known macroeconomic regularities and evidence provided by Dávila-Fernández et al. (2020) and Mealy and Teytelboym (2020). However, given that we are not calibrating a "real" economy, their studies were only used to give an idea of the magnitudes involved. Our reference values are:

$$\begin{aligned} a &= 0.03;\ b = 0.1 \\ f &= 0.1;\ c = 0.06;\ g = 0.1 \end{aligned}$$

and

$$\alpha = 0.25$$

such that, in every period, 75% of agents do not change their views on environmental regulation.

As mentioned in our previous discussion, an important parameter capturing the interaction between attitudes towards climate policies and the (macro)economy is β. Fig. 3 reports the 1D bifurcation diagrams for e_1^* and Φ_1^* and the Maximum Lyapunov Exponent (MLE) with β varying from 100 to 500.[5] From Eq. (19), it is not difficult to see that analogous diagrams can be obtained by instead varying parameter γ, which in the following simulations we assume to be 50. As employment conditions become increasingly important when forming an opinion about climate change, the unique non-trivial equilibrium point loses stability and a *Neimark-Sacker* bifurcation occurs. An equilibrium set still might exist because an attracting invariant closed curve coexists with the unstable fixed point. The orbits that result are a combination of points whose motion is either periodic or quasi-periodic. This is confirmed by a MLE that oscillates slightly above and below zero.

Fig. 4 depicts the periodic orbits in the (e, Φ) plane when $\beta = 250$ and $\gamma = 50$. The cycle is clock-wise oriented and results from the interaction between two main forces. During good times, when the economy is growing, firms hire more workers, increasing the rate of employment. Consequently, agents' basic needs are fulfilled and they become more inclined to accept environmental challenge. As attitudes towards climate policies become more favourable, policy makers are able to implement a more stringent environmental regulation, thus increasing efficiency through PH. There is an immediate reduction in energy-output ratios, resulting in higher output. This reinforces the initial impulse:

$$Y \uparrow \Rightarrow e \uparrow \Rightarrow \Phi \uparrow \Rightarrow Z \uparrow \Rightarrow \vartheta \downarrow \Rightarrow Y \uparrow$$

Such an unstable force is counterbalanced by the interaction between technical change and the labour market. An increase in the rate of growth of output results in firms' demanding more labour only up to certain point. This happens for two reasons. First, while technical change is to a large extent capital embodied, workers must be using the new machinery and equipment in order for productivity gains to be effectively incorporated. Second, as unemployment rates fall, a shortage of workers increases their bargaining power leading in

[5] The MLE is a prominent measure for stability evaluation in dynamic systems. It describes the phase speed at which two different points approach or depart. For each dimension of the system, a Lyapunov exponent exists forming the Lyapunov spectrum. A positive MLE is usually taken as an indication that the system is chaotic. In our simulations, we set the number of pre-iterates to 15000 and then considered 1000 interactions in Fig. (3) and 5000 interactions in Fig. (5).

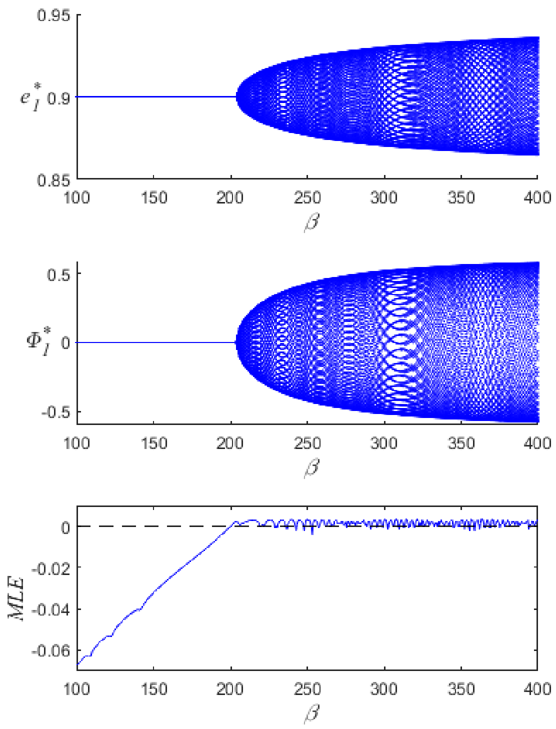


Figure 3: Bifurcation diagrams and Max. Lyapunov Exponent.

some cases to real-wages growing above the rate of growth of labour productivity. In both situations, firms respond by increasing their search for labour saving productivity techniques. As the rate of growth of labour productivity recovers, there is a reduction in the rate of employment:

$$e \uparrow \Rightarrow q \uparrow \Rightarrow e \downarrow$$

We thus have a representation in which endogenous cycles arise from the interaction between attitudes and the (macro)economy in line with the findings of Scruggs and Benegal (2012) and Shum (2012), among others. They have shown that during recessions, attitudes towards climate change in Europe and in the United States have become significantly less environmentally-friendly. While our results should be interpreted as long-run waves, the main message remains the same: not only are people's sentiments influenced by the state of the economy, but fluctuations are an intrinsic property of this relationship. This result is conditioned by the intensity of the *confirmation bias* which has proved to be a crucial element behind the dynamics we obtained.

4 Introducing a "group effect"

The model developed in the previous Section provides a formalisation of the interplay between attitudes towards climate policies and the economic system. However, an important element has been overlooked, that is, the fact that sentiments are to a large extent context dependent. People do not form their opinions and worldviews in a vacuum. In fact, agents are influenced

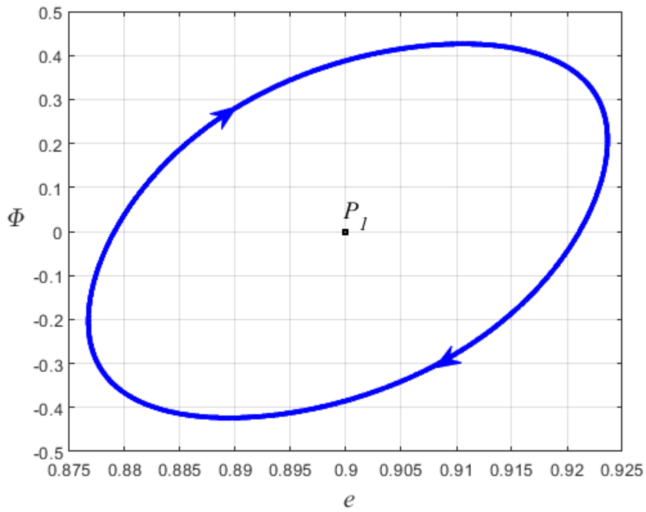


Figure 4: Emergence of a periodic attractor when $\beta = 250$ and $\gamma = 50$.

by the viewpoints of those surrounding them (e.g. Antoci et al., 2018). Scholars such as Kahan et al. (2009) have shown that individuals form perceptions and opinions influenced by the values of the group with which they identify themselves (see also Hart et al., 2015; Efferson et al., 2020).

As far as climate change, specifically, is concerned, scholars such as Hart (2011) have shown that people tend to support more environmental policies when exposed to a thematic frame than when exposed to an episodic frame. This evidence reinforces the idea that motivated reasoning at the individual level depends on aggregate attitudes at the collective level, while obviously the latter are a function of the former. In the real world, it is impossible to separate them from each other and our model should be compatible with this attribute.

As demonstrated by Dávila-Fernández and Sordi (2020), such a "group effect" is responsible for generating the coexistence of a basin of attraction in which the majority of the population supports environmental regulation and another in which most agents have the opposite attitude. Abstracting from this last element has allowed us, in the previous Section, to identify in a clear way that the interaction between employment conditions and the confirmation bias is the main source of endogenous cyclical trajectories. It is our intention to evaluate the robustness of this result with respect to a more realistic specification of the probability functions. Our main interest lies in the possibility of coexistence of periodic attractors. We proceed by modifying them as follows:

$$
\begin{aligned}
p_{t-1}^{+} &= \frac{\exp\left(\mu\Phi_{t-1} + \beta\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right)}{\exp\left(\mu\Phi_{t-1} + \beta\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right) + \exp\left(-\mu\Phi_{t-1} - \beta\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right)} \\
p_{t-1}^{-} &= \frac{\exp\left(-\mu\Phi_{t-1} - \gamma\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right)}{\exp\left(-\mu\Phi_{t-1} - \gamma\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right) + \exp\left(\mu\Phi_{t-1} + \gamma\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right)}
\end{aligned}
\tag{20}
$$

where $\mu > 0$ captures the strength of the "group effect". For values of this parameter close to zero, individuals pay little attention to the collective opinion. On the other hand, higher values of μ are associated with a greater interaction and influence between agents.

The first expression above indicates that the greater the share of the population that believes in climate change and supports environmental regulation, the greater the probability of adopting a favourable attitude. The opposite case also holds. If the majority of individuals oppose climate policies, this increases p^-.

4.1 Dynamic system

Our new dynamic system is a 2-dimensional nonlinear map similar to (17). The only difference lies in the probability functions that now are given by Eq. (20). We continue to have the interaction between different attitudes and the rest of the economy but there is an extra inertial component in the former that is given by the so-called "group effect":

$$\begin{aligned} e_t &= (1 + a + bf\Phi_{t-1} + c - ge_{t-1})\, e_{t-1} \\ \Phi_t &= \Phi_{t-1} + \alpha \left[\frac{\exp\left(\mu\Phi_{t-1} + \beta\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right)}{\exp\left(\mu\Phi_{t-1} + \beta\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right) + \exp\left(-\mu\Phi_{t-1} - \beta\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right)} \right. \\ &\quad \left. - \frac{\exp\left(-\mu\Phi_{t-1} - \gamma\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right)}{\exp\left(-\mu\Phi_{t-1} - \gamma\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right) + \exp\left(\mu\Phi_{t-1} + \gamma\left(\frac{e_t - e_{t-1}}{e_{t-1}}\right)\right)} - \Phi_{t-1} \right] \end{aligned} \tag{21}$$

In steady-state, the equilibrium conditions are such that:

$$\begin{aligned} a + bf\Phi &= -c + ge \\ \Phi &= \tanh(\mu\Phi) \end{aligned} \tag{22}$$

Contrary to the previous case, it is not possible to say in advance if the steady-state requires or not an equal division of the population between the two groups. As expected, the hyperbolic tangent function opens the door to the possibility of multiple equilibria. We thus recover one of the main results of Dávila-Fernández and Sordi (2020) and Dávila-Fernández et al. (2020), which can be appreciated by stating and proving the following Proposition:

Proposition 3 *When the "group effect" is weak enough, $\mu \leq 1$, the dynamic system has a unique non-trivial equilibrium solution that satisfies (18). On the other hand, when $\mu > 1$, the dynamic system has two additional non-trivial equilibrium solutions, $P_2 = (e_2^*, \Phi_2^*)$ and $P_3 = (e_3^*, \Phi_3^*)$, that satisfy:*

$$\begin{aligned} e_i^* &= \frac{c + a + bf\Phi_i^*}{g} \\ \Phi_i^* &= \tanh(\mu\Phi_i^*) \end{aligned}$$

where $i = [1, 2]$ while $\Phi_2^* > 0$ and $\Phi_3^* < 0$.

Proof. See Mathematical Appendix A3. ■

A weak interaction between agents leads to a unique equilibrium point in which the population is equally divided between those who support and those who oppose climate policies.

At the threshold $\mu = 1$, however, a *Pitchfork bifurcation* occurs and the system transitions from one to three fixed points. The two additional non-trivial equilibrium solutions are such that the majority of the population either believes in or is skeptical of the environmental challenge. In the first case, the adoption of a more stringent regulation leads to a higher rate of employment. On the contrary, in the second case, given that most individuals do not support the adoption of climate policies, there is a reduction in energy efficiency that harms output. As a result, the rate of growth of the economy is reduced, leading to lower employment rates.

Since the most interesting case occurs when there is sufficient interaction between agents, we will limit the remainder of our analysis to the case in which $\mu > 1$. Regarding the stability of equilibrium solutions, we can state and prove the following Proposition:

Proposition 4 *When the group effect is strong enough, $\mu > 1$, the equilibrium points P_2 and P_3 are locally asymptotically stable in the region of the parameter space defined as:*

$$1 + \frac{ge}{\alpha(1 - ge)} + \frac{bfe}{(1 - ge)}\left(\theta_e^+ - \theta_e^-\right) - \left(\theta_\Phi^+ - \theta_\Phi^-\right) > 0$$

where

$$\begin{aligned}
\theta_\Phi^+ &= (\mu + \beta bf)\, 2\kappa \\
\theta_\Phi^- &= (\mu + \gamma bf)\, 2\kappa \\
\theta_e^+ &= 2g\beta\kappa \\
\theta_e^- &= 2g\gamma\kappa \\
\kappa &= \frac{\exp(-\mu\Phi_i^*)\exp(\mu\Phi_i^*)}{\left[\exp(-\mu\Phi_i^*) + \exp(\mu\Phi_i^*)\right]^2}
\end{aligned}$$

A violation of this condition is associated with the occurrence of a super-critical Neimark-Sacker bifurcation.

Proof. See Mathematical Appendix A4. ■

Despite some obvious similarities with previous studies in the field, particularly concerning the number and determination of equilibria, there are important differences worth stressing. This is so, besides the fact that none has specifically addressed the confirmation bias issue. On the one hand, Dávila-Fernández and Sordi (2020) developed a continuous-time model that is incapable of generating persistent dynamics. On the other hand, Dávila-Fernández et al. (2020) acknowledged the possibility of a sequence of period doubling bifurcations, but did not explore the implications of this result nor provided an economic interpretation for them. The dynamic system developed in the present paper does not admit a *Flip* or *Fold* bifurcations. Still, under certain conditions, we might have an invariant set on which many different orbits exist.[6]

Propositions 2 and 4 are equivalent in terms of their economic content. Both fundamentally highlight the emergence of endogenous waves in public opinion that interact with the rest

[6] Another important difference lies in the direction of the underlying mechanism. In the aforementioned contributions, the rate of growth of output was determined by Thirlwall's law, only afterwards impacting energy consumption. On the other hand, our model endogenises the energy-output ratio in a more general closed economy set-up.

of the economy in a persistent way. Taking β as the controlling parameter, a *Neimark-Sacker* bifurcation occurs when:

$$\beta_{NS} = \frac{\alpha\left(1-4\mu\kappa\right)\left(1-ge_i^*\right)+ge_i^*}{2\kappa\alpha bf} - \gamma \tag{23}$$

such

$$\begin{aligned}
\lim_{\alpha\to 0}\beta_{NS} &= +\infty \\
\lim_{\alpha\to 1}\beta_{NS} &= \frac{\left(1-4\mu\kappa\right)\left(1-ge_i^*\right)+ge_i^*}{2\kappa bf} - \gamma
\end{aligned}$$

In the case in which there is no "group-effect", $\mu = 0$, it is easy to see that Eq. (23) collapses into (19). The correspondence with the *confirmation bias* is also the same. If the bias is maximum, the bifurcation parameter goes to infinity and the system is stable. This is an interesting result because it provides an evolutionary justification for the existence of the bias. Notice, nonetheless, that if a society is in the "bad" equilibrium, a higher bias makes it more difficult to change. Notice that the magnitude of α does not affect the equilibrium values but plays a decisive role in their local stability properties.

Two relevant questions remain to be answered. The first is related to the boundaries between basins of attraction when $\beta < \beta_{NS}$. This is important because it indicates how difficult it is to move from the "bad" equilibrium to the "good" one. The second question concerns the case in which $\beta \geq \beta_{NS}$. Given that the dynamic system admits multiple equilibrium solutions, we might have a unique cycle or the coexistence of periodic attractors. The latter stands as a representation of a type of path dependence since, depending on initial conditions, many different orbits around different equilibrium points can potentially exist. We shall investigate both issues by means of numerical simulations.

4.2 Numerical simulations

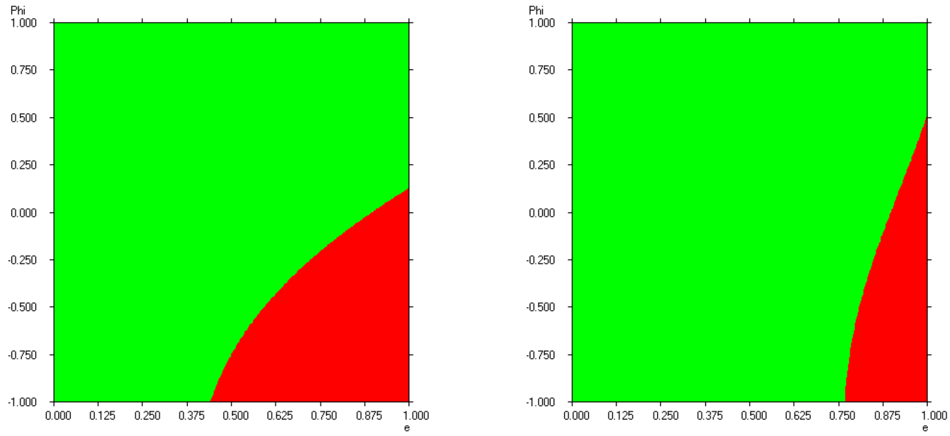
Maintaining the calibration parameters chosen in the previous Section, we still have to determine the intensity of the "group effect". In this baseline scenario, we suppose:

$$\mu = 1.025$$

With the chosen values for the parameters, the two additional equilibrium points are such that $P_2 = (0.9268, 0.26)$ and $P_3 = (0.8732, -0.26)$. Fig. 5 presents their basins of attraction when $\beta < \beta_{NS}$. In green, we have all initial conditions that converge to the more desirable state with favourable attitudes, less GHG emissions, and higher growth, P_2. In red, it is possible to appreciate the basin of attraction of the opposite case, P_3.

Under the current calibration, our model brings a message of hope. Most initial conditions lead to the "good" equilibrium point. Moreover, as we increase the response of sentiments to employment conditions, the green region expands while the red area narrows. Still, notice that it is enough to have a combination of high employment rates with negative or slightly positive attitudes to converge to the "bad" equilibrium. This is due to the fact that β was supposed to be sufficiently small so that sentiments' response to current macroeconomic conditions is low. Hence, the labour market effect is not enough to overcome initial negative attitudes.

Paradoxically, initially low employment rates seem to be associated with a green-growth equilibrium. At first, this result might sound counter-intuitive because a low e increases the



(a)

(b)

Figure 5: Basins of attraction when (a) $\beta = \gamma = 5$ and (b) $\beta = \gamma = 20$.

probability of opposing environmental regulation. Notice, however, that it is not the level of the employment rate that matters but the direction of change. Very low employment rates will inevitably converge to a higher equilibrium rate. In this process, $e_t > e_{t-1}$, thus giving impulse to positive attitudes and enlarging the basins of attraction of the green equilibrium point.

Increasing the response of sentiments to the rate of employment eventually leads to a *Neimark-Sacker bifurcation*. The equilibrium points P_2 and P_3 become unstable and a periodic orbit emerges. Still, since β_{NS} depends on e_i^*, this does not happen simultaneously. Fig. 6 reports the 1D bifurcation diagram when we set $\gamma = 50$. Persistent fluctuations first arise around the anti-environment position and only later around the pro-environment solution. For values of β beyond a critical point, the limit cycle loses its stability and an attracting torus is born including both states. In fact, the MLE is very similar to the one reported in the previous Section, indicating the existence of more complex dynamics.

Fig. 7 presents the clockwise cycles in the (e, Φ) plane. In diagram (a), we chose a value slightly above the first bifurcation threshold, $\beta = 34.8$. For initial conditions very close to P_3, there is convergence to a small orbit depicted in red. On the other hand, initial conditions in the neighbourhood of P_2 continue to converge to the corresponding equilibrium point. In all remaining cases, we have convergence to a larger orbit, in blue, that includes the three equilibria. In panel (b), with a higher β, the red orbit enlarges and merges with the blue one. When $\beta = 39.4$, the second equilibrium point, P_2, loses stability and we have a cycle around the green-growth solution. The coexistence of periodic attractors stands as a representation of path dependence in which different initial conditions are associated with different trajectories.

Further increasing β results in the fusion of the green orbit with the red one. At this point, we obtain a unique cycle that includes all three equilibrium points. The size of the area inside the cycle depends on the response of sentiments to labour market conditions. Panels (c) and (d) indicate that doubling the value of β and γ enlarges the cycle. It is possible to appreciate how colours overlap and this is the reason why we refrain from including arrows to indicate the clockwise motion of the cycle.

The rationality of the resulting persistent and bounded fluctuations follows very closely the description presented in Section 3. The interaction between employment and labour

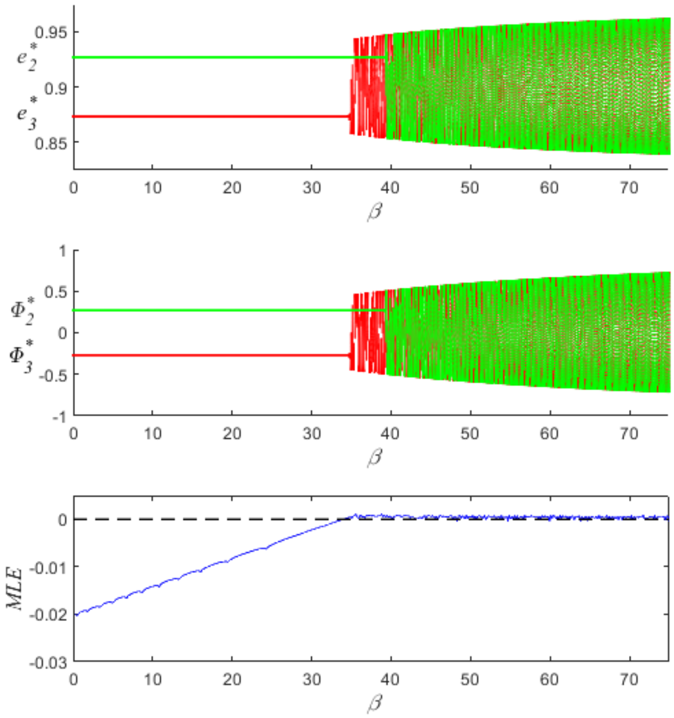


Figure 6: Bifurcation diagram and Max. Lyapunov Exponent when $\mu = 1.025$.

productivity plays a stabilising role given that firms respond to increases in employment by increasing their search for labour saving production techniques, which in turn reduces e. For the cycle to emerge, a destabilising force is necessary, which in our case comes from the interaction between employment dynamics with sentiments and environmental efficiency. An increase in the employment rate is related to a higher probability of adopting favourable attitudes towards climate policies. Through PH, this implies higher environmental efficiency, resulting in an increase in production and, consequently, in higher employment rates.

There are, however, important differences with the system presented in the first part of the paper. The most evident concerns the coexistence of periodic orbits. A less obvious difference is related to the magnitude of β_{NS}. In Fig. 3, it is possible to see that, in the first model, a $\beta \approx 200$ was required for the *Neimark-Sacker* bifurcation to occur. Without modifying our choice of parameters, Fig. 6 shows that, in the second case, a similar result can be obtained when $\beta \approx 34$. The introduction of the "group effect" implies the addition of another source of inertia – besides the confirmation bias – at the collective level. This increases the strength of the destabilising force in the system, thus reducing the magnitude of β required for the bifurcation to occur.

5 Final considerations

Psychologists among other behavioural scientists refer to *confirmation bias* as one of the most problematic aspects of human reasoning. While there are evolutionary reasons for the existence of such a mechanism, it can cause serious issues in different contexts. When it

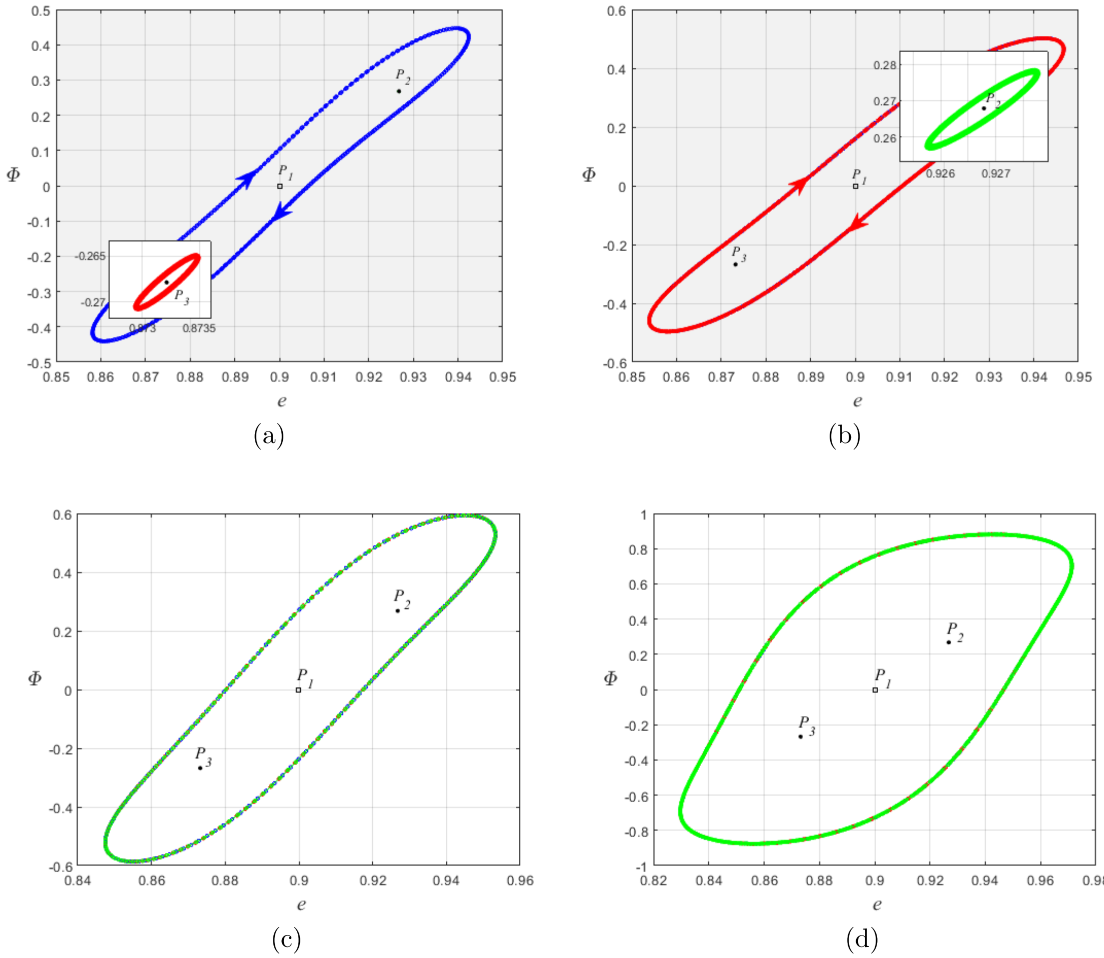


Figure 7: Emergence and coexistence of periodic attractors when (a) $\beta = 34.8$ and $\gamma = 50$, (b) $\beta = 39.4$ and $\gamma = 50$, (c) $\beta = \gamma = 50$, and (d) $\beta = \gamma = 100$.

comes to climate change, the tendency to favour information that supports one's prior beliefs increases the difficulty of tackling the environmental challenge.

By analysing the engagement on two different Twitter trend topics, #ClimateChangeIsReal *vs* #ClimateChangeHoax, prior to Greta Thunberg speech in the European Parliament, we showed how social media is a polarised battlefield. Agents intensively rely and interact with sources that confirm their prior beliefs. A simple sentiment analysis suggests that environmental supporters are equally divided in terms of gender while climate change deniers are mostly men. Tweets of the latter group were also found to be more aggressive with a higher proportion of negative sentiments.

We proceeded by developing an agent-based model to investigate how employment conditions affect attitudes towards climate policies under such a cognitive bias. We showed that endogenous fluctuations might emerge in the form of a super-critical Neimark-Sacker bifurcation. The system admits the coexistence of periodic attractors depending on the individual's response to the so-called "group effect". The emerging cycle is the outcome of the interaction between a stable and an unstable force. On the one hand, the former is related to induced-technical change. On the other hand, the latter refers to the positive response of environmental sentiments to the rate of employment.

In terms of policy implications, we highlight that the adoption of a successful green-agenda depends on the ability of policy makers to take advantage of favourable employment rates while appealing to different framing strategies. This last result is in line with studies indicating that bias effects might be contoured by focusing on the gains of climate change mitigation. In addition, more important than the level of the employment rate, our analysis suggests that policy makers should act when employment is increasing. The obtained cycles should be interpreted as long-run dynamics linked to processes of institutional and structural change. Still, if successfully tamed, they may actually be the path towards a more desirable green-equilibrium.

A Mathematical Appendix

A.1 Proof of Proposition 1

The equilibrium conditions of the dynamic system (17) are given by:

$$\begin{aligned} a &= -c + ge \\ \Phi &= 0 \end{aligned}$$

Rearranging the first expression, it immediately follows that:

$$\begin{aligned} e_1^* &= \frac{c+a}{g} \\ \Phi^{E_1} &= 0 \end{aligned}$$

is the unique non-trivial equilibrium point.

A.2 Proof of Proposition 2

In order to prove Proposition 2, we need to compute the partial derivatives of the probability functions with respect to the employment rate and to environmental attitudes. Define an auxiliary variable, s, as given by:

$$s_{t-1} = \frac{e_t - e_{t-1}}{e_{t-1}}$$

On the one hand, it is easy to see that:

$$\frac{\partial s_{t-1}}{\partial e_{t-1}} = -g$$

The probability of supporting the environment is such that:

$$\begin{aligned}
p_{t-1}^{+} &= \frac{\exp\left(\beta s_{t-1}\right)}{\exp\left(\beta s_{t-1}\right) + \exp\left(-\beta s_{t-1}\right)} \\
\frac{\partial p_{t-1}^{+}}{\partial e_{t-1}} &= \frac{\beta \frac{\partial s}{\partial e_{t-1}} \exp\left(\beta s_{t-1}\right)\left[\exp\left(\beta s_{t-1}\right) + \exp\left(-\beta s_{t-1}\right)\right]}{\left[\exp\left(\beta s_{t-1}\right) + \exp\left(-\beta s_{t-1}\right)\right]^2} \\
&\quad - \frac{\left[\beta \frac{\partial s}{\partial e_{t-1}} \exp\left(\beta s_{t-1}\right) - \beta \frac{\partial s}{\partial e_{t-1}} \exp\left(-\beta s_{t-1}\right)\right] \exp\left(\beta s_{t-1}\right)}{\left[\exp\left(\beta s_{t-1}\right) + \exp\left(-\beta s_{t-1}\right)\right]^2} \\
\left.\frac{\partial p_{t-1}^{+}}{\partial e_{t-1}}\right|_{e_t = e_{t-1}} &= \frac{\beta \frac{\partial s}{\partial e_{t-1}}}{2} = -\frac{\beta g}{2} < 0
\end{aligned}$$

In turn, the probability of opposing climate policies is given by:

$$\begin{aligned}
p_{t-1}^{-} &= \frac{\exp\left(-\gamma s_{t-1}\right)}{\exp\left(-\gamma s_{t-1}\right) + \exp\left(\gamma s_{t-1}\right)} \\
\frac{\partial p_{t-1}^{-}}{\partial e_{t-1}} &= \frac{-\gamma \frac{\partial s}{\partial e_{t-1}} \exp\left(-\gamma s_{t-1}\right)\left[\exp\left(-\gamma s_{t-1}\right) + \exp\left(\gamma s_{t-1}\right)\right]}{\left[\exp\left(-\gamma s_{t-1}\right) + \exp\left(\gamma s_{t-1}\right)\right]^2} \\
&\quad - \frac{\left[-\gamma \frac{\partial s}{\partial e_{t-1}} \exp\left(-\gamma s_{t-1}\right) + \gamma \frac{\partial s}{\partial e_{t-1}} \exp\left(\gamma s_{t-1}\right)\right] \exp\left(-\gamma s_{t-1}\right)}{\left[\exp\left(-\gamma s_{t-1}\right) + \exp\left(\gamma s_{t-1}\right)\right]^2} \\
\left.\frac{\partial p_{t-1}^{-}}{\partial e_{t-1}}\right|_{e_t = e_{t-1}} &= \frac{-\gamma \frac{\partial s}{\partial e_{t-1}}}{2} = \frac{\gamma g}{2} > 0
\end{aligned}$$

On the other hand, the derivative of s with respect to attitudes is such that:

$$\frac{\partial s_{t-1}}{\partial \Phi_{t-1}} = bf$$

Thus, we can compute:

$$\begin{aligned}
p_{t-1}^{+} &= \frac{\exp\left(\beta s_{t-1}\right)}{\exp\left(\beta s_{t-1}\right) + \exp\left(-\beta s_{t-1}\right)} \\
\frac{\partial p_{t-1}^{+}}{\partial \Phi_{t-1}} &= \frac{\beta \frac{\partial s}{\partial \Phi_{t-1}} \exp\left(\beta s_{t-1}\right)\left[\exp\left(\beta s_{t-1}\right) + \exp\left(-\beta s_{t-1}\right)\right]}{\left[\exp\left(\beta s_{t-1}\right) + \exp\left(-\beta s_{t-1}\right)\right]^2} \\
&\quad - \frac{\left[\beta \frac{\partial s}{\partial \Phi_{t-1}} \exp\left(\beta s_{t-1}\right) - \beta \frac{\partial s}{\partial \Phi_{t-1}} \exp\left(-\beta s_{t-1}\right)\right] \exp\left(\beta s_{t-1}\right)}{\left[\exp\left(\beta s_{t-1}\right) + \exp\left(-\beta s_{t-1}\right)\right]^2} \\
\left.\frac{\partial p_{t-1}^{+}}{\partial \Phi_{t-1}}\right|_{\Phi_t = \Phi_{t-1}} &= \frac{\beta \frac{\partial s}{\partial \Phi_{t-1}}}{2} = \frac{\beta bf}{2} > 0
\end{aligned}$$

while the probability of opposing climate policies is such that:

$$
\begin{aligned}
p_{t-1}^{-} &= \frac{\exp\left(-\gamma s_{t-1}\right)}{\exp\left(-\gamma s_{t-1}\right)+\exp\left(\gamma s_{t-1}\right)} \\
\frac{\partial p_{t-1}^{-}}{\partial \Phi_{t-1}} &= \frac{-\gamma \frac{\partial s}{\partial \Phi_{t-1}} \exp\left(-\gamma s_{t-1}\right)\left[\exp\left(-\gamma s_{t-1}\right)+\exp\left(\gamma s_{t-1}\right)\right]}{\left[\exp\left(-\gamma s_{t-1}\right)+\exp\left(\gamma s_{t-1}\right)\right]^{2}} \\
&\quad -\frac{\left[-\gamma \frac{\partial s}{\partial \Phi_{t-1}} \exp\left(-\gamma s_{t-1}\right)+\gamma \frac{\partial s}{\partial \Phi_{t-1}} \exp\left(\gamma s_{t-1}\right)\right] \exp\left(-\gamma s_{t-1}\right)}{\left[\exp\left(-\gamma s_{t-1}\right)+\exp\left(\gamma s_{t-1}\right)\right]^{2}} \\
\left.\frac{\partial p_{t-1}^{-}}{\partial \Phi_{t-1}}\right|_{\Phi_{t}=\Phi_{t-1}} &= \frac{-\gamma \frac{\partial s}{\partial \Phi_{t-1}}}{2}=-\frac{\gamma b f}{2}<0
\end{aligned}
$$

We are now ready to compute the Jacobian matrix of the dynamic system (17), which is defined and given by:

$$
J=\left[\begin{array}{cc}
1-g e & b f e \\
-\frac{\alpha g}{2}(\beta+\gamma) & 1-\alpha+\frac{\alpha b f}{2}(\beta+\gamma)
\end{array}\right]
$$

The coefficients of the characteristic equation are equal to:

$$
\begin{aligned}
C_{1} &= -\operatorname{tr} J \\
&= -1+g e-1+\alpha-\frac{\alpha b f}{2}(\beta+\gamma) \\
&= -2+g e+\alpha-\frac{\alpha b f}{2}(\beta+\gamma)
\end{aligned}
$$

$$
\begin{aligned}
C_{2} &= \operatorname{det} J \\
&= (1-g e)\left[1-\alpha+\frac{\alpha b f}{2}(\beta+\gamma)\right]+\frac{\alpha b f}{2}(\beta+\gamma) g e \\
&= (1-g e)(1-\alpha)+\frac{\alpha b f}{2}(\beta+\gamma)
\end{aligned}
$$

while the eigenvalues are such that:

$$
\begin{aligned}
\lambda_{1,2} &= \frac{2-g e-\alpha+\frac{\alpha b f}{2}(\beta+\gamma)}{2} \\
&\quad \pm \frac{\sqrt{\left[-2+g e+\alpha-\frac{\alpha b f}{2}(\beta+\gamma)\right]^{2}-4\left[(1-g e)(1-\alpha)+\frac{\alpha b f}{2}(\beta+\gamma)\right]}}{2}
\end{aligned}
$$

The necessary and sufficient conditions for the local stability of a given equilibrium point require that all eigenvalues of the Jacobian matrix, determined as roots of the characteristic equation, are less than unity in modulus:

$$
\begin{aligned}
1+C_{1}+C_{2} &> 0 \\
1-C_{1}+C_{2} &> 0 \\
1-C_{2} &> 0
\end{aligned}
$$

Through direct computation, we find that:

$$\begin{aligned} & 1 + C_1 + C_2 \\ = & \ 1 - 2 + ge + \alpha - \frac{\alpha b f}{2}(\beta + \gamma) + (1 - ge)(1 - \alpha) + \frac{\alpha b f}{2}(\beta + \gamma) \\ = & \ \alpha g e > 0 \end{aligned}$$

while

$$\begin{aligned} & 1 - C_1 + C_2 \\ = & \ 1 + 2 - ge - \alpha + \frac{\alpha b f}{2}(\beta + \gamma) + (1 - ge)(1 - \alpha) + \frac{\alpha b f}{2}(\beta + \gamma) \\ = & \ \alpha b f (\beta + \gamma) + (2 - ge)(2 - \alpha) > 0 \end{aligned}$$

This means that the first two conditions are always satisfied and we rule out the possibility of Fold and Flip bifurcations, respectively. Regarding the final condition:

$$\begin{aligned} & 1 - C_2 \\ = & \ ge + (1 - ge)\alpha - \frac{\alpha b f}{2}(\beta + \gamma) \gtreqless 0 \end{aligned}$$

Taking $\beta = \beta_{NS}$ as bifurcation parameter, we have that:

$$\beta_{NS} = \left[\frac{ge + (1 - ge)\alpha}{\alpha b f}\right] 2 - \gamma$$

For values of $\beta \geq \beta_{NS}$ the equilibrium point loses stability and a Neimark-Sacker bifurcation occurs. To prove that the emerging periodic orbit is stable, the transversality, the nonresonancy, and the nondegeneracy conditions need to be satisfied.

With regard to the first of them, notice that the eigenvalues are such that:

$$\begin{aligned} \left.\frac{\partial \lambda_{1,2}}{\partial \beta}\right|_{\beta = \beta_{NS}} &= \frac{\alpha b f}{2} \pm \frac{1}{2} \frac{\alpha g e \frac{\alpha b f}{2}}{\sqrt{ge\alpha (ge\alpha - 4)}} \\ &= \frac{\alpha b f}{2}\left(1 \pm \frac{1}{2}\sqrt{\frac{\alpha g e}{\alpha g e - 4}}\right) \neq 0 \end{aligned}$$

Euler's formula provides a means of conversion between cartesian coordinates and polar coordinates

$$\lambda_{1,2} = r(\beta)\exp(i\varphi)$$

where r is the magnitude of the eigenvalue and φ is the angle of a line connecting the origin with a point on the unit circle. Because $\left.\frac{\partial \lambda_{1,2}}{\partial \beta}\right|_{\beta=\beta_{NS}} \neq 0$, it follows that $\left.\frac{\partial r}{\partial \beta}\right|_{\beta=\beta_{NS}} \neq 0$ and the transversality condition is satisfied.

The eigenvalues when $\beta = \beta_{NS}$ are equal to

$$\begin{aligned} \lambda_{1,2}\big|_{\beta=\beta_{NS}} &= \frac{2 - \alpha g e \pm \sqrt[2]{(-2 + \alpha g e)^2 - 4}}{2} \\ &= 1 - \frac{\alpha g e}{2} \pm i\sqrt[2]{1 - \left(1 - \frac{\alpha g e}{2}\right)^2} \end{aligned}$$

This means

$$\begin{aligned}
r\left(\beta_{NS}\right) &= \left|\lambda_{1,2}\right| = \sqrt[2]{\left(1-\frac{\alpha ge}{2}\right)^2 + \left[\sqrt[2]{1-\left(1-\frac{\alpha ge}{2}\right)^2}\right]^2} \\
&= \sqrt[2]{\left(1-\frac{\alpha ge}{2}\right)^2 + 1 - \left(1-\frac{\alpha ge}{2}\right)^2} \\
&= 1
\end{aligned}$$

Moreover,

$$\begin{aligned}
\varphi &= \arctan\left(\frac{\sqrt[2]{1-\left(1-\frac{\alpha ge}{2}\right)^2}}{1-\frac{\alpha ge}{2}}\right) \\
&= \arctan\left(\sqrt[2]{\frac{1-\left(1-\frac{\alpha ge}{2}\right)^2}{\left(1-\frac{\alpha ge}{2}\right)^2}}\right) \\
&= \arctan\left(\sqrt[2]{\frac{1}{\left(1-\frac{\alpha ge}{2}\right)^2}-1}\right)
\end{aligned}$$

$$0 < \sqrt[2]{\frac{1}{\left(1-\frac{\alpha ge}{2}\right)^2}-1} < \sqrt[2]{3} \Rightarrow 0 < \varphi < 60°$$

Therefore:

$$\begin{aligned}
\lambda_{1,2}\big|_{\beta=\beta_{NS}} &= r\left(\beta_{NS}\right)\exp\left(i\varphi\right) \\
&= \exp\left(i\varphi\right) \neq 1
\end{aligned}$$

and the nonresonant condition is satisfied.

The last condition is the nondegeneracy one. Given the extremely long algebraic steps, this final part of the demonstration is available on request.

A.3 Proof of Proposition 3

The equilibrium conditions of the dynamic system (21) are given by:

$$\begin{aligned}
a + bf\Phi &= -c + ge \\
\Phi &= \tanh\left(\mu\Phi\right)
\end{aligned}$$

From the properties of the hyperbolic tangent function, we know that, when $\mu \leq 1$, the second expression will be satisfied iff $\Phi = 0$. In this case, we have:

$$\begin{aligned}
e_1^* &= \frac{c+a}{g} \\
\Phi^{E_1} &= 0
\end{aligned}$$

as the unique nontrivial equilibrium solution.

On the other hand, when $\mu > 1$, from the properties of the hyperbolic tangent function, we know there are two additional values of Φ that satisfy the second equilibrium condition. Accordingly, rearranging the first equation, the other two equilibrium points are such that:

$$\begin{aligned} e_i^* &= \frac{c + a + bf\Phi_i^*}{g} \\ \Phi_i^* &= \tanh\left(\mu\Phi_i^*\right) \end{aligned}$$

where $i = [1, 2]$ while $\Phi_2^* > 0$ and $\Phi_3^* < 0$.

A.4 Proof of Proposition 4

In order to prove Proposition 4, we need to compute the partial derivatives of the probability functions with respect to the employment rate and to environmental attitudes. Define an auxiliary variable, s, as given by:

$$s_{t-1} = \frac{e_t - e_{t-1}}{e_{t-1}}$$

On the one hand, it is easy to see that:

$$\frac{\partial s_{t-1}}{\partial e_{t-1}} = -g$$

Through direct computation, notice that:

$$\begin{aligned} p_{t-1}^{+} &= \frac{\exp\left(\mu\Phi_{t-1} + \beta s_{t-1}\right)}{\exp\left(\mu\Phi_{t-1} + \beta s_{t-1}\right) + \exp\left(-\mu\Phi_{t-1} - \beta s_{t-1}\right)} \\ \frac{\partial p_{t-1}^{+}}{\partial e_{t-1}} &= \frac{\beta\frac{\partial s}{\partial e_{t-1}}\exp\left(\mu\Phi_{t-1} + \beta s_{t-1}\right)\left[\exp\left(\mu\Phi_{t-1} + \beta s_{t-1}\right) + \exp\left(-\mu\Phi_{t-1} - \beta s_{t-1}\right)\right]}{\left[\exp\left(\mu\Phi_{t-1} + \beta s_{t-1}\right) + \exp\left(-\mu\Phi_{t-1} - \beta s_{t-1}\right)\right]^2} \\ &\quad - \frac{\left[\beta\frac{\partial s}{\partial e_{t-1}}\exp\left(\mu\Phi_{t-1} + \beta s_{t-1}\right) - \beta\frac{\partial s}{\partial e_{t-1}}\exp\left(-\mu\Phi_{t-1} - \beta s_{t-1}\right)\right]\exp\left(\mu\Phi_{t-1} + \beta s_{t-1}\right)}{\left[\exp\left(\mu\Phi_{t-1} + \beta s_{t-1}\right) + \exp\left(-\mu\Phi_{t-1} - \beta s_{t-1}\right)\right]^2} \\ \left.\frac{\partial p_{t-1}^{+}}{\partial e_{t-1}}\right|_{e_t = e_{t-1}} &= \frac{2\beta\frac{\partial s}{\partial e_{t-1}}\exp\left(\mu\Phi_{t-1}\right)\exp\left(-\mu\Phi_{t-1}\right)}{\left[\exp\left(\mu\Phi_{t-1}\right) + \exp\left(-\mu\Phi_{t-1}\right)\right]^2} = \theta_e^{+} < 0 \end{aligned}$$

while

$$\begin{aligned} p_{t-1}^{-} &= \frac{\exp\left(-\mu\Phi_{t-1} - \gamma s_{t-1}\right)}{\exp\left(-\mu\Phi_{t-1} - \gamma s_{t-1}\right) + \exp\left(\mu\Phi_{t-1} + \gamma s_{t-1}\right)} \\ \frac{\partial p_{t-1}^{-}}{\partial e_{t-1}} &= \frac{-\gamma\frac{\partial s}{\partial e_{t-1}}\exp\left(-\mu\Phi_{t-1} - \gamma s_{t-1}\right)\left[\exp\left(-\mu\Phi_{t-1} - \gamma s_{t-1}\right) + \exp\left(\mu\Phi_{t-1} + \gamma s_{t-1}\right)\right]}{\left[\exp\left(-\mu\Phi_{t-1} - \gamma s_{t-1}\right) + \exp\left(\mu\Phi_{t-1} + \gamma s_{t-1}\right)\right]^2} \\ &\quad - \frac{\left[-\gamma\frac{\partial s}{\partial e_{t-1}}\exp\left(-\mu\Phi_{t-1} - \gamma s_{t-1}\right) + \gamma\frac{\partial s}{\partial e_{t-1}}\exp\left(\mu\Phi_{t-1} + \gamma s_{t-1}\right)\right]\exp\left(-\mu\Phi_{t-1} - \gamma s_{t-1}\right)}{\left[\exp\left(-\mu\Phi_{t-1} - \gamma s_{t-1}\right) + \exp\left(\mu\Phi_{t-1} + \gamma s_{t-1}\right)\right]^2} \\ \left.\frac{\partial p_{t-1}^{-}}{\partial e_{t-1}}\right|_{e_t = e_{t-1}} &= \frac{-2\gamma\frac{\partial s}{\partial e_{t-1}}\exp\left(\mu\Phi_{t-1}\right)\exp\left(-\mu\Phi_{t-1}\right)}{\left[\exp\left(-\mu\Phi_{t-1}\right) + \exp\left(\mu\Phi_{t-1}\right)\right]^2} = \theta_e^{-} > 0 \end{aligned}$$

On the other hand, the derivative of s with respect to attitudes is such that:

$$\frac{\partial s_{t-1}}{\partial \Phi_{t-1}} = bf$$

Therefore, the probability of having positive attitudes responds to the current state of sentiments as follows:

$$\begin{aligned}
p^+_{t-1} &= \frac{\exp\left(\mu\Phi_{t-1}+\beta s_{t-1}\right)}{\exp\left(\mu\Phi_{t-1}+\beta s_{t-1}\right)+\exp\left(-\mu\Phi_{t-1}-\beta s_{t-1}\right)} \\
\frac{\partial p^+_{t-1}}{\partial\Phi_{t-1}} &= \frac{\left(\mu+\beta\frac{\partial s}{\partial\Phi_{t-1}}\right)\exp\left(\mu\Phi_{t-1}+\beta s_{t-1}\right)\left[\exp\left(\mu\Phi_{t-1}+\beta s_{t-1}\right)+\exp\left(-\mu\Phi_{t-1}-\beta s_{t-1}\right)\right]}{\left[\exp\left(\mu\Phi_{t-1}+\beta s_{t-1}\right)+\exp\left(-\mu\Phi_{t-1}-\beta s_{t-1}\right)\right]^2} \\
&\quad -\frac{\left[\left(\mu+\beta\frac{\partial s}{\partial\Phi_{t-1}}\right)\exp\left(\mu\Phi_{t-1}+\beta s_{t-1}\right)-\left(\mu+\beta\frac{\partial s}{\partial\Phi_{t-1}}\right)\exp\left(-\mu\Phi_{t-1}-\beta s_{t-1}\right)\right]\exp\left(\mu\Phi_{t-1}+\beta s_{t-1}\right)}{\left[\exp\left(\mu\Phi_{t-1}+\beta s_{t-1}\right)+\exp\left(-\mu\Phi_{t-1}-\beta s_{t-1}\right)\right]^2} \\
\left.\frac{\partial p^+_{t-1}}{\partial\Phi_{t-1}}\right|_{\Phi_t=\Phi_{t-1}} &= \frac{2\left(\mu+\beta\frac{\partial s}{\partial\Phi_{t-1}}\right)\exp\left(\mu\Phi_{t-1}\right)\exp\left(-\mu\Phi_{t-1}\right)}{\left[\exp\left(\mu\Phi_{t-1}\right)+\exp\left(-\mu\Phi_{t-1}\right)\right]^2}=\theta^+_\Phi>0
\end{aligned}$$

while the probability of having a negative attitude is such that:

$$\begin{aligned}
p^-_{t-1} &= \frac{\exp\left(-\mu\Phi_{t-1}-\gamma s_{t-1}\right)}{\exp\left(-\mu\Phi_{t-1}-\gamma s_{t-1}\right)+\exp\left(\mu\Phi_{t-1}+\gamma s_{t-1}\right)} \\
\frac{\partial p^-_{t-1}}{\partial\Phi_{t-1}} &= \frac{\left(-\mu-\gamma\frac{\partial s}{\partial\Phi_{t-1}}\right)\exp\left(-\mu\Phi_{t-1}-\gamma s_{t-1}\right)\left[\exp\left(-\mu\Phi_{t-1}-\gamma s_{t-1}\right)+\exp\left(\mu\Phi_{t-1}+\gamma s_{t-1}\right)\right]}{\left[\exp\left(-\mu\Phi_{t-1}-\gamma s_{t-1}\right)+\exp\left(\mu\Phi_{t-1}+\gamma s_{t-1}\right)\right]^2} \\
&\quad -\frac{\left[\left(-\mu-\gamma\frac{\partial s}{\partial\Phi_{t-1}}\right)\exp\left(-\mu\Phi_{t-1}-\gamma s_{t-1}\right)+\left(\mu+\gamma\frac{\partial s}{\partial\Phi_{t-1}}\right)\exp\left(\mu\Phi_{t-1}+\gamma s_{t-1}\right)\right]\exp\left(-\gamma s_{t-1}\right)}{\left[\exp\left(-\mu\Phi_{t-1}-\gamma s_{t-1}\right)+\exp\left(\mu\Phi_{t-1}+\gamma s_{t-1}\right)\right]^2} \\
\left.\frac{\partial p^-_{t-1}}{\partial\Phi_{t-1}}\right|_{\Phi_t=\Phi_{t-1}} &= \frac{-2\left(\mu+\gamma\frac{\partial s}{\partial\Phi_{t-1}}\right)\exp\left(-\mu\Phi_{t-1}\right)\exp\left(\mu\Phi_{t-1}\right)}{\left[\exp\left(-\mu\Phi_{t-1}\right)+\exp\left(\mu\Phi_{t-1}\right)\right]^2}=\theta^-_\Phi<0
\end{aligned}$$

We are now ready to compute the Jacobian matrix of the dynamic system (21), which is defined and given by:

$$J=\begin{bmatrix} 1-ge & bfe \\ \alpha\underbrace{\left(\theta^+_e-\theta^-_e\right)}_{<0} & 1-\alpha+\alpha\underbrace{\left(\theta^+_\Phi-\theta^-_\Phi\right)}_{>0}\end{bmatrix}$$

The coefficients of the characteristic equation are:

$$\begin{aligned}
C_1 &= -\mathrm{tr}J \\
&= -1+ge-1+\alpha-\alpha\left(\theta^+_\Phi-\theta^-_\Phi\right) \\
&= -2+ge+\alpha-\alpha\left(\theta^+_\Phi-\theta^-_\Phi\right)
\end{aligned}$$

$$\begin{aligned}
C_2 &= \det J \\
&= (1-ge)\left[1-\alpha+\alpha\left(\theta^+_\Phi-\theta^-_\Phi\right)\right]-\alpha\left(\theta^+_e-\theta^-_e\right)bfe
\end{aligned}$$

The necessary and sufficient conditions for the local stability of a given equilibrium point require that all eigenvalues of the Jacobian matrix, determined as roots of the characteristic equation, are less than unity in modulus:

$$\begin{aligned}
1+C_1+C_2&>0\\
1-C_1+C_2&>0\\
1-C_2&>0
\end{aligned}$$

Through direct computation, we find that:

$$\begin{aligned}
& 1+C_1+C_2 \\
= \; & 1-2+ge+\alpha-\alpha\left(\theta_\Phi^+-\theta_\Phi^-\right)+(1-ge)\left[1-\alpha+\alpha\left(\theta_\Phi^+-\theta_\Phi^-\right)\right]-\alpha\left(\theta_e^+-\theta_e^-\right)bfe \\
= \; & \alpha ge-ge\alpha\left(\theta_\Phi^+-\theta_\Phi^-\right)-\alpha\left(\theta_e^+-\theta_e^-\right)bfe>0
\end{aligned}$$

and

$$\begin{aligned}
& 1-C_1+C_2 \\
= \; & 1+2-ge-\alpha+\alpha\left(\theta_\Phi^+-\theta_\Phi^-\right)+(1-ge)\left[1-\alpha+\alpha\left(\theta_\Phi^+-\theta_\Phi^-\right)\right]-\alpha\left(\theta_e^+-\theta_e^-\right)bfe \\
= \; & \alpha\left(\theta_\Phi^+-\theta_\Phi^-\right)+(2-\alpha)(2-ge)+\alpha\left(\theta_\Phi^+-\theta_\Phi^-\right)(1-ge)-\alpha\left(\theta_e^+-\theta_e^-\right)bfe>0
\end{aligned}$$

implying that the first two conditions are always satisfied.

Regarding the last one, we have:

$$\begin{aligned}
& 1-C_2 \\
= \; & 1-(1-ge)\left[1-\alpha+\alpha\left(\theta_\Phi^+-\theta_\Phi^-\right)\right]+\alpha\left(\theta_e^+-\theta_e^-\right)bfe \\
= \; & 1-(1-\alpha)(1-ge)-\alpha\left(\theta_\Phi^+-\theta_\Phi^-\right)(1-ge)+\alpha\left(\theta_e^+-\theta_e^-\right)bfe \gtreqless 0
\end{aligned}$$

The expression above will be positive as long as:

$$\left(\theta_\Phi^+-\theta_\Phi^-\right)=1+\frac{ge}{\alpha(1-ge)}+\frac{bfe}{(1-ge)}\left(\theta_e^+-\theta_e^-\right) \tag{A.1}$$

If a change in one of the parameters determines the violation of this last condition, a Neimark-Sacker bifurcation occurs. Define:

$$\kappa=\frac{\exp\left(-\mu\Phi_{t-1}\right)\exp\left(\mu\Phi_{t-1}\right)}{\left[\exp\left(-\mu\Phi_{t-1}\right)+\exp\left(\mu\Phi_{t-1}\right)\right]^2} \tag{A.2}$$

Substitute Eq. (A.2) into the definitions of θ_e^+, θ_e^-, θ_Φ^+, and θ_Φ^-. Insert the resulting expressions into Eq. (A.1).Taking β as our bifurcation parameter in the resulting expression, we have that the system admits a family of periodic solutions when:

$$\beta \;\geq\; \left[\frac{\alpha(1-4\mu\kappa)(1-ge)+ge}{2\kappa\alpha bf}\right]-\gamma$$

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