

Proposal for new multidimensional and fuzzy measures  
of poverty and inequality at national and regional level

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## Abstract

This paper provides a step-by-step account of how fuzzy measures of monetary poverty and non-monetary deprivation may be constructed based on survey data such as from EU-SILC. For non-monetary deprivation, meaning dimensions or groupings of initial items of deprivation are identified using explanatory and confirmatory factor analyses, and a weighting system is applied for the aggregation individual items into the dimension they represent. Some numerical results for EU countries are given using EU-SILC 2007 data.

## 1. Introduction

### 1.1 Traditional poverty approach

The traditional poverty approach is characterized by a simple dichotomization of the population into poor and non poor defined in relation to some chosen poverty line that represents a certain percentage (generally 50%, 60% or 70%) of the mean or the median of the equivalised income<sup>2</sup> distribution.

The traditional poverty method takes place in two different and successive stages: the first aims to identify who is poor and who is not according to whether a person's income is below a critical threshold, the poverty line; the second stage consists of summarising the amount of poverty in aggregate indices that are defined in relation to the income of the poor and the poverty line.

This approach presents two main limitations: firstly, it is unidimensional, i.e. it refers to only one proxy of poverty, namely low income or consumption expenditure, and secondly it divides the population into a simple dichotomy.

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<sup>2</sup> The equivalised income of a household is obtained by dividing its total disposable income by the household's equivalent size computed by using an equivalence scale which takes into account the actual size and composition of the household.

However, poverty is a complex phenomenon that cannot be reduced solely to monetary dimension but it must also take account of non-monetary indicators of living conditions; moreover it is not an attribute that characterises an individual in terms of presence or absence, but is rather a vague predicate that manifests itself in different shades and degrees.

## 1.2 Fuzzy and multidimensional approach

Nowadays the multidimensional nature of poverty is a widely recognised fact, not only by the international scientific community, but also by many official statistical agencies (e.g. Eurostat, Istat) and by international institutions (United Nations, World Bank). This fact implies a more complete and realistic vision of this phenomenon and also an increased complexity at both the conceptual and the analytical levels. Such a complexity determines the need for adequate tools of analysis and the availability of statistical data that have to be adequate too, complete and reliable.

The fuzzy approach considers poverty as a matter of degree rather than an attribute that is simply present or absent for individuals in the population. In this case, two additional aspects have to be introduced:

- i. The choice of *membership functions* (m.f.), i.e. quantitative specification of individuals' or households' degrees of poverty and deprivation;
- ii. The choice of rules for the manipulation of the resulting fuzzy sets, as complements, intersections, union and aggregation.

### 1.2.1. Fuzzy monetary

In the conventional approach, the m.f. may be seen as  $\mu(y_i) = 1$  if  $y_i < z$ ,  $\mu(y_i) = 0$  if  $y_i \geq z$  where  $y_i$  is the equivalised income of individual  $i$  and  $z$  is the poverty line.

An early attempt to incorporate the concept of poverty as a matter of degree at methodological level was made by Cerioli and Zani (1990) who drew inspiration from the theory of *Fuzzy Sets* initiated by Zadeh (1965). They proposed the introduction of a transition zone ( $z_1 - z_2$ ) between the two states, a zone over which the m.f. declines from 1 to 0 linearly:

$$\mu_i = 1 \text{ if } y_i < z_1; \quad \mu_i = \frac{z_2 - y_i}{z_2 - z_1} \text{ if } z_1 \leq y_i \leq z_2; \quad \mu_i = 0 \text{ if } y_i > z_2 \quad (1)$$

Subsequently, Cheli and Lemmi (1995) proposed the so called *Totally Fuzzy and Relative* (TFR) approach in which the m.f. is defined as the distribution function  $F(y_i)$  of income, normalised (linearly transformed) so as to equal 1 for the poorest and 0 for the richest person in the population. In order to make this mean equal to some specified value (such as 0.1) so as to facilitate comparison with the conventional poverty rate, Cheli (1995) takes the m.f. as normalized distribution function, raised to some power  $\alpha \geq 1$ . Formally:

$$\mu_i = FM_i = (1 - F_{(M),i})^\alpha = \left( \frac{\sum_{\gamma=i+1}^n w_\gamma | y_\gamma > y_i}{\sum_{\gamma=2}^n w_\gamma | y_\gamma > y_1} \right)^\alpha, i = 1, 2, \dots, n; \mu_n = 0 \quad (2)$$

where  $y_i$  is the equivalised income of the  $i$ -th individual,  $F_{(M),i}$  is the value of the income distribution function  $F(y_i)$  for the  $i$ -th individual,  $(1 - F_{(M),i})$  is the proportion of individuals less poor than the person concerned with mean  $\frac{1}{2}$  by definition,  $w_\gamma$  is the sample weight of individual of rank  $\gamma$  in the ascending income distribution and  $\alpha$  is a parameter.

The value of  $\alpha$  is arbitrary, but Cheli and Betti (1999) have chosen the parameter  $\alpha$  so that the mean of the m.f. is equal to the head count ratio computed for the official poverty line. Increasing the value of this exponent implies giving more weight to the poorer end of the income distribution.

Betti and Verma (1999) have used a somewhat refined version of the expression (2) in order to define what they called Fuzzy Monetary indicator (FM):

$$\mu_i = FM_i = (1 - L_{(M),i})^\alpha = \left( \frac{\sum_{\gamma=i+1}^n w_\gamma y_\gamma | y_\gamma > y_i}{\sum_{\gamma=2}^n w_\gamma y_\gamma | y_\gamma > y_1} \right)^\alpha, i = 1, 2, \dots, n; \mu_n = 0 \quad (3)$$

where  $y_\gamma$  is the equivalised income and  $L_{(M),i}$  represent the value of the Lorenz curve of income for individual  $i$ ; then  $1 - L_{(M),i}$  represents the share of the total equivalised income received by all individuals who are less poor than the person concerned. It

varies from 1 for the poorest to 0 for the richest individual. The mean of  $1 - L_{(M),i}$  values equals  $(1+G)/2$ , where  $G$  is the Gini coefficient of the distribution.

### 1.2.2. Fuzzy supplementary

In addition to the level of monetary income, the standard of living of households and individuals can be described by a host of indicators, such as housing conditions, possession of durable goods, perception of hardship, expectations, norms and values.

To quantify and put together diverse indicators of deprivation several steps are necessary. Specially, decisions are required to assigning numerical values to the ordered categories, weighting the score to construct composite indicators, choosing their appropriate distributional form and scaling the resulting measures in a meaningful way.

Firstly, from the large set which may be available, a selection has to be made of indicators which are substantively meaningful and useful for a given analysis. Secondly, it is useful to identify the underlying dimensions and to group the indicators accordingly (these steps will be described in details in the next sections).

Whelan *et al.* (2001) suggest, as the first stage in an analysis of life-style deprivation, examining systematically the range of deprivation items to see whether the items cluster into distinct groups. Factor analysis can be used to identify such clusters of interrelated variables. In the same way as the FM indicator, a *Fuzzy Supplementary* ( $FS_{hi}$ ) index for dimension  $h$  can be defined in two alternative manners:

- i. The proportion of individuals who are less deprived than  $i$ :

$$\mu_i = FS_{hi} = (1 - F_{(S),hi})^\alpha \quad (4)$$

where  $F_{(S),hi}$  is the distribution function of  $S$  evaluated for individual  $i$  dimension  $h$ .

- ii. The share of the total non-deprivation  $S$  assigned to all individuals less deprived than  $i$ :

$$\mu_i = FS_{hi} = (1 - L_{(S),hi})^\alpha \quad (5)$$

where  $L_{(S),hi}$  is the value of the Lorenz curve of  $S$  for individual  $i$  in dimension  $h$ . The parameter  $\alpha$  is determined so as to make the overall non-monetary deprivation rate numerically identical to the monetary poverty rate  $H$ .

## 2. Proposal for new multidimensional and fuzzy

### 2.1 Fuzzy Monetary Indicator

In order to calculate the Fuzzy Monetary Indicator (FM) we consider the distribution of household equivalised disposal income (variable HX090 in EU-SILC) assigned to each individual. The distribution of the equivalised disposal income is trimmed taking as low bound 15% of the median of the same distribution. This distribution is referred as  $y$ .

The proposed FM Indicator is defined as combination of the  $(1 - F_{(M),i})$  indicator, the proportion of individuals less poor than the person concerned, proposed by Cheli and Lemmi (1995), and of the  $(1 - L_{(M),i})$  indicator, the share of the total equivalised income received by all individuals less poor than the person concerned, proposed by Betti and Verma (1999). Formally:

$$\mu_i = FM_i = (1 - F_{(M),i})^{\alpha-1} (1 - L_{(M),i}) = \left( \frac{\sum_{\gamma=i+1}^n w_\gamma | y_\gamma > y_i}{\sum_{\gamma=2}^n w_\gamma | y_\gamma > y_1} \right)^{\alpha-1} \left( \frac{\sum_{\gamma=i+1}^n w_\gamma y_\gamma | y_\gamma > y_i}{\sum_{\gamma=2}^n w_\gamma y_\gamma | y_\gamma > y_1} \right) \quad (6)$$

where, as in section 1.2,  $y_\gamma$  is the equivalised income,  $F_{(M),i}$  is the income distribution function,  $w_\gamma$  is the sample weight of individual of rank  $\gamma$  ( $\gamma = 1, \dots, n$ ) in the ascending income distribution,  $L_{(M),i}$  represent the value of the Lorenz curve of income for individual  $i$ .

The parameter  $\alpha$  is estimated so that the mean of the FM indicator is equal to the head count ratio computed for the official poverty line (60% of the median).

### 2.2 Fuzzy Supplementary Indicator

In addition to the level of monetary income, the standard of living of households and individuals can be described by a host of indicators, such as housing conditions, possession of durable goods, perception of hardship, expectations, norms and values.

To quantify and put together diverse indicators several steps are necessary.

1. Identification of items;
2. Transformation of the items into the  $[0, 1]$  interval;
3. Exploratory and confirmatory factor analysis;

4. Calculation of weights within each dimension (each group);
5. Calculation of scores for each dimension;
6. Calculation of an overall score and the parameter  $\alpha$  ;
7. Construction of the fuzzy deprivation measure in each dimension (and overall).

### 2.2.1. Calculation of the deprivation score for each dimension

Aggregation over a group of items in a particular dimension  $h$  ( $h = 1, 2, \dots, m$ ) is given by a weighted mean taken over  $j$  items:  $s_{hi} = \sum w_{hj} \cdot s_{hj,i} / w_{hj}$  where  $w_{hj}$  is the weight of the  $j$ -th deprivation variable in the  $h$ -th dimension (see Section 6).

### 2.2.2. Calculation of an overall score and the parameter $\alpha$

An overall score for the  $i$ -th individual is calculated as the unweighted mean:

$$s_i = \frac{\sum_{h=1}^m s_{hi}}{m} \quad (7)$$

Then, we calculate the FS indicator for the  $i$ -th individual over all dimensions as:

$$FS_i = (1 - F_{(S),i})^{\alpha-1} (1 - L_{(S),i}) \quad (8)$$

As for FM indicator, the parameter  $\alpha$  is determined so as to make the overall non-monetary deprivation rate numerically identical to the head count ratio computed for the official poverty line (60% of the median).

The parameter  $\alpha$  estimated is used to calculate the FS indicator for each dimension of deprivation separately.

### 2.2.3. Construction of the fuzzy deprivation measure in each dimension

The FS indicator for the  $h$ -th deprivation dimension for the  $i$ -th individual is defined as combination of the  $(1 - F_{(S),hi})$  indicator the  $(1 - L_{(S),hi})$  indicator .

$$\mu_i = FS_{hi} = (1 - F_{(S),hi})^{\alpha-1} (1 - L_{(S),hi}) = \left[ \frac{\sum_{\gamma=i+1}^n w_{h\gamma} | s_{h\gamma} > s_{hi}}{\sum_{\gamma=2}^n w_{h\gamma} | s_{h\gamma} > s_{h1}} \right]^{\alpha-1} \left[ \frac{\sum_{\gamma=i+1}^n w_{h\gamma} s_{h\gamma} | s_{h\gamma} > s_{hi}}{\sum_{\gamma=2}^n w_{h\gamma} s_{h\gamma} | s_{h\gamma} > s_{h1}} \right], \quad (9)$$

$$h = 1, 2, \dots, m; i = 1, 2, \dots, n; \mu_{hm} = 0$$

The  $(1 - F_{(S),hi})$  indicator for the  $i$ -th individual is the proportion of individuals who are less deprived, in the  $h$ -th dimension, than the individual concerned.  $F_{(S),hi}$  is the value of the score distribution function evaluated for individual  $i$  in dimension  $h$  and  $w_{h\gamma}$  is the sample weight of the  $i$ -th individual of rank  $\gamma$  in the ascending score distribution in the  $h$ -th dimension.

The  $(1 - L_{(S),hi})$  indicator is the share of the total lack of deprivation score assigned to all individuals less deprived than the person concerned.  $L_{(S),hi}$  is the value of the Lorenz curve of score in the  $h$ -th dimension for the  $i$ -th individual. The parameter  $\alpha$  is calculated only once as shown in section 2.2.2.

### **3. EU\_SILC data set and identification of items**

In the present work we use data from the European Survey on Income and Living Conditions (EU-SILC), distributed by Eurostat. The EU-SILC survey was designed to collect detailed information on the income of each household member, and on various aspects of the material and demographic situation of the household. A representative random sample of households throughout the country is approached to provide the required information. Data are available at cross-sectional level for years 2004, 2005, 2006 and 2007. In round 2004 only EU 15 countries are present; in rounds 2005 and 2006, 26 countries are present and in round 2007 27 countries. Below in Table 1 we report the number of households interviewed for each country.

Firstly, from the large set of EU-SILC variables, a selection has been made of indicators which are substantively meaningful and useful for the construction of Fuzzy Supplementary Indicators.

For our purpose, we have identified a set of items which could serve as indicators of concept of life-style deprivation. All these items are considered at household level, even if some of them are taken from the individual dataset and then aggregated to household level.



Table 1. EU-SILC household sample sizes. Waves 2004-2007

<b>Country</b>	<b>2004</b>	<b>2005</b>	<b>2006</b>	<b>2007</b>
AT	4,521	5,148	6,028	6,806
BE	5,275	5,137	5,860	6,348
CY		3,746	3,621	3,505
CZ		4,351	7,483	9,675
DE		13,106	13,799	14,153
DK	6,866	5,957	5,711	5,783
EE	3,993	4,169	5,631	5,146
ES	15,355	12,996	12,205	12,329
FI	11,200	11,229	10,868	10,624
FR	10,273	9,754	10,036	10,498
GR	6,252	5,568	5,700	5,643
HU		6,927	7,722	8,737
IE	5,477	6,085	5,836	5,608
IS	2,907	2,928	2,845	2,872
IT	24,270	22,032	21,499	20,982
LT		4,441	4,660	4,975
LU	3,571	3,622	3,836	3,885
LV		3,843	4,315	4,471
MT				3,477
NL		9,356	8,986	10,219
NO	6,046	5,991	5,768	6,013
PL		16,263	14,914	14,286
PT	4,989	4,615	4,367	4,310
SE	5,748	6,133	6,803	7,183
SI		8,287	9,478	8,707
SK		5,147	5,105	4,941
UK		10,826	9,902	9,275
<b>TOT</b>	<b>116,743</b>	<b>197,657</b>	<b>202,978</b>	<b>210,451</b>

The first set of items regards the lack of possession of a widely-desired item. These are:

- A telephone including mobile phone;
- A colour TV;
- A computer;
- A washing machine;
- A car.

In all these cases we consider a household to be deprived only if the lack of the item is enforced, in the sense that the household would like to have the item but cannot afford it. A second set of items relates to the lack of ability to afford items that are considered as basic:

- Keeping home adequately warm;

- Paying for one week annual holiday away from home;
- Eating a meal with meat, chicken, fish (or vegetarian equivalent) every second day;
- Being able to meet unexpected financial expenses.

A third set relates to absence of housing facilities, considered so basic that one can presume all household to wish to have them:

- A bath or shower in dwelling;
- An indoor flushing toilet for sole use of the household.

The fourth set of items relates to problems with accommodation and the environment, with the implicit assumption that the households wish to avoid such difficulties:

- Leaking roof, damp walls/floors/foundation, or rot in window frames or floor;
- Too dark, not enough light in dwelling;
- Noise from neighbours or from the street;
- Pollution, grime or other environmental problems;
- Crime violence or vandalism in the area.

The fifth set relates to arrears in paying bills that the household has experienced in the last 12 months;

- Arrears on mortgage or rent payments;
- Arrears on utility bills;
- Arrears on hire purchase instalments or other loan payments.

The sixth set is just one item related to the capacity of the household to make ends meet.

The seventh set relates to the health condition of the household. These items are from individual variables that have been aggregated at household level. We consider this dimension because we think that, in dealing with life-style deprivation, a lack of good health is also important. The items considered are:

- General health;
- Suffer from any chronic (long-standing) illness or condition;
- Limitation in activities because of health problems;
- Unmet need for medical examination or treatment;
- Unmet need for dental examination or treatment.

This dimension is not comparable for register countries, for which the unit of analysis is just the selected respondent.

The eighth set relates to the education. For this set we have constructed two composite indicators:

- Households with early school leavers not in education or training;  
Households with at least one person aged 18-24 with only lower secondary education or less (PE040: ISCED level currently attended: value 2 or less), and who at the same time is not in education or training leading to a qualification at least to upper secondary level (PE010: current education activity: value 2)
- Households with persons with low educational attainment.  
Households with at least one person aged 25-64 who has only lower secondary education or less (PE040).

The least dimension concerns the labour market. Also for this set we have constructed two composite indicators:

- Jobless households;  
This indicator identifies the worklessness of the household, using variable PL030. For details about the construction see next section.
- Intensity or duration of unemployment at household level.  
This indicator is constructed using variables PL070, PL072, PL080, PL085, PL087, PL090. For details about the construction see next section.

The variables used are listed below:

HH040: LEAKING ROOF, DAMP WALLS/FLOORS/FOUNDATION, OR ROT IN WINDOW FRAMES OR FLOOR  
HH050: ABILITY TO KEEP HOME ADEQUATELY WARM  
HH080: BATH OR SHOWER IN DWELLING  
HH090: INDOOR FLUSHING TOILET FOR SOLE USE OF HOUSEHOLD  
HS010: ARREARS ON MORTGAGE OR RENT PAYMENTS  
HS020: ARREARS ON UTILITY BILLS  
HS030: ARREARS ON HIRE PURCHASE INSTALMENTS OR OTHER LOAN PAYMENTS  
HS040: CAPACITY TO AFFORD PAYING FOR ONE WEEK ANNUAL HOLIDAY AWAY FROM HOME  
HS050: CAPACITY TO AFFORD A MEAL WITH MEAT, CHICKEN, FISH (OR VEGETARIAN EQUIVALENT) EVERY SECOND DAY  
HS060: CAPACITY TO FACE UNEXPECTED FINANCIAL EXPENSES  
HS070: DO YOU HAVE A TELEPHONE (INCLUDING MOBILE PHONE)?  
HS080: DO YOU HAVE A COLOUR TV?  
HS090: DO YOU HAVE A COMPUTER?  
HS100: DO YOU HAVE A WASHING MACHINE?  
HS110: DO YOU HAVE A CAR?  
HS120: ABILITY TO MAKE ENDS MEET  
HS160: PROBLEMS WITH THE DWELLING: TOO DARK, NOT ENOUGH LIGHT  
HS170: NOISE FROM NEIGHBORS OR FROM THE STREET  
HS180: POLLUTION, GRIME OR OTHER ENVIRONMENTAL PROBLEMS

HS190: CRIME VIOLENCE OR VANDALISM IN THE AREA  
PE010: CURRENT EDUCATION ACTIVITY  
PE040: HIGHEST ISCED LEVEL ATTAINED  
PH010: GENERAL HEALTH  
PH020: SUFFER FROM ANY A CHRONIC (LONG-STANDING) ILLNESS OR CONDITION  
PH030: LIMITATION IN ACTIVITIES BECAUSE OF HEALTH PROBLEMS  
PH040: UNMET NEED FOR MEDICAL EXAMINATION OR TREATMENT  
PH060: UNMET NEED FOR DENTAL EXAMINATION OR TREATMENT  
PL030: SELF-DEFINED CURRENT ECONOMIC STATUS  
PL070: NUMBER OF MONTHS SPENT AT FULL-TIME WORK  
PL072: NUMBER OF MONTHS SPENT AT PART-TIME WORK  
PL080: NUMBER OF MONTHS SPENT IN UNEMPLOYMENT  
PL085: NUMBER OF MONTHS SPENT IN RETIREMENT  
PL087: NUMBER OF MONTHS SPENT STUDYING  
PL090: NUMBER OF MONTHS SPENT IN INACTIVITY

#### 4. Transformation of the items into the [0, 1] interval

When the item is constituted by a fixed number of categories, then it is transformed using the following procedure. For each item we determine a deprivation score as follows:

$$d_{j,i} = \frac{1 - F(c_{j,i})}{1 - F(1)}; j = 1, 2, \dots, k; i = 1, 2, \dots, n \quad (10)$$

where  $c_{j,i}$  is the value of the category of the  $j$ -th item for the  $i$ -th individual and  $F(c_{j,i})$  is the value of the  $j$ -th item cumulation function for the  $i$ -th individual.

We transform the deprivation score to a positive score as follows:

$$s_{j,i} = 1 - \frac{1 - F(c_{j,i})}{1 - F(1)} = \frac{F(c_{j,i}) - F(1)}{1 - F(1)}; j = 1, 2, \dots, k; i = 1, 2, \dots, n \quad (11)$$

In the special, but the common case, where the variable is a dichotomy, the deprivation index  $d$  is 1 for deprivation and 0 otherwise, while the positive score  $s$  is 0 for deprivation and 1 otherwise. For a polychotomous item we assign to each household instead of the real value of the category, a value corresponding to the percentage of households that are “better off” than that household.

In the few cases in which the indicator is a composite one (a set of dichotomies indicating the presence or absence of an experience by household members), the score  $s$  represents the proportion of people in the household that experienced it.

The indicator concerning the worklessness of the household is constructed as follows. First we exclude households consisting only of persons who are aged 18-24 in full-time education or are older than a country-specific retirement age. In order to choose an appropriate retirement age we have proceeded as follows. Among people that have ever worked, we consider the distribution of the ones that are retired (PL030=5) by age and gender. Looking at the ratio of people that at a particular age are retired among all the people in that age, we look for the age where a large jump in this proportion is found to occur. Once this point has been found, we confirmed it by examining its relationship to the legal age of retirement for a specific country.

Among the remaining households we classify the people as employed or not employed using variable PL030. We thus identify the degree of worklessness of an household, by constructing a ratio where in the numerator there are all the people in the household for which variable PL030 takes value 1, 2 or 7, and the denominator is the sum of the people of the household for which PL030 takes value 1, 2, 3, 6, 7, 9, and values 5 and 8 only if the age of the person is less than the retirement age chosen above. So at household level we construct an index reflecting the degree of which eligible household members are engaged in work: a zero indicates a workless household with some of its members in working age; a one indicates that all its working age members are working. To construct the indicator concerning the duration of unemployment, we calculate at household level the ratio:

$$1 - \frac{\sum_{ind=1}^{HH-size} PL080_{ind}}{\sum_{ind=1}^{HH-size} (PL070 + PL072 + PL080 + PL085 + PL087 + PL090)_{ind}} \quad (12)$$

The variable for general health, PH010, is aggregated as follows. To the categories 1-2-3 is assigned value 1 and to categories 4-5 value 0. Then this variable is aggregated at household level so that a household is considered deprived for that indicator if at least one person in the household is deprived for the item. So the score  $s$  assumes value 1 if no one in the household is deprived concerning that item, and it assumes value 0 if at least one person is deprived.

The same kind of household aggregation is done for all the personal variables concerning the health and the educational status.

## 5. Factor analysis

In order to investigate on life-style deprivation we have followed the procedure from the Economic and Social Research Ireland (ESRI), as described in Whelan *et al.* (2001).

In proceeding to construct a summary index of deprivation employing different items, we begin by identifying and investigating the dimension of deprivation. By ‘dimension’ we mean a distinct group of individual items of deprivation. Exploratory and confirmatory factor analyses allow us to achieve this objective. The procedure consists in an exploratory factor analysis to give a preliminary framework of the dimensions. We then proceed to rearrange some factors in the dimensions found in order to create more meaningful groups. Finally, we do a confirmatory factor analysis to test the goodness of the model hypothesised.

The exploratory factor analysis identifies 9 dimensions as reported in Table 2.

Table 2. Results of exploratory factor analysis.

INDICATORS	DIMENSIONS
- Meals with meat, fish or chicken - Household adequately warm - Holiday away from home - Inability to cope with unexpected expenses - Ability to make ends meet	1
- General health - Chronic illness - Mobility restriction	2
- Pollution - Crime, Violence, vandalism - Noise	3
- Bath or Shower - Indoor flushing toilet	4
- Car - PC - Telephone - Washing Machine - TV	5
- Worklessness - Duration of unemployment - Early school leavers - Low education	6
- Arrears on mortgage or rent payments - Arrears on utility bills - Arrears on hire purchase instalments	7
- Unmet need for medical exam. - Unmet need for dental exam.	8
- Leaking roof and damp - Rooms to dark	9

Then we decided to rearrange the dimensions in order to achieve substantially more meaningful groupings, as reported in Table 3 below.

Table 3. Dimensions after rearrangement and confirmatory factor analysis.

INDICATORS	REARRANGEMENT of the dimensions	Name
<b>1</b> Meals with meat, fish or chicken <b>2</b> Household adequately warm <b>3</b> Holiday away from home <b>4</b> Ability to make ends meet	<b>1</b>	Basic lifestyle
<b>5</b> Car <b>6</b> PC <b>7</b> Telephone <b>8</b> Washing Machine <b>9</b> TV	<b>2</b>	Consumer durables
<b>10</b> Bath or Shower <b>11</b> Indoor flushing toilet <b>12</b> Leaking roof and damp <b>13</b> Rooms to dark	<b>3</b>	Housing amenities
<b>14</b> Inability to cope with unexpected expenses <b>15</b> Arrears on mortgage or rent payments <b>16</b> Arrears on utility bills <b>17</b> Arrears on hire purchase instalments	<b>4</b>	Financial situation
<b>18</b> Crime, Violence, vandalism <b>19</b> Pollution <b>20</b> Noise	<b>5</b>	Environment
<b>21</b> Early school leavers <b>22</b> Low education <b>23</b> Worklessness <b>24</b> Duration of unemployment	<b>6</b>	Work & Education
<b>25</b> General health <b>26</b> Chronic illness <b>27</b> Mobility restriction <b>28</b> Unmet need for medical exam. <b>29</b> Unmet need for dental exam.	<b>7</b>	Health related

Subsequently we applied the confirmatory factor analysis to the dimensions rearranged as above.

The results of the analysis are very good; in fact all the indicators of goodness of the model are significant. Below, we report measures of absolute, relative and parsimonious fit as follows:

- The Goodness of Fit Index (GFI) is 0.94. It is based on the ratio of the sum of squared discrepancies to the observed variances; it ranges from 0 to 1 with values above 0.9 indicating a good fit.
- The Adjusted Goodness of Fit Index (AGFI) is 0.93. It is the GFI adjusted for degrees of freedom of the model, that is the number of the fixed parameters. It can be interpreted in the same manner.
- The Parsimonious GFI is 0.86. It adjusts GFI for the number of estimated parameters in the model and the number of data points.
- The Root Mean Square Residual (RMR) is 0.06. The fit is considered really good if RMR is equal or below 0.06.
- The Root Mean Squared Error of Approximation (RMSEA) is 0.0475. It is based on the analysis of residuals, with small values indicating a good fit. Values below 0.1, 0.05 and 0.01 indicate a good, very good and outstanding fit respectively.

## 6. Calculation of weights within each dimension

The weights to be given to items are determined within each dimension separately and the set of weights are taken to be item-specific, i.e. for a given item they are common to all individuals in the population. Such weights comprise two factors: the dispersion of deprivation indicator and its correlation with other deprivation indicators in the given dimension:

$$w_{hj} = w_{hj}^a \cdot w_{hj}^b, h = 1, 2, \dots, m; j = 1, 2, \dots, k_h \quad (13)$$

where  $h$  is a particular dimension and  $j$  a particular deprivation indicator.

In a previous work, the first factor  $w_{hj}^a$  has been taken as proportional to the coefficient of variation of deprivation score for the variable concerned,  $w_{hj}^a \propto cv_{hj}$  (Betti and Verma, 1999).

Here the indicators were in terms of deprivation indexes  $d$ , defined above. This means that when an item of deprivation affects only a small proportion, the weight given to it varies inversely to the square-root of the proportion. Thus deprivation affecting a small proportion of the population is treated as more intense at the individual person's level



but, of course, its contribution to the average level of deprivation in the population as a whole is correspondingly smaller.

Now our analysis is carried on using the deprivation scores  $s$ , so the previous formula should be modified as follows:

$$w_{hj}^a \propto \frac{std_{hj}}{1 - mean_{hj}} \quad (14)$$

The second factor, as a measure of the correlation, can be computed in the following form:

$$w_{hj}^b \propto \left( \frac{1}{1 + \sum_{j=1}^{k_h} r_{e_{hj},hj'} \mid r_{e_{hj},hj'} < r_{e_{hj}}^*} \right) * \left( \frac{1}{1 + \sum_{j=1}^{k_h} r_{e_{hj},hj'} \mid r_{e_{hj},hj'} < r_{e_{hj}}^*} \right) \quad (15)$$

where  $r_{e_{hj},hj'}$  is the correlation coefficient between deprivation indicators corresponding to items  $j$  and  $j'$  in the  $h$ -dimension and  $r_{e_{hj}}^*$  is the critical value of the correlation coefficient.

Below in Table 4 we report the results from our data for  $w_{hj}$ , where, as noted earlier,  $h$  refers to the dimension and  $j$  to a particular item of deprivation in it.

Table 4. The weighting system.

Country	W11	W12	W13	W14	W21	W22	W23	W24	W25	W31	W32	W33	W34	W41	W42	W43	W44	W51	W52	W53	W61	W62	W63	W64	W71	W72	W73	W74	W75
AT	1.56	3.78	0.84	0.40	2.22	2.41	11.60	8.27	10.11	5.13	4.42	2.48	3.01	1.06	3.92	3.85	5.23	2.11	1.91	1.25	5.08	1.56	0.60	2.20	1.21	0.58	0.52	3.43	2.96
BE	2.67	1.31	0.80	0.38	1.60	1.69	10.85	2.92	6.82	5.07	6.44	1.93	2.55	1.11	3.37	2.50	6.40	1.36	1.51	1.15	5.28	1.05	0.47	1.49	1.20	0.57	0.60	5.67	3.18
CY	1.75	0.64	0.43	0.32	4.32	3.02	12.64	5.25	11.34	3.12	3.14	1.09	3.04	0.69	2.86	1.79	1.46	0.98	1.81	0.74	4.19	0.89	0.45	2.07	0.93	0.43	0.56	1.47	1.15
CZ	1.28	2.06	0.66	0.37	1.49	1.75	5.49	7.06	8.67	5.15	4.24	1.68	3.34	0.86	2.68	2.55	5.03	1.25	1.54	1.26	6.18	1.96	0.54	1.86	0.84	0.44	0.46	1.94	2.33
DE	1.35	2.15	0.82	0.39	1.96	2.64	9.66	7.69	7.06	8.43	6.34	2.19	3.89	0.87	3.68	2.75	4.59	0.99	1.62	0.88	7.32	2.55	1.11	1.94	1.32	0.52	0.58	1.78	1.61
DK	3.62	1.94	1.56	0.51	1.72	3.73		4.59	7.27	9.01		2.60	4.06	1.29	3.36	3.45	2.65	2.21	1.82	1.28	5.55	1.33	0.62	4.06					
EE	2.02	2.98	0.45	0.47	0.91	1.16	3.36	2.05	5.51	0.90	1.04	1.04	2.09	1.27	6.81	2.76	7.94	1.07	1.38	1.14	3.80	1.89	0.57	2.41	0.74	0.34	0.37	1.05	1.08
ES	4.27	2.00	0.71	0.41	3.41	2.52	9.73	13.07	16.68	9.68	11.06	1.67	2.32	1.05	3.65	3.12	4.58	1.50	1.50	1.08	2.94	0.59	0.41	1.59	0.91	0.54	0.55	4.28	1.87
FI	3.31	6.45	1.17	0.47	1.29	1.90	14.13	3.27	4.94	4.92	5.92	3.73	3.92	0.88	3.16	3.25	4.06	1.61	1.71	1.48	6.12	1.49	0.56	2.00					
FR	1.97	2.46	0.73	0.39	3.21	2.30	7.64	5.56	14.94	5.64	5.47	1.90	2.44	0.84	2.09	1.91	3.61	1.41	1.51	1.26	5.07	1.07	0.57	1.92	1.19	0.49	0.63	2.74	1.87
GR	1.69	1.13	0.46	0.31	1.71	1.71	6.06	3.23	8.20	4.75	2.96	1.22	2.40	0.86	2.22	1.24	2.10	1.12	1.73	1.07	5.68	0.72	0.44	1.74	0.94	0.50	0.56	1.41	1.52
HU	0.82	1.55	0.37	0.40	0.97	1.21	2.98	2.79	6.59	2.07	1.83	1.10	1.68	0.54	3.49	1.24	2.85	1.53	1.70	1.41	4.66	1.21	0.51	1.97	0.52	0.33	0.40	1.14	1.28
IE	4.09	2.98	1.05	0.42	2.06	2.16	8.18	6.13	12.24	5.85	6.24	1.67	2.28	0.78	2.34	1.88	3.55	2.04	1.57	1.57	4.44	0.69	0.41	2.04	2.51	0.56	0.67	2.83	2.54
IS	2.97	2.45	1.38	0.46	5.87	6.55		49.91	16.61	30.94	11.13	3.03	7.09	0.99	1.90	2.06	1.53	2.01	4.09	1.86	3.29	1.23	0.56	4.86					
IT	1.95	1.43	0.59	0.38	2.72	2.06	4.81	6.46	8.75	9.60	13.82	1.35	2.31	0.87	3.13	1.64	3.77	1.03	1.38	0.95	4.11	0.69	0.48	2.04	0.92	0.58	0.50	1.61	1.36
LT	1.01	0.92	0.37	0.39	1.19	1.50	2.43	1.62	3.98	0.84	0.84	1.03	1.81	0.85	8.01	2.18	8.28	1.29	2.31	1.14	6.30	1.97	0.55	2.24	0.63	0.39	0.45	1.10	1.32
LU	3.86	6.67	1.49	0.48	3.54	3.16	16.45	9.25	21.62	7.11	6.45	1.98	3.25	1.33	3.60	3.37	8.78	1.31	1.93	1.12	4.24	0.92	0.52	2.76	1.41	0.65	0.68	2.85	2.82
LV	0.64	0.83	0.32	0.32	0.75	0.92	2.38	1.52	4.23	0.78	0.87	0.92	1.67	0.60	3.95	1.96	6.97	0.78	0.95	1.17	3.53	1.45	0.54	2.13	0.57	0.34	0.36	0.62	0.74
NL	4.31	4.13	1.18	0.54	2.41	4.62		17.66	41.24	12.64	30.72	1.89	3.51	1.09	2.95	3.42	4.94	1.83	1.59	0.98	5.32	1.13	0.55	2.54					
NO	2.78	5.69	1.73	0.46	2.13	3.83	15.94	10.36	8.47	11.73	8.88	2.76	4.06	1.27	1.89	1.82	2.62	2.19	3.27	1.61	3.68	1.53	0.63	4.71					
PL	0.77	0.82	0.34	0.32	1.21	1.27	3.15	4.95	5.34	1.38	1.53	0.77	1.82	0.65	7.17	1.43	3.36	1.42	2.00	1.11	3.70	1.23	0.44	1.65	0.67	0.40	0.50	1.09	1.31
PT	3.34	0.57	0.38	0.37	1.57	1.59	2.33	2.65	6.25	2.20	2.34	1.27	1.58	1.42	3.58	2.54	5.01	1.15	1.82	1.00	2.61	0.42	0.42	1.59	0.55	0.37	0.39	1.13	1.78
SE	2.75	4.21	1.32	0.44	3.19	4.60		14.87	8.37	10.03		3.45	3.28	1.24	3.46	2.85	3.28	2.54	1.99	1.74	7.88	1.92	0.63	3.11					
SI	1.45	2.70	0.72	0.40	2.29	2.02	5.82	8.37	5.87	4.31	4.46	1.66	2.40	0.77	4.75	1.69	2.94	1.22	2.02	1.24	6.84	1.15	0.50	2.13					
SK	0.71	2.87	0.44	0.41	0.98	1.21	3.71	5.13	5.60	3.71	2.54	2.39	2.97	0.76	3.03	2.20	4.56	1.23	2.20	1.18	7.12	2.16	0.54	1.88	0.60	0.45	0.43	1.97	2.02
UK	2.63	2.55	0.99	0.43	2.80	3.02	17.27	10.59	27.69	23.09	10.72	2.14	2.46	1.00	2.74	2.49	4.15	1.82	1.18	1.32	7.59	1.65	0.59	4.24	1.62	0.46	0.64	2.71	2.89

## 7. Empirical analysis

Table 5 shows indicators of deprivation in various dimensions estimated with the methodology described above, using EU-SILC 2007 data.

The objective of illustrating those results is both substantive and methodological. It shows the relative situation of EU countries in terms of levels of overall deprivation (monetary and non-monetary), and also in terms of different dimensions of deprivation.

At the same time, the table illustrates the type of numerical values obtained with the above procedure, thus further clarifying details of the methodology.

The first column, FS0, is the overall deprivation rate. It is in fact the conventional poverty rate (HCR) for each country. The values of the FM (fuzzy monetary) and FS (fuzzy supplementary) deprivation indices are simply scaled for each country to numerically equal the conventional HCR. Those overall poverty or deprivation rates show large differences among EU countries, from the low value of 9.5% in CZ to the high of 21.2% in LV. In six countries the rate is below 11% (CZ, IS, NL, SK, SE, SI), it exceeds 19% in seven (LV, GR, IT, ES, EE, LT, UK). The average over countries is close to 15%.

We note that there is fairly strong correlation between the ranking of countries according to the overall and dimension-specific indices of deprivation. However, quite large differences in the rankings according to different dimensions are also present.

Numerically, deprivation rates for individual dimensions are not scaled in the methodology described above to equal – individually or even in the average over dimensions – the overall poverty or deprivation rate FS0. In fact, over countries, in these data the average of rates for individual dimensions (at 11%) is lower than the average of overall rates (15%).

In certain dimensions, the average over countries is 12-14%, which is quite close to that for the overall index (15%). This group includes:

FS1 – basic life-style

FS5 – environment

FS6 – work and education

FS7 – health related

For the remaining dimensions, the average values obtained are much lower (7-9%). These dimensions are:

FS2 – consumer durables

FS3 – housing amenities

FS4 – financial situation

Table 5. Fuzzy measures at Country level, SILC 2007 wave.

Country	Rate of deprivation by dimension of deprivation									'Normalised rates'								
	FS0	FS1	FS2	FS3	FS4	FS5	FS6	FS7	mean	FS0	FS1	FS2	FS3	FS4	FS5	FS6	FS7	mean
CZ	0.095	0.092	0.061	0.055	0.045	0.106	0.087	0.085	0.076	1.00	1.17	1.34	0.97	0.86	1.26	1.11	0.95	1.093
IS	0.100	0.087	0.021	0.041	0.084	0.071	0.083		0.065	1.00	1.05	0.45	0.70	1.53	0.81	1.02		0.927
NL	0.102	0.080	0.040	0.051	0.051	0.097	0.087		0.068	1.00	0.95	0.82	0.84	0.92	1.08	1.04		0.943
SK	0.105	0.087	0.063	0.059	0.055	0.103	0.094	0.095	0.079	1.00	1.01	1.26	0.96	0.96	1.11	1.09	0.96	1.050
SE	0.107	0.085	0.040	0.058	0.065	0.085	0.089		0.070	1.00	0.96	0.79	0.92	1.10	0.90	1.01		0.946
SI	0.109	0.094	0.052	0.066	0.075	0.100	0.093		0.080	1.00	1.04	1.00	1.02	1.26	1.04	1.04		1.068
DK	0.117	0.099	0.057	0.064	0.062	0.100	0.093		0.079	1.00	1.03	1.02	0.93	0.97	0.97	0.97		0.980
AT	0.120	0.098	0.058	0.070	0.047	0.102	0.105	0.088	0.081	1.00	0.98	1.01	0.98	0.72	0.96	1.06	0.78	0.929
NO	0.123	0.082	0.044	0.058	0.085	0.084	0.100		0.076	1.00	0.80	0.75	0.80	1.27	0.78	0.99		0.900
HU	0.124	0.127	0.085	0.096	0.083	0.112	0.106	0.140	0.107	1.00	1.24	1.44	1.31	1.22	1.03	1.04	1.20	1.212
FI	0.130	0.097	0.067	0.063	0.075	0.112	0.110		0.087	1.00	0.90	1.08	0.83	1.05	0.98	1.03		0.979
FR	0.131	0.101	0.058	0.078	0.078	0.126	0.111	0.107	0.094	1.00	0.93	0.94	1.00	1.08	1.09	1.02	0.87	0.988
LU	0.135	0.092	0.028	0.071	0.055	0.119	0.110	0.106	0.083	1.00	0.82	0.43	0.89	0.74	1.00	0.99	0.84	0.816
BE	0.151	0.131	0.071	0.087	0.081	0.141	0.127	0.102	0.105	1.00	1.04	0.98	0.97	0.98	1.05	1.02	0.72	0.966
DE	0.152	0.124	0.058	0.079	0.063	0.145	0.119	0.130	0.103	1.00	0.99	0.81	0.88	0.76	1.08	0.95	0.92	0.912
CY	0.155	0.140	0.058	0.075	0.117	0.146	0.128	0.143	0.115	1.00	1.09	0.79	0.81	1.37	1.06	1.00	0.98	1.014
PL	0.173	0.200	0.105	0.113	0.094	0.135	0.146	0.167	0.137	1.00	1.39	1.27	1.10	0.99	0.88	1.03	1.03	1.097
IE	0.175	0.128	0.083	0.095	0.086	0.133	0.143	0.124	0.113	1.00	0.89	1.00	0.92	0.90	0.86	0.99	0.76	0.902
PT	0.181	0.130	0.115	0.119	0.097	0.158	0.151	0.154	0.132	1.00	0.86	1.33	1.10	0.97	0.99	1.01	0.90	1.024
UK	0.191	0.143	0.060	0.103	0.105	0.162	0.146	0.137	0.122	1.00	0.91	0.66	0.91	1.00	0.96	0.93	0.76	0.875
LT	0.191	0.167	0.124	0.158	0.082	0.143	0.152	0.176	0.143	1.00	1.05	1.36	1.39	0.78	0.84	0.97	0.98	1.054
EE	0.194	0.126	0.114	0.149	0.090	0.183	0.155	0.181	0.143	1.00	0.79	1.23	1.30	0.85	1.07	0.97	1.00	1.030
ES	0.197	0.145	0.073	0.103	0.095	0.172	0.163	0.143	0.128	1.00	0.89	0.78	0.88	0.88	0.99	1.00	0.77	0.884
IT	0.198	0.164	0.064	0.100	0.117	0.192	0.155	0.169	0.137	1.00	1.00	0.68	0.85	1.08	1.10	0.95	0.91	0.937
GR	0.203	0.165	0.109	0.113	0.152	0.169	0.160	0.165	0.148	1.00	0.99	1.13	0.94	1.37	0.94	0.96	0.87	1.028
LV	0.212	0.219	0.136	0.171	0.081	0.224	0.169	0.246	0.178	1.00	1.25	1.35	1.37	0.70	1.20	0.97	1.24	1.154
average	0.149	0.123	0.071	0.088	0.081	0.132	0.122	0.140	0.108	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

NOTES FS0 stands for "HCR = FM = FS"  
'Normalised rates'  $N_{ij}$ : all values scaled such that:  
(1) for each dimension (j), average over countries rescaled to = 1.0; and  
(2) for each country (i),  $FS_j$  values scaled to correspond to  $FS_0 = 1.0$ .

$$N_{ij} = \left( \frac{FS_{ij}}{FS_{.j}} \right) / \left( \frac{FS_{i0}}{FS_{.0}} \right)$$

FS1 – FS7 refer to the seven dimensions of deprivation defined in Table 3.

We believe that the indices for individual dimensions represent a mixture of relative and absolute levels of deprivation, even if the relative aspect predominates. However, values observed for dimensions 2-4 imply that, compared to overall deprivation and to other dimensions, deprivation in these dimensions may be less severe in the absolute sense in EU countries on the average.

The second panel of table 5 examines the pattern of variation across countries and dimensions more closely, bringing out the relationship in scores across different dimensions in relative terms.

The figures shown are ‘normalised’, meaning that we have rescaled them to remove the effect of variations among countries in the overall deprivation (or poverty) rates  $FS_0$ , and also to remove the effect of differing average values for the various dimensions.

The last column shows the average over the dimensions (FS1-FS7) of the ‘normalised’ values. This average, by definition, is 1.0 over all countries.

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