

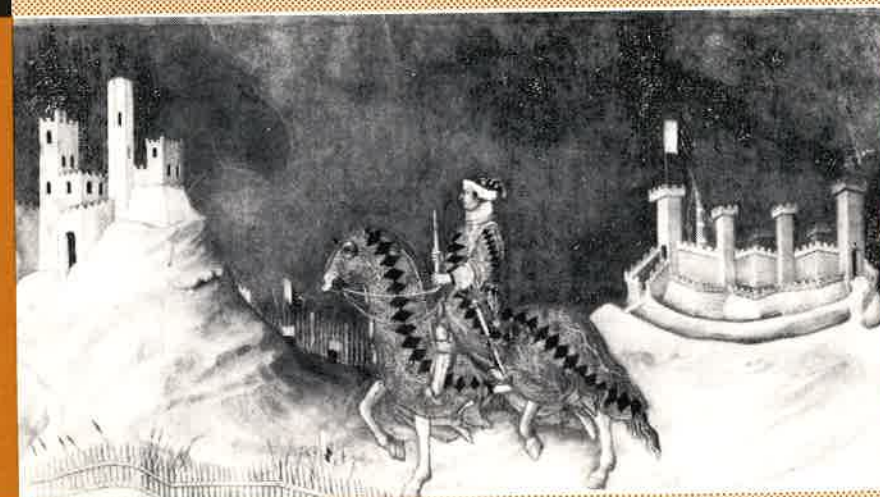
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QUADERNI DEL DIPARTIMENTO
DI ECONOMIA POLITICA

Stefano Vannucci

ON CONSISTENCY PROPERTIES
OF SOME STRONGLY IMPLEMENTABLE
SOCIAL CHOICE RULES WITH ENDOGENOUS
AGENDA FORMATION



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Stefano Vannucci

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Siena, aprile 1989

Abstract

A suitable consistency criterion for efficient agenda formation is newly formulated. Then it is shown that the core correspondences of nicely stable effectivity functions (the main class of strongly implementable social choice rules) do satisfy such a consistency criterion. The relevance of this result in order to assess the robustness of the well-known Moulin-Peleg's "possibility theorem" w.r.t. issues of agenda-manipulation is emphasized. Connections with some related literature on non-binary social choice are also discussed.

1. Introduction and summary of results (*)

A social choice rule (SCR) is best seen as a conflict resolution device. The most common interpretation goes as follows. The players agree on some general principles: say, some criteria of efficiency and symmetry. Then, they identify a SCR which meets such criteria. Finally, they agree to use it as a collective decision device in all relevant circumstances. Two major game-theoretic problems arise in that connection:

i) are the socially best outcomes likely to be selected when the players, and their coalitions as well, behave strategically? This is the implementation problem. A SCR is implementable w.r.t. a suitable equilibrium concept if and only if a game form exists such that for any relevant profile of players' characteristics the SCR-optimal outcomes at that profile arise as the equilibrium outcomes of the induced game.

ii) To what extent are the socially best outcomes robust w.r.t. changes in the agenda, i.e. in the set of actually feasible alternatives?

This is a subtle question, because in a sense socially best outcomes do obviously depend on the specification of the set of feasible alternatives. However, one would like to rule out any change in the optimal set due to mere addition or deletion of suboptimal alternatives. The main reason for this requirement is, again, a strategic one. If it is the case that such "suboptimal" changes in the agenda do modify the optimal set, then the choice of the agenda may be a source of conflict (perhaps, of a conflict stronger than the one over the final choice). Thus, a SCR vulnerable to that kind of agenda manipulation would lose much of its significance.

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There is no easy solution to the above problems, especially when tackled at the same time. In his classic work (Arrow (1951)) Arrow deliberately disregarded the implementation problem. On the other hand, the combination of the arrowian "independence" with the "social ordering" requirement is best understood as an attempt to overcome the agenda manipulation problem. The same is true for later mixtures of "independence" and "consistency" -or "collective rationality"- requirements in a choice-functional format (see, for example, Fishburn (1973) or Sen (1986)).

The impressive sequence of impossibility results discovered along this line of research shows that escaping the agenda manipulation problem is a very difficult task, if the SCR is to be fairly symmetric and efficient.

On the positive side, Nakamura's theorem (see Nakamura (1979), Moulin (1983) or Peleg (1984)) and its extensions to smooth manifolds of alternatives (see Schofield (1986)) entail that restrictions on the admissible pairs of societies and outcome-sets have to be introduced, provided that exclusive reliance on unanimity rules is to be avoided.

Another strand of the social choice-theoretic literature deals with the implementation problem instead. Fixed agenda models are typically used in this research area. Thus, the agenda manipulation problem is tacitly disregarded (but see Moulin (1983) for explicit observations on this point). A major positive result in the implementation literature, due to Moulin, Peleg (1982), relies heavily on effectivity functions (EFs). An EF is a formal representation of the allocation of power among coalitions under a given game form. Symmetry properties of an EF can be defined in a natural way. At any profile of individual preferences the core of an EF can also be easily defined. The Moulin-Peleg's theorem entails that there are symmetric EFs such that: i) at any profile of linear orderings their core is non-empty and ii) the resulting core correspondence -a SCR, by definition- is implementable w.r.t. strong Nash equilibrium (or, simply, strongly implementable; see Moulin, Peleg (1982), Moulin (1983), Peleg (1984)). The main

limitation of this important "possibility" theorem is that the above mentioned symmetric EFs embody a strong restriction on the cardinalities of the players' set and of the agenda. Namely (sticking to the finite case) these cardinalities are bound to be a pair of relatively prime numbers.

This is the point of departure of the present paper. In fact, the aim of the following analysis is to scrutinize the credentials of the core correspondences mentioned above as a solution to the agenda manipulation problem.

First, we need to assess the relevance of the foregoing restrictions on the cardinalities of the outcome-set and of the players's set. Actually, it seems to us that this restriction is only disturbing if the agenda formation process is assumed to be totally independent of the parameters of the ensuing voting game.

If some modelling of the agenda formation process is allowed, however, the dependence of the outcome-set's cardinality on the number of players seems to be a sensible assumption.

One may introduce an explicit agenda formation rule (AFR) which specifies, for any set of players, a set of admissible agendas.

Then a compelling criterion of symmetry -w.r.t.- the players (for an AFR) dictates that their number- and only their number- counts in order to establish the admissible agendas. A similar extension to an AFR of a natural criterion of symmetry-w.r.t.-the-outcomes requires that two agendas of the same cardinality are bound to be either both admissible or both inadmissible.

An AFR which satisfies the first criterion of symmetry -and a weakened version of the second one- is the following. For any society of n players, the admissible agendas are the sets of outcomes containing exactly $kn+1$ outcomes, to be interpreted as k proposals for each player (k being a positive integer) plus one "status quo" outcome.

(To keep things as simple as possible, we shall in fact stick to the

$k=1$ case, i.e. to the AFR which entitles each player to one proposal).

Under the present modelling of the collective choice process -i.e. via a SCR plus an AFR- the most prominent issue consists in ascertaining whether the agenda formation process can be expected to be efficient. Therefore, a (mild) criterion of efficiency for the agenda formation process is formulated below. It requires that, whenever a player discovers a feasible outcome which is both a) preferred by him/her to any alternative in the previous choice set and b) Pareto-improving w.r.t. his/her previous proposal, then substituting the "new" outcome for the old proposal can only be to his/her own advantage. Hence, the criterion is labelled "dramatic improvements pay" (DIP).

The main result of this paper is that the core correspondences of stable EFs meet the DIP criterion. As a consequence, the core correspondences of (maximal, monotonic) stable and symmetric EFs do qualify as a solution to the agenda manipulation problem.

The paper is organized as follows: section 2 contains the formal description of the model and the results. Section 3 is devoted to a brief discussion of the results of section 2 and to comments on some related literature.

2. Definitions and results

We denote by Ω (the set of natural numbers) the universal set of players, and by X the universal set of outcomes. A finite subset of Ω is also called a society. For any society N we denote by N^* the set $N \cup \{\Omega\}$, and by $X^*(N)$ the set $X \times N^*$ of extended outcomes (we shall also write X^* for $X^*(N)$ whenever the underlying society is clearly identified).

An agenda for N is a non-empty subset of $X^*(N)$. An agenda formation rule (AFR) is a function $f: PF(\Omega) \rightarrow \bigcup_{N \in PF(\Omega)} 2^{X^*(N)}$ s.t. for any

society N , $\emptyset \neq f(N) \subseteq 2^{X^*(N)}$ (throughout this paper, we denote by $P(Y)$, $PF(Y)$ and 2^Y , respectively, the power set of Y , the set of all finite subsets of Y , and the set of all non-empty subsets of Y , for any set Y).

Thus, an AFR specifies -for any society- a set of admissible agendas. Each agenda consists of a-possibly empty-set of proposal for any player of the given society, plus a-possibly empty- set of exogenously fixed, or "status quo", outcomes, indexed by $\{\Omega\}$.

An AFR f is intersociety-anonymous iff $f(N) = f(M)$ whenever $\#N = \#M$, for any pair of societies N, M . An AFR f is intrasociety-anonymous iff for any society N , for any $i, j \in N$, and for any agenda $A \in f(N)$, $A_i = A_j$, where $A_h = \{x \in A / x \in X \times \{h\}\}$ for any $h \in N$.

Moreover, an AFR is said to be strictly anonymous whenever it is both intersociety-anonymous and intrasociety-anonymous.

An AFR f is neutral (nearly neutral, respectively) iff a $B \in 2^{X^*(N)}$ exists s.t., for any society N , $f(N) = \{A \subseteq X^*(N) / \#A = \#B \text{ and } x \in A \text{ for some } x \in X \times \{\Omega\}\}$.

For any positive integer k and for any "status quo" outcome $s \in X \times \{\Omega\}$, a nearly neutral and strictly anonymous AFR $f(k, s)$ can be defined as follows: $f(k, s)(N) = \{A \subseteq X^*(N) / A_\Omega = \{s\} \text{ and for all } i \in N : A_i = k\}$.

Hence, under $f(k, s)$ a "status quo" s is fixed, and each player is entitled to k distinct proposals to be chosen in X (in the sequel s will also be used as a shorthand for $s \in X \times \{\Omega\}$).

Let N be a society. We assume (w.l.o.g.) that $N = \{1, \dots, n\}$. We denote by $L = L(X, N)$ the set of all total preorderings on $X^*(N)$ (a total preordering is a total and transitive binary relation). Moreover, the following notation will be used, for any total preordering $r \in L$: $I(r)$ denotes the symmetric part of r , whereas $P(r)$ denotes the asymmetric part of r . We shall restrict our analysis to the subset $R \subseteq L$ defined as follows: $R = \{r \in L / ((x, i), (y, j)) \in I(r) \text{ iff } x = y, \text{ for any } (x, i), (y, j) \in X^*(N)\}$.

All this amounts to denying any welfare implication to the identity of proponents, in spite of the fact that the latter has been previously used in order to distinguish the relevant outcomes. This admittedly roundabout procedure is introduced as a matter of technical convenience, as will be apparent in the sequel.

A profile for N is a function $r^N: N \rightarrow R$. We denote by R^N the set of all profiles (for N). Let $D \subseteq 2^{X^*(N)}$ be a non-empty set of agendas. Then, a social choice rule (SCR) (for N on $R^N \times D$) is a function $F: R^N \times D \rightarrow 2^{X^*(N)}$ s.t. $F(r^N, A) \subseteq A$, for any $(r^N, A) \in R^N \times D$. In words, a SCR selects a non-empty subset of "optimal" extended outcomes for any profile of individual preferences and for any "admissible" agenda. If f is an AFR, the SCR $F: R^N \times D \rightarrow 2^{X^*(N)}$ is consistent with f iff $D = f(N)$. SCRs with a fixed agenda correspond to the case $D = (Y \times N^*)$, for any $Y \subseteq X$ (i.e. they are consistent with any AFR f s.t. $f(N) = (Y \times N^*)$). "Unconstrained" SCRs - such as those most commonly used in the literature focussing on "consistency" and agenda-manipulation issues - correspond to the case $D = 2^{X^*(N)}$ (hence, "unconstrained" SCRs are consistent with any AFR f s.t. $f(N) = 2^{X^*(N)}$).

In the sequel we shall be mainly concerned with an intermediate case (namely, with SCRs which are consistent with the AFR $f(k, s)$: see definition above).

We now turn to the implementation side of our model. To begin with, it is tacitly assumed that the players are aware of each other's preferences (perhaps due to a small-dimension-effect and/or to a repetition-effect). Also, binding agreements are feasible (perhaps due to a repetition-effect).

Thus, a cooperative environment is assumed to prevail. As a consequence, we shall be concerned with coalitional -or strong (Nash)- implementation.

A game form (for N and $A \in 2^{X^*(N)}$) is a $n+1$ -tuple $G = ((S_i)_{i \in N}, g)$, where S_i denotes the set of strategies of player i ($i=1, \dots, n$) and

$g: S_1 \times \dots \times S_n \rightarrow A$ is the outcome-function.

A strong equilibrium of G at profile $r^N \in R^N$ is a multistrategy $s \in S_1 \times \dots \times S_n$ s.t. for any non-empty coalition $T \in 2^N$, and for any $s'_T = (s'_i)_{i \in T}$ (where $s'_i \in S_i$, for any $i \in T$), $(g(s), g(s'_T, s_{N \setminus T})) \in r_j$, for some player $j \in T$. We denote by $SE(G, r^N)$ the set of all strong equilibria of G at r^N .

A SCR (for N) $F: R^N \times D \rightarrow 2^{X^*(N)}$ is strongly implementable iff for any $A \in D$ a game form G (for N and A) exists s.t. for all $r^N \in R^N$, $g(SE(G, r^N)) = F(r^N, A)$. Obviously, a strongly implementable SCR is Pareto-efficient as well.

An effectivity function (EF) on N and $A \in 2^{X^*(N)}$ is a function $E: P(N) \rightarrow P(P(A))$ s.t.: i) $E(\emptyset) = \emptyset$ ii) $\emptyset \notin E(S)$, for any $S \in 2^N$ iii) $A \in E(S)$, for any $S \in 2^N$ iv) $B \in E(N)$, for any $B \in 2^A$. We denote by $EF(N, A)$ the set of all EFs on N and A (for any $A \in 2^{X^*(N)}$).

The core of $E \in EF(N, A)$ at profile $r^N \in R^N$ - written $\text{Core}(E, r^N)$ - is the set of all (extended) outcomes $x \in A$ satisfying the following condition: for no $B \in A - \{x\}$ and $S \in 2^N$ both 1) $B \in E(S)$ and 2) for all $y \in B$: $(y, x) \in \bigcap_{i \in S} P_i(r^N)$ hold true.

An $E \in EF(N, A)$ is said to be (core-)stable - on R^N - iff $\text{Core}(E, r^N) \neq \emptyset$, for any profile $r^N \in R^N$. An $E \in EF(N, A)$ is said to be monotonic iff it is both N -monotonic (i.e. for all $S, T \in 2^N$ and for all $B \in 2^A$: $S \subseteq T$ and $B \in E(S)$ entail $B \in E(T)$) and A -monotonic (i.e. for all $S \in 2^N$ and for all $B, C \in 2^A$: $B \subseteq C$ and $B \in E(S)$ entail $C \in E(S)$).

An $E \in EF(N, A)$ is maximal iff for all $S \in 2^N$ and $B \in 2^A$: $B \in E(S)$ entails $A \setminus B \in E(N \setminus S)$. The typical symmetry properties of an EF are defined as follows: $E \in EF(N, A)$ is anonymous iff for all $S, T \in 2^N$ and all $B \in 2^A$: $\#S = \#T$ and $B \in E(S)$ entail $B \in E(T)$; $E \in EF(N, A)$ is neutral iff for all $S \in 2^N$ and for all $B, C \in 2^A$: $\#B = \#C$ and $B \in E(S)$ entail $C \in E(S)$.

Clearly enough, an anonymous (neutral) EF treats the players (the outcomes) in a fairly equal way. An anonymous and neutral EF will also be called symmetric in the sequel.

Two EFs $E \in \text{EF}(N, A)$, $E' \in \text{EF}(N, A')$ are said to be equivalent (w.r.t. a bijective function $\pi: A \rightarrow A'$) iff for all $S \in 2^N$ and all $B \in 2^A: B \in E(S)$ iff $\pi B \in E'(S)$. If $E \in \text{EF}(N, A)$ and $E' \in \text{EF}(N, A')$ are two equivalent EFs as in the foregoing definition we also write $E' = E(A' \setminus A, \pi)$, $E = E'(A \setminus A', \pi^{-1})$.

The relevance of stable EFs to the strong implementation problem is apparent from the following fundamental result:

Theorem 1 (Moulin, Peleg (1982): Let A be a finite agenda (i.e. $A \in \text{PF}(X^*(N)) \setminus \{\emptyset\}$), and let $E \in \text{EF}(N, A)$ be monotonic and stable. Then the SCR (with fixed agenda) $\text{Core}(E, \cdot): R^N \times A \rightarrow 2^A$ is strongly implementable iff E is maximal as well.

On the other hand, EFs which are stable, monotonic, maximal and symmetric do actually exist (the typical example being the EFs attached to some suitable voting-by-veto method: see Moulin (1983), Peleg (1984)). Hence, the foregoing theorem may be regarded as an outstanding positive result w.r.t. the implementation problem. Apart from the finiteness of A , the main limitation of this positive result is the following: the symmetry requirement entail the restriction to pair of societies and agendas whose cardinalities are relatively prime numbers.

However restrictive, the latter requirement is consistent with a variable-agenda setting. Actually, it is easily shown that the following proposition holds true:

Proposition 1: Let $f = f(k, s)$ the strictly anonymous and nearly neutral AFR defined above (for some arbitrarily fixed $s \in X \times \{\Omega\}$ and integer $k > 0$). Let $A \in f(N)$, and let $\pi = \{\pi_B\}_{B \in f(N)}$ a family of bijective functions $\pi_B: A \rightarrow B$ s.t. $\# \pi = \# f(N)$ (that is, π contains exactly one function for each $B \in f(N)$). Moreover, let $E \in \text{EF}(N, A)$ be stable, monotonic, maximal, anonymous and neutral and let $H: R^N \times f(N) \rightarrow 2^{X^*(N)}$ be the function

defined by the following rule: for any $r^N \in R^N$ and any $B \in f(N)$

$$H(r^N, B) = \text{Core}(E(B \setminus A, \pi_B), r^N).$$

Then H is a well-defined and strongly implementable SCR.

Proof. It is easily shown that if E, E' are equivalent EFs and E is monotonic (respectively maximal, anonymous, neutral) then E' is monotonic (respectively maximal, anonymous, neutral) as well: the proof is straightforward and thus it is left to the reader. Also, any E' equivalent to E is stable. In fact, we know from the previous statement that E' is monotonic, maximal, anonymous and neutral. On the other hand, we know from Moulin, Peleg (1982) that i) a monotonic, maximal, anonymous and neutral $E \in \text{EF}(N, B)$ exists iff $(\#N, \#B)$ is a pair of relatively prime numbers; ii) furthermore, if this is the case, there is only one stable, monotonic, maximal, anonymous and neutral $E \in \text{EF}(N, B)$, defined as follows: for all $S \in 2^N$ and all $C \in 2^B$, $C \in E(S)$ iff $(\#S / \#N) > 1 - (\#C / \#B)$ (E is also called the proportional EF of $\text{EF}(N, B)$).

Thus, if E is the proportional EF of $\text{EF}(N, A)$ - it follows, by definition of equivalence - that E' is the proportional EF of $\text{EF}(N, B)$. Therefore E' is stable as well.

As a consequence, for any $B \in f(N)$, $\text{Core}(E(B \setminus A, \pi_B), \cdot)$ is a well-defined and strongly implementable SCR.

Hence, the proposition follows.

It should be noticed that - for any agenda $B \in f(N)$ - $\text{Core}(E(B \setminus A, \pi_B), \cdot)$ retains the nice symmetry properties of $\text{Core}(E, \cdot)$.

Therefore, the foregoing proposition makes precise the obvious suggestion that the Moulin-Peleg's solution to the strong implementation problem is by no means confined to fixed-agenda SCRs.

In other words, we are now confronted with a SCR (namely, H of Proposition 1) which may be envisaged as a positive solution to the strong

implementation problem in a variable agenda setting. Then, it is worth asking what is the performance of H w.r.t. the agenda manipulation problem.

The most relevant issue is the efficiency of the agenda formation process under $H = \bigcup_{B \in f(k,s)} \text{Core}(E(B \setminus A, \pi_B), \cdot)$.

We assume that each player has partial information about feasible alternatives. Borrowing from Simon's favourite terminology (see, for example, Simon (1972)), we may well say that the players have a state-description of any alternative in X , whereas they only have a process-description of alternatives in a proper, possibly small, subset of X .

As a minimal guarantee against socially inefficient agenda manipulation we require that whenever a player happens to discover a new feasible alternative which is both 1) Pareto-improving w.r.t. an old proposal of his/her own and 2) regarded by him/her as better than any current optimal outcome, then substituting the new outcome for the old proposal is a profitable move.

All this motivates the following definitions.

Definition (d.i. pairs): Let $F: R^N \times D \rightarrow 2^{X^*(N)}$ a SCR for N . A pair $(x, y) \in (X \times \{i\})^2$ is a dramatic improvement (d.i.) at $(r^N, A) \in R^N \times D$ and w.r.t. the SCR F iff i) $x \notin A$ and $y \in A$ ii) $(x, y) \in \bigcap_{i \in N} r_i$ and iii) $(x, a) \in P_i(r^N)$ for any $a \in F(r^N, A)$.

Definition (DIP criterion). Let $F: R^N \times D \rightarrow 2^{X^*(N)}$ be a SCR for N . F is said to satisfy the "dramatic improvements pay"- criterion (DIP) iff for all $(r^N, A) \in R^N \times D$, for all $i \in N$, and for all $(x, y) \in (X \times \{i\})^2$: (x, y) is a d.i. pair w.r.t. F at (r^N, A) implies $(z, w) \in r_i$ for any $z \in F(R^N, A \cup \{x\} \setminus \{y\})$ and any $w \in F(r^N, A)$.

According to the classification of social choice axioms proposed by Fishburn (1973), DIP is an active intraprofile condition. Broadly speaking, DIP amounts to a consistency condition, since it is a restriction on choice sets when the agenda changes while the profile remains fixed. However weak, the DIP criterion ensures that the agenda formation process

enjoys a minimal degree of "dynamic" efficiency.

A couple of remarks are in order here. First, it should be noticed that the consequent of the DIP criterion amounts to the most typical condition imposed on extensions of orderings to powersets (see, for example, Gardenfors (1979)). Furthermore, the consequent of the DIP criterion closely resembles the "ideal" criterion of nonmanipulability invoked and regrettably dismissed in Demange (1987).

We are now ready to prove the main result of this paper.

Theorem 2. Let $E \in EF(N, A)$ be stable (on R^N), let $f=f(k, s)$ be an AFR as in Proposition 1, and let $A \in f(N)$ be an f -admissible agenda. Let $H=H(E, \pi)$ be a SCR for N as defined in Proposition 1. Then H is DIP.

Proof. Let $(x, y) \in (X \times \{i\})^2$ be a d.i. pair at $(r^N, A) \in (R^N, f(N))$ and w.r.t. H (see the definition above), and let $A' = A \cup \{x\} \setminus \{y\}$. We denote by $\bar{\pi}: A \rightarrow A'$ the bijective function defined by the following rules: $\bar{\pi}(y)=x$, and $\bar{\pi}(a)=a$ for any $a \in A \setminus \{y\}$. The following notation will also be used: $H_1 = \text{Core}(E, r^N)$, $H_2 = \text{Core}(E(A' \setminus A, \bar{\pi}), r^N)$. Thus, it has to be shown that for all $z \in H_2 \setminus H_1$ and for all $w \in H_1 \setminus H_2$: $(z, w) \in r_i$.

We have to distinguish the following cases: i) $z=x, w=y$. In this case, there is nothing to prove, since $(x, y) \in \bigcap_{i \in N} r_i$ (by definition of a d.i. pair).

ii) $z=x, w \in A \setminus \{y\}$. Since $w \in H_1$, it follows that $(z, w) \in P_i(r^N)$ by definition of a d.i. pair.

iii) $z \neq x$. Since $z \in H_2 \setminus H_1$, it follows that a pair (S, B) exists s.t. $S \in 2^N$, $B \in 2^A$, $B \in E(S)$ and for all $b \in B$: $(b, z) \in \bigcap_{i \in S} P_i(r^N)$. There are two subcases: either $w=y \in B$, or $w=y \notin B$. If $y \in B$ then $B \in E'(S)$ (where $E'=(A' \setminus A, \bar{\pi})$, by definition of E' . If $y \notin B$, then $B' = B \cup \{x\} \setminus \{y\} \in E'(S)$. Moreover, by hypothesis, by definition of a d.i. pair, and by transitivity of individual preferences: $(x, z) \in \bigcap_{i \in S} P_i(r^N)$. Hence, in either case, a $C \in 2^A$ exists (namely, $C=B$, or $C=B'$) s.t. $C \in E'(S)$ and $(c, z) \in \bigcap_{i \in S} P_i(r^N)$ for any $c \in C$. As a consequence, $z \notin \text{Core}(E', r^N) = H_2$, contrary to the hypothesis. Therefore, case iii)

is to be excluded. It follows that the theorem holds true.

As mentioned above, theorem 2 entails that the SCR consisting of the core correspondences of nicely stable EFs enjoys an encouraging degree of robustness against inefficient agenda manipulation.

3. Discussion

The suggested interpretation of Theorem 2 is the following:

the agenda formation process associated to the SCR consisting of the core correspondences of stable EFs (henceforth, EF-cores-SCR) may be expected to be efficient.

Thus, such an SCR does qualify as a solution to the agenda-manipulation problem (provided that the involved EFs enjoy a suitable set of nice properties).

A proper assessment of the previous claim entails an evaluation of the performance of such an SCR with respect to the typical criteria invoked as a guarantee against "bad" agenda manipulation. As mentioned above, the relevant criteria are the following:

1) Independence of Irrelevant Alternatives (IIA):

a SCR $F: R^N \times D \rightarrow P(X^*(N)) \setminus \{\emptyset\}$ satisfies IIA iff for any $A \in D$ and for any pair (r^N, q^N) of profiles in R^N s.t. for all $i \in N$ $r_i \cap A^2 = q_i \cap A^2$, $F(r^N, A) = F(q^N, A)$.

2) Weak Axiom of Revealed Preference (WARP):

a SCR $F: R^N \times D \rightarrow P(X^*(N)) \setminus \{\emptyset\}$ satisfies WARP iff for all profiles in R^N and for all $A, B \in D$: $A \subseteq B$ and $F(r^N, B) \cap A \neq \emptyset$ imply $F(r^N, A) = F(r^N, B) \cap A$ (it should be recalled that WARP amounts to a contraction-consistency condition - $F(r^N, B) \cap A \subseteq F(r^N, A)$ - plus an expansion-consistency condition - $F(r^N, A) \subseteq F(r^N, B) \cap A$).

IIA rules out the relevance of any information about outcomes

(possibly feasible outcomes) which are not in the actual agenda.

Hence, there is no way to manipulate the choice set through a change in the description of the feasible set, whenever the actual agenda remains fixed.

Moreover, WARP - under the usual "unconstrained"-domain assumption- entails the "rationalizability" of social choices through an ordering (i.e. for any agenda the choice set may be represented as the set of elements of the agenda which are maximal w.r.t. that ordering; but see Hansson (1968) for a more general result).

As a consequence, addition or deletion of outcomes which are sub-optimal w.r.t. the "social" ordering can not affect the choice set.

It is easily checked that the following proposition holds true.

Proposition 2: the EF-cores-SCR H of Theorem 2 satisfies both IIA and WARP.

The fact that H satisfies both IIA and WARP is a straightforward consequence of the strong restriction imposed by the agenda formation rule $f(k, s)$ on the set of admissible agendas. Namely, only trivial (i.e. non proper) subsets of admissible agendas are admissible as well.

Thus IIA and WARP are trivially satisfied. Therefore, in this highly particular variable-agenda setting, both IIA and WARP lose any force as a safeguard against "bad" agenda manipulation.

Hence the role and the significance of other "new" criteria such as DIP follow.

Another implication of the present analysis to be emphasized is the utmost importance of "structural" domain restrictions. Of course, the general relevance of "structural" restrictions is well known.

However, it should be noticed that the proposed model focuses on a "new" class of domain restrictions which are explicitly rationalized

as the outcome of a suitable agenda formation rule (AFR).

This is at variance with the standard treatment of structural domain restrictions as a completely "exogenous" matter.

The very definition of our EF-cores-SCR embodies the foregoing restrictions, as dictated by the underlying AFR.

In fact, the proposed AFR - $f(k,s)$ - imposes a strong regularity condition on the admissible agendas and, thus, on the relevant EFs.

This is why any disturbing volatility of choice sets can be ruled out.

The possibility that, in principle, EF-cores-SCRs may exhibit such a volatility of choice sets was recently emphasized by Strnad (1987).

This Author maintains that the alleged volatility of the choice set under EF-cores-SCRs shows that they are only viable when the agenda is exogenously fixed.

Obviously, this amounts to a strong criticism of EF-cores-SCRs. However, the above mentioned volatility phenomena are easily ruled out by a sensible choice of an explicit AFR. Therefore, Strnad's conclusion should be reversed. Namely, EF-cores-SCRs are exactly the kind of SCR which strongly need an endogenous agenda formation process guided by a suitable AFR (whereas other relevant SCR may perhaps admit exogenously fixed agendas as well).

Furthermore, the main result of this paper shows that the relevant AFR can be expected to be fairly efficient.

We conclude with some comments about related literature.

First, there is a rich literature on the so-called nonbinary approach to social choice theory, started by Hansson and Fishburn (see Hansson (1969), Fishburn (1973), Fishburn (1974)), and subsequently pursued by Grether, Plott (1982) among others.

Clearly enough, the common focus of this literature is on the role of structural assumptions on the domain of admissible agendas.

However, two main lines of research are currently pursued under the same "nonbinary social choice" heading.

The original topic of nonbinary social choice was the attempt to generalize impossibility theorems of the arrowian variety through a relaxation of domain conditions.

The aim of this research program is to single out the axiom, if any, which is the ultimate source of Arrow's theorem. (Incidentally, the most widely accepted conclusion is that such a role should be ascribed to "consistency" - or "collective rationality"- requirements: see Fishburn (1974) and Grether, Plott (1982)).

As a consequence of this concern for suitable generalizations of Arrow's theorem, no agenda is really excluded (either binary or not). The emphasis is on restrictive sufficient conditions for the admissibility of agendas (typically, these conditions establish the admissibility of any agenda which contains at least k distinct alternatives, for some integer $k > 2$).

More recently, another strand of the nonbinary social choice -theoretic literature has addressed the problem of finding positive results by introducing explicit restrictions on the domain of admissible agendas (see Grether, Plott (1982)).

The underlying rationale of these restrictions consists in the observation that in a standard economic environment feasible sets of outcomes are typically infinite.

Moreover in this literature one can hardly find attempts to provide an explicit model of the agenda formation process. Hence the admissible agendas are straightforwardly identified with the conceivable feasible sets.

As a result, all finite agendas (not only the binary ones) are excluded from the relevant domain.

This is the most striking difference with respect to the model proposed in the present paper, in spite of the common focus on structural

restrictions on the domain of agendas.

Concern for the outstanding role of the status quo in social choice problems is the major motivation of some other recent work on agenda-domain restrictions (see Richelson (1984) and Gibbard, Hylland, Weymark (1987)).

These works point out that some scope for positive results arises when the admissible agendas are bound to contain the status quo outcome. Namely, using such a domain restriction, Richelson identifies a pair of SCRs which approximately satisfy Arrow's conditions: however, the first rule fails on efficiency grounds, whereas the second fails on consistency grounds.

Furthermore, Gibbard, Hylland and Weymark show that there is no SCR satisfying symmetry, independence, efficiency and consistency requirements whenever all the agendas containing the status quo are admissible.

Once again, a comparison with our own results suggests the central role of agenda-domain assumptions.

The literature on "agenda control" is also relevant to the issue of agenda manipulation. However, the focus of this literature is on the effects of alternative orderings of a given agenda on the final choice, under a sequential elimination procedure. A recent notable exception is Banks (1987) who provides an explicit analysis of the agenda formation process, in a highly specialized setting.

An earlier attempt to provide an explicit analysis of the agenda formation process is due to Dutta and Pattanaik (see Dutta, Pattanaik (1975), Pattanaik (1978), Dutta (1981)). These Authors envisage a two-stage game: the first stage consists in a sponsoring game which determines the agenda, in view of the second stage. The second stage consists in a voting game on the previously chosen agenda. Building on an ingenious example due to Majumdar (1956), Dutta and Pattanaik are able to prove

that the agenda formation process is likely to involve manipulative, i.e. "insincere", sponsoring of outcomes. Namely, they prove that the foregoing result holds true under a wide class of decision procedures, including all the majoritarian ones, and under various assumptions on the relation between individual preferences over alternatives and individual preferences over agendas. To put it in other terms, under such assumptions the players are to be expected to sponsor alternatives which are not their preferred ones (in the feasible set). However, Dutta and Pattanaik are very careful to emphasize that this kind of manipulation may well turn out to be an advantageous occurrence.

In a similar vein, Mueller (1978) quotes as a major virtue of his own version of the voting-by-veto method the fact that this decision procedure induces the players to take into account each other's preferences while making their proposals.

The present paper reinforces Mueller's contention, from a different perspective. In fact, the EF-cores-SCRs can be seen as a representation of the equilibrium behaviour of suitable voting-by-veto schemes. On the other hand, the results presented above show that the EF-cores-SCRs enjoy, besides their well-known strong-implementability- property, another nice strategic property. Namely, these rules provide the players with the incentive to propose any feasible dramatic improvement they envisage. To the extent that such incentives are well captured by the DIP criterion, the main theorem of the present paper may well be interpreted as an encouraging positive result towards a simultaneous solution of the implementation problem and of the agenda manipulation problem.

References

ARROW K.J. (1951), *Social Choice and Individual Values*. (Wiley, New York).

- BANKS J.S. (1987), Endogenous Agenda Formation in Three-Person Committees. *Social Choice and Welfare*, 4, 133-152.
- DEMANGE G. (1987), Nonmanipulable Cores. *Econometrica*, 55, 1057-1074.
- DUTTA B. (1981), Individual Strategy and Manipulation of Issues. *Mathematical Social Sciences*, 1, 169-176.
- DUTTA B., PATTANAIK P.K. (1975), On Strategic Manipulation of Issues in Group Decision Making. Reprinted in PATTANAIK P.K. (1978).
- FISHBURN P.C. (1973), *The Theory of Social Choice*. (Princeton U.P., Princeton N.J.).
- FISHBURN P.C. (1974), On Collective Rationality and a Generalized Impossibility Theorem. *Review of Economic Studies*, 41-445-458.
- GARDENFORS P. (1979), On Definitions of Manipulation of Social Choice Functions, in J.J. Laffont (ed.): *Aggregation and Revelation of Preferences* (North Holland, Amsterdam).
- GIBBARD A., HYLLAND A., WEYMARK J.A. (1987), Arrow's Theorem with a Fixed Feasible Alternative. *Social Choice and Welfare*, 4, 105-115.
- GRETHER D.M., PLOTT C.R. (1982), Nonbinary Social Choice: An Impossibility Theorem. *Review of Economic Studies*, 49, 143-149.
- HANSSON B. (1968), Choice Structures and Preference Relations. *Synthese*, 18, 423-442.
- HANSSON B. (1969), Voting and Group Decision Functions. *Synthese*, 20, 526-537.
- MAJUMDAR T. (1956) Choice and Revealed Preference. *Econometrica*, 24, 71-73.
- MOULIN H. (1983), *The Strategy of Social Choice*. (North Holland, Amsterdam).
- MOULIN H., PELEG B. (1982), Cores of Effectivity Functions and Implementation Theory. *Journal of Mathematical Economics*, 10, 115-145.
- MUELLER D.C. (1978), Voting by Veto. *Journal of Public Economics*, 10, 57-75.

- NAKAMURA K. (1979), The Vetoers in a Simple Game with Ordinal Preferences. *International Journal of Game Theory*, 8, 55-61.
- PATTANAIK P.K. (1978), *Strategy and Group Choice*. (North Holland, Amsterdam).
- PELEG B. (1984), *Game Theoretic Analysis of Voting in Committees*. (Cambridge U.P., Cambridge U.K.).
- RICHELSON J.T. (1984), Social Choice and the Status Quo. *Public Choice*, 42, 225-234.
- SCHOFIELD N. (1986), Existence of a 'Structurally Stable' Equilibrium for a Non-Collegial Voting Rule. *Public Choice*, 51, 267-284.
- SEN A.K. (1986), Social Choice Theory, in K.J. Arrow, M.D. Intriligator (eds.): *Handbook of Mathematical Economics*, vol. 3, (North Holland, Amsterdam).
- SIMON H.A. (1972), *The Theory of Problem Solving*. Information Processing, 71, 261-277.
- STRNAD J. (1987), Full Nash Implementation of Neutral Social Functions. *Journal of Mathematical Economics*, 16, 17-37.

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