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Secular Stagnation and innovation dynamics:
an agent-based SFC model. Part I

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Secular Stagnation and innovation dynamics: an agent-based SFC model. Part I

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Abstract

The paper fills a gap in the Secular Stagnation literature and develops an agent-based SFC model to analyse the deep relationship between income distribution and productivity through the channel of innovation. With a steady gaze on US macro-economic data since 1950, we put forth the idea that the continuous shift of income from wages to profits may have resulted in a smaller incentive to invest in R&D activity, with the decline in productivity performances that characterizes Secular Stagnation in the USA. The paper is the first step toward the growth model that will be developed in Part II.

JEL Code: E10, O31, O38, O43, P16.

Keywords: Secular Stagnation, Innovation dynamics, Income distribution, Agent-based SFC models.

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1 Introduction

Prof. Larry Summers re-evoked in recent times the old concept of Secular Stagnation to describe a situation in which changes in the economic fundamentals after the Great Recession of 2007 might have caused a significant shift in the natural balance between savings and investments, making the achievement of adequate growth, capacity utilisation and financial stability increasingly difficult [Summers \(2014\)](#). Many economists dealt with that phenomenon thus far, each underlining a peculiar aspect.¹ However, the debate paid little attention to the deep relationship between income distribution, innovation and productivity.

The paper fills that gap in the literature and sets Secular Stagnation into an agent-based framework. We focus on the US capitalistic evolution of the last fifty years and study in which way the distribution of income between wages and profits can determine the rate of innovative activity and then further attainments in productivity. In particular, we depart from Summers' definition and look at Secular Stagnation in the USA as the tendency to the long-term slowdown in the growth rates of labour and total factor productivities which starts in the early Seventies and reaches the trough with the Great Recession in 2007. Moreover, we consider other major features of the US post-1972 economy like the progressive worsening of the functional distribution of income at the expense of the labour share and, on the other hand, a slower growth in R&D activity.

In this contribution, we develop an agent-based SFC model in the line of [Dosi et al. \(2010\)](#), [Caiani et al. \(2016a\)](#) and [Godley and Lavoie \(2006\)](#). The model involves a one-good two-class closed economy with no government sector. The good can be used either for consumption or for investment purposes. Households are divided between workers, that supply labour inelastically at the going wage share, and capitalists, which own the firms and act as entrepreneurs. The latter invest in innovative research activity a percentage revenue from past sales while trying to reach a normal profit rate. In our model, the micro-foundation of the endogenous innovation process is essential to avoid isomorphism between micro and macro phenomena and to remark the evolutionary character of the theoretical base. The adoption of an

¹Eichengreen wrote that Secular Stagnation is like Rorschach test: it means different things to different people [Eichengreen \(2015\)](#).

agent-based perspective needs to be justified. One may ask, indeed, whether such an evolutionary framework is really necessary or, in other terms, if that modeling allows to show up insights not visible with the standard representative-agent modeling or with a more aggregate perspective, if any. We reply that both standard representative-agent models or the aggregate perspective suffer from, among the others, a lack of micro-heterogeneity, interpreted as the multiplicity of interactions among agents with no a priori commitment towards any reciprocal consistency (Dosi et al., 2010). Agent-based models are particularly suitable to the task since the user knows by construction the micro data-generating process and can explore the features of macro-variables as properties emerging out of the evolutionary dynamics (Dosi et al., 2018). In other words, to explain aggregate outcomes we cannot sum the predicted individual behaviours, because the actions taken by the single agent influence the behaviour of the others (Bowles, 2009). This kind of modeling recognizes, on the one hand, the importance of Solow (2008)'s call for micro-foundations more realistic than usual. On the other hand, micro-foundation is absent in many macro-aggregate models, which do not enable the researcher to fully understand processes occurring at the micro- and meso-level of economic activity.

The contribution of the paper to the literature on Secular Stagnation lies on its capability to show the way phenomena at the macro-level affect the dynamic path of variables at the micro level. More precisely, it is interesting to show that the shrinkage of the labour share impacts negatively on firms' propensity and ability to innovate. Wages indeed sustain consumption and, indirectly, investments. The lower aggregate demand after a fall in the wage share reduces capitalist's incentive to invest either on tangible capital or on innovative search at the micro-economic level. The result will be an overall bad economic performance on aggregate. We advance the idea that the continuous shift of income from wages to profits may have resulted in a smaller incentive to invest in R&D activity, entailing the evident decline in productivity performances that marks the US Secular Stagnation. We have to admit, of course, that this is not the only valid explanation for the long-run tendency of productivity growth to fall. Non-technological motives, like lower top marginal tax rates, increased low-skill immigration, rising trade with China and low-cost manufacturing countries or the rise of superstar firms (Autor et al., 2020)

are equally plausible.

Back to the model, we test additionally the role exerted by the rate of interest and the loosening of barriers to innovation and imitation. On the one hand, the decrease in the interest rate leads to particular results: it helps increase aggregate production and employment levels but impacts negatively on labour productivity, since the entrepreneurs reach more easily a normal profit rate and prefer organizing the production process to less labour-saving techniques. On the other hand, loosening the barriers to the interaction among firms and increasing the possibility to exchange ideas through imitation allow for further innovation and better economic performances as a whole, but the effect is circumscribed to the long run only.

The paper is organised as follows: Section II deals with the literature; Section III presents empirical evidence on income distribution, innovation and productivity; Sections IV to VI are about the model and related experiments; Section VII concludes. The Appendices contain tables, figures and convey some information on the main matching processes. Although the model approaches to a stationary state and does not refer to growth questions (yet), it represents the first step toward the development of a growth model. The latter will be developed in Part II.

2 Relation with the literature

Several fields of research contribute to define the background literature of the present work. First and foremost, the paper inserts into the literature of Secular Stagnation, here defined as the tendency to the long-term slowdown in the growth rates of labour and multi-factor productivities, which starts in the early Seventies and reaches the trough with the Great Recession of 2007.² Albeit for a different context, the concept was introduced with the pioneering work by [Hansen \(1939\)](#) to describe the somber situation in which the US economy fell after the Great Depres-

²The productivity-growth decline is a well-documented fact in the literature. We nonetheless present some empirical evidence on that in the following Section.

sion in 1929.³ To date, the concept was re-evoked by [Summers \(2014\)](#) to outline a situation in which changes in the economic fundamentals, after the Great Recession, might have led to a significant shift in the natural balances between savings and investments. The equilibrium natural interest rate associated to full employment of labour would have reached negative values. The related outcome is a situation in which the achievement of adequate growth, capacity utilisation and financial stability appears increasingly difficult [Summers \(2014\)](#).⁴ However, his analysis is limited in scope in that he focuses on Secular Stagnation through the lens of the Great Recession only. Summers examines the very recent past and the remarkable decline in productivity growth finds no place in that framework.⁵

Many economists recovered the concept after him: we find [Gordon \(2015\)](#), [Eichengreen \(2015\)](#) and [Hein \(2016\)](#), among the others. Their approach is historical data-driven. On the one hand, [Gordon \(2015\)](#) and [Eichengreen \(2015\)](#) adopt a supply-side view to analyze the long-period determinants of productivity growth and disregard cyclical influences. They suggest that mounting inequality impacted negatively on the accumulation of human capital, since students are ever more burdened by the loans they take to pay their college tuition. Furthermore, they are concerned to the revolution started by digital electronics, which ran out of steam, with the electronics facing diminishing returns. Their idea is that innovation achievements of the last fifty years had a relatively smaller impact on productivity than, for example, innovations at the turn of the XXth century. On the other hand, [Hein \(2016\)](#) points out that the redistribution of income at the expense of the labour share squeezed the investments in capital stock through an accelerator mechanism. Moreover, the author claims that stagnating-demand policies and the overall surge of personal income inequality depressed investments and productivity growth.⁶

³The author looked at the high unemployment as the principal problem for Americans and the expression of Secular Stagnation stood for “sick recoveries which die in their infancy and depressions which feed on themselves and leave a hard and seemingly immovable core of unemployment” ([Hansen, 1939](#)).

⁴[Eggertsson et al. \(2019\)](#) draws upon [Summers \(2014\)](#) and provides a general setting for the natural rate hypothesis. They develop an analytic overlapping generation model, the steady state of which is characterized by a negative full-employment real interest rate.

⁵Additionally, the (negative) natural rate hypothesis suffers from important theoretical weaknesses: see [Palley \(2019\)](#) and [Di Bucchianico \(2020\)](#).

⁶We should remark that [Hein \(2016\)](#) contrasts the concept of Secular Stagnation to that of Stagnation Policy. We believe that such a juxtaposition could be misplaced: the former should concern to some stylized fact or empirical evidence, while the latter to the rationales. Anyway, we do not discuss that since it is beyond the scope of the present work.

The second stream of research is about the Schumpeterian and evolutionary tradition (Aghion and Howitt, 2008; Bowles, 2009; Schumpeter, 1982). It deals with “dynamic processes causing qualitative transformation of economies driven by the introduction of innovation in their various and multifaceted forms and the related co-evolutionary process” (Hanusch and Pyka, 2007).⁷ Innovation turns out to be the most important force driving productivity and economic growth. Moreover, it is strongly related with uncertainty in its Knightian sense, causing complex modes of behaviour. Although innovation occurs at the micro-level of the economy through the creation of novelties and many entrepreneurial decisions, its potentiality manifests at the industry or meso-level of the economic activity (Dopfer et al., 2004).

Third, we refer to the agent-based (AB, hereafter) literature, that considers economic systems as populated by many heterogeneous interacting agents without any central coordination (Caiani et al., 2016b).⁸ Several works study the interplay between innovation, income distribution and economic performances. Dosi et al. (2010, 2016, 2018) and Napoletano et al. (2012) are key contributions to the topic. The family of their $K + S$ models investigates the way innovations affect macro-variables, through endogenous generation of supply shocks at the micro and meso levels. Their models are structural since they build on a representation of what agents do. An important characteristic is that they link the Schumpeterian tradition of innovation-driven economic growth with the Keynesian theories of demand generation. In other terms, the Schumpeterian engine fuels growth only in the presence of Keynesian policies, which do contribute to reduce output volatility and unemployment rates. On the same line of research is Wirkierman et al. (2018), focused on the distributional impacts of innovation. The public sector invests directly in R&D and licenses to private firms access to the new technology to produce the final good. Increasing the wage share allows the public sector to drive the process

⁷An eminent precursor of evolutionary and complex economics is Von Hayek (1937) with the notion of spontaneous order. Briefly, chaotic processes at the micro-economic level may entail some form of regularity at the macro-aggregate perspective. Markets are viewed as places for learning and discoveries, hence a place for innovation and imitation. In that framework, the evolution of institutions is the product of countless interactions, the aggregate outcome of which is often unintended (Bowles, 2009).

⁸Economies are seen as complex dynamic systems, whereby a multitude of micro-agents locally interact and give rise to the multifaceted stylized facts for growth rates, employment, income distributions and institutions (Tsfatsion, 2006). Additionally, aggregate outcomes condition local interaction patterns.

of innovative search toward an outcome in which the distributional impacts of innovation reflect the distribution of contributions to the innovative process.

The afore-mentioned methodology suffers from some drawbacks. Many former AB models violated accounting consistency requirements, with some financial flows arising out of nowhere. [Caiani et al. \(2016a\)](#) starts from this point and builds a fully decentralised stock-flow consistent model with heterogeneous interacting agents, where consistency is applied since the micro-economic level to account for the structural interrelatedness of agents. Although the model does not concern to growth questions, it is promising in the field of bank regulation and macro-prudential issues. Additionally, this contribution offers interesting guideposts to calibrate, validate and adapt the basic framework to alternative research questions. Based on the above consolidated framework, [Caiani et al. \(2019\)](#) investigates the nexus between inequality and growth, assessing the impact of several distributive regimes on innovation dynamics and economic development. The crucial feature is the segmentation of the labour markets in four tiers, according to the role assumed by each worker in the hierarchical organisation of the firm. The distributive regimes concern to the implementation of more, or less, progressive tax schemes and higher, or lower, downward wage rigidity of lower-tier workers. The results are in tune with the literature that emphasizes wage-led growth regimes in a closed economy. In other words, more progressive tax systems and measures to sustain low and middle income households help foster economic development and innovation.

3 Statistics on wages, productivity and innovation since 1950

We said that the literature on Secular Stagnation paid little attention to the interplay between income distribution, innovation and productivity developments. The mounting shift of income from wages to profits, on the one hand, and the reductions in productivity growth, on the other, have manifested since the end of the Golden Age of capitalism (1950-1973). Hence, what we represent in [Tab. I](#) and in [Fig. I](#) is a well-documented fact in the economic debate. Starting from the late Sixties,

when it was near to 70%, the adjusted wage share keeps decreasing to the current minimum value around 60%. For what regards to labour productivity, there has been a slow and steady decline in the growth rate over the period of interest. The rate of growth exhibits a timid recovery in the Nineties, before the new and long-lasting collapse in the aftermath of 2007 crisis. The same holds for TFP growth, that follows the same qualitative trajectory. TFP grows 1.7% on average during 1950-1973, then it shrinks to one-third of that value in 1973-1995. It rebounds in the Nineties and eventually reaches the bottom 0.6% in the post-2007 decade. As above, [Gordon \(2015\)](#) suggests that soaring inequality impacted negatively on the accumulation on human capital and then on productivity, since students are ever more burdened by the loans they take to pay their college tuition. On the demand side, [Hein \(2016\)](#) points out that the redistribution of income at the expense of the labour share lowered investments in capital stock through an accelerator mechanism. For what concerns to the relation between innovation and productivity, the literature refers to [Eichengreen \(2015\)](#) and still to [Gordon \(2015\)](#). Their idea is that the innovation achievements of the last fifty years had a relatively smaller impact on productivity than, for example, innovations at the turn of the XX^{th} century. What is missing on the analysis around Secular Stagnation is the demand-side channel between functional distribution of income, innovation and productivity. [Sylos-Labini \(1983\)](#) first, and [Allen \(2009, 2011\)](#) and [Carnevali et al. \(2020\)](#) later, explain the role of a distribution favourable to the wage share in triggering a process of economic development, through continuous investments in innovative activities and further achievements in productivity. Is it possible that the same process occurred in the opposite way? In other terms, does any positive relation exist between wage share, investments in R&D and productivity enhancements? Tab. II to VI and Fig. II present data on the evolution of R&D expenditures in United States since 1950, whenever possible⁹. Tabs. II- III and the upper panels of Fig. II evidence the striking decline in growth of R&D expenditure since late 1960s, either by source or by function.¹⁰ The careful observer may object that the Golden Age in US was a particular period marked by the necessity of winning the Cold War against the So-

⁹We worked on AAAS and BEA data.

¹⁰Source refers to federal, industry, non-profits, universities, other and total. Function refers to federal expenditures in defense and non-defense, energy, general science, health, natural resources, space and other. Categories are established by the AAAS database.

viet Union. That would explain why, for instance, space expenditure growth rates surged toward extraordinary values until the end of the Sixties and then fell sharply after the first moon landing. Therefore, we analyzed the time trend of each variable since 1973, finding that the majority of sources and functions exhibited a steadily downward trend in growth.¹¹ Tabs. IV- V and the lower panels in Fig. II concern to the evolution of R&D shares. It is interesting to notice that private R&D as percentage of Fixed Investments kept increasing throughout the period, the decline in growth showed above notwithstanding. Furthermore, Arora et al. (2018) notes a shift away from science by large corporations between 1980 and 2006. Although science remains an important input for innovation, their empirical evidence points to a reduction of the private benefits of internal research, which leads to closing and downsizing their labs¹². Therefore, albeit firms are committing a higher share of investments in R&D, it does correspond to the redirection of resources and attention from more exploratory scientific research toward more commercially-oriented projects (Arora et al., 2018). By the same token, federal R&D shares kept decreasing from late 1960s onwards, either as share of discretionary outlays or as share of total budget¹³. Moreover, they kept decreasing regardless to the destination, whether defense or non-defense.

To sum up, Tab. VI shows that wage share, innovation rates and productivity measures are significantly and positively correlated. We advance the idea that the ongoing shift of income from wages to profits may have resulted in a smaller incentive to invest in R&D, entailing the evident decline in productivity growth. Secular Stagnation, as defined above, might have originated from that. In what follows we develop an agent-base SFC model to highlight whether that hypothesis grounds on a micro-founded framework too. AB models are particularly suitable to the task since one knows by construction the micro-economic data-generating process and

¹¹For the sake of brevity, we did not report tables on trend regressions, since what asserted is visible in Tabs. II- III and Fig. II. Anyway, they are available upon request.

¹²Although Bloom et al. (2020) agree on this point too, we must point however that innovative investments in the private sector is now very disseminated in a multiplicity of small-size firms and start-ups, often unrelated to the investing firm from a corporate point of view. That may invalidate, at least partially, the empirical evidence by Arora et al. (2018).

¹³The US Senate defines discretionary spending the spending budget authority and outlays controlled in annual appropriation acts. Total outlays or total budget identify the amount of expenditure set out by the federal government.

can explore the characteristics of macro-economic variables as properties emerging out of the evolutionary dynamics (Dosi et al., 2018).

4 The model

The analysis concerns to the role of income distribution and demand in affecting the economic performance. We focus on a one-good two-class closed economy with no government sector that approaches to and gravitates around a stationary state.¹⁴ We describe the economy as populated by heterogeneous interacting agents with the help of Fig. III.¹⁵ Precisely:

- A collection N_s of households: on the one hand, $N_s - F$ agents offer labour inelastically at the going wage rate; on the other hand, the remaining households are capitalists. Regardless of their status, households spend part of their income in the purchase of the (consumption) good. Savings are held in the form of bank deposits, only and always. Moreover, they own the bank proportionally to their wealth and receive banking profits as dividends.
- A collection F of firms owned by the entrepreneurs organizing the production process and taking investment decisions. They produce a homogeneous good that can be used either for consumption or for investment purposes. Additionally, they may apply for loans to finance production and investment.
- A consolidated bank, whose activity is limited to providing firms with loans and households with deposits at given interest rates. So doing, the big bank is an input supplier.

During each period of the simulation, agents interact on five markets:

- The (capital) goods market: firms interact with each other to buy and sell (capital) goods.

¹⁴Following Caverzasi-Godin (2015), we define a stationary state as a logical construction where all stocks and flows do not change over time and that can be reached if all the behaviours were fixed after a transition period.

¹⁵To be precise, the model is complex, adaptive and structural in the spirit of Tesfatsion (2006): complex because the system involves interacting units; adaptive since concerns to environmental changes; structural because built on what agents do.

- The (consumption) goods market: households purchase (consumption) goods from firms.
- The labour market: capitalists interact with workers through hiring and firing.
- The credit market: the consolidated bank provides firms with loans.
- The deposit market: the consolidated bank gathers households' deposits.

The behavioural equations for households and firms are in line with the agent-based SFC literature as in [Caiani et al. \(2016a,b\)](#) and [Godley and Lavoie \(2006\)](#). We further assume no population growth; however, labour supply is exogenous and unbinding, since in a mature capitalist economy as the USA are there is usually a pocket of unemployment, while episodes of labour shortages, if any, are solved through exogenous migration flows. A crucial feature of the model is the role assumed by innovation, which turns out to be the driver of productivity and economic development. Though it occurs at the micro-level of the economy through several entrepreneurial decisions, its potentiality gets fully fledged at the industry or meso-level of the economic activity. The model exhibits evolutionary roots since it envisages path dependencies and irreversibilities. ¹⁶

For the sake of simplicity, we split the exposition as follows: timeline of events, production firms, labour market, households, innovation and imitation processes, banking system, prices and inflation expectations.

4.1 Timeline of events

Production firms are endowed with a unit of (capital) good at $t = t_0$. After that, micro-economic decisions occur with this sequential order any given period t :

1. Firms compute their target level of capital.
2. Capitalists draw from previous accumulated wealth, if any, and borrow from the banking system in order to have enough fund to hire workers and buy the (capital) goods they need. Once this has been done, they set up production

¹⁶The heterogenous agents show close and frequent interactions and the “outcome of evolutionary processes is determined neither ex ante nor as a result of global optimizing, but rather is due to true uncertainty underlying all processes of novelty generation” ([Hanusch and Pyka, 2007](#)). Italics in the original text.

to build the (capital) goods they are ordered by the other firms and to satisfy the demand for (consumption) goods from households.

3. Workers receive a wage. Regardless of their status, agents purchase the (consumption) good with part of the received income and save as money deposits what remains. Businessmen earn a profit as residual claim, if any.
4. The aggregate bank gathers interest payments from firms and pays interests on households' deposits. Then, it distributes profits to households.
5. Firms update their production plans according to the demand they face. Moreover, they invest on capital stock and on R&D to improve their technology level, save manpower and earn further – extra – profits. New machines and productivity enhancements due to the R&D activity, if any, will be available at $t + 1$.

4.2 Production firms

We start describing how production takes place and how entrepreneurs take their decisions. The economy produces a single good that can be used either for consumption or for investment purposes. There are no inventories and production adapts to demand. Output components are all expressed at constant prices. The first equality is about production at firm level:

$$y_j = c_{f,j} + i_{s,j} + i_{rd,j} \quad (1)$$

where y is the amount of good produced by the single firm, split into production of (consumption) and (capital) goods, and innovative activity respectively, while j always refers to the single firm if not otherwise specified. The production technology employs labour and capital in fixed proportions, following the usual Leontief production function:

$$y_j^P = \min [\varphi \cdot k_j; a_j \cdot N_s] \quad (2)$$

where y^P is the productive capacity of the i -th firm, k its capital endowment, φ the inverse of the capital-output ratio and a is the labour productivity within the same firm. Entrepreneurs target a certain capital stock k^T .¹⁷ For simplicity,

¹⁷Even if production adapts to demand, firms maintain excess capacity and this does not reflect a

we assume that a constant proportion δ of the existing stock of capital depreciates period-by-period and that capitalists set aside an amount of funds exactly sufficient to replace the used-up equipment:

$$da_j = \delta \cdot k_{-1,j} \quad (3)$$

$$af_j = da_j \quad (4)$$

da and af define the depreciation allowances and the amortization fund, respectively.

Let us turn on the investment decisions. The entrepreneurs distinguish between investments on tangible capital – i.e. machines – and intangible capital – i.e. R&D. Investments on tangible capital increase the productive capacity but do not improve technology and labour productivity, whereas investments on R&D do. In other words, since inventive activity is costly, capitalists have two alternatives: capital accumulation and innovation. Both types of investment raise total earnings, but in different ways: innovation reduces unit labour costs in production, while capital accumulation increases the size of a firm’s business.¹⁸ Gross investments on tangible capital consist of a modified version of the standard partial-adjustment accelerator model:

$$i_{k,j} = i_0 + i_{1,j} \cdot (k_j^T - k_j) + af_j \quad (5)$$

where i_k , i_0 and i_1 represent the investment in physical capital, the autonomous investment or animal spirits and the adjustment coefficient, respectively.

Firms invest in innovative search to save labour and to earn extra-profits. In line with the Schumpeterian literature, we posit the amount invested in innovative activity is made up of two components:

$$i_{rd,j} = \vartheta_{0,j} \cdot c_{f,av,j} + \vartheta_{1,j} \cdot (\bar{q} - q_j) \quad (6)$$

wrongful process of expectations formation, but rather the rational decision of the firm to be able to accommodate fluctuations in demand (Ciccone, 1986). In addition to this, a further clarification is apt: we hypothesize productivity improvements result in different technologies. That allows to keep the capital productivity as constant through time; the same holds for the capital to labour ratio along the same technology. In other words, the adoption of new technologies leads to discrete jumps in the capital to labour ratio, which keeps its constancy with respect to a given technology.

¹⁸For simplicity, there is no trade-off between different types of investment.

The first component on the right-hand-side captures firm's expectations about future demand that are equal to the average revenue from past sales of consumption goods.¹⁹ The other component reflects the cost-side of the expected profit rate, provided that the actual profit rate can be seen as an indicator for expected profitability.²⁰ Firms' profits are sales minus amortization fund, interest payments on past loans and wages:

$$f_j = y_j - af_j - r_l \cdot ld_{-1,j} - wb_j \quad (7)$$

where r_l is the given interest rate, ld_{-1} the stock of loans from the past and wb the wage bill.

An important clarification is now necessary: i_k and i_{rd} represent the expenditure each firm does to ameliorate its technology. Since the expenditure related to the investment in capital stock is commissioned to other firms, we call (8) the investment demand. The random pattern of interactions among firms leads to a configuration in which the single firm produces an average amount of (capital) goods for the others, as in (9):

$$i_{d,j} = i_{k,j} + i_{rd,j} \quad (8)$$

$$i_{s,j} = \bar{i}_{k,j} \quad (9)$$

The capital stock, k , is the result of past (depreciated) equipment plus gross investments in physical capital i_k :

$$k_j = (1 - \delta) \cdot k_{-1,j} + i_{k,j} \quad (10)$$

To conclude this subsection, how do firms finance their (net) investments? We have three options. First, all net investment is financed out of new loans; second, all net investment is financed out of past accumulated wealth; third, the net investment is financed partly out of wealth and partly out of new loans. We adopt the third way

¹⁹“Firms in the capital-good industry “adaptively” strive to increase their market shares and their profits trying to improve their technology both via innovation and imitation. Both are costly processes: firms invest in R&D a fraction of their past sales” (Dosi et al., 2010). Moreover, $c_{f,av} = \frac{t-1}{t-1} c_{f,i}$.

²⁰We suppose firms wish to obtain a normal profit rate \bar{q} .

and suppose that the entrepreneur contributes to finance her investment decisions with part of past accumulated wealth, say $q \cdot mh_e$, while the remaining need will be financed out of new loans as follows:

$$dl_{d,j} = i_{d,j} - af_j - q \cdot mh_{-1,j,e} \quad (11)$$

where dl_d is the change in loans demand. Furthermore, the single firm can borrow whatever sum it needs from the banking system at a constant rate r_l for convenience.

4.3 The labour market

Each firm needs a certain amount of effective labour to set out the production process, that is it must consider the productivity of each worker within the enterprise. Denoting with a the effective labour productivity, the labour demand for the single firm is:

$$nd_j = \frac{y_j}{a_j} \quad (12)$$

The distribution of income at firm level is divided between profits and wages. The model follows what found in the literature: the worker is given a wage rate and entrepreneurial profits are determined as a residual. We can translate what said in the following equations:

$$w_r = (w_0 - w_1 \cdot u_{r,-1}) \cdot pr_t \quad (13)$$

$$wb_j = w_r \cdot nd_j \quad (14)$$

The first equation identifies the wage rate w_r as the result of the positive constant w_0 and it is a negative function of the unemployment rate u_r , since $w_1 > 0$. The wage rate is updated every period to account for inflationary pressures, as denoted by pr_t . (13) says the lower the unemployment rate and the higher the inflation expectations, the higher the wage rate.²¹ The wage bill at firm level wb is the simple product between the wage rate and the number of employees. We mentioned earlier that the labour supply is exogenous and unbinding. We integrate it with

²¹It reflects the logic assumed by the new-Keynesian Phillips curve.

the assumption that every worker is willing to accept a job at the going wage rate. Therefore, no firm faces (labour) supply constraints. In other words, the setting admits no over-employment but involuntary unemployment.

4.4 Households

Households consume and save. They are distinguished between capitalists and workers according to their propensity to save. The flows of income they may receive consist of four components: wage rate, entrepreneurial profits, an amount of bank's profits proportional to their wealth, $\sigma_{mh} \cdot F_{b,t}$, and interest payments on past deposits int_{mh} .²² We write the households disposable income $yd h_i$ as equal to:²³

$$yd h_i = \begin{cases} f_i + \sigma_{mh,i} \cdot F_{b,t} + int_{mh,i} & \text{if } i = e \\ w_r + \sigma_{mh,i} \cdot F_{b,t} + int_{mh,i} & \text{if } i = w \end{cases} \quad (15)$$

For simplicity, agents consume part of their disposable income and part of their accumulated wealth, that takes the form of deposits²⁴.

$$c_{inc,i} = \begin{cases} \alpha_0 + \alpha_{1,i} \cdot w_{r,-1} & \text{if } i \text{ is worker} \\ \alpha_0 + \alpha_{2,i} \cdot f_{i,-1} & \text{if } i \text{ is capitalist} \end{cases} \quad (16)$$

$$c_{wea,i} = \alpha_{3,i} \cdot (m_{h,-1,i} + \sigma_{mh,i} \cdot F_{b,t} + int_{mh,i}) \quad (17)$$

$$c_i = c_{inc,i} + c_{wea,i} \quad (18)$$

(16) says that the current consumption out of income is composed of an autonomous component α_0 and a portion of the past income: α_1 and α_2 are the marginal propensities to consume out of disposable income. (17) represents the

²²Where $\sigma_{mh} = \frac{m_h}{M_{h,t}}$.

²³ $i = e$ for capitalists; $i = w$ for workers.

²⁴For convenience, we suppose households differ in the way they consume the income out of work. No difference exists in the way they consume wealth, interest payments and banking profits, since the marginality of that consumption component.

current consumption out of wealth, dividends and interest payments, equal to a percentage α_3 applied to that sum. The consumption function c is the sum of c_{inc} and c_{wea} as in (18). What is not consumed is saved (dm_h) and accumulated to the stock of deposits:

$$dm_{h,i} = ydh_i - c_i \quad (19)$$

$$m_{h,i} = \begin{cases} m_{h,-1,i} + dm_{h,i} - q \cdot m_{h,-1,i} & \text{if } i = e \\ m_{h,-1,i} + dm_{h,i} & \text{if } i = w \end{cases} \quad (20)$$

4.5 Innovation dynamics

Innovation is very affected by uncertainty. Potential innovators do not know whether their effort and expenditures to promote technological improvements will succeed or not. A well established evolutionary tradition models firms' innovative activity as a two-step stochastic process (Caiani et al., 2019; Dosi et al., 2010)). We decided to depart from that tradition for two reasons: firstly, we want to keep the model as simple as possible; secondly, we want to respect some empirical regularity in the innovation process. Nevertheless, it does not mean that our way to analyze innovation does not reflect what found in the Schumpeterian literature: for instance, we may imagine innovation as it took place with the hiring of researchers devoted to the development of a software, the latter improving the technological apparatus of the firm.

To begin, we provide the reader with some basic definitions: we shall denote with a_{jj} the labour productivity of the j^{th} firm as result of its effort in R&D, with a_{ji} the labour productivity of the j^{th} firm as result of the imitation process, and a_j the effective labour productivity of the j^{th} firm at some point in time, that is equal to the maximum between a_{jj} and a_{ji} as below. For simplicity, we assume their equality at the very beginning of the analysis, precisely $a_j = a_{jj} = a_{ji} = 1$. Firms incur new loans to improve their technology levels. The literature often emphasizes the R&D expenditure as share of output as the determinant for the growth in productivity or for the innovation rate in the economy. In this contribution, however, we want to stress the role of the total amount of funds invested on innovative research. In fact, two firms may devolve the exact share but if the absolute amount differs, the

larger firm will have higher probability to innovate than the smaller one.²⁵ As said, the more a firm invests on innovative activities, the more probable it innovates. To represent this process, we can define a logistic probability distribution as an increasing function of the amount invested in R&D:²⁶

$$\lambda_j = \frac{1}{1 + \exp^{-\varepsilon \cdot i_{rd,av,j} \cdot t}} \quad (21)$$

$$i_{rd,av,j} = \frac{\sum_{i=0}^t i_{rd,j}}{t} \quad (22)$$

(21) is the probability to innovate and it is a sinusoidal function approaching to 1 as $t \rightarrow \infty$. The speed with which it tends to 1 is governed by the cumulated amount of resources invested in research and development. This means that the probability each firm has to innovate strictly depends on how much the same firm spends on average. The logistic function has been used quite often in the literature to illustrate the progress of creation and diffusion of an innovation through its life cycle.²⁷ Precisely, the introduction of new products or processes in the economies spurs an intense amount of research and development leading to strong improvements in cost reduction and quality. The mid-term outcome consists of a rapid growth of that industry. Clear examples from the past are railroads, urban electrification, cars, light bulbs and so on. However, once those improvements exhausted, new products or processes are so widespread that markets saturate. Back to (21), it is important to underline that λ changes from firm to firm, pointing that the ability and probability to introduce innovations are a direct function of the own R&D effort.²⁸

To know whether innovation occurs, every firm is assigned a random number drawn from a uniform distribution $p_{inn} = \zeta_1$, where $\zeta_1 \sim U[0;1]$. If this number is smaller than the threshold λ , the firm innovates. Innovation takes place in the economy as an improvement in labour productivity. Recalling the model focuses on levels

²⁵Think about the comparison between a large firm as Apple and a much smaller one. Suppose both of them invest twenty percent of profits, but for Apple this share amounts to million dollars. For the smaller one, it can amount to thousand dollars only, at best. Who will be the most probable innovator in the field?

²⁶Since there are several probability distributions that may do it for us, we tried an inverse exponential function and the Gumbel probability distribution. Results do not change significantly.

²⁷De Tarde (1903) was the first.

²⁸That is tantamount to introduce path dependency and irreversibility in the model.

and not on growth, labour productivity is a direct function of the average outlay in innovation activities:

$$a_{jj} = a_0 + a_1 \cdot i_{rd,av,j} \quad (23)$$

in this way we take into account firm's ability to learn from past achievements.

The imitation process is similar to the innovative one. Let us look at firms as if they were people walking in the street. The single person has got a certain probability to meet somebody. For simplicity, we assume that one person cannot meet more than three people in the same period. Moreover, meetings are fully random. We can image each meeting as the single possibility to copy the technology of the competitor. The imitation process occurs with the same law followed by the innovation process.²⁹ To formalize it, we define a $F \times F$ network matrix, called imi_{net} . Its cells take value 1 if a connection between two firms is established, and 0 otherwise. Once we got all the linkages, we record in a_{ji} all the potential productivity levels that a firm can reach by imitating the technology of its competitors. Then, the firm compares the productivity levels from imitation and home-innovation, choosing the best-performing technique and updating its productivity. As before, every firm is assigned a number drawn from a uniform distribution, $p_{imi} = \zeta_2$, which is compared to the λ threshold above. This represents an important feature of the model: the probability the firm has to imitate strictly depends on its amount of innovative investments. In other terms, we do exclude free-rider or opportunistic behaviours. Therefore, if $p_{imi} < \lambda$ a firm may imitate when $i_{rd} > 0$. Then:

$$a_j = \max [a_{jj}; a_{ji}] \quad (24)$$

4.6 The banking system

Schumpeter (1982) places the banking system side by side with the creative entrepreneur, as is the case of a symbiotic relationship. The former makes innovative investments possible through the opening of a credit line for the necessary expenditures, while at the same time the banker is offered a possibility to earn money by

²⁹We assume individuals make use of only local knowledge and make transaction with positive probability as long as it is beneficial to them.

the innovative businessman.³⁰

In reality banks discriminate between clients according to their credit worthiness by credit rationing. In the AB literature, it is quite common to assume banks discriminate through higher or lower interest rates on loans. Since we are not very concerned to banks' behaviours in financial markets, we suppose that the banking sector is composed of an aggregate bank and constitutes a pure accommodating agent. It provides production firms with loans to finance their investment plans and gathers whatever amount of deposits the public wishes to hold. For simplicity, households' accumulated wealth takes the form of bank-account deposits only. For the same reason above, the big bank sets constant interest rates: it finances loans at a rate r_l and rewards deposits with a rate r_h . Obviously, $r_l > r_h$ strictly holds. The equations describing the bank's behaviour are the following:

$$int_{ld,j} = r_l \cdot \sigma_{ld,j} \cdot L_{d,t-1} \quad (25)$$

$$int_{mh,j} = r_h \cdot \sigma_{mh,j} \cdot M_{h,t-1} \quad (26)$$

$$F_{b,t} = r_l \cdot L_{d,t-1} - r_h \cdot M_{h,t-1} \quad (27)$$

(25) describes the interest payments the bank picks from each firm according to its share on total loans, $\sigma_{ld} \cdot L_{d,t-1}$.³¹ (26) reflects how the bank rewards the single household's deposit, according to its share on total wealth, $\sigma_{mh} \cdot M_{h,t-1}$. (27) is the banking profits equation. To ensure consistency, bank's profits are distributed to households as in (15).

4.7 Prices, mark-up and inflation expectations

The model does not involve the production of public goods, hence the prices we should consider come from the unit price of private output. Firms set the price as

³⁰For simplicity, we assume no credit constraints.

³¹Where $\sigma_{ld} = \frac{l_d}{L_{d,t}}$.

a mark-up over unit labour costs:

$$p_j = (1 + \mu_j) \cdot \frac{w_r}{a_j} \quad (28)$$

The mark-up is set by the entrepreneurs according to the market-share differential:

$$\mu_j = \mu_0 + v \cdot (\sigma_{m,-1,j} - \bar{\sigma}) \quad (29)$$

where μ_0 and v are constant while $(\sigma_{m,-1,j} - \bar{\sigma})$ indicates that the mark-up increases when the market share is above the median market share and decreases in the opposite case.³²

The market share will be determined accordingly as follows:³³

$$\sigma_{m,j} = \frac{y_j}{Y_t} \quad (30)$$

The inflation rate is the percentage change in the (average) price level and it is obtained once average prices are computed:

$$\bar{p} = \frac{1}{F} \sum_j p_j \quad (31)$$

$$\pi_t = \frac{\bar{p}_t}{\bar{p}_{t-1}} - 1 \quad (32)$$

Inflation enters the model through its influences on investment and consumption decisions.³⁴ We define the expected inflation rate π^e as:

$$\pi^e = \psi_0 + \psi_1 \cdot (\pi^T - \pi_{t-1}) + \pi_{t-1} \quad (33)$$

where π^T is the target inflation rate while ψ_0 and ψ_1 are non-negative parameters. The expected price level p^e is:

$$p_t^e = (1 + \pi^e) \cdot \bar{p}_{t-1} \quad (34)$$

³²The assumption resembles and simplifies what found in [Dosi et al. \(2010\)](#).

³³ Y_t is the value of aggregate production in the economy.

³⁴We adopt a regressive inflation-expectations process since it “[...] provides a more accurate approximation of how economic agents make their decisions in the real world” ([Sawyer and Passarella, 2019](#)).

The final step consists of introducing the following term into the target-capital and wages functions, defined as the ratio between expected and actual prices:

$$pr_t = \frac{p_t^e}{\bar{p}_t} \quad (35)$$

5 Notes on the baseline model

The model is run through 450 periods on quarterly basis. It does not allow for analytical, closed-form solutions. The latter is a general characteristic of AB models and comes from the many non-linearities in the agent decision rules and patterns of interaction. Most coefficients and initial values of variables are either borrowed from the literature or given reasonable values. For instance, each firm is endowed with a single unit of (capital) good in the first period of the simulation. The symmetry condition is borrowed from [Caiani et al. \(2016b\)](#). However, key coefficients of key behavioural equations are given stochastic values varying agent by agent. Examples are the marginal propensities to consume out of income, the coefficient in the R&D investment function and so on. [Tab. VII](#) clarifies which parameter varies and which does not. It is important to underline that the symmetric condition of agents' initial characteristics does not prevent that heterogeneity emerges in subsequent stages of the model, as outcome of interactions among agents. The adoption of stock-flow norms since the very beginning dampens the arbitrariness of behavioural parameters and influences from purely stochastic factors. At the same time, we perform 100 Monte Carlo runs to wash away the variability across simulations. As clarified by [Dosi et al. \(2010\)](#): “Monte Carlo distributions are sufficiently symmetric and unimodal to justify the use of across-run averages as meaningful synthetic indicators”.³⁵

[Fig. IV](#) displays average trends surrounded by their standard deviations for the main variables of interest. The figure shows that the model first experiences the usual burn-in period, converging to a relatively stable configuration after 30 periods circa. We call this situation a stationary state as defined in the footnote above. However, convergence toward a stationary state does not imply stasis: a roller-coaster dynamics generates persistent fluctuations at the business-cycle fre-

³⁵We must nonetheless recognise that averages flatten differences within and between simulations, possibly hiding some interesting features occurring in each simulation.

quencies.³⁶ This is confirmed also by the amplitude of standard-deviation intervals around the average trend.³⁷ In addition to this, the model is stock-flow consistent as plotted in Fig. V: the adoption of stock-flow consistency norms since the very onset diminishes the arbitrariness of behavioural parameters and the influences from purely stochastic factors.

Output, consumption and the unemployment rate exhibit a unit root, so they are nonstationary: that can be ascertained through Tab. VIII in which we applied either the ADF or the KPSS test for unit roots. By contrast there is uncertainty for aggregate investments: the ADF test does not find a unit root in the time series, but the KPSS does.

Fig. VI compares the volatility structure of main aggregate variables in the model through a comparison of their cyclical components: consumption, investment, output, investment in physical capital and innovative research, productivity and the unemployment rate. We have separated trends and cyclical components using the Hodrick-Prescott filter. However, we are well aware that assuming trends and cycles as additive is a very simplifying hypothesis. Furthermore we normalized the cyclical component by the trend to allow for a comparison on same scales. The artificial time series replicate well-known empirical evidence such as in [Napoletano et al. \(2006\)](#), [Fagiolo et al. \(2008\)](#). In particular, investment components and the unemployment rate are indeed more volatile than output and consumption, while the latter is almost as volatile as output.³⁸

Fig. VII exhibits the auto-correlation function of our de-trended series, looking similar to what observed in empirical data.³⁹ Most variables possess positive and significant auto-correlations which do not go beyond the fifth lag, while labour pro-

³⁶A recent debate on the literature emphasizes the surge of Harrodian instability in agent-based models ([Botte, 2019](#); [Franke, 2019](#); [Russo, 2020](#)). More precisely, although firms strive to reach a normal capacity utilization rate at the micro-economic level, the accelerator effect from their investment schedule does not allow firms to satisfy their goal on aggregate [Botte \(2019\)](#). The present setting does not suffer from such an instability for several reasons: first and foremost, it does not deal with economic growth, so firms attain a stationary state in their rate of capacity utilization, and second, the heterogeneity among firms helps avoid the puzzle as highlighted by [Russo \(2020\)](#). We do not report the figure corresponding to the aggregate capacity utilization rate for brevity; it is none the less available on request.

³⁷A common practise in AB models is that of doing away the initial periods of the simulations. They concern to the transient phase before convergence and they are strongly affected by initial conditions. We nonetheless display also those periods for completeness of exposition and for the reasons in [Caiani et al. \(2016a\)](#).

³⁸Consumption should actually be a bit less volatile than output. This is not very clear in the model probably due to our assumption about consumption functions.

³⁹Dashed lines in the plots indicate whether correlations are significant.

ductivity does not show any significant auto-correlation. More precisely, aggregate investment, its components and the unemployment rate have significant first-order auto-correlations, while consumption and output extend the significance until the fifth lag. By contrast, the result for labour productivity is probably due to the imitation process, that allows for breaks with respect to preceding developments. Fig. VIII captures the cross-correlation function of the previous variables with respect to aggregate production. In tune with the literature above, consumption and aggregate investments are pro-cyclical and lead output. Productivity displays a clearly pro-cyclical and slightly leading pattern while unemployment is counter-cyclical and lagging. We have to spend nonetheless some words on the investments' behaviour at micro-economic level. There is an important body of literature showing that investment decisions are dictated by an opportunity-cost effect: if firms experience a sales boom and in the absence of tight credit constraints, they prefer allocating their human and physical assets to current production. Hence, longer-term (innovative) investments should be counter-cyclical, while short-term investments are pro-cyclical.⁴⁰ However, [Napoletano et al. \(2006\)](#) found empirical evidence that aggregate investments are pro-cyclical and synchronized with – or slightly leading – the business cycle. Our model does not explicit any remarkable trade-off between short-term or long-term investments or between investments in tangible and intangible assets. Additionally, we do not model any particular credit-market constraint. So, our results are consistent with either [Aghion et al. \(2010, 2012\)](#) or [Napoletano et al. \(2006\)](#), since R&D investments are counter-cyclical but aggregate investment is pro-cyclical. The reason lies in the greater amount of investments in tangible capital at firm level, which is pro-cyclical. This feature more than counterbalances the counter-cyclicity in R&D on aggregate. Anyhow, the debate is still open on that point and we deserve further attention in future research.⁴¹

Besides these macroeconomic stylized facts, the model is able to replicate micro- and meso-economic empirical evidence. First of all, we have properties about firms size distribution. The literature on the topic says that manufacturing industries are characterized by skewness and heavy-tailedness in firms distribution ([Bottazzi](#)

⁴⁰In contrast, the huge presence of credit constraints makes long-term investments pro-cyclical. For further detail, see [Aghion et al. \(2010, 2012\)](#), [Chiao \(2001\)](#) and [Rafferty and Funk* \(2004\)](#).

⁴¹Still, for further detail, check [Stock and Watson \(1999\)](#).

and Secchi, 2003, 2006). We consider three proxies for size: sales of consumption commodities, production of either consumption and capital goods, and the employment level. The threefold choice helps us to gain some robustness in the results. Fig. IX shows an interesting outcome: our proxies can be perfectly fitted by a gamma probability distribution, which is right-skewed and presents a tail heavier than normal distribution. In other words, the model leads to a configuration in which the economy is populated by many small firms and few big enterprises.⁴²

Bottazzi and Secchi (2003) deal with moments of firm size too; they argue that moments are generally stationary and trendless, with some exception about the mean. Tab. IX and Fig. X display our findings. Although moments are clearly stationary according to the standard ADF test, we cannot express a uniform opinion about trends. Precisely, there is no doubt about the trendless-ness of skewness and kurtosis for each proxy of firm size; however, either the means or the standard deviations seem to exhibit a significant trend, a trend which is positive for employment while negative for production and sales. We have to remark that, albeit statistically significant, the corresponding trend magnitude is very tiny.

A further feature out of the model is the heterogeneity in productivity that distinguishes our firms, as we may appreciate from Fig. XI. We are not able to provide a good probability distribution for productivity differentials; nevertheless, it is clear that heterogeneity takes the form of high skewness with the right tail heavier than in the normal distribution case. Bartelsman-Doms (2000) claim that productivity levels are quite dispersed and differentials reflect the differences in the outcomes of technological bets: even if the entrepreneurs bet the same, they may not reap the same rewards because of uncertainty.

Finally, investments are heterogenous and possibly lumpy as in Figs. XII and XIII, i.e. firms do experience investment spikes and co-exist with near-zero investment firms (Dosi et al., 2010). Abundant literature shows that investments in manufacturing is characterized by periods of intense activity interspersed with periods of much lower one (Caballero, 1999; Doms and Dunne, 1998). However, we should remark that lumpiness has been modeled in economics through (S,s) investment functions. This family of schedules are able to display discontinuities not visible

⁴²We will see in Part II that economic growth let firm size move from a gamma toward a log-normal distribution, which looks more skewed and heavier-tailed. Additionally, the latter is in line with the shapes described by Bottazzi and Secchi (2003).

with linear investment functions as ours. So, if our framework gets lumpiness, that arises out of two main determinants: the matching process between firms and consumers, and the process of creation and diffusion of innovations. Such mechanisms allow for discontinuities in the demand each firm faces, so they affect investment patterns leading to high-investment periods followed by a longer calm.

To conclude this Section, Tab. X reports to the wide spectrum of real stylized facts matched by the model. We remark again that our firm-level analysis has been possible through the adoption of the AB procedure. Standard macro-models, for instance, do not allow for such a deepening. Furthermore, the empirical validation gives robustness to our policy experiments. The outcomes we get suggest that the observed correlation structures are not simply dependent on specific parametrizations of the model: as explained by [Caiani et al. \(2016a\)](#), if we changed the parameters of the model, we would obviously get differences in the behaviour of the agents and consequently aggregate results would differ; however, the inherent properties of the model in term of correlation structure and the way variables impact on each other would be the same and not tied to a specific set of parameters.

6 Policy experiments

Once the model approaches to the stationary state, we shock it. In other terms, we modify the value of some parameter or exogenous variable to see how the economy reacts and then compare the different stationary states. Six policy options are tested in the next few pages, namely:

- a decrease in the exogenous coefficient of the wage equation, w_0 ;
- an increase in the exogenous coefficient of the wage equation, w_1 ;
- a cut in the interest rate on loans, r_l ;
- a cut in the interest rate on deposits, r_h ;
- an increase in the meetings per unit of time, $meet$;
- an increase in the parameter of the threshold function λ , ε .

The first and the third policies are the most important, since they concern to the role played by the functional distribution of income and the interest rate. The

second and fourth policies help check if the model works as expected and help us confirm previous results. Finally, the last two are about an enlargement of innovation and imitation possibilities.⁴³

6.1 The role of income distribution

The reason to test the role exerted by wages lies in the general disagreement found in the literature.⁴⁴ On the one hand, some can argue that high wages squeeze profits and reduce investments, while keeping them in check frees resources and helps increase output and employment. On the other hand, high wages foster aggregate demand, enhancing investment outlets and providing incentive for a dynamic mechanization of the productive process. Positive effects are then reflected on higher profits.

The parameter w_0 can be seen as the balance of the social conflict between workers and entrepreneurs in the economy as a whole. In contrast, w_1 allows for the endogenization of the wage rate and represents the influence exerted by the unemployment rate of precedent period. Fig. XIV displays the results of a social compromise more favourable to capitalists, hence a lower w_0 . If compared to the baseline scenario, lower wages result in worse economic performances. Indeed, wages sustain consumption and, indirectly, investments. A lower aggregate demand reduces capitalist's incentive to invest either on capital stock or on innovation activity at the micro-economic level. The reason lies in the fact that firms try to adjust the capital stock to reach a normal capital-output ratio (see (2)). If target output declines, so does target capital and firms start disinvesting. On the same line, the decrease in sales, on the one hand, and the concomitant smaller discrepancy between the normal and the actual profit rate, on the other hand, do not provide the incentive to perform R&D and save labour. Firms will find more convenient to adapt the productive process to less labour-saving techniques. The result will be a worse general economic performance on aggregate.⁴⁵

⁴³A caveat for the reader: Fig. XIV through XIX exhibit policy results in values relative to the baseline surrounded by the standard deviation, as common to the literature.

⁴⁴Check Stockhammer (2017) and Onaran and Galanis (2012) for further detail.

⁴⁵The shift toward less labour-saving techniques could have resulted in higher employment rates, which in turn might have prompted an increase in aggregate demand. In that case, a distribution of income more favourable to profits would have resulted in better economic performance. However, the negative effect of low wages more than compensated that positive effect, as clear in the bottom-left panel of Fig. XIV.

The same holds in Fig. [XV](#) from an increase of w_1 : as in [\(12\)](#), that policy is another way to shifting income toward profits. Although results are qualitatively similar to the above, the impact of this change is very tiny, if compared to the former.

In conclusion, we trace out from the first set of policy experiments the positive influence of higher wages in triggering a process of economic development and innovation achievements. To put things differently, the improvement of labour market regulation, the centralization of the industrial relation system or other pro-worker measures help achieve better results in terms of long-run performances, such as lower unemployment rates or higher capital accumulation and productivity. This is line with [Allen \(2009, 2011\)](#) and [Dosi et al. \(2018\)](#).

6.2 The role of interest rates

The economic literature always asked whether, and how, the interest rate spurs the economic activity. The neoclassical belief is that a cut in the rate of interest stimulates the expansion of production since capitalists are less burdened by the service of debt. Theoretical arguments in the line of [Petri \(2003\)](#) and [Girardi \(2016\)](#) assume the rate of interests does not directly influence investment decisions. Although we follow this line of reasoning, there are yet several channels through which the interest rate could affect investments and the economic performance as a whole: an example is the impact of a lower service of debt as mentioned earlier. [Fig. XVI](#) shows the effect of a decrease in the rate of interest on loans applied to firms, r_l . As we can see, the result is interesting. A lower service of debt increases the amount of profits in capitalists' pockets; that translates into higher consumption levels out of capitalist income, which contribute to sustaining aggregate demand. Investments in capital stock will rise accordingly at the micro-economic level, since the entrepreneurs will adjust the capital stock to fulfill a higher target capital requirement. Nevertheless, we notice a different pattern for the innovative search and for productivity. The decrease in the interest rate on loans raises the profit rate of the individual firm, so it eases the entrepreneur to reach a normal profit rate. Facing lower competitive pressures, the capitalist reduces her innovative investment and labour productivity will be lower than in the baseline scenario. To put that differently, we have a disentanglement: although the economic activity stands in higher levels of aggregate production and employment, the innovative rate at

firm level is not encouraged since entrepreneurs prefer adapting production to less labour-saving techniques.⁴⁶

In contrast, Fig. XVII displays the effect of a cut in the interest rate on money deposits, r_h . It is easy to see that our variables of interest are affected in no way. Lower interest rates on savings diminish the load of interests paid to households and increase bank profits accordingly. The outcome is a reduction in the households' disposable income. However, on the other hand, we assumed that banking profits are fully distributed to households according to their wealth share. Moreover, we posited in (17) that the marginal propensity to consume out of interest payments and banking profits is the same, α_3 . Therefore, what comes out of door, as interests, falls through the window, as profits. The economic system performs as nothing happened.

6.3 Experiments on the innovation possibilities

The last set of experiments consists of loosening the barriers to innovation and imitation. The economic theory spent a lot of effort to judge whether the protection of intellectual property rights is a vehicle, or not, for further innovation attainments, coming to heterogeneous conclusions. Our simple setting does not allow for very complex analyses, but it could nonetheless provide some insights. In what follows we test an increase in the maximum number of meetings per unit of time, *meet*, and an increase of a parameter in the threshold function λ , ε . Both of them may affect the innovation and the imitation rates in the economy, increasing the flow of ideas at the meso-economic level.

Fig. XVIII is about an increase in the parameter *meet*. In Section IV we described it as the measure of the network size around the single capitalist. More meetings per unit of time consist of more potential competitors from which to imitate and, at the same time, more exposure toward competitor's imitation. However, the first hypothesis seems prevailing: the higher capability to imitate raises labour productivity at firm level and profits. More profits, more spending out of capitalist income and hence more sales. Further achievements in labour productivity are then possible through (21). The new stationary state is higher than the baseline.

⁴⁶We should however say that the reduction in the innovative search is quantitatively little, albeit qualitatively important.

However, the positive effects are evident in the very long run only, while in the short-to-medium run results are very uncertain, because of the increased volatility around the average trends. Furthermore the impact on the unemployment rate is doubtful in the long term too.

Something similar occurs through a slight increase in ε . In (21) it represents the speed with which the logistic function λ converges to 1. We recall that λ is the probability to innovate and imitate according to the level of R&D outlays. The greater ε , the greater λ , the greater the labour productivity and the firm's profits. The economy gravitates around higher stationary states as in Fig. XIX. Anyway, the positive outcomes, if any, are circumscribed to the very long period.

7 Conclusions

The aim of Part I was to set Secular Stagnation into the agent-based perspective and to provide some insights on the matters affecting the US economy since the end of the Golden Age of capitalism (1950-1973). Crucial features of the American economy are the very remarkable slowdown in growth of federal R&D expenditures and the redirection, by many leading firms, of resources and attention from more exploratory scientific research towards more commercially-oriented projects (Arora et al., 2018). They accompany the mounting retrenchment of the wage share and the decreasing productivity growth noticeable since 1973.

We developed a simple agent-based, stock-flow consistent model for a one-good two-class closed economy without government sector. The distribution of income between wages and profits is pivotal to determine the intensity of R&D activity within the economic system. Though the very simple framework, the model shows that distributions of income more favourable to wages, the improvement of the social protection system, the centralization of the collective bargaining structure or any other pro-labour policy result in better economic performances on aggregate, since production, capital accumulation and labour productivity would gravitate around higher stationary states. The American economy experienced a strong weakening of all these institutions in the last decades and that can help give a justification for the problem of Secular Stagnation as we define it. Obviously, we admit that this is not the only valid reason for the long-run tendency of productivity growth to fall.

Non-technological motives, like lower top marginal tax rates, increased low-skill immigration, rising trade with China and low-cost manufacturing countries or the rise of superstar firms ([Autor et al., 2020](#)) are equally plausible. As a side exercise, we tested also the role exerted by the rate of interest and the loosening of barriers to innovation and imitation. On the one hand, the decrease in the interest rate leads to particular results: it helps increase aggregate production and employment levels but impacts negatively on labour productivity, since the entrepreneurs reach more easily a normal profit rate and prefer organizing the production process to less labour-saving techniques. On the other hand, loosening the barriers to the interaction among firms and increasing the possibility to exchange ideas through imitation allow for further innovation and better economic performances as a whole, but the effect is circumscribed to the long run only.

To conclude, though we are aware of the limitations of the model, the adoption of an agent-based framework helps reply to Prof. Robert Solow’s call for more realistic micro-foundations ([Solow, 2008](#)). On the one hand, agent-based models allowed us to get and study the emergence of skewness and heterogeneity in firm’s size distribution and productivity differentials; moreover, the firm-level analysis with its implications would not have been possible in standard economic models otherwise. As in [Bowles \(2009\)](#): “An adequate theory must illuminate the process by which group structure emerges in a population of individuals, how the boundaries among the resulting higher-level entities are maintained, and how they pass out of existence”. Nonetheless, the model cannot deal with growth questions yet. We promise therefore we would improve the model in future research to address these topics, namely in Part II.

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A Tables and Figures

Time - Variable	Adjusted wage share	Labour Productivity	TFP
1950 – 73	0.674	0.027	0.018
1973 – 95	0.656	0.017	0.006
1995 – 07	0.642	0.026	0.012
2007 – 19	0.613	0.015	0.005

Note: author's own calculations on Ameco and BLS data. Data on wage share are available since 1960. We applied the HP-filter to focus on the trend component only.

Table I. Average wage share and average growth rates for labour productivity and TFP

Time - Variable	Federal	Other - Gov't	Industry	Non-profits	University	Total
1953 – 73	0.028	0.032	0.027	0.035	0.036	0.028
1973 – 95	0.004	0.014	0.022	0.023	0.028	0.014
1995 – 07	0.008	0.016	0.016	0.030	0.023	0.014
2007 – 17	-0.001	0.005	0.014	0.021	0.019	0.010

Note: author's own calculations on AAAS data. Data are available since 1953 for each variable. We applied the HP-filter to focus on the trend component only.

Table II. Average growth rates in US R&D expenditures by source

Time - Variable	Defense	Energy	General Science	Health	Natural Resources	Non-defense	Other	Space
1953 – 73	0.027	0.066	0.043	0.065	0.042	0.062	0.046	0.082
1973 – 95	0.008	0.003	0.005	0.018	0.005	0.002	-0.001	-0.011
1995 – 07	0.012	-0.024	0.023	0.025	-0.004	0.013	0.003	-0.001
2007 – 18	-0.020	0.019	0.006	0.001	0.002	0.002	-0.004	0.002

Note: author's own calculations on AAAS data. Data are available since 1953 for each variable. We applied the HP-filter to focus on the trend component only.

Table III. Average growth rates in US Federal R&D expenditures by function

Time - Variable	R&D as % of GDP	Private R&D as % of Fixed Investments	Federal R&D as % of Discretionary Outlays	Federal R&D as % of Total Outlays
1950 – 73	0.022	0.078	0.144	0.092
1973 – 95	0.026	0.104	0.117	0.052
1995 – 07	0.027	0.137	0.124	0.045
2007 – 18	0.029	0.155	0.109	0.038

Note: author's own calculations on AAAS and BEA data. Data are available since 1962 for variables referred to Federal R&D. We applied the HP-filter to focus on the trend component only.

Table IV. R&D shares, I

Time - Variable	Defense as % of Defense Outlays	Non-defense as % of Total Budget	Non-defense as % of Non-defense Outlays
1962 – 73	0.119	0.041	0.197
1973 – 95	0.120	0.022	0.114
1995 – 07	0.141	0.019	0.107
2007 – 18	0.116	0.017	0.103

Note: author's own calculations on AAAS data.

Table V. R&D shares, II

Ordinary Correlation	Adjusted Wage Share	Labour Productivity	TFP	Federal	Industry	Non-profits	Other - Gov't	Universities	Total
Adjusted Wage Share	1								
Labour Productivity	0.354***	1							
TFP	0.482***	0.940***	1						
Federal	0.319***	0.294***	0.172	1					
Industry	0.512***	-0.349***	-0.123	0.185	1				
Non-profits	-0.040	0.174	0.054	0.409***	0.196	1			
Other - Gov't	0.555***	0.445***	0.457***	0.602***	0.417***	0.680***	1		
Universities	0.702***	0.286**	0.475***	0.496***	0.710***	0.221	0.770***	1	
Total	0.158	-0.207	-0.205	0.739***	0.655***	0.564***	0.522***	0.538***	1

Note: author's own calculations. Star significance: *** $p < 0.01$, ** $p < 0.05$ and * $p < 0.1$.

Table VI. Ordinary correlation among some variables

Notation	Description	Value
$Time$	Time span	450
MC	Monte Carlo runs	100
F	Firms	100
N_s	Workers-Consumers	500
α_0	Autonomous consumption	0.001
α_1	Worker's marginal propensity to consume out of income	[0.75;0.9]
α_2	Capitalist's marginal propensity to consume out of income	[0.5;0.7]
α_3	Marginal propensity to consume out of wealth	[0;0.1]
a_0	Labour-productivity initial value	1
a_1	Coefficient in the productivity equation	0.75
δ	Capital depreciation	0.05
ε	Parameter in the threshold function	0.05
φ	Inverse normal capital-output ratio	1
i_0	Autonomous investment	0.8
i_1	Partial-adjustment coefficient	[0.15;0.2]
μ_0	Coefficient in the mark-up equation	0.075
$meet$	Meetings per unit of time	5
ψ_0	Coefficient in the price expectations function	0
ψ_1	Coefficient in the price expectations function	0.01
q	Share of capitalist wealth re-invested	0.0027
ϱ	Normal profit rate	0.05
r_l	Interest rate on loans	0.0075
r_h	Interest rate on deposits	0.0025
θ_0	Coefficient in the R&D investment function	[0.007;0.008]
θ_1	Coefficient in the R&D investment function	0.15
v	Coefficient in the mark-up function	0.02
w_0	Coefficient in the wage equation	0.7
w_1	Coefficient in the wage equation	0.005
ζ_1	Stochastic component from a uniform distribution	[0;1]

Note: shaded lines denote variables whose value differs between agents.

Table VII. Time span, number of agents, parameter setting and exogenous variables

	ADF test	KPSS test
Output	-1.7895 (0.3855)	2.1682 (0.739)
Consumption	-1.5761 (0.4938)	2.1783 (0.739)
Investment	-6.9708 (0.000)	1.6494 (0.739)
Unemployment rate	-1.7992 (0.3807)	2.1657 (0.739)

Note: ADF test assumes unit root in the null hypothesis, while the KPSS test supposes time-series are stationary. We delete the first one hundred period simulations to focus entirely on the stationary state.

Table VIII. Unit root test on selected aggregate variables

	Sales		Production		Employment	
	Trend β	ADF test	Trend β	ADF test	Trend β	ADF test
Mean	$-6.80E - 05^*$	-8.1309***	-0.0001***	-17.8379***	0.0007***	-11.4832***
Standard deviation	-0.0003***	-14.7695***	-0.0005***	-17.27723***	$-2.50E - 05$	-6.4327***
Skewness	-0.0001	-19.7903***	-0.0001	-19.5577***	-0.0001	-20.4145***
Kurtosis	0.0003	-12.6002***	$7.17E - 05$	-19.0745***	$5.45E - 05$	-19.3862***

Note: moments are computed after $t = 100$, so when the model already gravitates around the stationary state. ADF test assumes unit root in the null hypothesis, while the KPSS test supposes time-series are stationary. Star significance: *** $p < 0.01$, ** $p < 0.05$ and * $p < 0.1$.

Table IX. Moments of firms size distribution

Stylized facts	Tables - Figures	References
Micro-economic level (firms)		
Skewness and heavy tailed-ness firm size distribution	Fig. IX	Bottazzi and Secchi (2003, 2006)
Moments of size distribution are stationary	Tab. IX, Fig. X	Bottazzi and Secchi (2003); Dosi et al. (2010)
Heterogeneous productivity across firms	Fig. XI	Bartelsman and Doms (2000); Bottazzi and Secchi (2003)
Investment heterogeneity and lumpiness	Figs. XII- XIII	Caballero (1999); Doms and Dunne (1998)
Macro-economic level (aggregate)		
Fluctuations at business-cycle level	Fig. IV	Caiani et al. (2016a); Stock and Watson (1999)
Stock-flow consistency	Fig. V	Godley and Lavoie (2006)
Output components and unemployment are non-stationary series	Tab. VIII	Blanchard and Summers (1986); Hamilton (2020); Nelson and Plosser (1982)
Volatility of output, investment, consumption and unemployment	Fig. VI	Stock and Watson (1999)
Cross-correlations among macro-variables	Figs. VII- VIII	Stock and Watson (1999)

Table X. Stylized facts matched by the model.

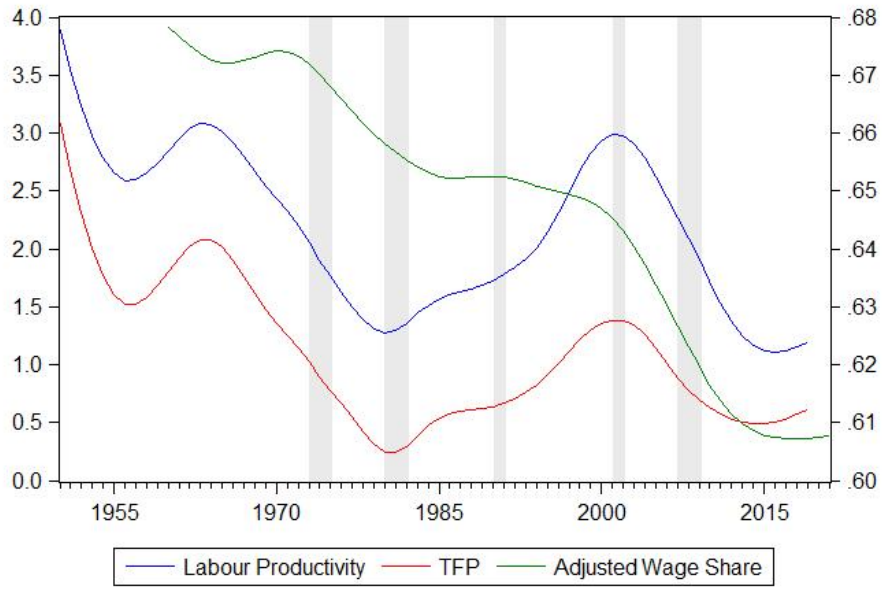


Figure I. US adjusted wage share and productivity growth rates, 1950-2018

Note: left axis refers to productivity growth rates, right axis to the wage share; shaded areas indicate major crises; we reported results of the HP-filter trend component of real time series so to focus on the long-run component. Source: author's calculations on Ameco and BLS data.

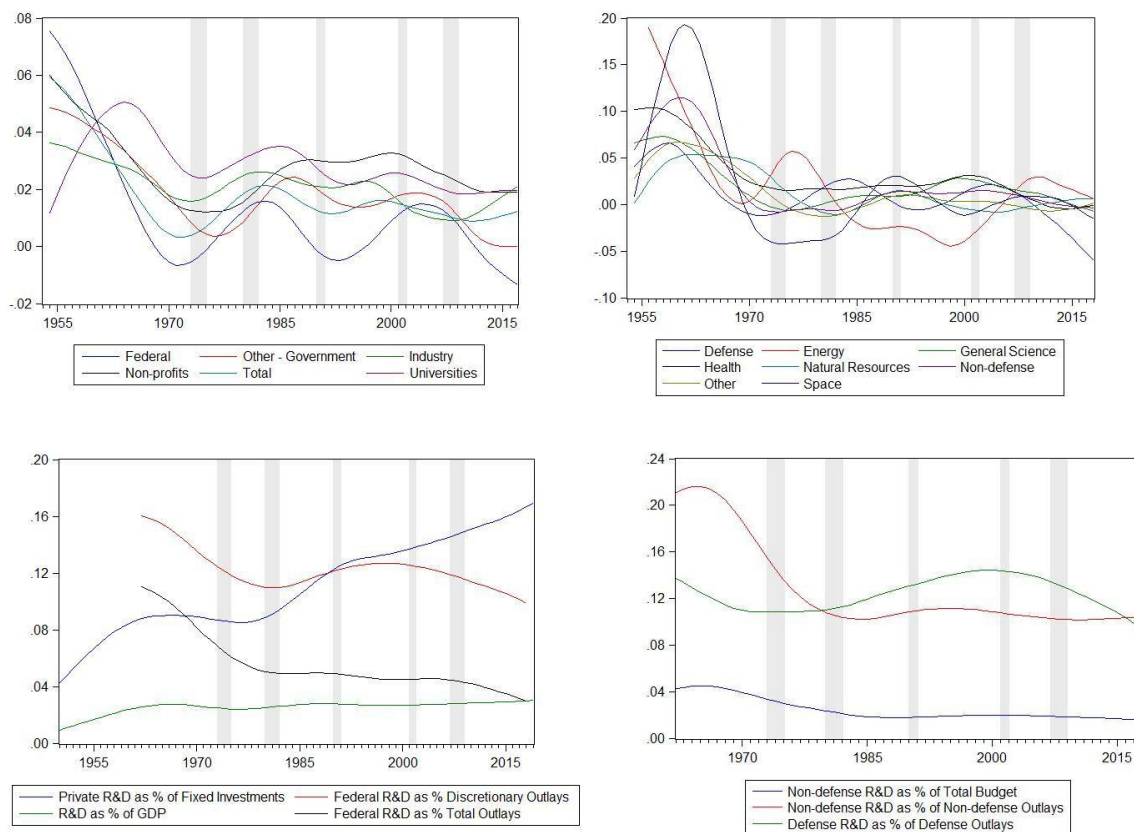


Figure II. US R&D expenditures, 1953-2018

Note: top-left graph points to R&D expenditure by source, top-right graph refers to R&D expenditure by function, bottom graphs point to R&D shares in some aggregate; shaded areas indicate major crises; we reported results of the HP-filter trend component of real time series so to focus on the long-run component. Source: AAAS data.

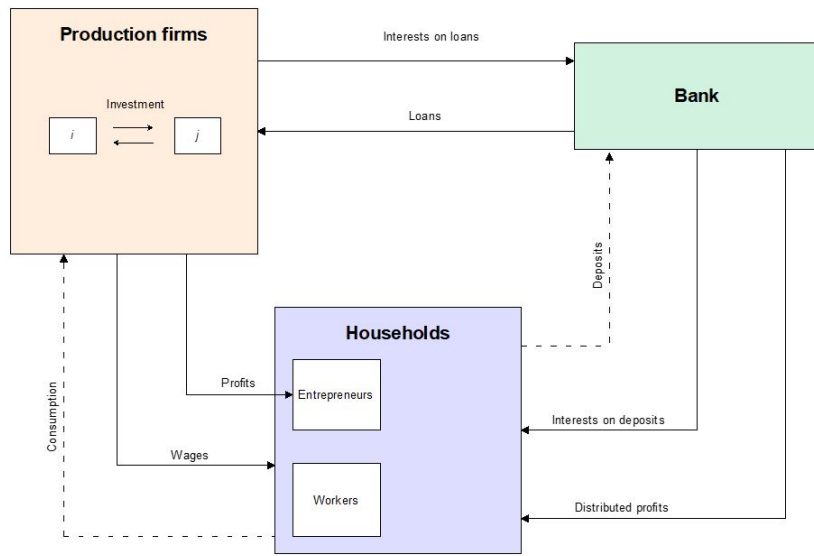


Figure III. Flow diagram of the model

Note: arrows point from paying sectors to receiving sectors.

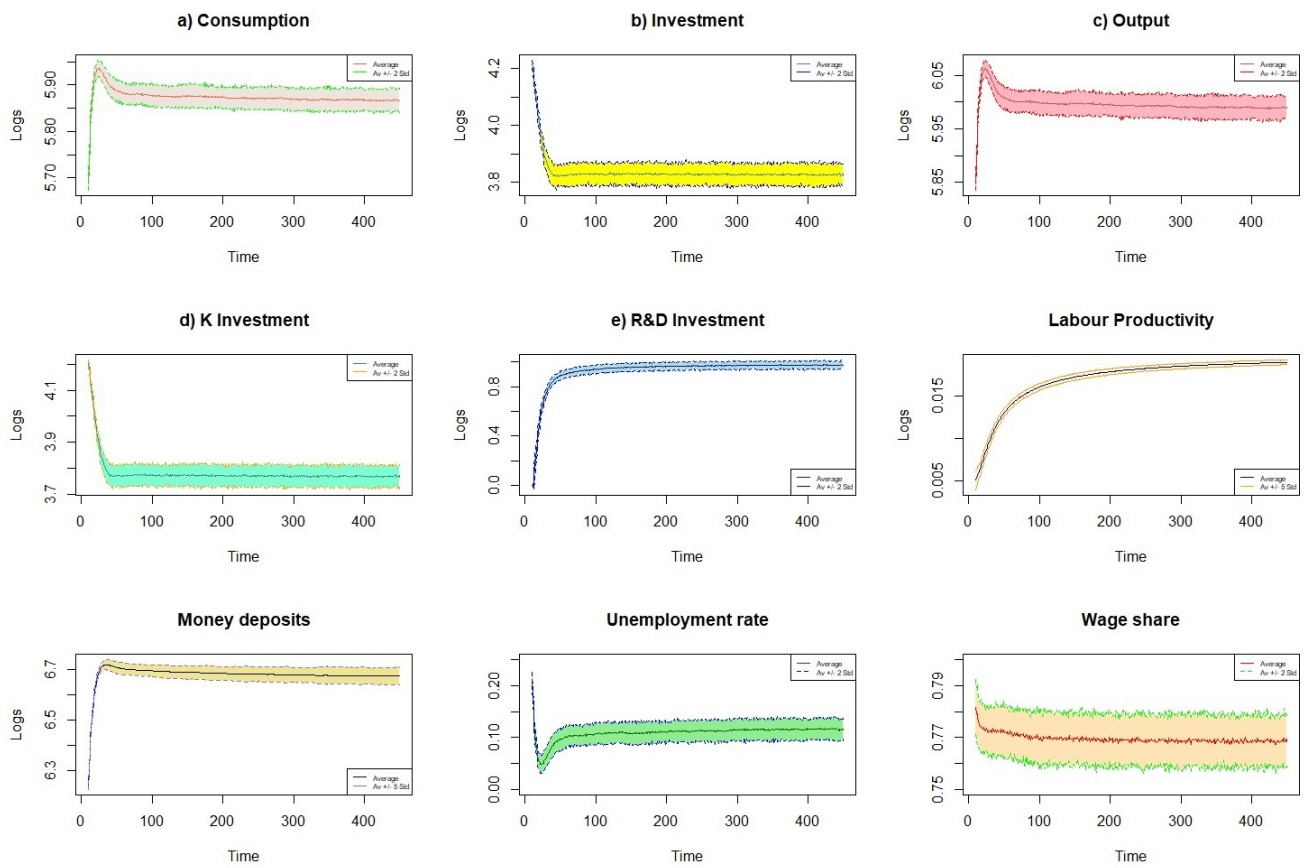


Figure IV. Baseline model

Note: levels in log terms.

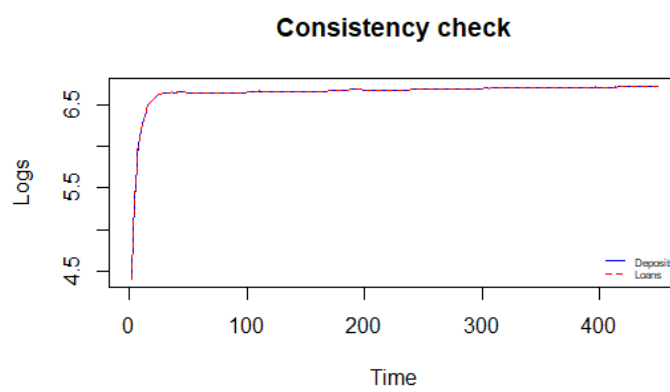


Figure V. Consistency check

Note: deposits equal loans every period.

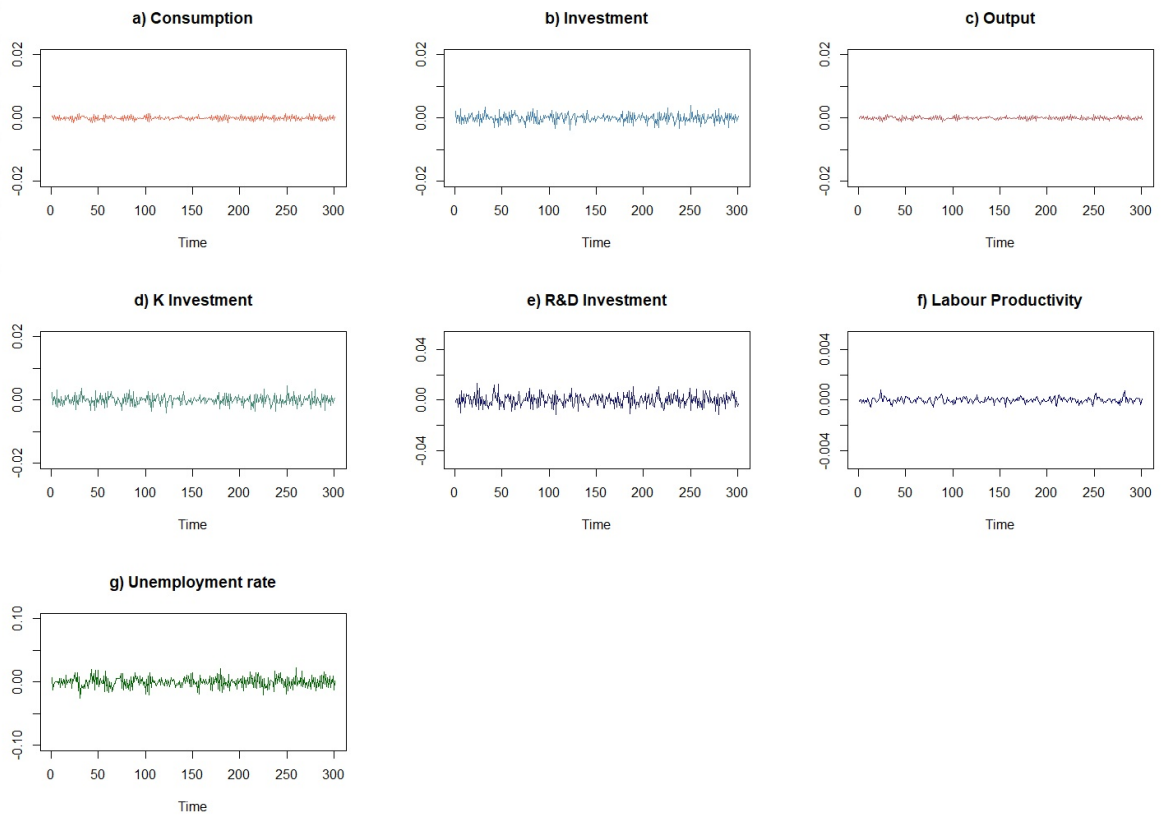


Figure VI. Cyclical components of simulated time series for some aggregate variables

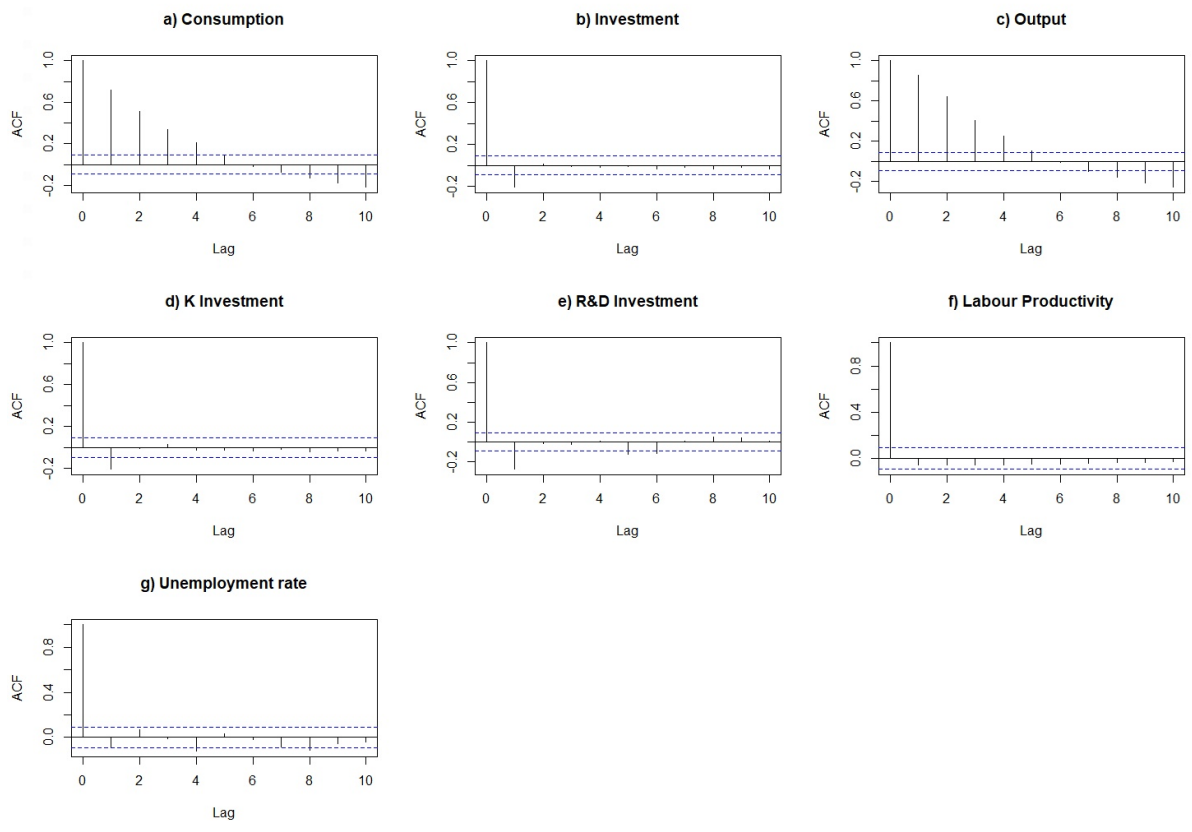


Figure VII. Baseline model: auto-correlations

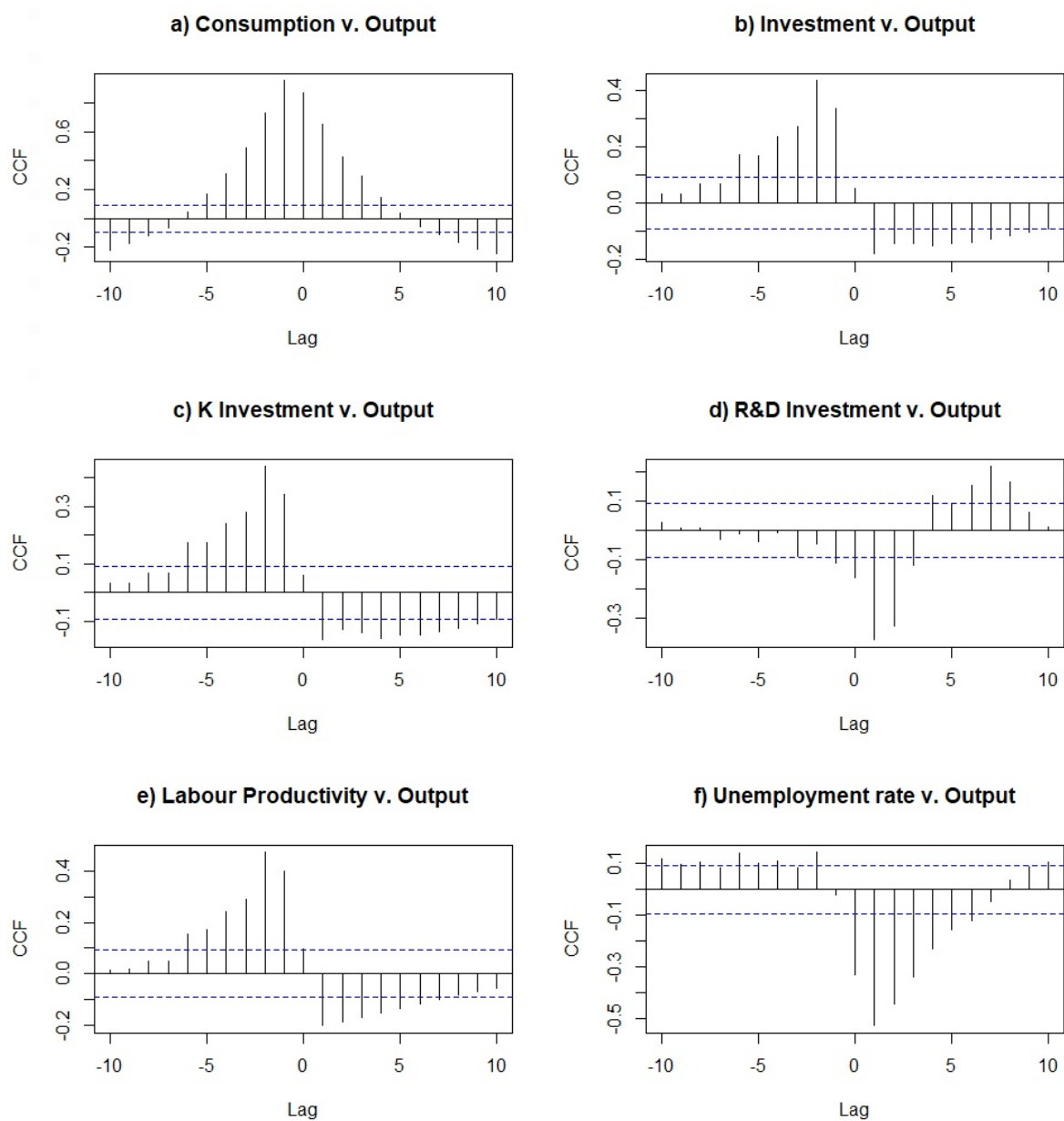


Figure VIII. Baseline model: cross-correlations with respect to aggregate output

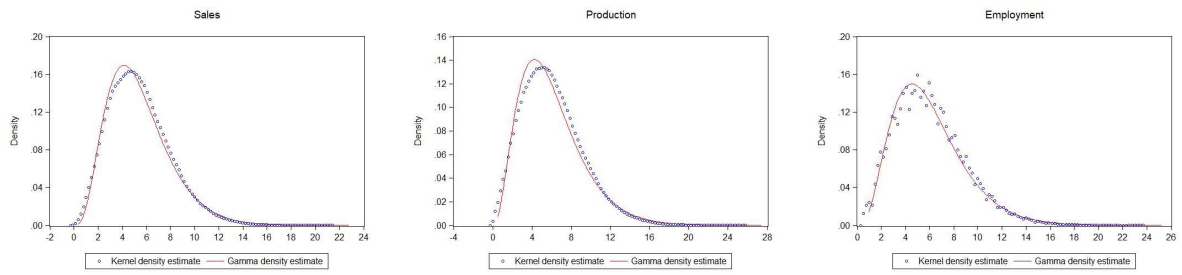


Figure IX. Firm size distribution

Note: sales refer to shipments of consumption good, while production is about shipments of consumption and investment goods.

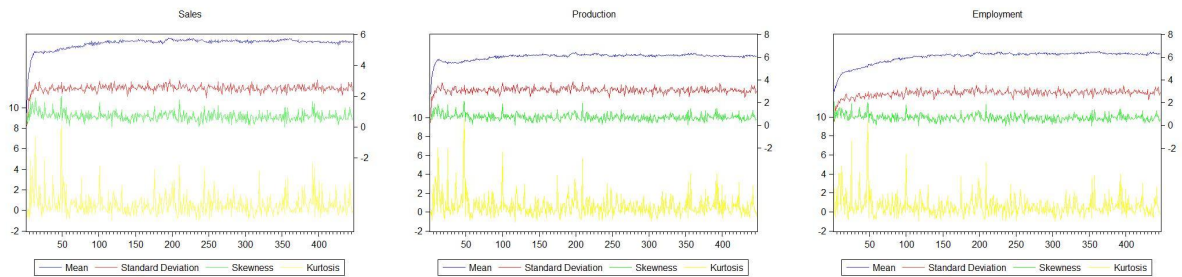


Figure X. Moments of size distribution

Note: sales refer to shipments of consumption good, while production is about shipments of consumption and investment goods.

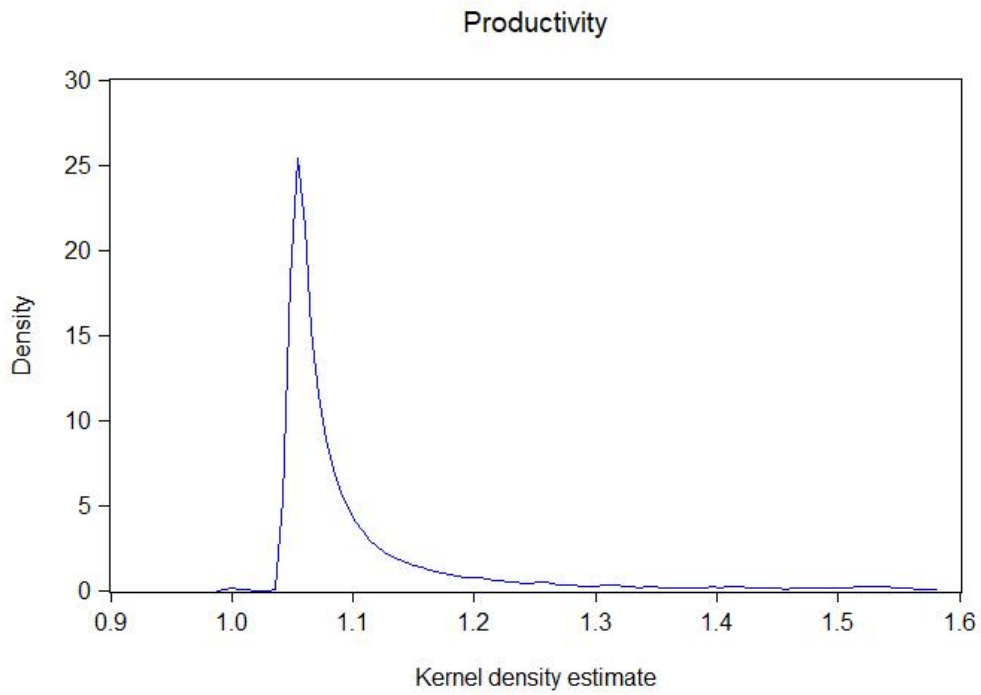


Figure XI. Productivity differentials at firm level

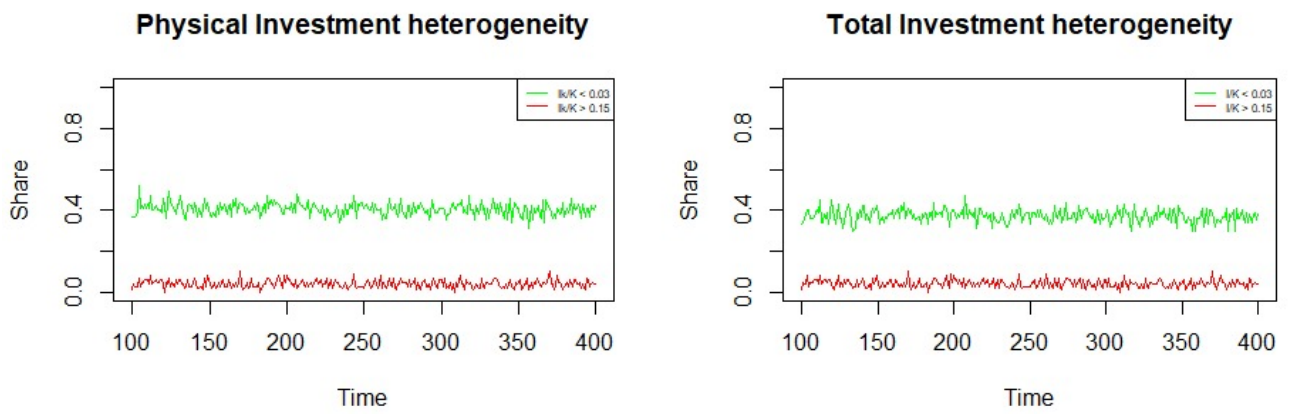


Figure XII. Investment heterogeneity at firm level

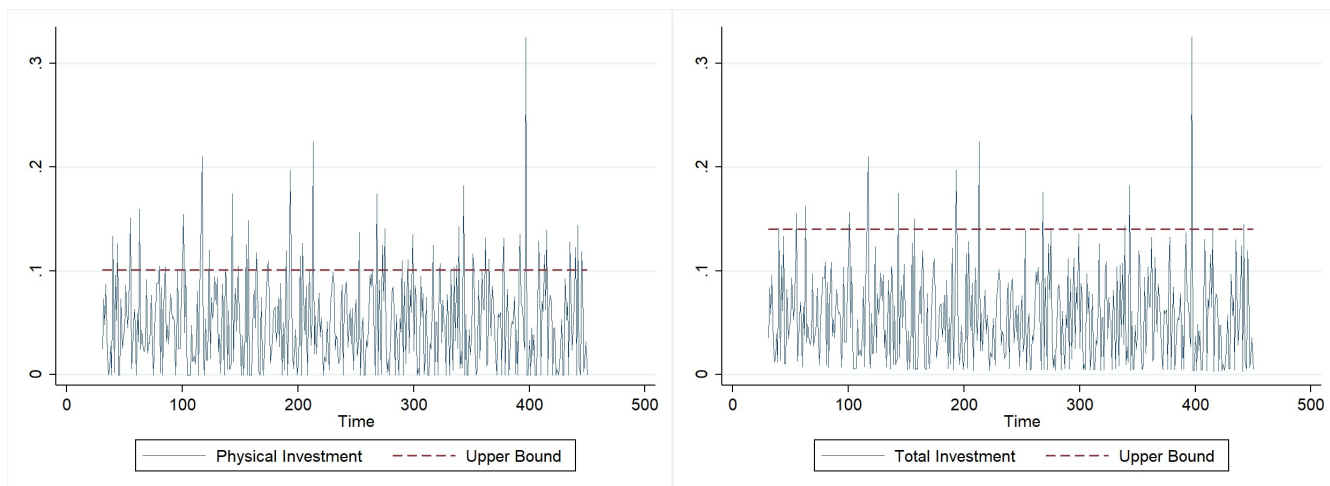


Figure XIII. Investment lumpiness?

Note: investment patterns from a selected firm; the upper bound is determined as median value plus one standard deviation.

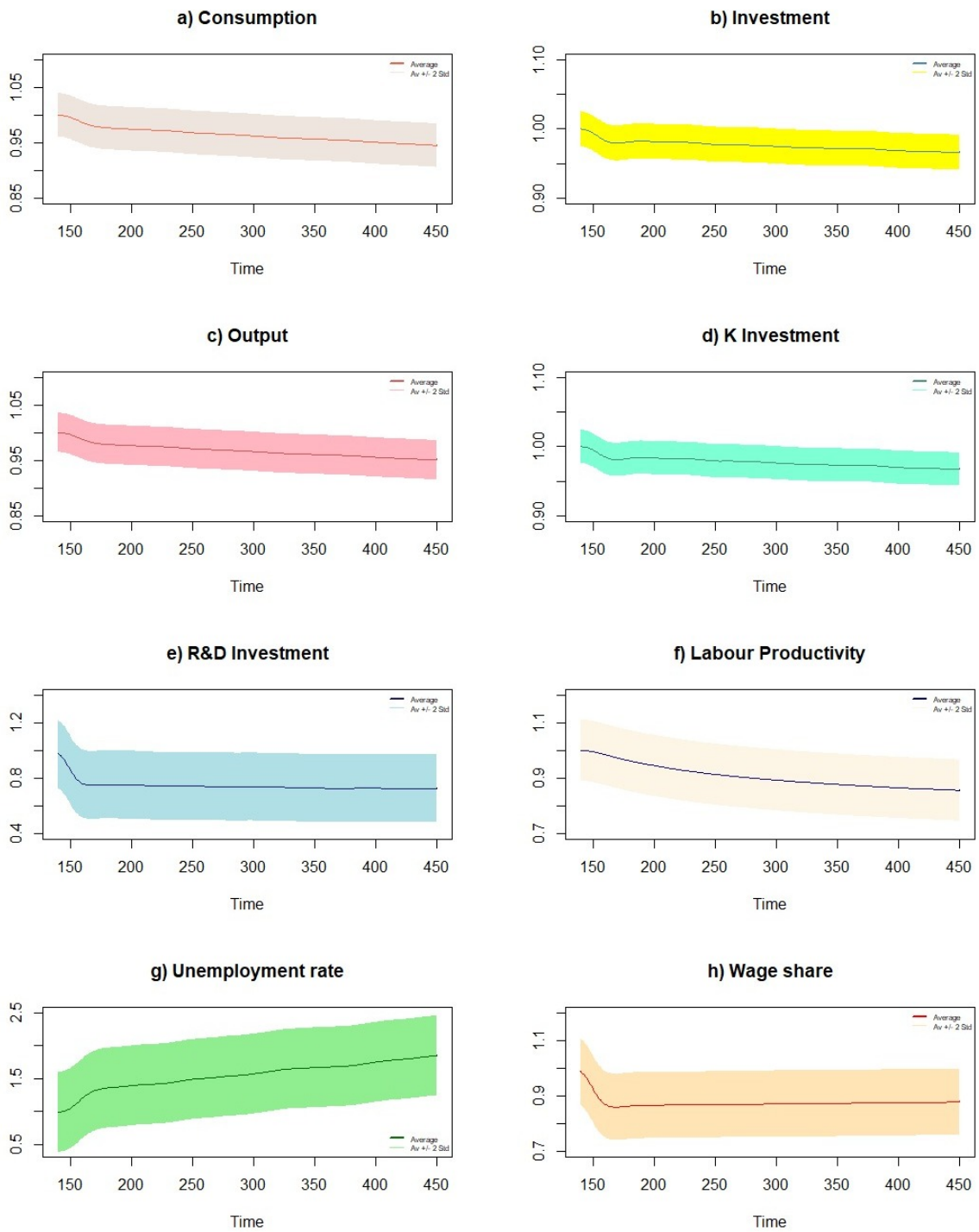


Figure XIV. Policy experiment: a decrease in the parameter w_0

Note: values relative to the baseline surrounded by the standard deviation; ratios are computed between HP-trend components.

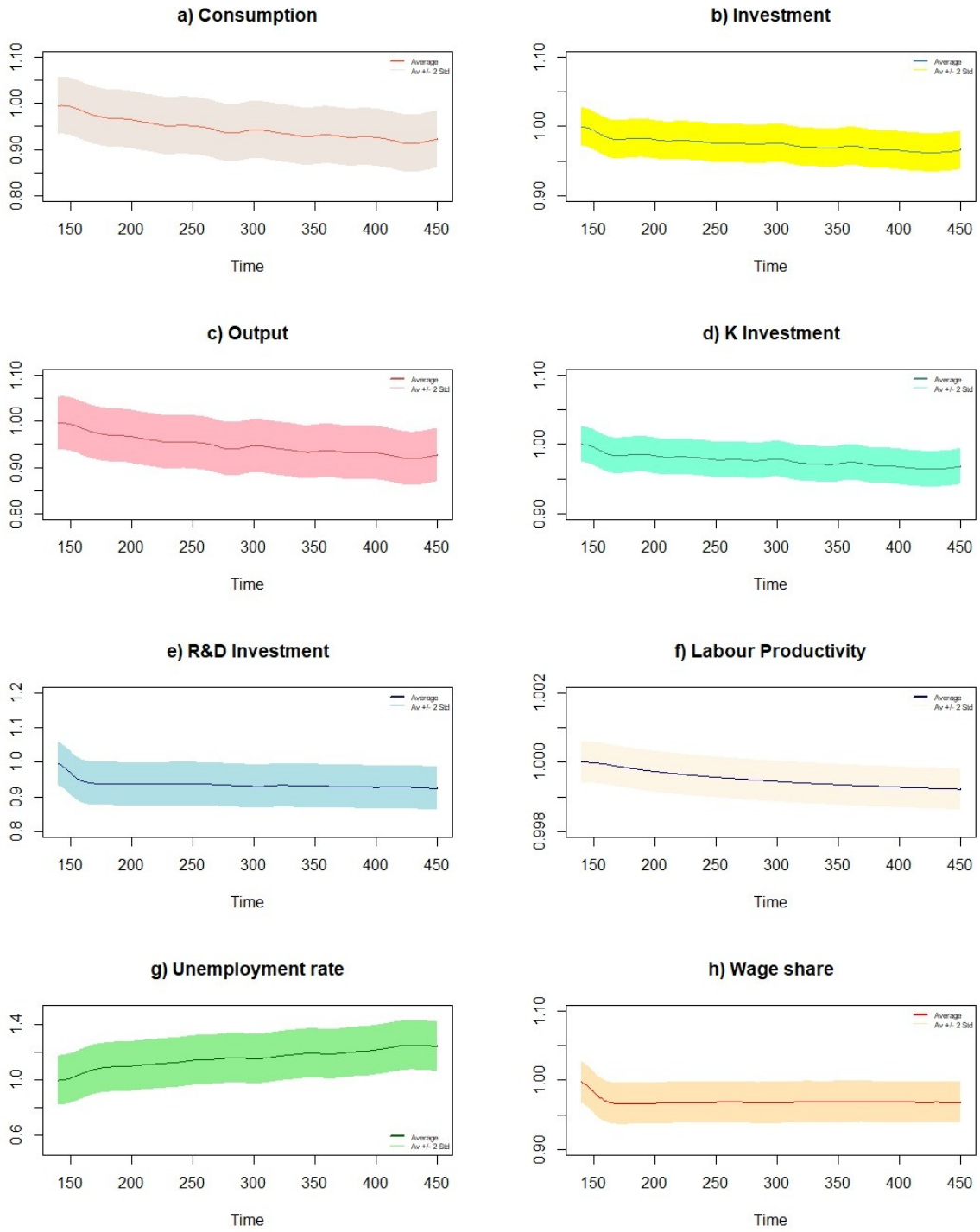


Figure XV. Policy experiment: an increase in the parameter w_1

Note: values relative to the baseline surrounded by the standard deviation; ratios are computed between HP-trend components.

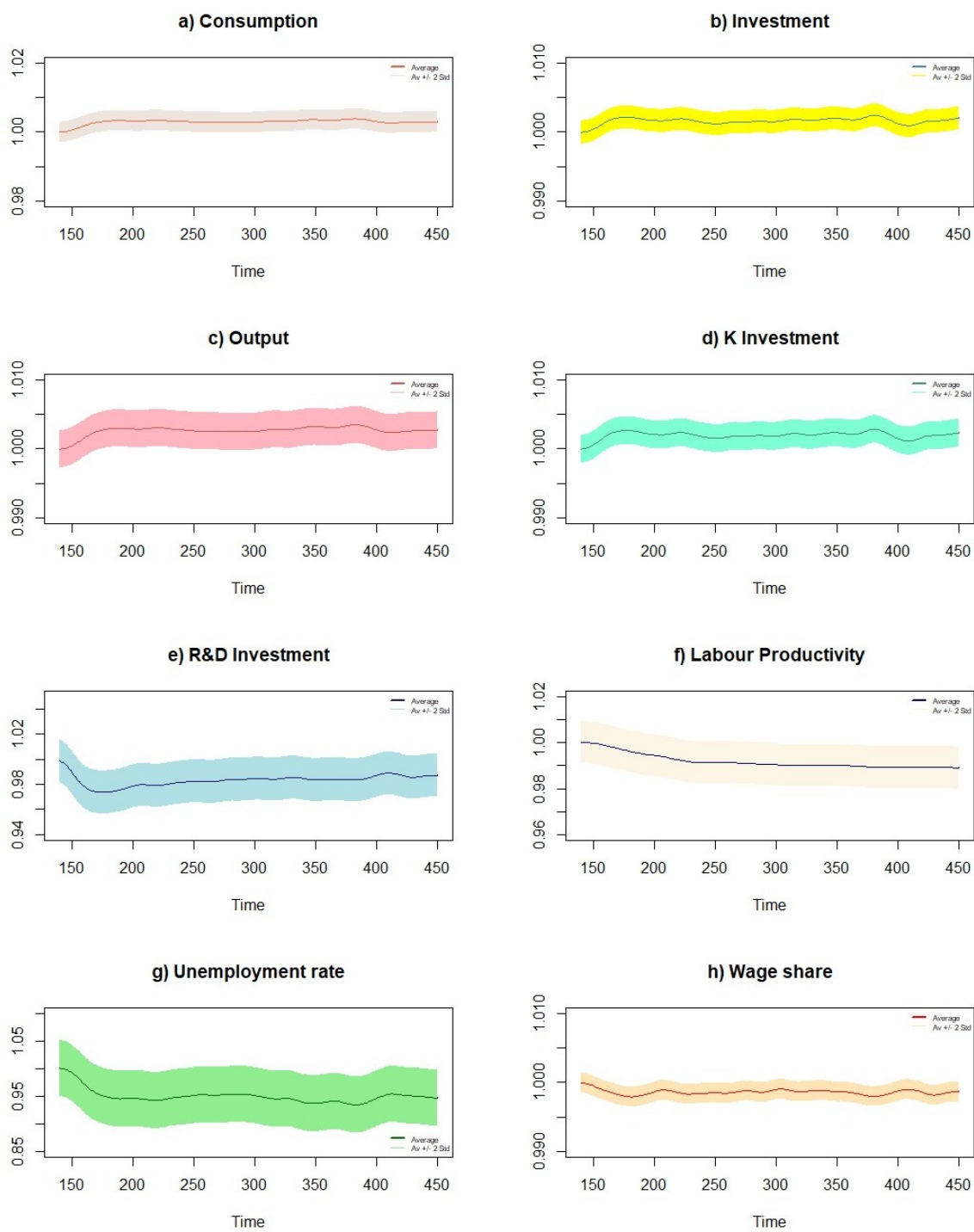


Figure XVI. Policy experiment: a decrease in the rate of interest r_l

Note: values relative to the baseline surrounded by the standard deviation; ratios are computed between HP-trend components.

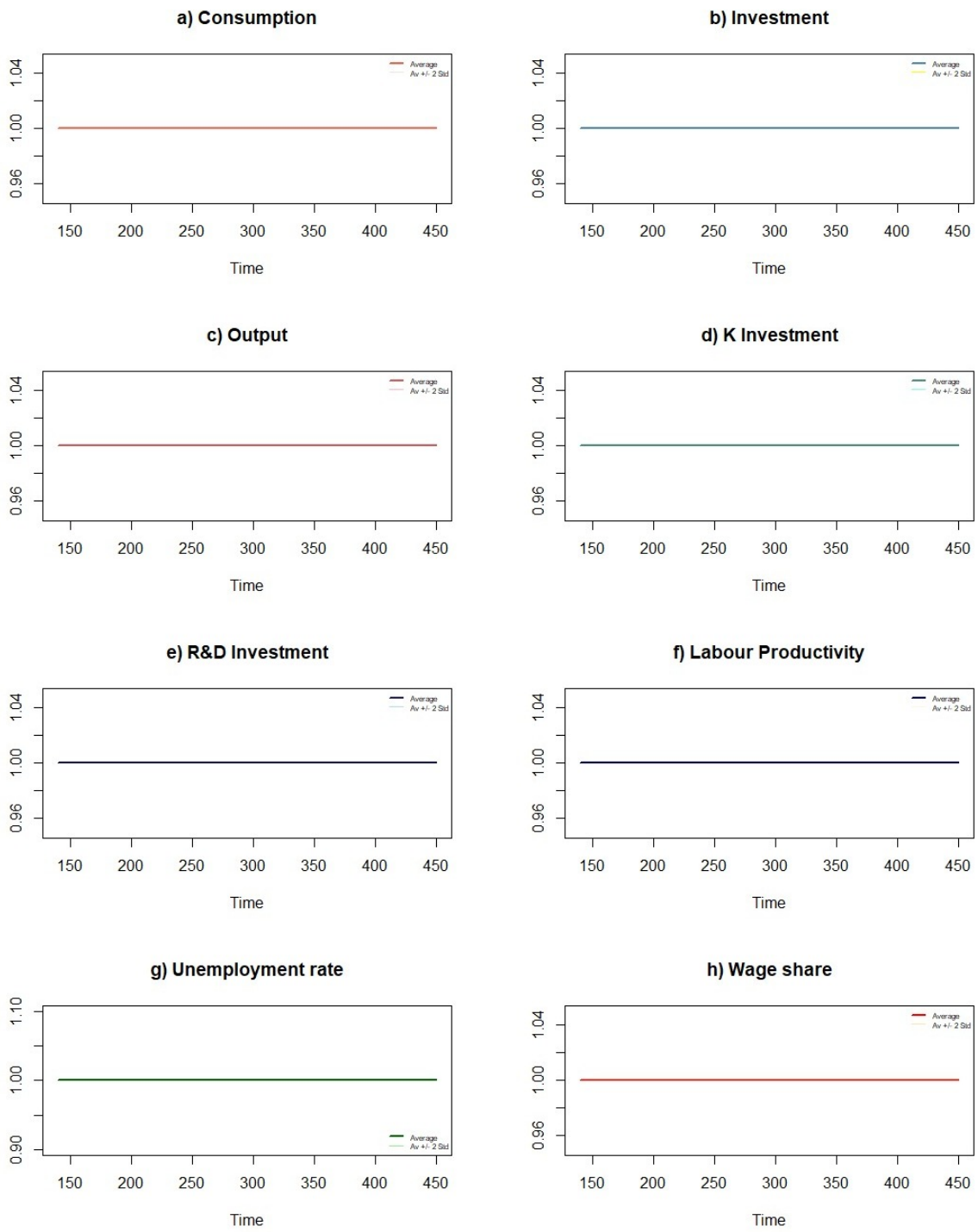


Figure XVII. Policy experiment: a decrease in the rate of interest r_h

Note: values relative to the baseline surrounded by the standard deviation; ratios are computed between HP-trend components.

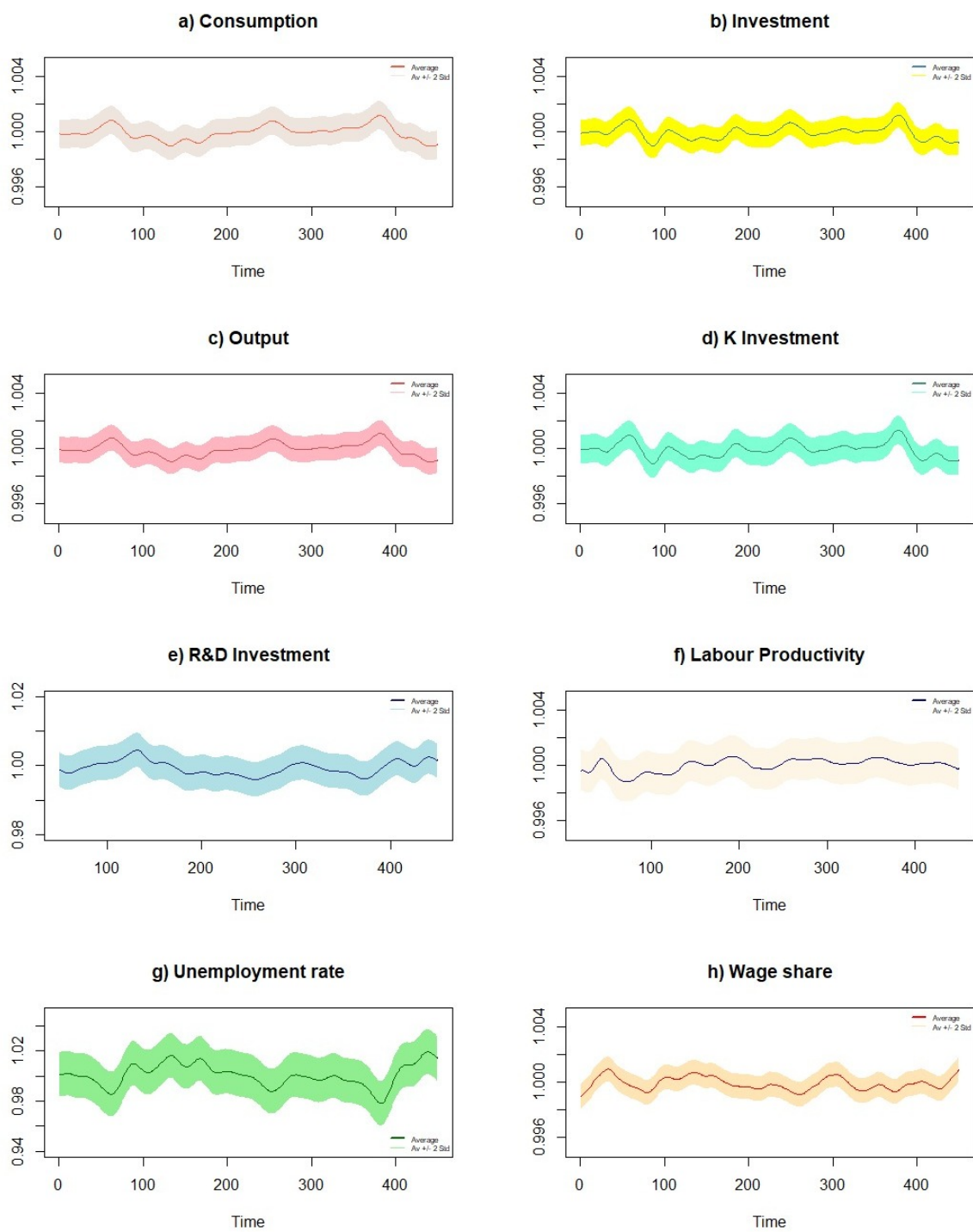


Figure XVIII. Policy experiment: an increase in the parameter *meet*

Note: values relative to the baseline surrounded by the standard deviation; ratios are computed between HP-trend components.

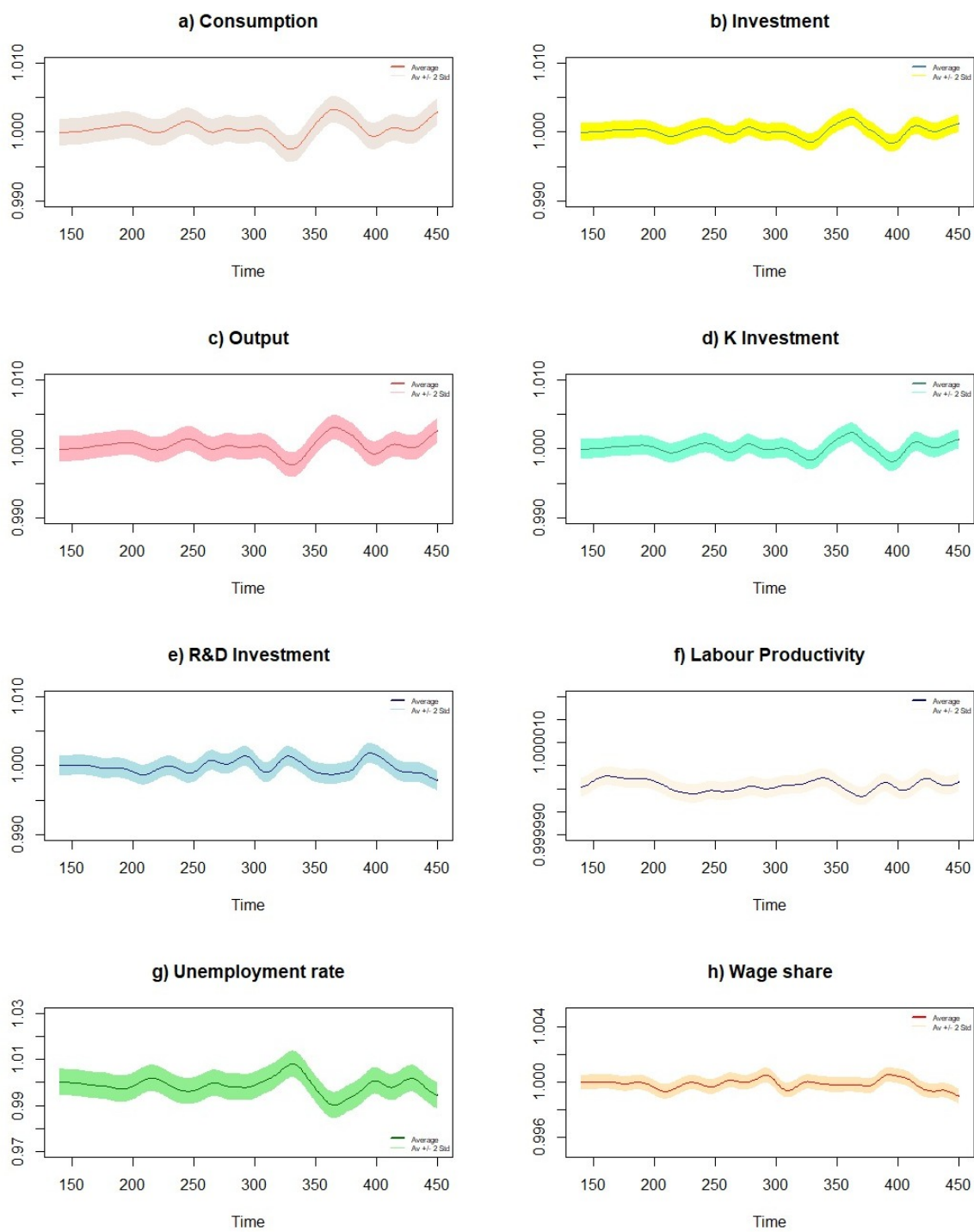


Figure XIX. Policy experiment: an increase in the parameter ε

Note: values relative to the baseline surrounded by the standard deviation; ratios are computed between HP-trend components.

B More on the model ...

B.1 Notes on the matching process

This subsection provides a few notes on the matching process between firms and workers-consumers, and how it is possible to get the demand faced by each firm. The first step of our procedure creates a $F \times N_s$ matrix, called fw_{net} . It represents the firms-workers network. The cells take value 1 if a link between a firm and a worker is established and 0 otherwise. We sample random cells and set them equal to 1: precisely, every row will count a number of 1s equal to the labour demanded by the single firm. For instance, if the first row of the network contains ten 1s, then the employment in the first firm amounts to ten workers, and so on.⁴⁷ The procedure allows for the establishment of a simple random matching, repeated across time.⁴⁸ Let us deal with the formation of the demand schedule.⁴⁹ We discern from each period the wage vector $\mathbf{w}'_r = [w_{r,1}, \dots, w_{r,i}, \dots, w_{r,F}]$, being $w_{r,i}$ the wage paid by the i -th firm. We can transform the wage vector in a $F \times F$ diagonal matrix as below:

$$\mathbf{w}'_r \rightarrow \mathbf{w}_{r,diag} = \begin{bmatrix} w_{r,1} & \dots & 0 \\ \vdots & w_{r,i} & \vdots \\ 0 & \dots & w_{r,F} \end{bmatrix}$$

The approach allows to determine the $F \times N_s$ matrix of disposable incomes \mathbf{ydh} referred to each worker for every period:

$$\mathbf{w}_{r,diag} \times \mathbf{fw}_{net} = \begin{bmatrix} w_{r,1} & \dots & 0 \\ \vdots & w_{r,i} & \vdots \\ 0 & \dots & w_{r,F} \end{bmatrix} \times \begin{bmatrix} 1 & 0 & \dots & 1 \\ \vdots & 1 & 0 & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} = \begin{bmatrix} w_{r,1} & 0 & \dots & w_{r,1} \\ \vdots & w_{r,i} & 0 & \vdots \\ 0 & 0 & \dots & w_{r,F} \end{bmatrix}$$

$$\mathbf{w}_{r,diag} \times \mathbf{fw}_{net} = \mathbf{ydh}$$

We may imagine a $F \times N_s$ matrix as something observed from the point of view of the firms. Its $N_s \times F$ transpose matrix is the same object looked from the point

⁴⁷To be precise, it amounts to nine workers and one manager, the entrepreneur.

⁴⁸To avoid the matching be invariant across simulations, we limit to define the network inside the for-loop. However, capitalists do not change over time.

⁴⁹We do not consider firms' and bank's profits or interest payments in what follows to streamline the discussion. Anyway, the same argument holds as well.

of view of the workers. Why is the reversal perspective useful? If every worker were assigned a random marginal propensity to consume out of income, we could adopt the same diagonalization as above to derive the consumption demand out disposable income, c_{ydh} . In particular, the transpose matrix makes clearer that the propensity to consume varies between workers but not with respect to the single firm: that is, if the j -th worker is supposed to have a marginal propensity to consume equal to 0.6, then this value is the same regardless of the firm to which the agent decides to consume. Formally we have:

$$\mathbf{ff}'_1 = [\alpha_{11}, \dots, \alpha_{1i}, \dots, \alpha_{1N_s}] \rightarrow \mathbf{ff}_{1,\text{diag}} = \begin{bmatrix} \alpha_{11} & \dots & 0 \\ \vdots & \alpha_{i1} & \vdots \\ 0 & \dots & \alpha_{1N_s} \end{bmatrix}$$

$$\mathbf{ff}_{1,\text{diag}} \times \mathbf{ydh}^T = \begin{bmatrix} \alpha_{11}w_{r,1} & \dots & 0 \\ 0 & \alpha_{1i}w_{r,i} & 0 \\ \vdots & 0 & \vdots \\ \alpha_{1N_s}w_{r,1} & \dots & \alpha_{1F}w_{r,F} \end{bmatrix} = \mathbf{c}_{ydh}$$

We repeat the same procedure to obtain the consumption out of wealth and the total consumption function c . Updating every time the fw_{net} network allows that each agent works potentially for every firm during the simulation span, and then \mathbf{c} represents a $N_s \times F$ matrix with the amount spent by the single agents in consumption goods, hence a full demand schedule. Transposing and row-aggregating the consumption matrix \mathbf{c} return the amount of consumption demand faced by the single enterprise, c_f .⁵⁰

For what regards to the process matching firms with consumers, we re-adapt that in Ricetti et al. (2015) and Caiani et al. (2016b). Agents meet on the (consumption) good market and act following the same protocol: potential consumers observe a subset of prices from a restricted and random set of suppliers, reflecting their imperfect information. They choose the best seller according to the lowest selling price. Each period agents have the opportunity to switch to another supplier with

⁵⁰This overall procedure is very little time-consuming since it requires no for-loops at all. Every operation is performed through the help of matrix algebra.

a certain probability, the latter depending on the price differential:

$$Prob = \begin{cases} 1 - e^{\chi_1 \cdot \frac{p_{new} - p_{old}}{p_{new}}} & \text{if } p_{new} < p_{old} \\ 0 & \text{otherwise} \end{cases} \quad (36)$$

Eq. 36 says that the larger the price differential between the old and the new supplier, the higher the probability to switch to the new. The assumption considers the empirical fact that consumers establish a durable relationship based on trust and reciprocity to solve problems from asymmetric information.⁵¹

⁵¹The literature on behavioural economics is endless. We suggest [Bowles \(2009\)](#).