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Aggregation of Experts Opinions and the Assessment of Tipping Points

Catastrophic Forecasts for Higher Temperature Changes

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Abstract

This paper assesses the probability of occurrence of tipping points conditional on a given temperature scenario by combining probability intervals from elicited experts opinions using the data of Kriegler et al. (2009). The computation of such conditional probabilities is based on the aggregation of imprecise probability judgments through the Steiner point. In addition, the probability of a tipping point can be updated via the standard Bayes rule to generate tipping point scenarios. Our results suggest that tipping events may happen with relatively large probabilities, in contrast with the view that tipping points are low-probability-high-impact events.

Keywords: Bayesian updating; aggregation; global warming; judgmental forecasting; Steiner point; tipping points.

JEL classification: Q54; D81; C10.

1 Introduction

On the basis of estimates and information from paleo-climate data, new model simulations and combined evidences about human influence on climate variables, the Sixth Assessment Report of the Intergovernmental Panel on Climate Change (IPCC), is definitive and clear: "It is unequivocal that human influence has warmed the atmosphere, ocean and land. Widespread and rapid changes in the atmosphere, ocean, cryosphere and biosphere have occurred" (IPCC, 2021, p. 4). Greenhouse gas (GHG) concentrations have been increasing since 1750, reaching annual averages of 410 ppm for CO2 (carbon dioxide), 1866 ppm for CH4 (methane) and 332 ppm for N2O (nitrous oxide) in 2019: this situation is unequivocally due to human activities. The scale of such changes in the climate system as a whole has been never found in the history of the Earth that could induce unpredictable rare and extreme climate events. IPCC's recommendations are unequivocal: "From a physical science perspective, limiting human-induced global warming to a specific level requires limiting cumulative CO2 emissions, reaching at least net zero CO2 emissions, along with strong reductions in other greenhouse gas emissions. Strong, rapid

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and sustained reductions in CH4 emissions would also limit the warming effect resulting from declining aerosol pollution and would improve air quality"(IPCC, 2021, p. 27). In spite of such recommendations, the leaders of the World's richest economies and biggest polluters struggle to find a common ground and take actions that may effectively change the current state of affairs (see for example the conclusions of the recent G20 and COP26 meetings G20, 2021; COP26, 2021).

As of today, there is a large literature suggesting that the consequences of climate change may involve abrupt changes and tipping points (e.g. Lenton et al., 2019).[1] The concept of tipping point was introduced into the scientific debate in the '80s of the last century by the IPCC to represent large scale discontinuities in the climate system. At that time, experts believed that tipping points would be crossed if global warming had exceeded $5°C$. In the 2018 IPCC report (IPCC, 2018b) experts suggested that tipping points could be crossed even between 1 and $2°C$ of global warming. Lenton et al. mention that some "models suggest that the Greenland Ice sheet could be doomed at $1.5°C$ of warming, which could happen as soon as 2030" (Lenton et al., 2019, p. 592). This seems to point to the fact that, even if experts' judgments are obtained under irreversibility and uncertainty, such extreme events, characterised by low probabilities and catastrophic consequences, could be more likely than it was once thought.

Providing a realistic assessment of the probability of a tipping event may have a fundamental impact in influencing policy decisions concerning climate change. Unfortunately, there are very few models able to simulate abrupt climate changes as consequences of realistic external forcing and, even when sophisticated climate models show effects, in particular a temperature response, they are weak with respect to Dansgaard-Oeschger events.[2] To manage the complexity and the uncertainty implied in climate models, policy makers (PMs) have no alternative but to resort to experts and obtain experts' elicitations; that is, expert probabilistic judgments (Colson & Cooke, 2018). In order to elicit experts' opinions about tipping points, it is customary to interview (face-to-face) leading climate scientists about the possible effects of global climate change and response of the climate system to future trajectories of radiative forcing (i.e., Zickfeld et al., 2007; Kriegler et al., 2009; Zickfeld et al., 2010; IPCC, 2017). Once the elicitation is completed, the data are aggregated using a suitable combination approach. Using a reliable method to provide a synthesis of the various opinions seems of paramount importance, also given the fact that no clear dominant approach exists.

The main objective of this paper is to provide an assessment of the occurrence of tipping points using the elicitation data in Kriegler et al. (2009). In Kriegler et al.'s seminal paper, experts are asked to provide probability intervals about three temperature scenarios. The low temperature scenario (Low) considers an increase between about $1°C$ and $2°C$ by year 2200 in comparison with year 2000. The medium temperature scenario ($Medium$) considers an increase between about $2°C$ and $4°C$ by year 2200 in comparison with year 2000. The high temperature scenario ($High$) considers an increase between about $4°C$ and $8°C$ by year 2200 in comparison with year 2000. This particularly long time horizon differs from that considered by the IPCC. The IPCC uses the cumulative CO2 emissions from 2020 to 2050 for six illustrative scenarios SSP1-1.9, SSP1-2.6, SSP2-4.5, SSP3-7.0, and SSP5-8.5 about the projected global warming that also

[1] A tipping point is defined as "[a] level of change in system properties beyond which a system reorganizes, often abruptly, and does not return to the initial state even if the drivers of the change are abated. For the climate system, it refers to a critical threshold when global or regional climate changes from one stable state to another stable state" (IPCC, 2018b).

[2] Dansgaard-Oeschger events are rapid climate fluctuations such as the Greenland ice melting occurred in the Eemian interglacial. This event took the form of rapid warming episodes, followed by gradual cooling periods, that increased the average annual temperature on the Greenland ice sheet of 8 $°C$ over 40 years (Heinrich, 1988; Dansgaard et al., 1993).

includes effects of anthropogenic forces. Kriegler et al. have different goals and the longer horizon, up to 2200, is consistent with transition processes that take place in the cryosphere.

The aggregation approach considered in this paper uses a recently introduced method (Basili & Chateauneuf, 2020) that allows a researcher to recover the probability distribution of the event under investigation from opinions expressed as probability intervals. Using a standard Bayesian updating rule we can provide posterior probabilities that may be used to generate scenarios or hypotheses for future studies once more data become available.[3]

In this paper we assume that opinions are expressed through different probability distributions and that there exists a PM who adopts a multiple priors decision model. The set of probability distributions or lotteries of all experts can be considered a reflection of the PM's assessment of the reliability of available information about the underlying uncertainty, that is, her perception of uncertainty; the optimal aggregation rule incorporates the PM's attitude about scanty and vague information. Facing the set of all probability distributions attached by experts to possible events, the PM considers the weighted expected value of their common probability set, or their probability intersection. Such a weighted average opinion is the Steiner point of the convex capacity that emerges from the aggregation of experts' opinions (see Appendix B for more details on the aggregation approach).

In general, our results suggest that the interpretation of a tipping point as a high-impact-low-probability event seems to be incorrect (see also Lenton et al., 2019; Lenton & Ciscar, 2013). We find, in fact, that the probability of crossing a tipping point threshold is non trivially larger than zero in nearly all cases, even for low climate change scenarios. Furthermore, it has been recently claimed (Lenton & Ciscar, 2013) that a more adequate picture for the representation of tipping points would be to provide the (joint) probability distribution for tipping each element. Due to limited data availability, we can only recover the marginal distribution of tipping points. In spite of this limitation, this paper is an effort towards that direction. We update the resulting probabilities using a standard Bayes rule for all the possible combinations of the likelihood of occurrence of a certain temperature scenario. This approach generates posterior probability surfaces that may be interpreted as tipping points scenarios. The results suggest that some tipping points (the dieback of the Boreal forest, the reorganization of the Atlantic meridional overturning circulation, the decline of the ocean carbon sink and the melt of the West Antarctic and Greenland ice) are extremely likely to occur under the high temperature scenario. On the other hand, only committing to low climate change temperature corridors would substantially reduce the probability of occurrence of most tipping points.

The paper proceeds as follows. Section 2 shows the related literature. Section 3 introduces the fundamental aspects of our theoretical framework. In Section 4 we present the conditional probabilities of occurrence of various tipping points computed via the Steiner point as well as the results of the Bayesian updating. Finally, Section 5 concludes the study.

2 Related Literature

Communicating uncertainty about climate change is crucial to effectively influence policy decisions and shape public opinion. In this context, the IPCC special report Global Warming

[3]It seems that there are no follow up studies on experts' assessment of tipping points. This is also confirmed in the correspondence with leading researchers in the field. Other recent studies use the data and results in Kriegler et al. (2009), see, e.g.,Wunderling et al. (2021) and Gaucherel & Moron (2017).

of 1.5°C (IPCC, 2018a) is an important example. The IPCC special report is based "on the assessment of around 6,000 peer-review publications, most of them published in the last few years" (IPCC, 2018a, p. v). The report aggregates multiple forms of knowledge and uses a specific language to address and communicate the degree of certainty (or lack thereof) of specific findings.

In general, however, some critical issues emerge in the treatment of uncertainty: findings are based on multiple lines of evidence and are expressed using confidence qualifiers and many of them depend on certain model assumptions. In addition to that, findings have to be updated if new information becomes available. Borsuk & Tomassini (2005), Tomassini et al. (2007), Knutti et al. (2008), Zickfeld et al. (2007), Zickfeld et al. (2010), Kriegler et al. (2009) highlight that aggregation of probabilistic projections with a variety of statistical models is an unsolved problem. One fundamental challenge of the assessment process is to summarize such information into one single quantity. However, due to the heterogeneity in sources and quality of the information, obtaining a unique synthetic measure may become a daunting task.

Furthermore, when formal statistical procedures are unavailable, expert judgment approaches are often employed to provide an assessment of uncertainty (see e.g. Mastrandrea et al., 2011).[4] Significantly, "one option is to resort to imprecise probability (e.g., Kriegler & Held (2005); Hall et al. (2007); Tomassini et al. (2007)), that is, consider an uncertainty in PDFs [probability distribution functions] or sets of PDFs" (Knutti et al., 2008, p. 2658). If each expert has a set of probability distributions, the mean value for each scenario is evaluated along with all the individual PDFs (Tomassini et al., 2007).

Experts' qualitative judgments are often elicited in face-to-face interviews. Then, using a range of different procedures, mean (averaged over experts) ranks are computed (Kriegler et al., 2009; Zickfeld et al., 2010) . In Zickfeld et al. (2010), 14 experts (leading climate scientists) discuss about three scenarios (high, medium, low) of net radiative forcing at the top of the atmosphere from anthropogenic sources through the year 2200. Experts use a cardinal scale from 0 (no chance) to 1 (definite chance) for each of the three forcing trajectories. Experts elicit probabilities for each scenario and in the following step they are asked to estimate the median of the mean trajectories of warming between 2000 and 2050. What they find is that it falls between 0.16°$C/decade$ and 0.36°$C/decade$. These results are qualitatively similar to those of Kriegler et al. (2009).

From a more general point of view, the problem of eliciting information from experts may take various forms (Garthwaite et al., 2005). Suppose, for example, that the PM wants to learn about a certain phenomenon characterized by a random variable X and that there is a set of m experts equipped with probability $P^{(j)}$ with $j \in \{1, \ldots, m\}$. The PM may supply the experts with a probability for the phenomenon under investigation, say, $p^{(PM)}$ and in turn the experts would provide a value for the quantile x, say, $x^{(j)}$. This definition of the problem is similar to what one may find in the analysis of Zickfeld et al. (2010) and, more recently, of Bamber et al. (2019). Alternatively, the generic expert j may be asked to assess the probability of an event associated to X where $x^{(PM)}$ is a value provided by the PM and $p^{(j)}$ is expert j's assessment. This is similar to the case considered for example in Kriegler et al. (2009).

[4] In 1975, the U.S. Nuclear Regulatory Commission (NCR) introduced for the first time a procedure for the elicitation process and, since then, techniques and methods have spread to other areas such as volcanology, public health, ecology, aeronautics, climatology etc. (Cooke, 2013).

3 Theoretical Framework

The aggregation process of probabilistic opinions (opinion pooling) entails a function, known as a pooling rule, that elicits a consensus distribution. Such a consensus distribution is determined among not necessarily independent and fully competent experts when each of them has multiple priors on future states of the World.[5]

Under uncertainty or deep uncertainty experts have partial, incomplete or fuzzy knowledge and their beliefs cannot be represented by a unique, additive and fully reliable probability distribution, but either by a finite set of them, an interval of probabilities or by a non necessarily additive measure (e.g. a capacity) (e.g. Basili & Chateauneuf, 2011, 2016; Basili & Pratelli, 2015, and references therein).[6]

When experts' opinions are measured as probability intervals, their consensus distribution is elicited by means of the Steiner point of the intersection (core) of all the experts' probability distributions. This type of aggregation assumes that experts have imprecise information, but nothing about their competence, experience and independence. It is interesting to notice that when the set of states of the World is finite, the Steiner point coincides with the Shapley value (Shapley, 1971; Pechersky, 2015; Basili & Chateauneuf, 2020). The Shapley value is defined as

$$\Pi_i^v = \sum_{s_i \in \mathcal{A} \subseteq \mathcal{S}} \frac{(|\mathcal{A}|-1)!(n-|\mathcal{A}|)!}{n!} \left(v(\mathcal{A}) - v(\mathcal{A}\backslash\{s_i\}\right), \; i = 1, \ldots, n \tag{1}$$

where $v(\cdot)$ is a capacity (see equation B.2 in Appendix B) and $\mathcal{S} = \{s_1, \ldots s_n\}$ is a set of states of the World. Our problem consists of two states of the World ($n = 2$), this is, whether a tipping point occurs (Tip) or it does not occur ($No\ Tip$). The two states of the World occur with probability Π_1^v and $\Pi_2^v = 1 - \Pi_1^v$ respectively.

Let us now define the random variables $T \in \{Tip, No\ Tip\}$ denoting the occurrence (or not) of a tipping point and $C \in \{Low, Medium, High\}$ representing possible temperature scenarios as defined in Section 1. Hence, we have the following Bayes rule

$$P(T = Tip|C) = \frac{P(C|T = Tip)P(T = Tip)}{P(C|T = No\ Tip)P(T = No\ Tip) + P(C|T = Tip)P(T = Tip)}. \tag{2}$$

The Bayes rule in equation 2 can be used to update the probability of occurrence of a tipping point obtained via the Steiner point. Specifically, we choose Π_1^v for $P(T = Tip)$ as a prior probability, while $P(C|T = Tip)$ and $P(C|T = No\ Tip)$ can be obtained from expert knowledge. Alternatively, they can be left unspecified. In this last case the posterior probability $P(T = Tip|C)$ can be represented as a surface. For the empirical analysis in Section 4 we opt for the latter.

4 Eliciting the Probability of a Tipping Point by the Steiner Point

Kriegler et al. (2009) elicited beliefs of experts about the probability of triggering major changes

[5] Further details on the theoretical features of the aggregation approach can be found in Appendix B.

[6] Deep uncertainty includes "situations in which we are still able (or assume) to bound the future around many possible plausible futures and situations in which we only know that we do not know" (Marchau et al., 2019). See also Rohmer et al. (2019) and Frederikse et al. (2020) for some recent discussion of the concepts of uncertainty in the context of climate change issues.

in the Earth system associated to seven tipping points, (see Table 1 for the definition of the tipping points under analysis) for three different global median temperature scenarios. The elicitation was developed in multiple rounds, including revisions of previously stated opinions. The revisions also included consistency checks.

The data are summarized in the dumbbell plots in Figures A.1 to A.7 in Appendix A. Each figure consists of three plots and each plot refers to one of the three temperature scenarios. To every row in the plots we attach one single expert. By direct inspection, we note, as also stressed in Kriegler et al. (2009), the tendency of the experts to place high probability in the high temperature scenarios.

AMAZ	Dieback of the Amazon rainforest.
BOFO	Dieback of Boreal forests.
AMOC	Reorganization of the Atlantic meridional overturning circulation.
DAIS	Disintegration of the West Antarctic ice sheet.
MGIS	Melt of the Greenland ice sheet.
DOCS	Decline in ocean carbon sink.
NINO	Shift to a more persistent El Niño regime.

Table 1: Definition of the tipping points considered in Kriegler et al. (2009).

In order to provide policy relevant information Kriegler et al. (2009) combine, using an appropriate combination rule (see Clemen & Winkler, 1999; Nau, 2002), the probability intervals supplied by the experts for every tipping point and every temperature scenario.[7] Their aggregation rule produces upper and lower probabilities. Our approach is substantially different. In fact, the Shapley value described in equation 1 returns the probability of occurrence of a tipping point conditional on a given temperature scenario.

In Figure 1 we see that, in general, high climate change scenarios (red triangles) are more likely to trigger a tipping point. While this result is not too surprising, we also observe that there is a certain degree of heterogeneity across scenarios and tipping points. Specifically, AMOC has a very low chance to occur for the low climate change scenario. All the other tipping points have, for the same scenario a non trivial (say, larger than 10%) probability to occur. As we consider the higher climate change scenarios, $Medium$ and $High$, the probability of having a tipping point increases. It is remarkable that for $High$, the probability of occurrence for BOFO, DAIS, DOCS and MGIS is 50% or higher.

To provide further intuition for the interpretation of the plots in Figure 1, we adapt the terminology used in Kriegler et al. (2009). This is, we label as *remote* the prospect of having a tipping point if $\Pi_1^v < 0.1$, *significant* if $\Pi_1^v \geq 0.1$ and *large* if $\Pi_1^v \geq 0.5$. Under these criteria only AMOC for the low temperature scenario has a remote chance of being triggered. DAIS and MGIS have a large probability of being triggered under the high temperature scenario, while BOFO and DOCS have a large probability of being triggered under both the medium and high temperature scenario. The remaining scenarios have a significant probability of triggering a tipping point.

By applying the Bayes rule in equation 2 we update the results obtained via the Steiner point to generate tipping point scenarios for different combinations of $P(C|T = 0)$ and $P(C|T = 1)$. Every point in the resulting surfaces (figures 2 to 8) is the conditional probability of crossing a

[7] Kriegler et al. (2009) introduce a novel aggregation rule called forced consensus pooling.

tipping point given the likelihood of the (conditional) occurrence of the temperature scenario. We notice that the high temperature scenario produces large probabilities for the occurrence of tipping points. This is result is particularly clear for BOFO (figure 3 (c)), AMOC (figure 4 (c)), DAIS (figure 5 (c)), DOCS (figure 6 (c)) and MGIS (figure 7 (c)). Committing to low climate change scenarios may produce remote probabilities of tipping, at least for AMOC (figure 4 (a)) and DAIS (figure 5 (a)), yet in most of the remaining cases the probabilities of triggering a tipping point are generally at least significant.

5 Conclusions

This paper considers the aggregation of experts' opinions about uncertain climate change scenarios. The aggregation rule based on the Steiner point provides a probability of the occurrence of a tipping point. Our results suggests, as expected, that tipping points have higher probabilities to occur under high temperature scenarios. Nonetheless, the probability of crossing a tipping point threshold is non trivially different from zero, even under lower climate change scenarios and for the majority of tipping points under investigation. Our analysis, consistently with the recent literature on the topic, highlights the fact that these type of events may turn out to be one of the main climatic emergencies in the upcoming years. Future research can further contribute to the understanding of these issues by addressing problems concerning elicitation data and the construction of joint distributions for tipping events.

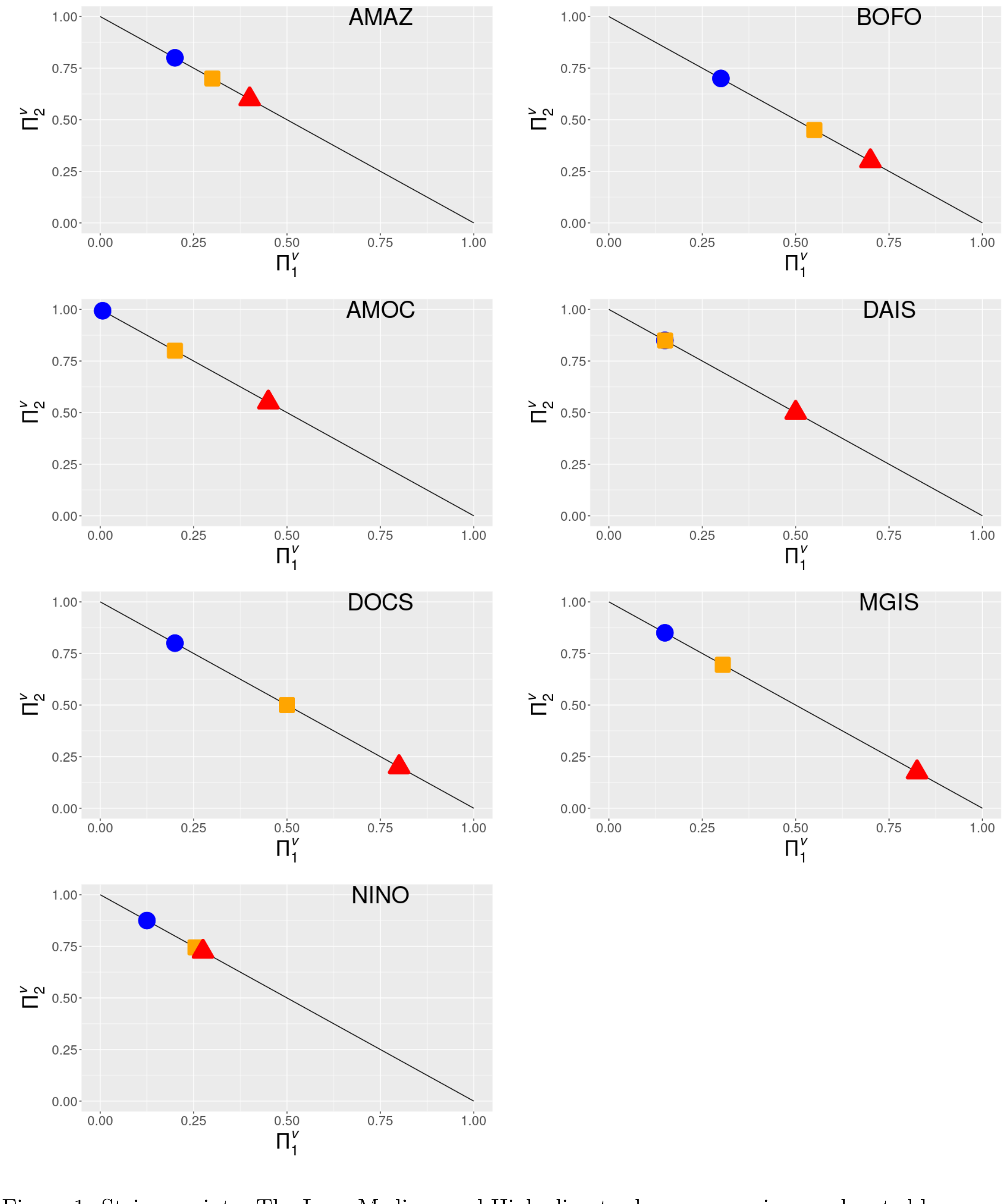
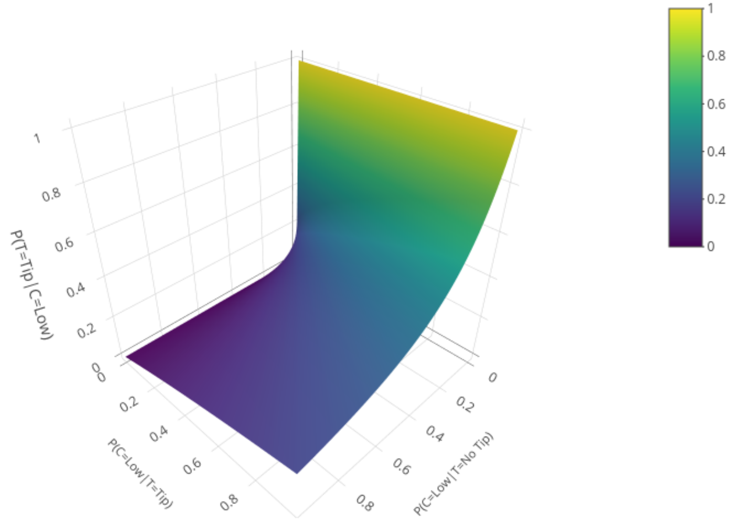
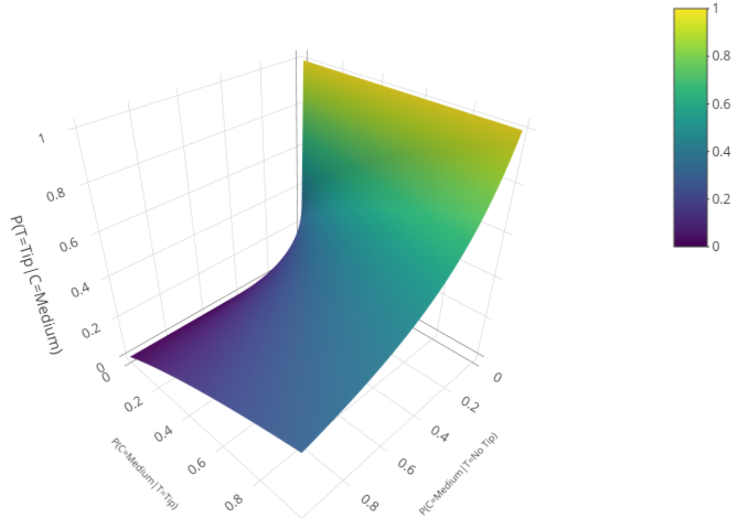


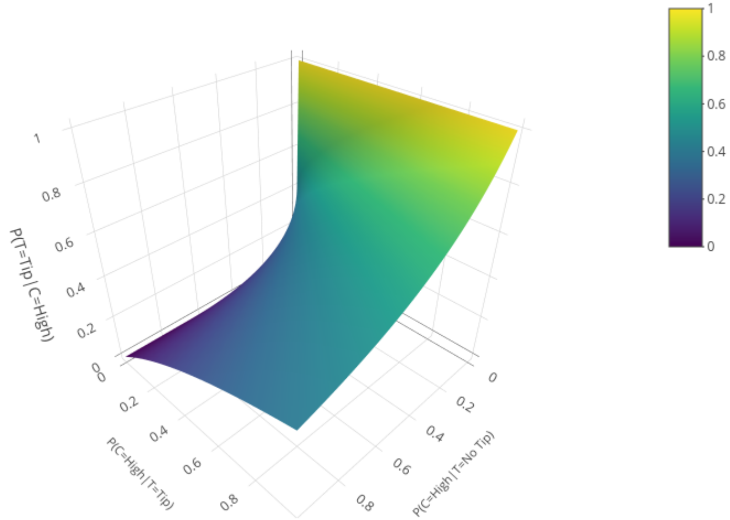
Figure 1: Steiner points. The Low, Medium and High climate change scenarios are denoted by a circle, square and triangle respectively. Π_1^v is the probability that the tipping point is triggered, while Π_2^v is the probability that the tipping point is not triggered.



(a) Low temperature scenario.

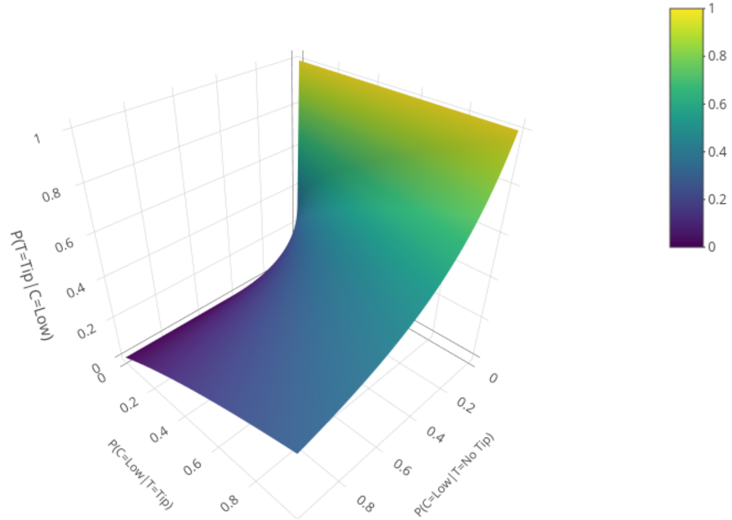


(b) Medium temperature scenario.

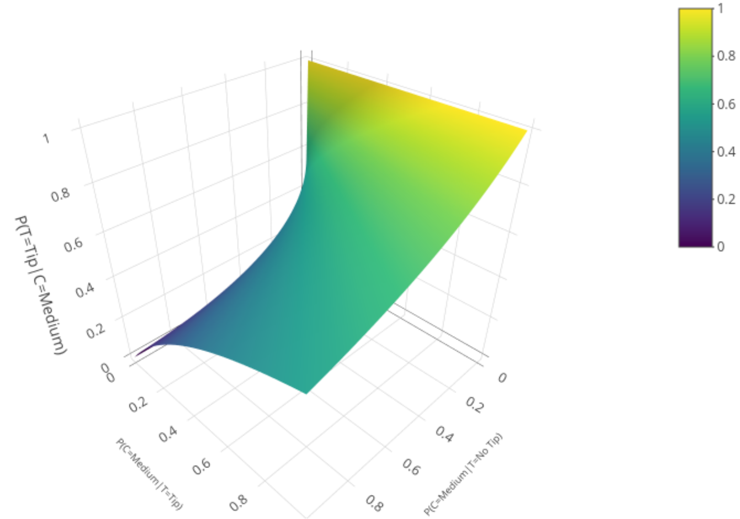


(c) High temperature scenario.

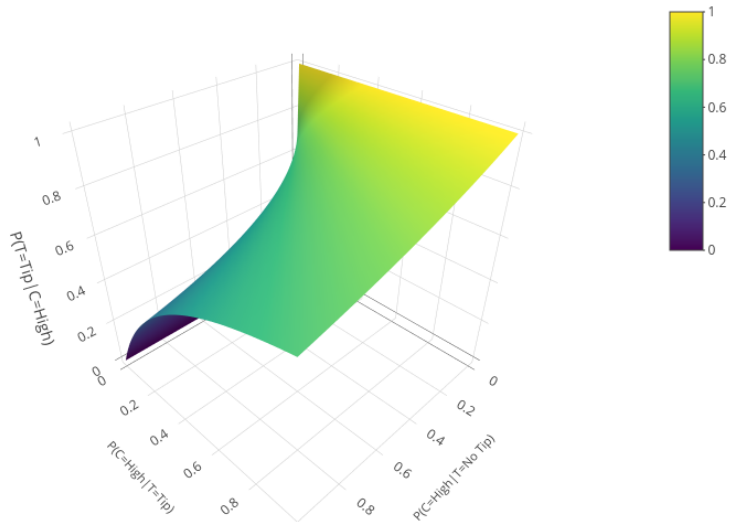
Figure 2: Posterior surfaces for the AMAZ tipping point.



(a) Low temperature scenario.

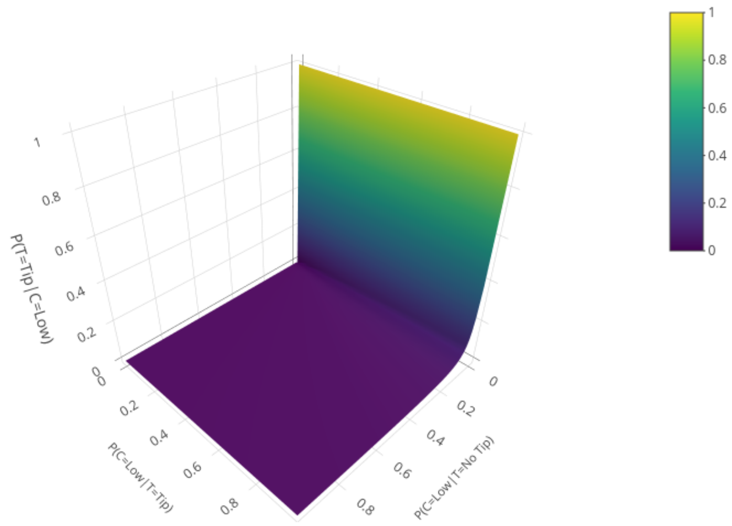


(b) Medium temperature scenario.

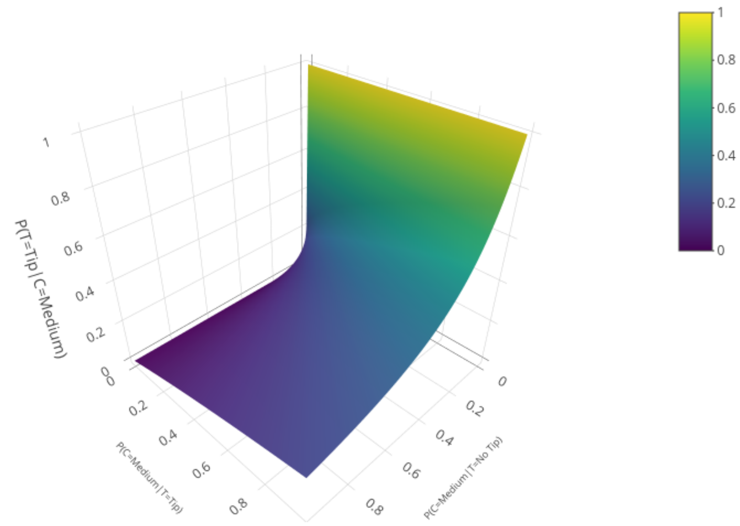


(c) High temperature scenario.

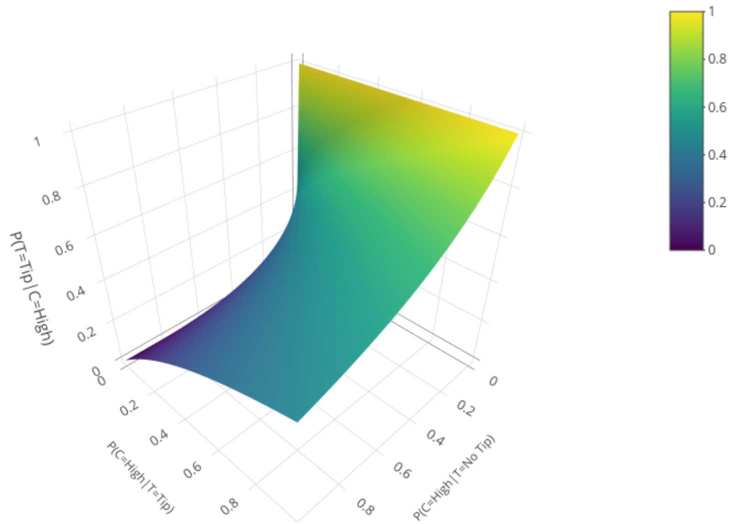
Figure 3: Posterior surfaces for the BOFO tipping point.



(a) Low temperature scenario.

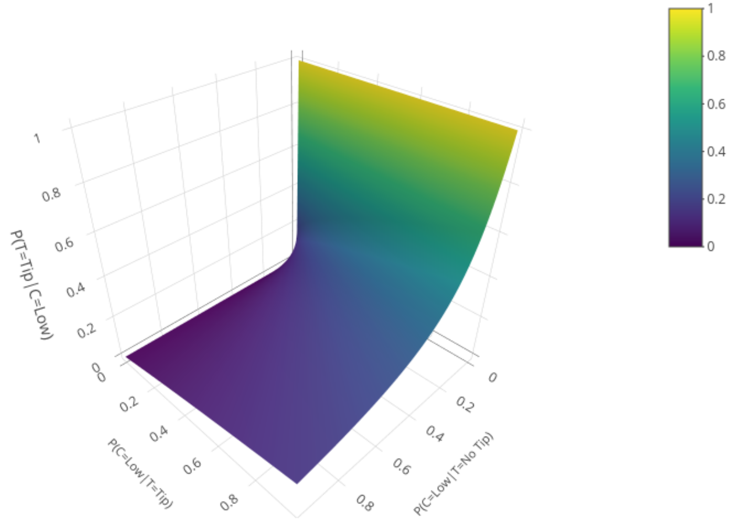


(b) Medium temperature scenario.

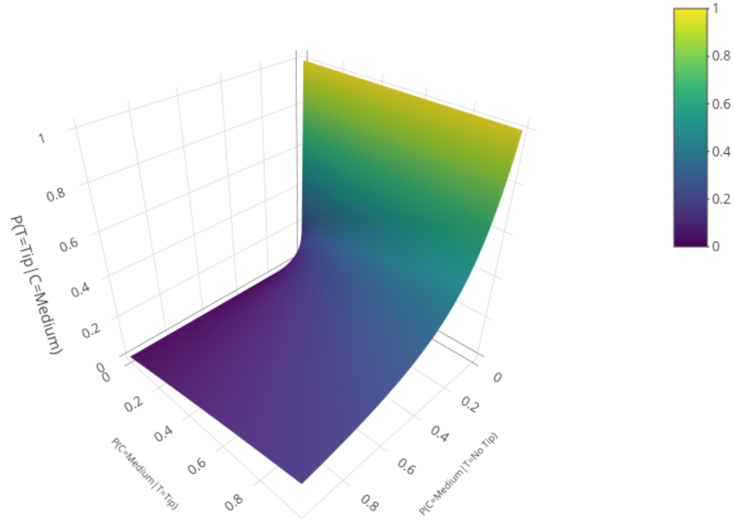


(c) High temperature scenario.

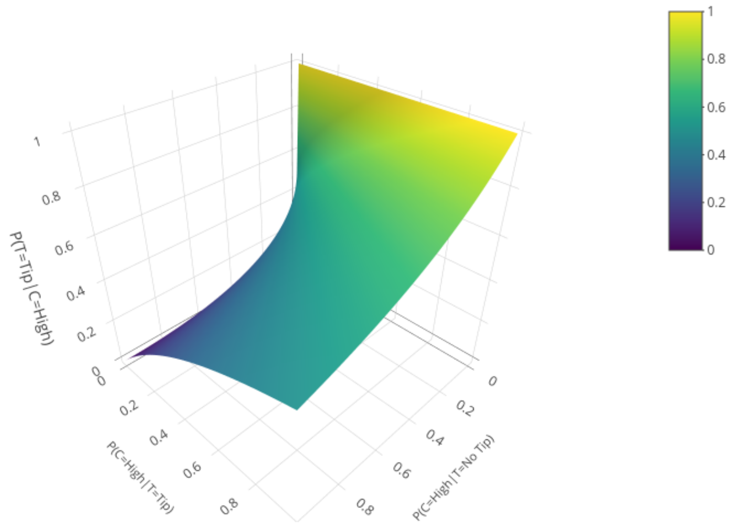
Figure 4: Posterior surfaces for the AMOC tipping point.



(a) Low temperature scenario.

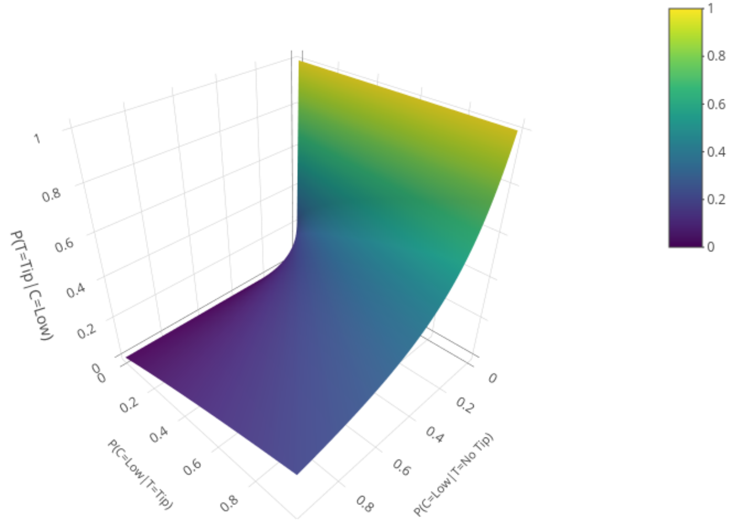


(b) Medium temperature scenario.

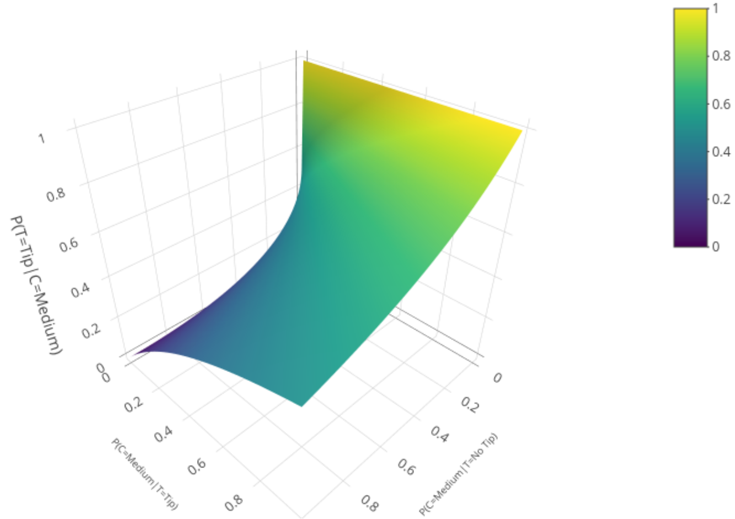


(c) High temperature scenario.

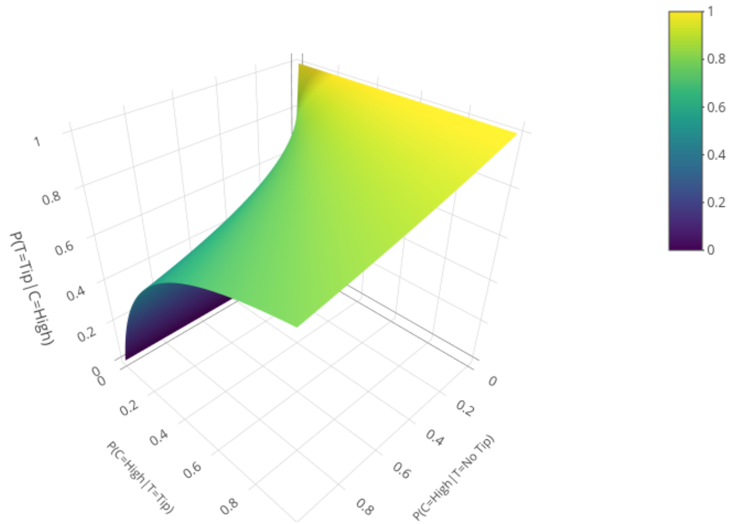
Figure 5: Posterior surfaces for the DAIS tipping point.



(a) Low temperature scenario.

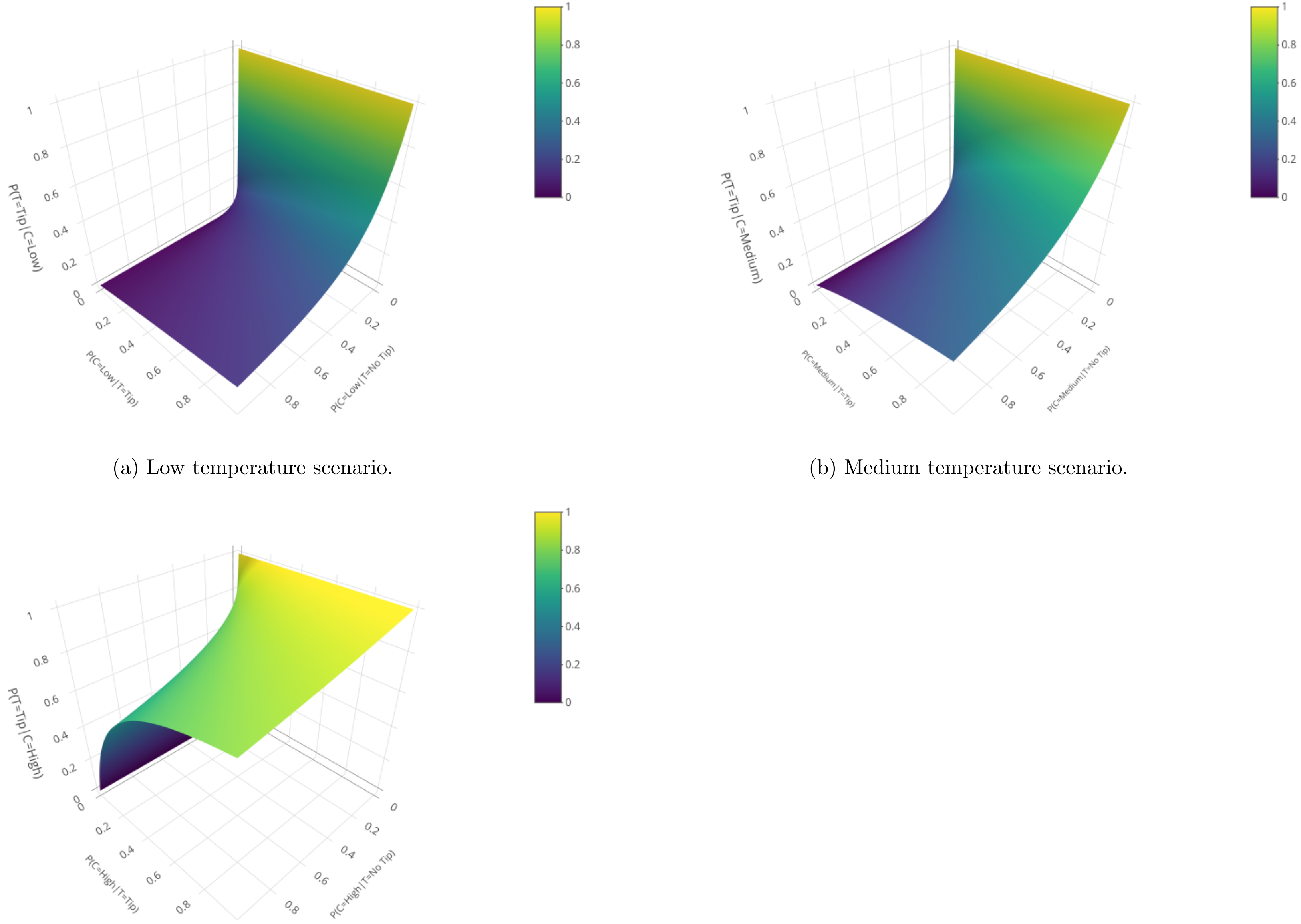


(b) Medium temperature scenario.



(c) High temperature scenario.

Figure 6: Posterior surfaces for the DOCS tipping point

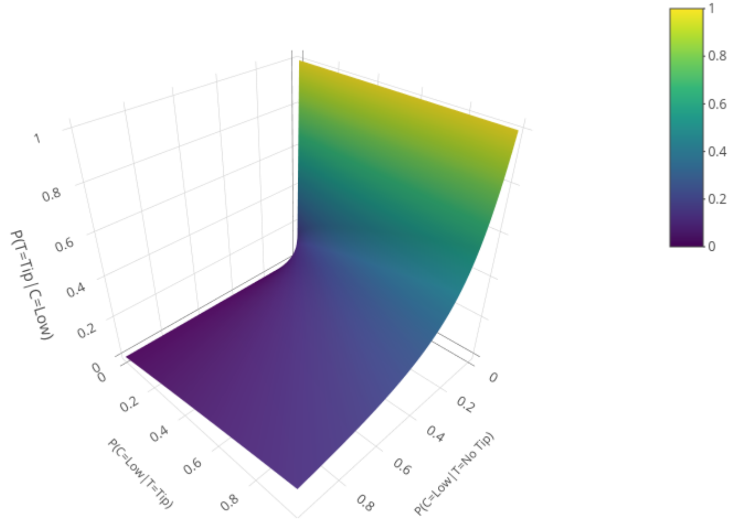


(a) Low temperature scenario.

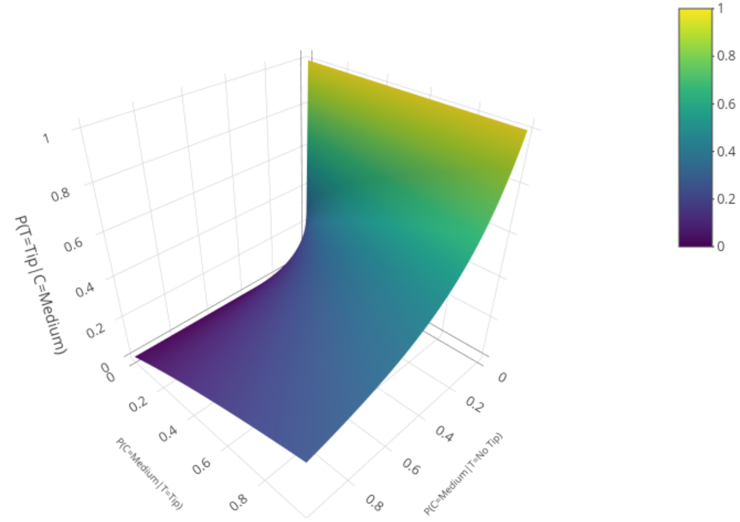
(b) Medium temperature scenario.

(c) High temperature scenario.

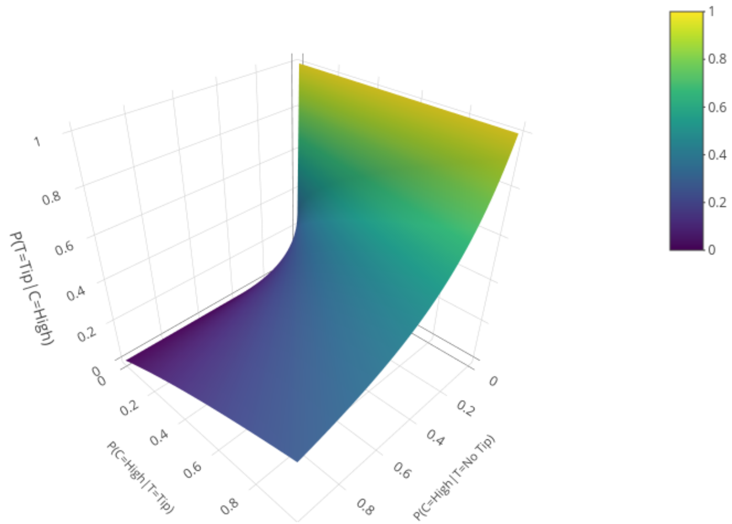
Figure 7: Posterior surfaces for the MGIS tipping point.



(a) Low temperature scenario.



(b) Medium temperature scenario.

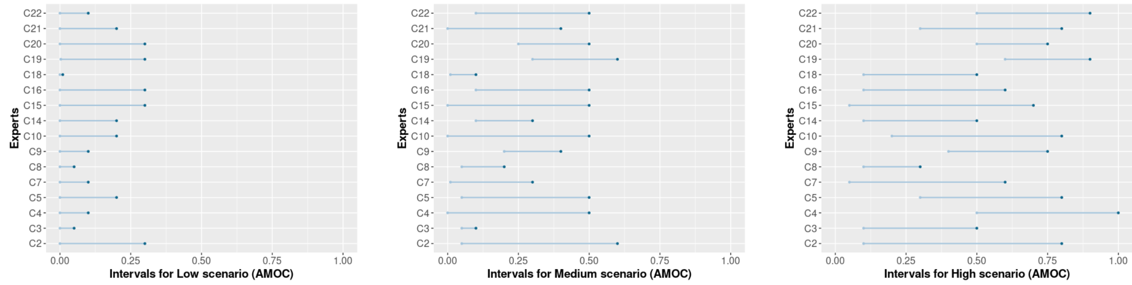


(c) High temperature scenario.

Figure 8: Posterior surfaces for the NINO tipping point.

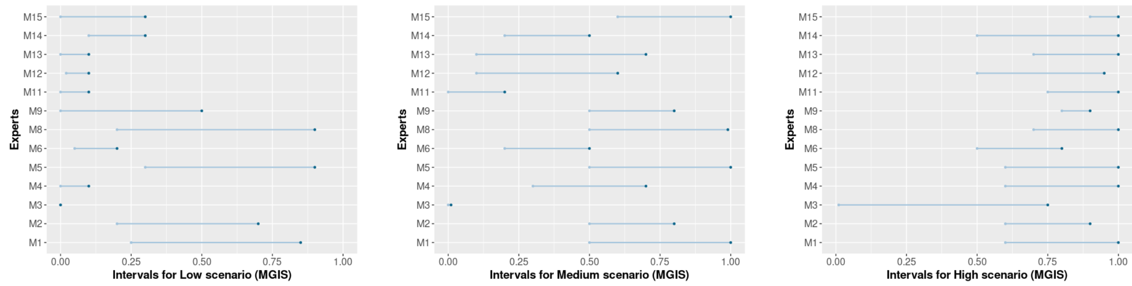
A Figures

This section displays a set of dumbbell plots that describe the data used in the analysis. The plots are similar to those in Figure 1 of Kriegler et al. (2009). Furthermore, in Kriegler et al. some of the experts are recognized as *core experts*. The caption of each plot indicates which experts are not core experts.



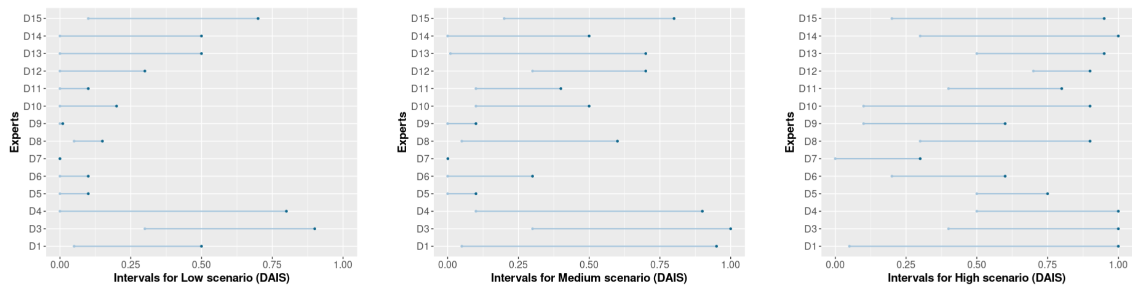
(a) Low (b) Medium (c) High

Figure A.1: Elicited probability intervals for the AMOC tipping point. In Kriegler et al. (2009) C9, C20 and C22 are not classified as core experts.



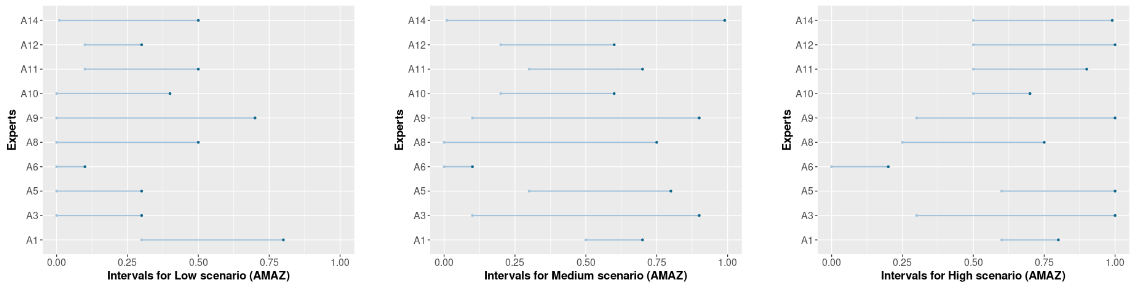
(a) Low (b) Medium (c) High

Figure A.2: Elicited probability intervals for the MGIS tipping point. In Kriegler et al. (2009) M1, M3 and M14 are not classified as core experts.



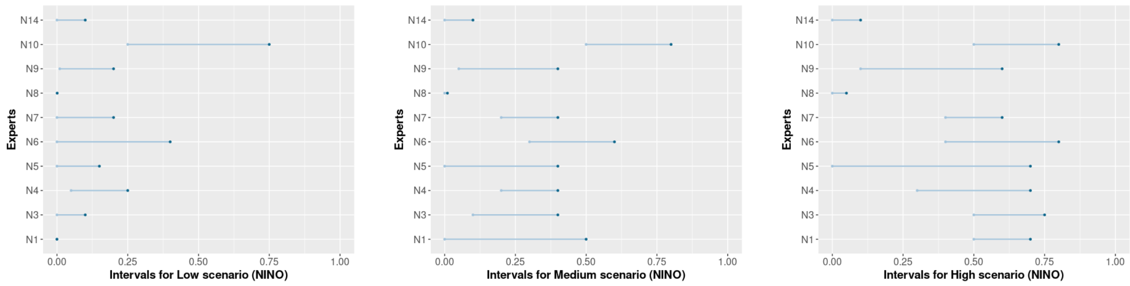
(a) Low (b) Medium (c) High

Figure A.3: Elicited probability intervals for the DAIS tipping point. In Kriegler et al. (2009) D7 and D10 are not classified as core experts.



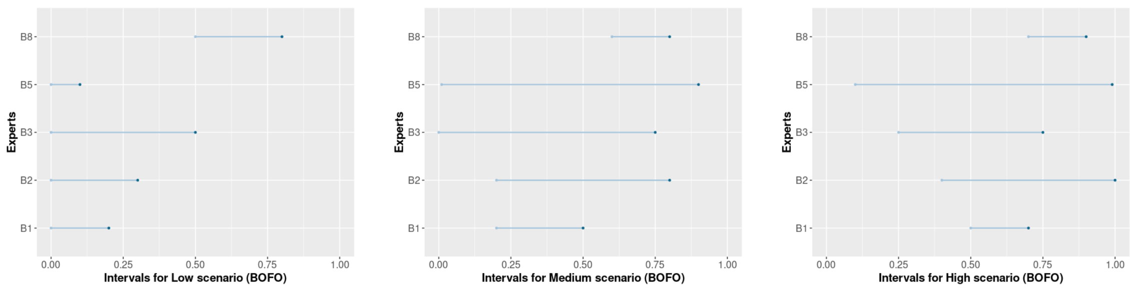
(a) Low (b) Medium (c) High

Figure A.4: Elicited probability intervals for the AMAZ tipping point. In Kriegler et al. (2009) A1, A6, A10 and A12 are not classified as core experts.



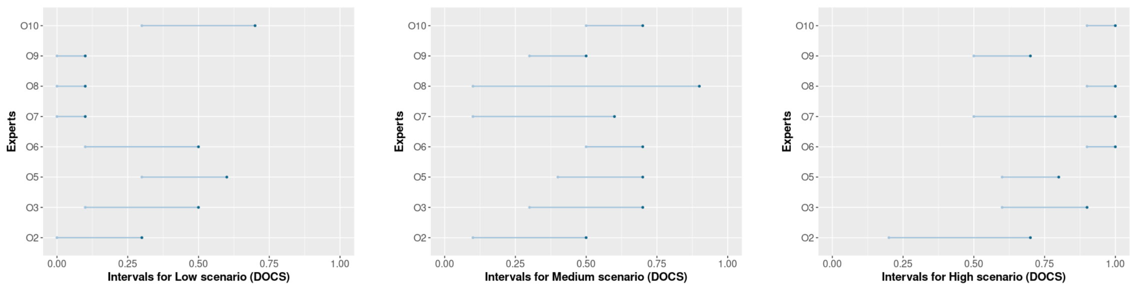
(a) Low (b) Medium (c) High

Figure A.5: Elicited probability intervals for the NINO tipping point. In Kriegler et al. (2009) N3, N6 and N10 are not classified as core experts.



(a) Low (b) Medium (c) High

Figure A.6: Elicited probability intervals for the BOFO tipping point. In Kriegler et al. (2009) B1 is not classified as a core experts.



(a) Low (b) Medium (c) High

Figure A.7: Elicited probability intervals for the DOCS tipping point. In Kriegler et al. (2009) O2 is not classified as a core experts.

B Aggregation of Opinions and the Steiner Point

Let us consider a finite set $\mathcal{S} = \{s_1, \ldots, s_n\}$ of states of the World and let $\Sigma = 2^{\mathcal{S}}$ be the σ-algebra associated to the set $\mathcal{S}$. Since the study deals with discrete events, P is a probability mass function (pmf) on $(\mathcal{S}, \Sigma)$ such that $P : \Sigma \to [0, 1]$. For a specific pmf, $p(s_i) = P(S = s_i)$ for any $i \in \{1, \ldots, n\}$ and $\sum_{i=1}^{n} p(s_i) = 1$.

Consider now a non-negative set function $v : \Sigma \to \mathbb{R}$. Such a function is a capacity on $(\mathcal{S}, \Sigma)$ if, for any $\mathcal{A}, \mathcal{B} \in \Sigma$, $\mathcal{A} \subseteq \mathcal{B} \Rightarrow v(\mathcal{A}) \leq v(\mathcal{B})$, $v(\emptyset) = 0$ and $v(\mathcal{S}) = 1$, where $\emptyset$ is the empty set. The capacity $v(\cdot)$ is convex if $v(\mathcal{A} \cup \mathcal{B}) + v(\mathcal{A} \cap \mathcal{B}) \geq v(\mathcal{A}) + v(\mathcal{B})$. The corresponding dual capacity is defined as $\bar{v}(\mathcal{A}) = 1 - v(\mathcal{A}^c)$. The dual capacity $\bar{v}(\mathcal{A})$ is concave. Uncertainty is modeled via the core of the convex capacity v. The core $\mathcal{C}(v)$ is a set of probability distributions P on $(\mathcal{S}, \Sigma)$ such that $P(\mathcal{A}) \geq v(\mathcal{A})$ $\forall \mathcal{A} \in \Sigma$.

The PM's problem is to learn the distribution of a certain event by aggregating experts' opinions. Let P_0 denote the unobserved pmf that governs the phenomenon under study. She asks expert j, $(j = 1, \ldots, m)$, to provide a lower and upper bound for the probability $p_i = p_0(s_i) = P_0(S = s_i)$. The set of possible probabilities considered by expert j is defined as

$$\mathcal{P}^j = \left\{ P^j = \left(p_1^j, \ldots, p_i^j, \ldots, p_n^j \right) : a_i^j \leq p_i^j \leq b_i^j, i = 1, \ldots, n \right\}.$$

The PM will accept the expert's opinion if two consistency conditions are met. Specifically, for the expert's opinions to make sense the bounds a_i^j and b_i^j must meet the following conditions of individual consistency (*I-consistency*).

Condition 1. [**I-consistency**] The bounds a_i^j and b_i^j satisfy the conditions $0 \leq a_i^j \leq b_i^j \leq 1$ and $\sum_{i=1}^{n} a_i^j \leq 1 \leq \sum_{i=1}^{n} b_i^j$.

The set $\mathcal{P}^j$ is not empty if and only if the I-consistency condition is met. Furthermore, $\mathcal{P}^j$ can be seen as the core $\mathcal{C}(v^j)$ of a given convex capacity v^j where

$$v^j(\mathcal{A}) = \max \left(\sum_{i \in \{i : s_i \in \mathcal{A}\}} a_i^j, \ 1 - \sum_{i \in \{i : s_i \notin \mathcal{A}\}} b_i^j \right) \tag{B.1}$$

and the set $\mathcal{A} \subseteq \mathcal{S}$ is a set of states of the World (Chateauneuf & Cornet, 2018; De Campos et al., 1994). It is expected that the unknown distribution P_0 be in the the set $\mathcal{P}^j$ of the generic expert j, i.e. $P_0 \in \mathcal{P}^j$. In addition to that, it is expected also that P_0 be in the intersection of all $\mathcal{P}^j$. Hence, the following group consistency (*G-consistency*) condition is supposed to be met.

Condition 2. [**G-consistency**] The intersection of the probability sets associated with the pool of experts is non empty, i.e., $\mathcal{P} = \cap_{j=1}^{m} \mathcal{P}^j \neq \emptyset$ and that $P_0 \in \mathcal{P}$.

Whenever the intersection set $\mathcal{P} = \emptyset$, this is, when experts have conflicting opinions, the PM should either require a revision of the experts' opinions or change the expert panel. Notice that, however, even when the G-consistency condition is not met, the PM may still be able to extract valuable information by applying the G-consistency condition to a subset of experts. This approach may reveal, for example, whether some experts are more or less optimistic about a given phenomenon.

The consensus distribution set is associated to a convex capacity $v(\cdot)$, defined as

$$v(\mathcal{A}) = \max \left(\sum_{i \in \{i: s_i \in \mathcal{A}\}} a_i,\ 1 - \sum_{i \in \{i: s_i \notin \mathcal{A}\}} b_i \right) \tag{B.2}$$

where $a_i = \max_j a_i^j$ and $b_i = \min_j b_i^j$. In this context the Steiner point is particularly relevant, since the Steiner point of $\mathcal{C}(v)$ is the center of the core of the convex capacity v and represents the consensus probability for the given set of experts. The Steiner point is denoted as $\Pi^v \in \mathcal{C}(v)$.

In our framework the Steiner point coincides with the Shapley value and it is easily computed via the following expression

$$\Pi_i^v = \sum_{s_i \in \mathcal{A} \subseteq \mathcal{S}} \frac{(|\mathcal{A}| - 1)!(n - |\mathcal{A}|)!}{n!} \left(v(\mathcal{A}) - v(\mathcal{A} \backslash \{s_i\}) \right),\ i = 1, \ldots, n. \tag{B.3}$$

Equation (B.3) shows how the Shapley value represents the average marginal individual contribution over all the possible different permutations in which the grand coalition $\mathcal{S}$ may be formed (Basili & Chateauneuf, 2020).[8]

[8] Recent research suggests that, for independent inputs, the Steiner point-Shapley value is bracketed between two different Sobol' indices. This result seems to hold also for the case of dependent inputs or expert judgments (Song et al., 2016, see also Owen & Prieur (2017) for further uses of the Shapley value in the context of ANOVA). Sobol' index provides a measure of the importance of inputs to a function and is defined in terms of the functional analysis variance decomposition (Sobol', 1990, 1993).

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