UNIVERSITÀ DI SIENA
1240

QUADERNI DEL DIPARTIMENTO
DI ECONOMIA POLITICA E STATISTICA

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How robust is the natalist bias of pollution control?

n. 895 – Gennaio 2023

How robust is the natalist bias of pollution control?*

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January 2023

Abstract

This paper assesses the robustness of the so-called "natalist bias" of pollution control. The latter suggests that taxing emissions encourage agents to shift from production to tax-free activities such as procreation, further deteriorating the environment and gradually impoverishing the next generations. We relax the assumptions that human capital does not depend on environmental quality and that society does not allocate resources to pollution control. Using a similar Overlapping Generations (OLG) growth model, our findings indicate that taxation does not necessarily encourage agents to permanently shift away from production because living under better environmental conditions enhances productivity through human capital formation. As the government increases the emissions price, agents reduce consumption and education spending, hurting output in the short term. However, in the long run, the reduction in emissions that follows taxation more than compensates for the initial adverse effects, provided that the sensitivity of human capital accumulation to environmental degradation is strong enough. Furthermore, as we increase the coefficient capturing such pollution externality, a Neimark-Sacker bifurcation occurs, making the system compatible with persistent endogenous fluctuations.

Keywords: Climate change, Natalist bias, OLG, Growth, Neimark-Sacker bifurcation

JEL: Q56, Q57, O11, C62

*An earlier version of this paper was presented at the 11th Workshop Dynamic Models in Economics and Finance, Urbino, Italy, at the 46th Meeting of the Italian Association for Mathematics Applied to Social and Economic Sciences, Palermo, Italy, and at the Workshop Climate Change and Ecological Behavior, Florence, Italy. We are grateful to the participants for their insightful comments and suggestions. We are indebted to David de la Croix and Davide Radi for enlightening discussions while preparing an earlier draft of the article. Alessia gratefully acknowledges the Catholic University of Louvain-la-Neuve, in particular, Fabio Mariani, whose hospitality assisted her work on the completion of this paper. The usual caveats apply.

1 Introduction

The urgency of the climate change emergency has renewed a long-standing concern about population growth causing resource scarcity and environmental degradation (e.g. Marsiglio, 2011; Lawson and Spears, 2018; O'Sullivan, 2020). The existing literature has investigated different channels through which population dynamics influence carbon emissions, which might affect economic activity and the demographic structure (see also Marchiori and Schumacher, 2011; Lupi and Marsiglio, 2021). Among such recent studies, de la Croix and Gosseries (2012) have argued that controlling pollution through Pigovian taxes encourages agents to shift away from production to tax-free activities such as procreation. As a result, a *natalist bias* might emerge that further deteriorates the environment, entailing the need to impose more stringent environmental regulations. However, this gradually impoverishes the successive generations.

The natalist bias of pollution control depends on three main assumptions: there is no technical change, human capital does not depend on environmental quality, nor does society allocates resources to control environmental degradation. In this paper, we aim to assess the robustness of such a result by relaxing the last two of them. There is some empirical evidence suggesting air pollution may lead to reduced cognitive performance (Lavy et al., 2014). Using county-level data for the United States, Sapci and Shogren (2018) find that a one percentage point reduction in pollution increases the human capital stock by 0.10 per cent. The negative impact of emissions on growth is frequently recognised in theoretical models in the form of a "damage function" (e.g. Aloi and Tournemaine, 2013; Dafermos et al., 2018; Chen et al., 2021). Under those conditions, one of the main consequences of ecological degradation is the decline of individuals' health and learning abilities. Therefore, human capital accumulation is likely to be non-neutral to this externality.

Furthermore, de la Croix and Gosseries (2012) assume that the resources raised through pollution taxation are redistributed as a lump-sum transfer and cannot be used to revert environment degradation. Using a similar Overlapping Generations (OLG) growth model, authors such as John and Pecchenino (1994) and, more recently, Caravaggio and Sodini (2021, 2023) provide a valuable framework to tackle this limitation (see also, Antoci et al., 2016). Accepting that environmental quality is a public good, the main idea is that consumption from the "old" hurts the environment, while the "young" may take action to improve it. The present paper dwells on those contributions and considers the case in which taxes are applied to emission mitigation, maintaining the production technology unchanged. Agents do not explicitly include the environment in their utility function and only indirectly benefit from it through the quality of their offspring. Of course, the assumption of constant technology continues to be an extreme one, especially given the evidence showing climate mitigation fundamentally depends on increasing energy efficiency and adopting renewable energy sources (Nordhaus, 2019). Still, we maintain it to focus on the other two, perhaps less studied, mechanisms.

Our results suggest that taxing emissions do not necessarily encourage agents to permanently shift away from production because living under better environmental conditions enhances productivity through human capital formation. Instead of a natalist bias, we find a transitory "education spending bias". As the government increases the emissions price, agents reduce consumption and education expenditures, hurting output in the short term. However, in the long run, the reduction in emissions that follows taxation more than compensates for the initial adverse effects. Such a result holds conditional to the sensitivity of human capital accumulation to environmental degradation being strong enough. For example, when we suppose an elasticity of human capital with respect to pollution of -0.2, it is possible to increase the price of CO2 up to $p = 0.4$ obtaining lower emissions and higher output. The absence of this externality recovers the original negative and monotonic correspondence between taxes and output. Still, from its inclusion, we have that only for very high tax rates the bias prevails and leads to the complete collapse of the economy.

Finally, we carefully analyse the stability properties of the resulting dynamic system. As we increase the coefficient capturing the negative externality from pollution, a Neimark-Sacker bifurcation occurs, making the system compatible with persistent endogenous periodic fluctuations. We argue that such waves deserve careful analysis and should not be disregarded as mere mathematical curiosity. They reflect the intrinsic connection between short-term and long-term dynamics. Such endogenous cycles between environmental and economic variables have been documented in the climate mitigation literature (e.g. Zeppini, 2015; Cafferata et al., 2021) and have the following rationale. Suppose there is a reduction in human capital accumulation than brings production down. It follows that agents consume less, and pollution falls. Fewer pollutants altogether to the resources used to improve environmental quality increase output through its effect on human capital. Consumption, thus, recovers, leading to higher environmental pressure. More pollutants damage human capital, thus repeating this long-run low-frequency cycle.

The remainder of the paper is organised as follows. Section 2 brings a general overview of some empirical stylised facts linking pollutant emissions and human capabilities. Section 3 presents our OLG growth model with a detailed explanation of its two main modifications with respect to de la Croix and Gosseries (2012). It will be calibrated in Section 4 to provide a more concrete view of its main dynamic properties. Some final considerations follow.

2 Some stylised facts

Given the centrality of the mechanism for the results of the model we develop later on, our purpose in this Section is to provide some empirical insights into the importance of the human capital–pollution nexus. Several scholars have investigated the topic over the years. Air quality appears as one of the leading indicators of environmental quality (e.g. Mitchell, 2005; Farzi and Bond, 2006; Ganda, 2022) and is frequently proxied by the number of CO2 emissions per unit of GDP. For example, Salatin and Somea (2016), considering the 1998-2013 period, show that human capital has a negative and meaningful effect on CO2 emissions as an indicator of environmental quality. Voorheis (2017) discusses the effects that pollution exposure at birth has on individuals' college attendance at 19-22 years old and on their adult economic well-being. Liu et al. (2021) examining the impact of air pollution on a firm's adherence to the principles of Corporate Social Responsibility (CSR) also gives special attention to the role of human capital, finding that firms that improve their CSR performance are motivated to attract higher quality employees.

Along similar lines, Goetz et al. (2016) investigate the relationship between states with more highly educated populations and environmental conditions. They argue that the strategy of raising human capital stocks to maintain or improve environmental quality may be seen as a complement to direct government intervention. Furthermore, Jun et al. (2011) conclude that a higher degree of human capital could reduce the adverse effects that follow the negative association between environmental quality and the imbalance of income distribution observed in China.

On the other hand, investment in the skills and experience possessed by an individual or population, viewed in terms of their productive value, may take many forms (for a comprehensive review, see Abraham and Mallatt, 2022). Formal education is undoubtedly part of it, though comprehensive measures can broadly consider medical care and file expectancy. For example, the World Bank's human capital index includes the probability of survival to age five, expected years of school, harmonised test scores, the fraction of children under age five whose growth is not stunted, and adult survival rates. The present paper uses the Human Development Index (HDI) to illustrate the previous discussion. The HDI was introduced by the United Nations Development Programme. It is computed as the geometric mean of normalised indices for three dimensions: life expectancy at birth, years of schooling for adults aged 25 years, and gross national income per capita.

Fig. 1 reports its negative correlation with pollution. The adverse effects of pollution on people's health or living standards are not immediate, with different effects in the short-term or long-term. Public policies implemented to reduce emissions may require some time to be effective. Therefore, we report CO2 emissions for three different years, 2000, 2009, and 2019. Data refers to 37 European countries plus the United States.[1] The red dotted line indicates the direction of the documented correlation, while in blue, we have the 95% confidence interval. Overall, the negative correspondence between the variables seems relatively straightforward. Naturally, as our brief literature review suggests, causality likely runs in both directions. More educated societies are increasingly acknowledging the need to address the climate emergency. On the other hand, a better environment positively impacts human capital accumulation. This is equivalent to saying that high pollution negatively impacts the quality of human resources. The model we develop in the next Section explores this correspondence when assessing the robustness of the so-called natalist bias of pollution control suggested by de la Croix and Gosseries (2012).

3 The model

The urgency to address the climate emergency has highlighted the need to lower polluting emissions. If there is no technical change, two immediate ways can be undertaken to reach this goal: reducing production per capita or reducing population size. Population growth could be controlled through taxes or tradable quotas of population rights. Depending on its design, the latter comes with the disadvantage of potentially encouraging countries to support an increase in the population size to obtain a higher relative share in the quota allocation in the next period (see Garvey, 2008). In that case, a rise in the number of children risks making the positive impact of the initial measures in vain. Alternatively, one could think of taxes or tradable quotas for emissions, which is an implicit control over production. As anticipated in the Introduction, de la Croix and Gosseries (2012) raised a yellow flag to these measures as well. As the government increases the emissions price, agents turn to leisure and might result in a natalist bias of production control. An increase in population would further deteriorate the environment, entailing the need to impose additional taxation, and gradually impoverishing future generations.

[1] In alphabetic order, countries in the sample are Albania, Austria, Belgium, Bosnia and Herzegovina, Bulgaria, Canada, Switzerland, Cyprus, Czech Republic, Germany, Denmark, Spain, Estonia, Finland, France, United Kingdom, Greece, Croatia, Hungary, Iceland, Italy, Lithuania, Latvia, Moldova, North Macedonia, Montenegro, Netherlands, Norway, Poland, Portugal, Romania, Serbia, Slovak Republic, Slovenia, Sweden, United States and Turkey.

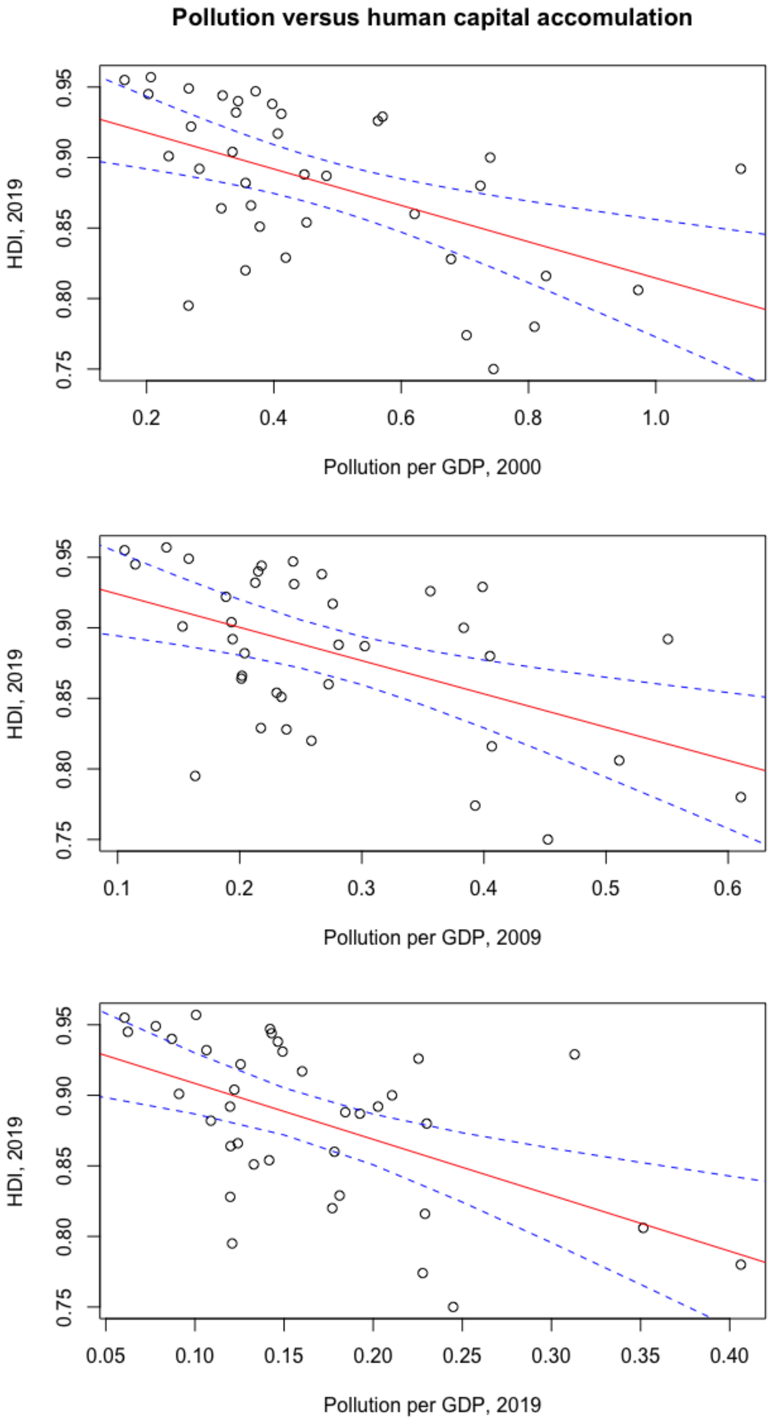


Figure 1: CO2 emissions per unit of GDP (PPP $) in 2000, 2009 and 2019 and human development in 2019. Sources: Human Development Index and World Bank.

The emergence of a natalist bias relies on three main assumptions:

- There is no technical change,
- Human capital does not depend on environmental quality,
- Society does not allocate resources to environmental degradation.

This Section develops an OLG discrete-time model that relaxes the last two hypotheses. We will assess whether such modifications in the original framework can overcome the apparent tension between capping emissions and preserving the population size. We first consider the household maximisation problem. Then we introduce the evolution of pollution versus environmental quality and, finally, the generated aggregate dynamics.

3.1 Households

At each time $t \in [0, \infty)$, the adult generation of households derives their utility from consumption (c), leisure (l), the number (n) and quality (k) of children. As in de la Croix and Gosseries (2012), we consider a logarithmic utility function:

$$\max \quad \ln c_t + \delta \ln l_t + \gamma \ln n_t k_{t+1}$$

where $\{\delta, \gamma\} \in \mathbb{R}^+$ capture the taste for leisure and the altruism factor. We refer to altruism as "impure" because parents do not care about the young's utility beyond that of their children. Notice that differently from John and Pecchenino (1994) and those afterwards, the utility does not directly depend on environmental quality. This modelling strategy is in line with our "impure" altruism hypothesis, given that agents do not care about the environment *per se*. They are willing to invest in conservation or mitigation policies only if those can positively impact consumption, leisure or their children.

The quality of the offspring is formulated with the evolution of human capital accumulation. Future human capital is formed by the following ingredients: education (e), the human capital of adults (k), and environmental degradation (P) which is approximated by CO2 pollutant emissions. This last component is one of our major innovations with respect to the original framework:

$$k_{t+1} = \tau e_t^{\eta} k_t^{v} P_t^{-\psi} \tag{1}$$

where $\tau > 0$ indicates the productivity of education technology, $\eta > 0$ captures the elasticity of human capital with respect to education, $v > 0$ states the strength of an externality from parents' human capital to children's human capital – the usual parental influence on child outcome – and $\psi > 0$ measures the negative externality of environmental degradation on human capital accumulation. Despite environmental quality does not explicitly appears in the utility function of the old, it directly influences the quality of future generations. All the coefficients must be economically meaningful, so we constrain the parameter space such that $\tau \in \mathbb{R}^+$ and $\eta + v + \psi < 1$.[2]

[2]These two assumptions are standard in economic literature. The first is necessary to obtain non-negative values of human capital. The second guarantees constant returns to scale. Allowing for $\eta + v + \psi \geq 1$ would imply an endogenous growth model. We avoid this route to be closer to de la Croix and Gosseries. For empirical estimations of those elasticities, the reader is referred, for example, to Glomm and Ravikumar (1992) and Krueger and Lindahl (2001).

3.2 Production and the environment

At any time t agents have to face their budget constraint given by their available income (y). They split their resources between consumption, education spending ($n \times e$) and the cost of preserving the environment ($p \times a \times y$). Thus, the budget constraint is given by:

$$c_t + n_t e_t + pay_t \leq y_t \tag{2}$$

where $p > 0$ stands for the price or tax rate of polluting and $a > 0$ is a technical coefficient capturing the level of pollution per unit of output produced. In this way, pay_t represents resources raised by the government by taxing emissions. Therefore, a crucial underlying assumption in Eq. (2) is that all money raised from polluters is reverted to environmental conservation activities. This is a relevant difference with respect to de la Croix and Gosseries, who assumed pay_t was just redistributed as a lump-sum transfer to households so that it did not appear in the budget constraint. It is possible to show that a Pigovian tax on emissions transferred back to consumers and a system of tradable pollution rights are equivalent. However, given that we do not have such lump-sum transfers, the compatibility between the two vanishes. Considering the initial statement linking the natalist bias to taxes, we will focus on that instrument and abstract from emission quotas.

Producing offspring (n) requires a time effort (u), represented as:

$$u_t = \phi N_t^{\alpha} n_t$$

where, for simplicity, $\phi > 0$ is a scaling constant, N is the total population, and $\alpha \in (0.1)$ corresponds to the weight in the space of generating children. As the population increases, the time necessary to raise children must be increased, but it happens at a decreasing rate.

Households face a total time endowment that is normalised to 1. They allocate their time to hours spent working (h), producing children, and on pure leisure (l). In formal terms:

$$h_t + u_t + l_t \leq 1 \tag{3}$$

As long as the time constraint is binding, an increase in working hours necessary implies a reduction in leisure and/or u. Analogously, less time dedicated to production results in the population shifting towards the two alternatives.

The production level is determined by the productivity of workers, and the time they dedicate to work. Furthermore, the amount of production for every working hour is given by the number of hours spent working (h) multiplied by the quality of workers, i.e. their human capital (k), that is:

$$y_t = h_t k_t \tag{4}$$

Thus, output can be increased either "intensively", through higher human capital, or "extensively", by working longer hours. In the extreme case where agents are fully dedicated to productive activities, $y = k$. Alternatively, if households do not invest in education or do not work, production will be zero.

Substituting the saturated constraints in Eqs. (2-4) into the objective function, we obtain the following agents' maximation problem:

$$\begin{aligned} \max_{l_t,\, n_t,\, e_t} \quad & \ln\left[(1-ap)\left(1-\phi N_t^{\alpha} n_t - l_t\right) k_t - n_t e_t\right] \\ & + \delta \ln l_t + \gamma \left(\ln n_t + \eta \ln e_t + \tau \ln E_t + v \ln k_t\right) \end{aligned}$$

We can easily derive the first-order conditions (FOCs) as:

$$\frac{-\left(1-ap\right)k_t}{\left(1-\phi N_t^{\alpha}n_t-l_t\right)k_t-n_te_t}+\frac{\delta}{l_t}=0$$
$$\frac{-\left(1-ap\right)\phi N_t^{\alpha}k_t-e_t}{\left(1-\phi N_t^{\alpha}n_t-l_t\right)k_t-n_te_t}+\frac{\gamma}{n_t}=0$$
$$\frac{-n_t}{\left(1-\phi N_t^{\alpha}n_t-l_t\right)k_t-n_te_t}+\frac{\gamma\eta}{e_t}=0$$

The maximation problem is convex, so the FOCs are necessary and sufficient for a maximum. They can then be solved, together with the constraints (2-4) with respect to l_t, c_t, e_t and n_t to obtain the following closed-form solutions:

$$l_t=\left(\frac{\delta}{1+\delta+\gamma}\right) \tag{5}$$

$$c_t=\frac{\left(1-ap\right)k_t}{1+\delta+\gamma} \tag{6}$$

$$e_t=\left(1-ap\right)\left(\frac{\eta}{1-\eta}\right)\phi N_t^{\alpha}k_t \tag{7}$$

$$n_t=\left(\frac{1-\eta}{1+\delta+\gamma}\right)\frac{\gamma}{\phi N_t^{\alpha}} \tag{8}$$

As in the original model, consumption and education are negatively related to p in the optimal choice. A higher tax pressure reduces resources available for those ends. However, notice that since we do not have pollution quotas, there is no initial endowment of emission rights (q). Hence, Eqs. (5) and (8) cease to depend on p and the natalist bias "disappears". This result is important because links the allocation of agents' time to a specific policy instrument that does not necessary have to be used. In the absence of emission rights, leisure decisions strictly depend on preference-related parameters.

Finally, population is supposed to evolve at each period t accordingly to the number of children conceived n_t:

$$N_{t+1}=n_tN_t \tag{9}$$

The model so far presented is fundamentally the same as the simplest case studied by de la Croix and Gosseries. The main difference is the introduction of an externality from pollution to human capital accumulation. Still, the accumulated impact of pollutant emissions on environmental quality remains obscure. The tax rate p on CO_2 discharges appears as implicitly penalising productive activities, but we still have to formalise how the environment side of the model changes over time.

Following John and Pecchenino (1994), we will suppose environmental degradation – proxied by emissions – depends on three main components (see also the discussion in McNicoll, 1984). First, nature has an intrinsic "recovery rate" capable of slowly absorbing emissions as part of the natural carbon cycle, $b \in [0,1]$, i.e. the so-called carbon sinks. A carbon sink absorbs more carbon from the atmosphere than it releases. Vegetation, the ocean and soil are typical examples. Second, consumption stands as a carbon source that increases emissions. Third, the resources raised by taxing emissions are supposed to be used to reduce pollution. As a result, suppose the following behavioural rule:

$$P_{t+1} = (1-b)\,P_t + \beta c_t - pay_t \tag{10}$$

where $\beta \in \mathbb{R}^+$ is the marginal effect from consumption to emissions. Unlike John and Pecchnino, we do not model environmental quality directly but notice that the latter and P are inversely related. We recall that pay_t is the price of emissions multiplied by the emissions-output ratio and the production level, i.e. a sort of Pigouvian tax on emissions mitigation. Furthermore, the production technology is fixed. Notice that consumption from the "old" generation negatively affects the environment, while the "young" may intervene to recover it.

3.3 Dynamic system

Substituting Eqs. (7) and (10) into Eq. (1), we obtain the relationship driving human capital accumulation. On the other hand, substituting Eq. (6) into Eq. (10) we have the evolution of environmental degradation depending upon the quality of children. Our 2-dimensional map is given by:

$$\begin{aligned} k_{t+1} &= \tau\left[(1-pa)\left(\frac{\eta}{1-\eta}\right)\phi N_t^{\alpha}\right]^{\eta} k_t^{\eta+v} P_t^{-\psi} \\ P_{t+1} &= (1-b)\,P_t + \left[\frac{\beta\,(1-pa) - pa\,(1+\eta\gamma)}{1+\delta+\gamma}\right] k_t \end{aligned} \tag{11}$$

Moreover, substituting Eq. (8) into (9) we have the law governing population dynamics:

$$N_{t+1} = \left(\frac{1-\eta}{1+\delta+\gamma}\right)\frac{\gamma}{\phi} N_t^{1-\alpha}$$

Notice that N_{t+1} does not depend on human capital or emissions. As indicated before, this result follows from our choice of abstracting pollution quotas and fully concentrating on the Pigovian tax component, which is the central piece linked to the natalist bias of pollution control. For this reason, the 3D full system is recursive, and we may refer to it as a $2D+1D$ map. Given that α, η lie between 0 and 1, and that N is very large, we have that:

$$\frac{\partial N_{t+1}}{\partial N_t} = (1-\alpha)\left(\frac{1-\eta}{1+\delta+\gamma}\right)\frac{\gamma}{\phi} N_t^{-\alpha} < 1$$

which means population dynamics are globally stable. Excluding the case in which the population is zero, it will converge to:

$$N^E = \left[\left(\frac{1-\eta}{1+\delta+\gamma}\right)\frac{\gamma}{\phi}\right]^{\frac{1}{\alpha}}$$

Thus, the remaining analysis will mainly focus on the interaction between pollution and human capital.

In steady state, $k_t = k_{t+1}$ and $P_t = P_{t+1}$. We can state and prove the following Proposition regarding the existence of two equilibrium points.

Proposition 1 *The dynamic system (11) admits a trivial solution* $(k_1^E,\ P_1^E) = (0,0)$ *and a unique non-trivial equilibrium point* $(k_2^E,\ P_2^E)$ *defined and given by:*

$$k_2^E = \tau^{\frac{1}{1-\eta-v+\psi}} \left[\frac{\beta(1-pa) - pa(1+\eta\gamma)}{b(1+\delta+\gamma)}\right]^{\frac{-\psi}{1-\eta-v+\psi}} \left[\frac{(1-pa)\eta\gamma}{1+\delta+\gamma}\right]^{\frac{\eta}{1-\eta-v+\psi}} \tag{12}$$

$$P_2^E = \tau^{\frac{1}{1-\eta-v+\psi}} \left[\frac{\beta(1-pa) - pa(1+\eta\gamma)}{b(1+\delta+\gamma)}\right]^{\frac{1-\eta-v}{1-\eta-v+\psi}} \left[\frac{(1-pa)\eta\gamma}{1+\delta+\gamma}\right]^{\frac{\eta}{1-\eta-v+\psi}} \tag{13}$$

Proof. See Mathematical Appendix. ■

Both the equilibrium rate of human capital and CO2 discharges depend upon the externality and the emissions price. A strong response of k to P as captured by ψ necessarily results in lower output. This is because agents become too sensitive to the quality of the environment and the accumulation of human capital suffers strongly from the related negative externality. The effect on pollution, on the other hand, is dubious. A strong reduction in output and consumption reduces emissions. However, it also means fewer resources dedicated to recovering the environment. Depending on which of the two forces prevails, P might be higher or lower. A similar reasoning applies to p. Higher taxation disincentives education spending. Human capital might improve if the pollution externality is sufficiently strong and taxes are not high enough. We will discuss in more detail these scenarios numerically. For the moment, we can state and prove the following Proposition regarding the local stability of the economically meaningful solution:

Proposition 2 *The equilibrium point* $(k_2^E,\ P_2^E)$ *is locally asymptotically stable in the region of the parameter space defined as:*

$$1 - \eta - v - b(\psi - \eta - v) > 0 \tag{14}$$

If a change in one of the parameters determines the violation of this condition, a Neimark-Sacker bifurcation occurs.

Proof. See Mathematical Appendix. ■

When (14) is not satisfied, the system violates the third Yury's condition, thus admitting a Neimark-Sacker bifurcation. From an economic perspective, this result paves the way for the emergence of endogenous and persistent cycles related to the strength of the negative externality of environmental degradation on human capital (ϕ) and the capacity of nature to absorb and process emissions in the carbon cycle (b). Persistent cycles are not mere mathematical curiosity. They come with important economic content. Suppose there is a reduction in human capital accumulation than brings production down. It follows that agents consume less, and pollution falls. Fewer pollutants plus tax revenues can be used to improve environmental quality. This will increase output through its positive externality on human capital. Consumption, thus, recovers, leading to higher environmental pressure. More pollutants damage human capital, thus repeating this long-run low-frequency cycle.

Still, to provide a more concrete view of the qualitative properties of the system and the economic intuition behind our narrative of cycles in environmental preservation, we will rely on a set of numerical examples. From a mathematical point of view, they are also necessary to show that the Neimark Sacker bifurcation is supercritical, resulting in a stable periodic orbit.

4 Numerical simulations

The parameter values are chosen to provide results that are economically meaningful. Most of them simply follow from the calibration provided by de la Croix and Gosseries (2012) and references therein. The technical coefficient corresponding to the emissions-output ratio was normalised to unity. On the other hand, the human capital to pollution elasticity is in accordance with the magnitudes reported by Sapci and Shogren (2018). For those parameters that are relevant to the bifurcation, we rely on sensitivity analysis. This is particularly the case of b, which we allow to vary from 0 to 1. Analogously, the response of emissions to consumption (β) is supposed to vary from zero up to 2, meaning that each unit of consumption can generate at most two units of pollution. Our reference values are reported in Table 1. Recall that the two endogenous variables are human capital accumulation and environmental degradation.

We begin by reporting in Fig. 2 the equilibrium values of k and P for different values of ψ. The red line indicates no externalities from pollution to human capital, $\psi = 0$. In this case, there is a clear-cut negative relationship between human capital and taxation. As p increases, the quality of the workforce falls because agents reduce education investments, bringing down production and emissions. Still, the latter will only decrease until the point at which resources devoted to reducing environmental degradation and the natural "recovering rate" are lower than pollution generated by consumption. After that threshold, higher taxes will continue to reduce output but paradoxically increase emissions. However, an increase of ψ to 0.1 or 0.2 significantly changes the picture. It introduces a region where higher taxes bring pollution down and improve human capital conditions, as depicted by the blue and yellow lines. Despite the relatively small value of this elasticity, agents now benefit from environmental quality. Thus, in their optimisation problem, a raise in p does not necessarily reduces e because k improves. As before, given the assumption of constant technology, higher production leads to more emissions. Nonetheless, tax revenues are immediately reverted to improve environmental quality by reducing emissions.

The positive correspondence between human capital and taxing CO2 discharges disappears for very high tax rates. Our numerical experiments indicate that we recover the previous negative relationship when p reaches the 0.4 ceiling. These findings suggest that as long as $\psi > 0$, we have two distinct regimes. The first, with $p < 0.4$, challenges the

Table 1: Choice of parameters in baseline scenario

Parameter	Value	Source
γ	0.470588	de la Croix and Gosseries (2012)
δ	2.27941	de la Croix and Gosseries (2012)
η	0.53125	de la Croix and Gosseries (2012)
v	0.0722147	de la Croix and Gosseries (2012)
α	0.5976	de la Croix and Gosseries (2012)
ϕ	0.0164	de la Croix and Gosseries (2012)
τ	33.7329	de la Croix and Gosseries (2012)
a	1	Normalised to unity
ψ	0.1	Sapci and Shogren (2018)
b	0.1	$\in [0, 1]$
β	0.8	$\in [0, 2]$

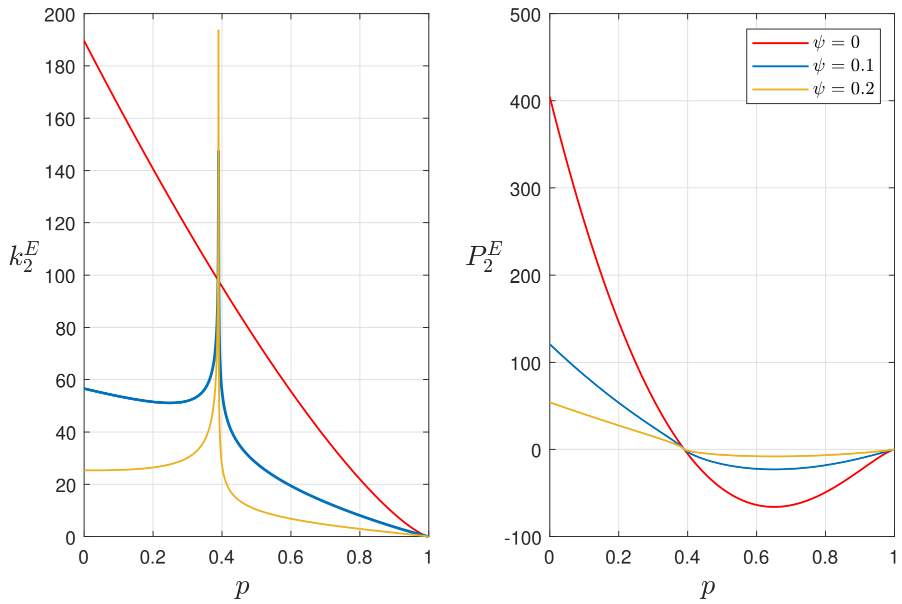


Figure 2: Changes in equilibrium values of k_2^E and P_2^E as we increase the negative externality of environmental degradation on human capital accumulation (ψ).

natalist or leisure bias of pollution control in the sense that raising taxes is compatible with improvements in economic performance. This is the case even under the strong assumption of no technical change. Still, the case remains that too intense fiscal pressure might be counterproductive, somehow saving the planet at the cost of impoverishing future generations.

An alternative visualisation of the discussion above can be found in Fig. 3, which plots the trajectories of human capital, environmental degradation, output per capita (y) and pollution per output (P/y) for different tax rates on pollution emissions for a given technology. It allows us to differentiate between short-run and long-run effects. The red line shows the dynamic of the four series in the absence of taxation using $\psi = 0.1$ as in Table 1. In the short run k, P and y increase given that polluting is cost-less. Still, at a certain point, human capital suffers from the harmful effects of CO2 emissions, thus reducing output that converges to its equilibrium value. Pollution stabilises at very high levels as the economy moves towards a steady-state. Given that pollution externalities are assumed to be relatively small, raising $p = 0.25$ does not change things very much. We can contrast these scenarios with the case in which $p = 0.375$, as in the green line. Even though taxes are higher, the economy continues to grow and converges to an equilibrium with higher output. Moreover, pollution will also be significantly lower, approaching the desirable zero emissions.

Proposition 2 discussed the conditions under which the non-trivial equilibrium solution loses stability through a Neimark Sacker bifurcation. Nonetheless, its proof leaves us in the dark regarding the nature of the emerging cycles that might be stable or unstable. The region in the parameter space for which such an oscillator might emerge is also essential from an economic point of view because it is related to the values of v, η, b and ψ. Fig. 4 presents the 2D bifurcation diagram for human capital accumulation and environmental degradation in the (ψ, b) space. The yellow area corresponds to the stability region, a combination of parameter values for which condition (14) is satisfied, and the solution is locally stable. On the other hand, the grey region is associated with persistent endogenous fluctuations. The system loses its stability after the occurrence of a Neimark-Sacker bifurcation, whose

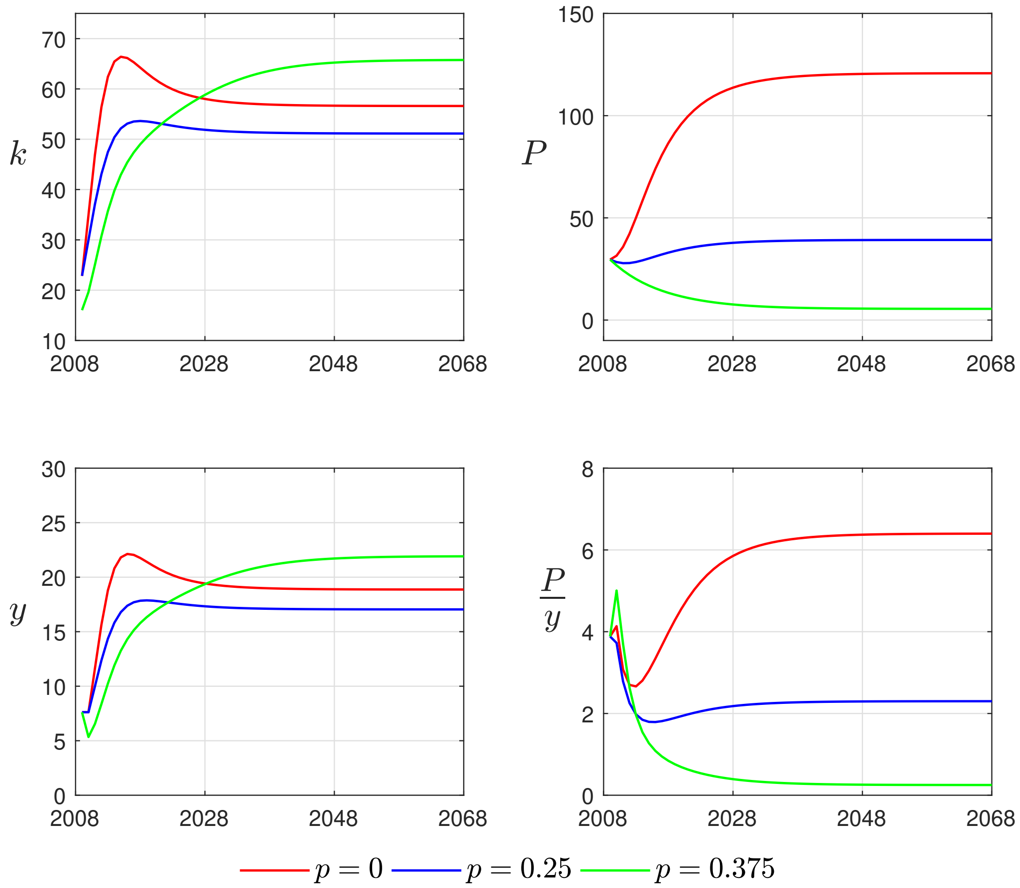


Figure 3: Trajectories of human capital (k), CO2 emissions per person (P), output per capita (y), and pollution per output (P/y) for different tax rates on pollution emissions.

parameter combinations are shown by the red line. Notice that the bifurcation is associated with high values of ψ that are not those used in our baseline parametrisation. The negative externality of environmental degradation on human capital accumulation has to be very strong for closed invariant curves to arise, transforming the non-trivial equilibrium into an unstable focus.

When the equilibrium point loses its stability, stable limit cycles are generated around it, as shown in Fig. 5. Taking b as the critical bifurcation parameter, panel (a) corresponds to the case in which we still have convergence through period-six cycles of decreasing amplitude. Increasing b from 0.6 to 0.7 leads to the emergence of the oscillator. Even though the values of the pollution externality required for the cycle to appear are excessively high, they still come with a non-neglectable economic intuition. Our model is very stylised and the forces behind the dynamics we obtain have qualitative value. They describe the possibility of a persistent and dynamic relationship between the relevant variables in the model. We believe the motions they describe are relevant from a policy perspective as bring insights into the sequence of events beyond equilibrium analysis.

First, notice that as we increase p, the cycle's centre moves towards the right, implying lower pollution and higher output. Second, their rationale sheds some light on the interaction between economic and environmental variables in the form of low-frequency waves. Running the risk of being repetitive, let us revisit the mechanisms involved. Suppose human capital

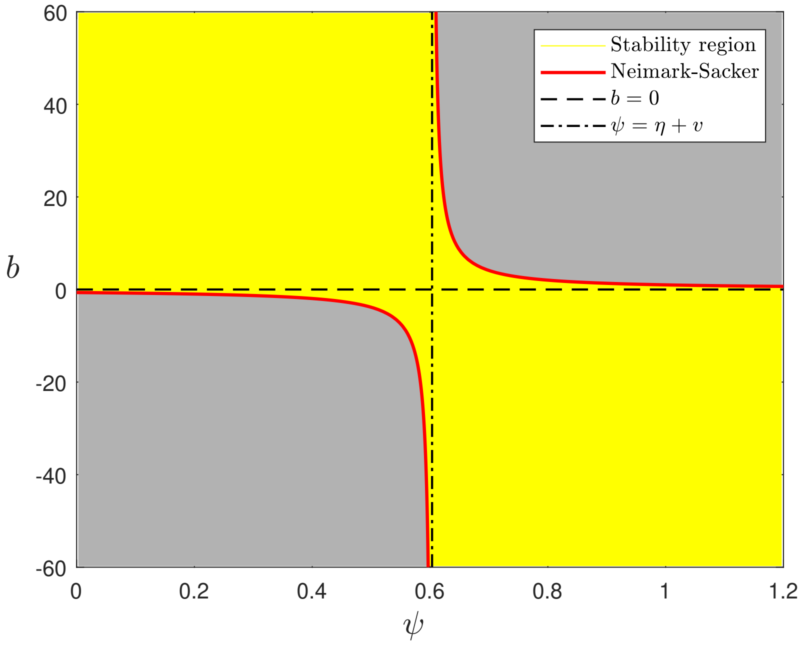


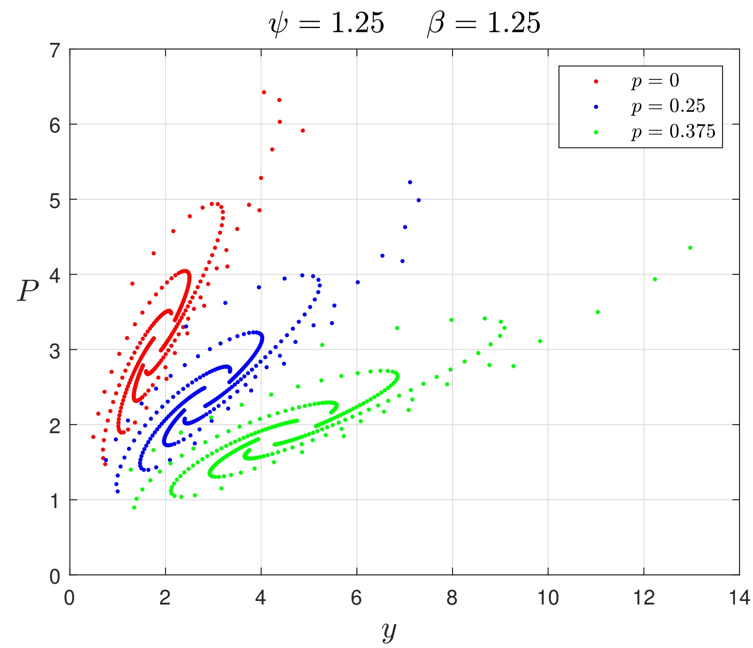
Figure 4: 2D bifurcation diagram showing the emergence of a Neimark-Sacker bifurcation in the (ψ, b) space.

accumulation is falling such that there is a deterioration in the productive capabilities of firms, and they are constrained to reduce production. As output falls, households also have to consume less, reducing pollutant emissions. Lower emissions plus conservation resources raised by the CO2 tax improve environmental quality. At this point, the positive externality on human capital reflects a higher quality of the labour force and greater output. As a subproduct of production and consumption, pollution increases with environmental pressure. Human capital will start deteriorating, restarting this long-run low-frequency cycle:

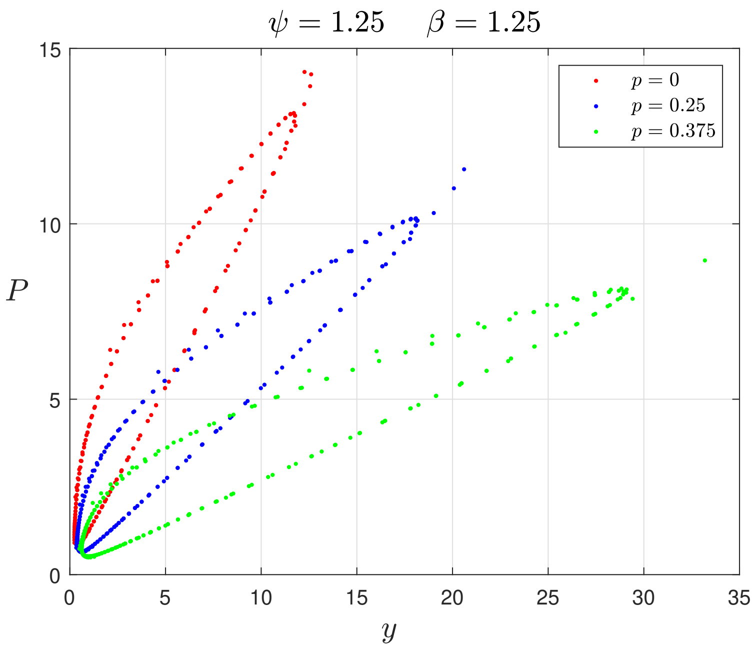
$$\downarrow k \Rightarrow \begin{array}{c} \downarrow y \\ \downarrow c \end{array} \overset{p}{\Rightarrow} \downarrow P \Rightarrow \uparrow k$$

$$\uparrow k \Rightarrow \begin{array}{c} \uparrow y \\ \uparrow c \end{array} \Rightarrow \uparrow P \Rightarrow \downarrow k$$

As a final test, we report in Fig. 6 the Maximum Lyapunov Exponent (MLE) in two different scenarios. A positive MLE is usually taken as an indication that the system depicts sensitivity to initial conditions, i.e. chaos. Still, when there is a Neimark-Sacker bifurcation, it oscillates around zero. We can confirm this is our case. First, we fix $\psi = 1.25$ and vary b from 0 to 1. Even under high values of ψ, a recovery rate $b < 0.6$ is safely related to a stable equilibrium point. In the next step, we allow ψ to change while fixing the parameter capturing the natural rate of carbon absorption. When $b = 0.8$, the bifurcation occurs only for an externality greater than one. After the bifurcation point, an attracting invariant closed curve coexists with the unstable fixed point. The resulting orbits combine points whose motion is periodic or quasi-periodic.



(a)



(b)

Figure 5: Emergence of a periodic orbit in the (y, P) space for different tax rates on pollution emissions. Panel (a) shows convergence to equilibrium when $b = 0.6$. Panel (b) depicts the cycle when $b = 0.7$.

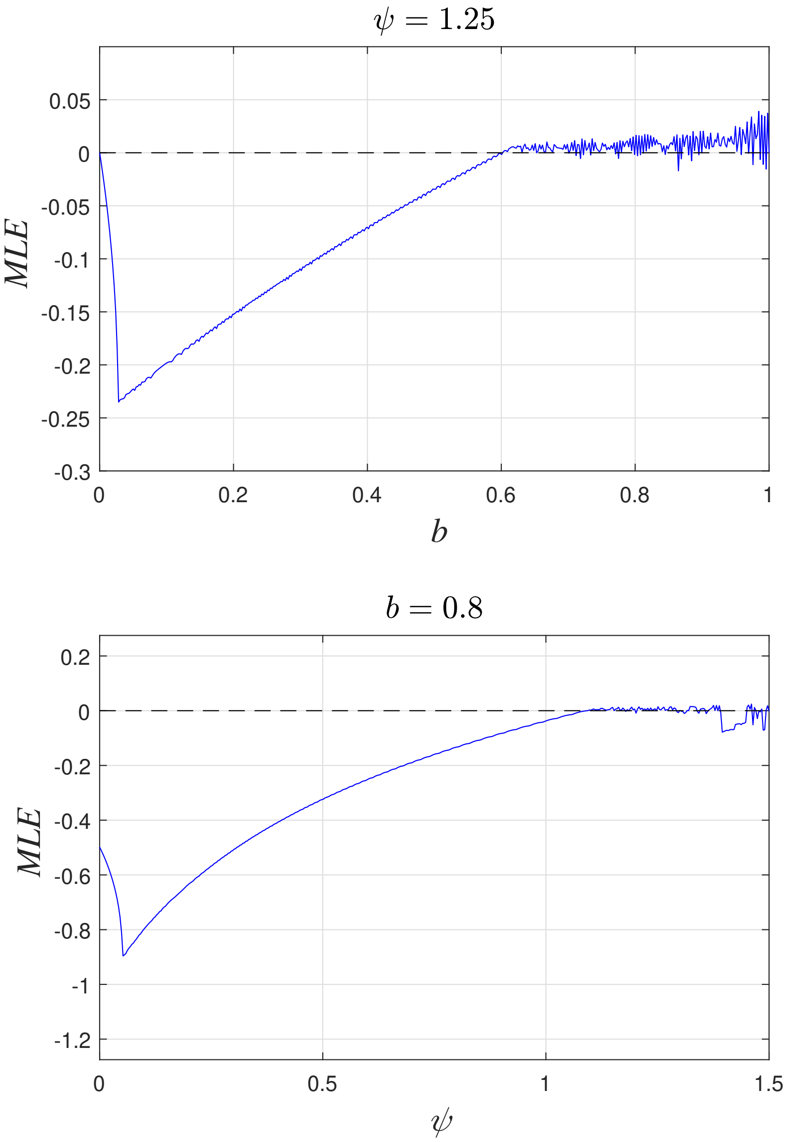


Figure 6: MLE confirming the occurrence of a Neimark-Sacker bifurcation for different combinations of parameters ψ and b. All the remaining parameters follow values reported in Table 1.

5 Final considerations

The urgency of the climate change emergency has renewed a long-standing concern about population growth causing resource scarcity and environmental degradation. As a result, several studies have investigated how population growth influences carbon emissions, including contributions concerning the apparent tension between reducing production or population size to lower pollution. In this paper, we revisit the robustness of the so-called natalist bias of pollution control. The argument was put forward not long ago by de la Croix and Gosseries (2012) in an elegant OLG framework. The main idea is that as the government increases the price of emissions, agents turn to tax-free activities such as procreation. But the increase in population that follows might further deteriorate the environment, entailing the need to raise taxes. In the long-run, however, this would gradually impoverish future generations.

Their model fundamentally depends on three central assumptions: there is no technical change, human capital does not depend on environmental quality, and society does not allocate resources to control pollutant emissions. Significant attention has been devoted to the technical change issue in recent years. The present paper focuses on the last two hypotheses, maintaining technology constant. After revisiting the relevant literature and some empirical insights from the relationship between pollution and human capital, we assess the robustness of the natalist bias. We allow for a negative externality from pollution to human capital and investments in environmental conservation. Using a similar OLG framework, we can show that taxation does not necessarily hurt production but might improve productivity through higher human capital accumulation due to better environmental conditions.

When the emission price is higher, we may only have a contraction of output in the short run. In the long term, adverse effects are more likely to be compensated if taxes are not excessively high. Our numerical experiments suggest a desirable range between 0.3 and 0.4. This result notably weakens the substitution effect between productive and unproductive activities. The stability of the economically relevant equilibrium point depends on pollution externalities and the impact of consumption decisions on emissions. As we increase these two parameters, a Neimark-Sacker bifurcation occurs, making the system compatible with persistent endogenous fluctuations.

Regarding policy implications, introducing pollution taxation may succeed in preserving the environment without shrinking production, provided that taxes are under certain limits. When adopting welfare measures of environmental policies, policymakers have room to increase emissions prices and redistribute them as a lump-sum transfer to improve the quality of the environment. For high emission levels, environmental degradation is such that it impoverishes the quality of children, lowering productivity. To overcome the trade-off between population size and the necessity of capping emissions, we should make emissions more costly, focusing more on the quality of the next generations.

Our model is very stylised and the analysis could be refined in many ways. In general, environmental degradation is not only consumption-based. Here, we focused only in the direction impact from consumtion but other linkages are worth studying. Furthermore, the model's setup is still framed in the dichotomy of number vs quality of children. Recent evidence suggests women's decisions to develop and advance professional careers, together with the distribution of unpaid domestic labour, are increasingly becoming more relevant for demographic trends. Future research is needed to incorporate those issues, analysing the role of human capital and productivity in determining the employment rate and considering a more substantial degree of altruistic behaviour.

A Mathematical Appendix

A.1 Proof of Proposition 1

Recall that in steady-state:

$$\bar{k} = \tau \left[(1 - pa) \left(\frac{\eta}{1 - \eta} \right) \phi \bar{N}^{\alpha} \right]^{\eta} \bar{k}^{\eta + v} \bar{P}^{-\psi}$$
$$\bar{P} = (1 - b) \bar{P} + \left[\frac{\beta (1 - pa) - pa (1 + \eta\gamma)}{1 + \delta + \gamma} \right] \bar{k}$$

and

$$\bar{N} = \left(\frac{1 - \eta}{1 + \delta + \gamma} \right) \frac{\gamma}{\phi} \bar{N}^{1 - \alpha}$$

Excluding the case in which population is zero, it follows that we have:

$$\bar{N} = \left[\left(\frac{1 - \eta}{1 + \delta + \gamma} \right) \frac{\gamma}{\phi} \right]^{\frac{1}{\alpha}}$$

Notice that

$$\frac{\bar{P}}{\bar{k}} = \frac{\beta (1 - pa) - pa (1 + \eta\gamma)}{b (1 + \delta + \gamma)}$$

where

$$\partial \bar{k} / \partial p \gtreqless 0, \ \partial \bar{P} / \partial p \gtreqless 0$$

A.2 Proof of Proposition 2

In order to prove Proposition 2 we need to linearize the dynamic system around the non-trivial steady state (k_2^E, P_2^E) leads to the following Jacobian matrix associated to the dynamic system (11):

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

where the elements of the Jacobian, defined by the partial derivatives of the prices of emissions with respect to human capital accumulation and environmental degradation, are given by:

$$J_{11} = (\eta + v) \tau \left[\frac{(1 - pa) \eta\gamma}{1 + \delta + \gamma} \right]^{\eta} \bar{k}^{\eta + v - 1} \bar{P}^{-\psi}$$
$$= \eta + v$$
$$J_{12} = -\psi\tau \left[\frac{(1 - pa) \eta\gamma}{1 + \delta + \gamma} \right]^{\eta} k^{\eta + v} P^{-\psi - 1}$$
$$= -\psi \bar{k} \bar{P}^{-1}$$

$$J_{21} = \frac{\beta (1 - pa) - pa (1 + \eta\gamma)}{1 + \delta + \gamma}$$
$$= b \bar{P} \bar{k}^{-1}$$
$$J_{22} = 1 - b$$

The characteristic equation can be written as:

$$\lambda^2 - \text{tr}J\lambda + \det J = 0$$

where the coefficients of the characteristic equation are equal to:

$$\text{tr}J = \eta + v + 1 - b$$

$$\det J = (\eta + v)(1 - b) + \psi b$$

The necessary and sufficient conditions for the local stability of a given equilibrium point require that all eigenvalues of the Jacobian matrix, determined as roots of the characteristic equation, are less than unity in modulus:

$$1 + \text{tr}J + \det J > 0 \quad \text{(I)}$$
$$1 - \text{tr}J + \det J > 0 \quad \text{(II)}$$
$$1 - \det J > 0 \quad \text{(III)}$$

Through a direct computation we find that:

$$\begin{aligned} 1 + \text{tr}J + \det J &= 1 + (\eta + v) + (1 - b) + (\eta + v)(1 - b) + \psi b \\ &> 0, \text{ which is always satisfied.} \end{aligned}$$

$$\begin{aligned} 1 - \text{tr}J + \det J &= 1 - (\eta + v) - (1 - b) + (\eta + v)(1 - b) + \psi b \\ &= (1 + \psi - \eta - v)\, b \\ &> 0, \text{ which is always satisfied.} \end{aligned}$$

while

$$\begin{aligned} 1 - \det J &= 1 - (\eta + v)(1 - b) - \psi b \\ &= 1 - \eta - v - b(\psi - \eta - v) \\ &\gtreqless 0 \end{aligned}$$

The first two conditions are always satisfied and we rule out the possibility of fold and flip bifurcations, respectively. The former is a local bifurcation in which two equilibrium points that are created simultaneously - one stable and one unstable - of a dynamical system collide and annihilate each other. The latter occurs when a slight change in a system's parameters causes a new periodic trajectory to emerge from an existing unstable periodic trajectory, the new one having double the period of the original and being stable. A violation of condition $1 - \det J$ is associated with the occurrence of a Neimark-Sacker bifurcation. It corresponds to the birth of a closed curve from an equilibrium point in a discrete dynamic system when two complex conjugated eigenvalues, cross the unit circle. The bifurcation can be stable or unstable. In our case the emerging cycle is stable.

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