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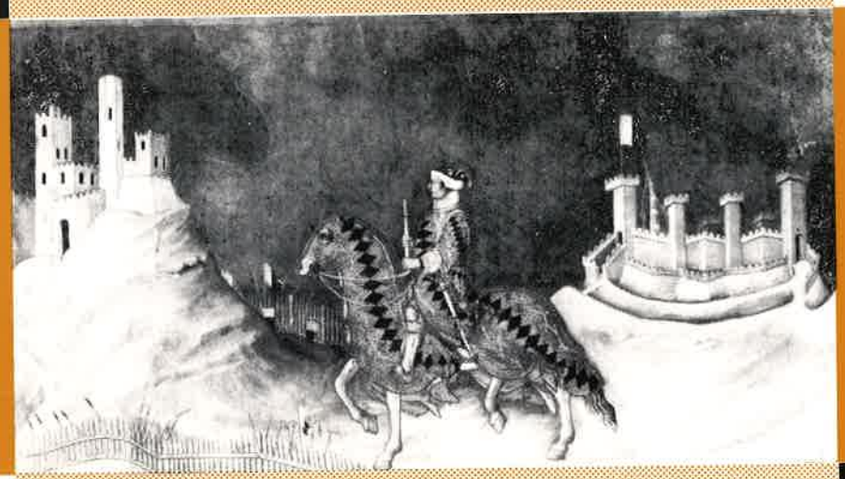
QUADERNI DEL DIPARTIMENTO
DI ECONOMIA POLITICA

Nicola Dimitri

ON THE ROBUSTNESS OF THE RATIONAL EXPECTATIONS HYPOTHESIS



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DI ECONOMIA POLITICA

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Nicola Dimitri

ON THE ROBUSTNESS OF THE RATIONAL EXPECTATIONS HYPOTHESIS

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Nicola Dimitri is a post graduate student in the Department of Economics of the University of Siena.

1. *Introduction*[1]

The rational expectations hypothesis certainly represents one of the most influencial theoretical proposals of the last few years. Although its *importance* has somehow declined recently, it is difficult to forget the impact that such *methodological tool* has had on the profession for about a decade or so after it appeared[2]. In this paper I shall try to evaluate (under well defined point of views) whether or not some of the original assumptions of the theory are still to be considered relevant.

In doing so I start by giving the following two definitions of rational expectations (R.E.) proposed by Pesaran (1987), who distinguishes between a *weak* and a *strong* version of the hypothesis[3]. The first considers agents whose expectations are based on an amount of information determined through the equality of the marginal expected benefit with the marginal cost of a further piece of information. The second assumes the existence of a *true* model for the economic system and postulates that agents know it, together with the probability distribution of the random variables of interest. In the latter case the objective and subjective probability distributions are the same.

The former version seems to be rather realistic since people pay for *costly* information. Moreover, being constrained by their available resources, they can not hold an unlimited amount of *data*. Information is not free implying that knowledge about the

[1] I would like to thank Prof. A. Vercelli for the very useful discussions we had on the topics of this paper. Furthermore I had interesting comments from Professors S. Fazzari, E. Greenberg and H. Minsky.

[2] I consider the early seventies as the period during which the idea started to be systematically introduced.

[3] As we will see the important factor here is the amount of information that agents have.

economy is partial and incomplete.By contrast,the latter definition is evidently more *strict* since it assumes that people know *everything* but the realizations of future variables;in any case,inferences on these can be made through the knowledge of their *true* probability distributions.

Although I consider the amount of information possessed by the agents to be a crucial issue ,it is not certainly the only important aspect of the theory;as a matter of fact it is well known that,among others,two further methodological assumptions play a very delicate role in the R.E.theory.The first one concerns the identification of rational behaviour with *optimizing* behaviour for every individual in the economy.According to this view people are supposed to *choose optimally* in every occasion and for any kind of economic problem .

The second point relates to the so called *market fundamentals* defined by preferences,technology and endowments. For R.E. theorists agents' choices are *driven* exclusively by these three factors ;in particular,no variable uncorrelated with them can explain the evolution of the economic system.As a consequence it is immediate to realize that,in this theoretical framework, phenomena like *self- fulfilling* expectations can come up only with reference to *intrinsic uncertainty*[4] as opposed to *extrinsic uncertainty*[5].

The three points mentioned give the background for the theoretical apparatus of the R.E. hypothesis . Their relevance may certainly be debatable and indeed there have been many discussions

[4] For a definition of *intrinsic uncertainty* see,among others,Cass-Shell (1983).In few words we could say that it is given by random variables which have some effect,directly or indirectly, on market fundamentals.
[5] See,Cass-Shell (1983),Azariadis-Guesnerie (1986) and others.

about it; however, in this paper I shall not deal with their theoretical importance ,focusing instead on the following point.Suppose that for some reason one would want to study the effects of *deviating*,in some specified sense, from those assumptions,to explore the difference between the configuration of the economic system under R.E. and the one under the *alternative* hypotheses.By forcing a little the terminology we may rephrase the question and formulate it by asking:is the R.E. theory *robust*[6] with respect to changes in some of its assumptions ? If it were true that different premises explain individual behaviour ,would the economic system show characteristics analogous to the ones defined by the R.E. approach ?

The reason why I consider this to be an important issue is simple enough;in case we would be certain that the R.E. theory works *reasonably* well under a number of alternative hypotheses, then its explanatory power would be strenghtened .If the *nature* of an economy does not *dramatically* change by varying some of the R.E. assumptions,so that the original theory can still, approximately, interpret the new features, then one should look at that theory as a *robust* theory.

In what follows I intend to examine this issue with reference to the three points that I have identified;in doing so I shall take some results offered by the literature in order to highlight what I consider to be a few interesting aspects.

2. *Learning of Rational Exptectations*

[6] The meaning of *robustness* will be seen later,in connection with the various topics treated.

In this section I shall mostly refer to the *strong* version of the hypothesis (in the sense of Pesaran (1987)).To introduce the problem,I am going to present now a simple example that should make the point of this paragraph clear.

Consider a very basic equation of the sort

$$y_t = a y^e_t + b + u_t \qquad (1)$$

where y_t is a particular economic variable (price for example), y^e_t are agents' exptectations on that variable,defined at time $(t-1)$ for the value of the variable at time t , a and b are constants and u_t is a stochastic process of i.i.d. random variables with finite variance.This simple equation could be seen as a version of the celebrated *cobweb model* ,for instance,where the equilibrium output is determined by the individual expectations on price.Suppose now that individuals do not know the structure of the economy neither as for the functional form of (1) is concerned,nor as for the numerical values of the parameters are concerned.Moreover let us assume that,not knowing *anything*, they follow some *ad hoc* rule when predicting; in particular, I shall consider the arithmetic mean of the past values of y [7],namely

$$y^e_t = \Sigma_i y_i/(t-1) \qquad i=1,2,..,t-1$$

Consider now the R.E. equilibrium of (1);noticing immediately that in this case $y^e_t = b/(1-a)$ for each t,we obtain that the R.E. solution of (1) is $y_t = b/(1-a)+u_t$.

Before going any further let us briefly comment on the specific expectational rule proposed;we may consider it to be a *reasonable* predictor since the R.E. is the mathematical conditional

[7] See Bray (1983).

expectation for the variable of interest[8].In case we *believe* that there is some sort of *natural* tendency, for the individuals,to use expectational criteria *close* to the R.E. predictions then averaging the past values does not seem to be a totally arbitrary hypothesis[9].

At this point we ask the following question:in this most simple model,as time passes,are there any chances for the agents to come to R.E. forecasts ? In other terms:is the economic system structured in such a way to converge to the R.E. solution ? Is it possible for the individuals of this economy to ever form predictions *as if* they would know the *true* model[10] ?

The different formulations of the same question that I have considered have a definite answer in our context under well specified conditions.If the structural form of the model remains unaltered for a *sufficiently* long period,if the numerical values of the parameters that appear in (1) also stay fixed and if, finally, $a<1$ then,asymptotically , the average of past prices will be the R.E. with probability one.

It must be immediately noticed that this simple result,proved by Bray (1983),does away, as other contributions in the literature[11] do,with the speed of convergence of the learning process.

As a matter of fact ,if the *approximation* becomes *good* rather fastly,then it is reasonable to think of agents that are sticking

[8] The conditioning is with respect to the amount of information that people have.An implicit assumption for the predictor proposed is that the past values of y are observable by the agents.

[9] Obviously,the motivation that I have put forward is not necessarily the only one nor the *most* likely.For example,one may simply observe that averaging is a *natural* way to forecast when individuals do not know anything about the model.

[10] Evidently in this most simple example there exists a *true* mathematical model for the economy.

[11] More on this can be found,among others, in Bray-Savin (1986),Fourgeaud-Gourieroux-Pradel (1986).

to that particular expectational rule over the necessary period of convergence.If this would not be true it is harder to imagine that people will keep using a certain predictor when they observe *serious* mistakes for a *sufficiently* long time[12].

The example considered is certainly too simple to catch appropriately the complexity of this problem;however,it certainly gives some useful insights at least as far as the functioning of a basic *self referential* mechanism is concerned[13].It is obviously quite difficult to enclose,in a single theoretical proposal,all the richness and the difficulties that a *learning* problem presents.As a matter of fact there are many assumptions to be made,one of the most important being (at least in my view) the type of forecasting rule adopted by the agents .

Before going any further with the discussion let us go back to the example examined to try to give an answer to the following question:is there any *automatic* tendency for the system to *approach* the R.E. equilibrium? Are disequilibrium phases,with respect to R.E.,going to be transitory,somehow *irrelevant* for the understanding of the nature of that particular economy ? From what we have seen the reply does not seem to be necessarily affirmative;the R.E. can be *stable*[14] but only under the particular condition given by $a<1$[15]. This conclusion is similar to the ones

[12] There are other considerations to be made;for instance,in this case the structure is linear and the stochastic driving process has very nice properties.
[13] The *self-reference* is given by the influence that agents' expectations have on the future values of the variables.
[14] In the sense that there can be asymptotic convergence (with probability one) to it.
[15] We obviously do not have to forget all the other hypotheses concerning the functional form of the equation.It must be noticed that Frydman (1983) shows that the average of past values does not converge to the conditional mathematical expectation,but rather to the unconditional expectation,once we model the exogenous variables as an ARMA model rather than as a constant.

found by other authors[16] in different contexts;as a matter of fact in the literature one comes across various assumptions concerning the forecast rules.These in turn are made dependent on the agents' *nature* , almost exclusively assumed to be either that of Classical Statisticians or of Bayesians.In the first instance people use least squares regressions to predict while in the other there are some prior probability distributions which are revised every time a new piece of information is observed.

In any case ,an important feature that these models often have is the time dependency of the structural coefficients sometimes due to agents misinterpreting the true economy[17] .Leaving aside the simplicity of most of these models ,it still remains open the problem that failure of the stability conditions may lead the system to cycles,explosions and so on,depending upon the particular values of the structural parameters.

One could certainly argue that people tend to learn from their mistakes but it seems really difficult for them to *control* (though crucial for the final result) the structure of the system.

The success of the learning process is not at all guaranteed *a priori* and certainly calls for a fairly good understanding of the economy. Furthermore we have to notice that the stochastic component,once appropriately defined,does not play any role in the stability conditions;this does not mean that it does not have any importance at all but rather that it must be appropriate right at the *beginning* for the convergence to occur.

[16] See,for instance,Fourgeaud-Gourieroux-Pradel (1986),Bray-Savin (1986).Notice that the deterministic version of (1),with agents' expectations given by $y(t-1)$,has a similar stability condition given by $|a|<1$.
[17] This,in turn,causes further technical problems.

These observations are certainly rather immediate but by no means trivial.Moreover,in presence of non linearities things may be more complicated;unlike the linear case there might be dependence on initial conditions which,even when the system is properly structured[18],are going to play a fundamental role for the evolution of the economy.

Another point to be noticed concerns the possible heterogeneity that could characterize agents;in fact,it is *reasonable* to admit individual differences as for the predictions are concerned,but in this case the system becomes more complex.Indeed the parameters of a particular model will probably have to be somehow linked in an appropriate way for the learning to be successful.

Consider for instance the following simple generalization of (1);suppose that a portion α of the population,with $0<\alpha<1$,uses the same forecast function as above and that the remaining $(1-\alpha)$ predicts *naively* with a constant rule β[19].Furthermore let us assume that there is also an *interaction* element which one could model[20],for instance,in the following fashion

$$a\alpha(1-\alpha)(\Sigma^t y_i/t)\beta \qquad (2)$$

This particular specification is quadratic in α so that goes to zero whenever α goes to one or zero,and it is maximum[21] when $\alpha=1/2$,namely when the two segments of the population are numerically the same.

Under these assumptions equation (1) becomes now

$$y_{t+1}=a(\alpha\Sigma^t y_i/t+(1-\alpha)\beta+a(1-\alpha)(\Sigma^t y_i/t)\beta)+b+u_{t+1} \qquad (3)$$

It is immediate to see that the R.E. solution is

[18] From the point of view of the parameters value.See,for instance,Dimitri (1988).
[19] In other words β is a fixed number.
[20] It is obviously not the only way to model it.
[21] As far as the absolute value is concerned.

$$y^*=(1-a\alpha(1+(1-\alpha)\beta)^{-1}(a(1-\alpha)\beta+b) \qquad (4)$$

and that the stability condition turns out to be $(a\alpha(1+(1-\alpha)\beta)<1$ which links together the *naive* expectations β,the structural parameter a[22] and the weight α.For a quick comparison of the two convergence conditions[23] let us establish when

$$\alpha(1+(1-\alpha)\beta)-1 \qquad (5)$$

is greater,equal or less than zero.In particular the two roots of the quadratic[24] equation

$$\alpha(1+(1-\alpha)\beta)-1=0 \qquad (6)$$

are $\alpha_1=1$ and $\alpha_2=1/\beta$.Consequently,if $\beta=1$ there is a repeated root and (5) is never positive implying that in this case the stability condition is less strict than for the original example.When $\beta>1$ (6) is non negative for $1/\beta\leq\alpha\leq1$, and,inside this interval,for the modified economy the convergence is more strict.Analogous considerations will have to be made for the cases when $0\leq\beta<1$, $-1\leq\beta<0$ and $\beta<-1$.

Finally we should not forget a most important point;if the structural parameter a depends on technology and,possibly preferences,it is bound to vary with time.In this case further complications enter into the picture since one needs to specify what sort of law is driving the evolution of these *fundamental* parameters.

To conclude this section I would like to make a final comment on what we have examined.The problem of modelling a learning process seems to be a very difficult task;the well known *feedback* from present to future, and viceversa, induces very complex

[22] Let us suppose that a is positive;similar considerations will hold when a is negative.
[23] The convergence conditions of the original model as compared with the modified one.
[24] The variable of interest is α.Notice that (5) ,when $\beta\geq0$,is never negative so that the sign of a will also be that of the stability condition .

dynamics whenever we are not dealing with the *fixed point* of this mapping[25].There are a number of factors that in problems of this sort play an important part and even in extremely simple models,as the one we examined, we have to be careful before stating something definite about our original question.

However there is at least one aspect that could be identified,namely that there does not seem to be any automatic mechanism which,in every economy,drives the system to the R.E. equilibrium,when agents lack the apropriate information.If the assumption of perfect knowledge is *perturbed* by allowing partial and incomplete information for the individuals,then the economy could evolve in such a way that the R.E. solution is *never* achieved;the R.E. hypothesis may lack robustness when information is less than *perfect*.

The following paragraph deals with another source of *theoretical non robustness* for the R.E. hypothesis;the possible existence of *sunspot equilibria*[26].

3. *Market Fundamentals and Sunspot Equilibria*

In the introduction we mentioned that one of the cornerstones of the R.E. theory is that agents' actions depend only upon *market fundamentals*;in particular whenever individuals are facing a problem involving uncertainty they do not consider signals which are uncorrelated with the *market fundamentals* (m.f.).Before going any further is best to point out that the definition of m.f. is

[25] The R.E. equilibrium of a certain model is nothing but a fixed point of the *self referential* mechanism,linking past with future.
[26] I shall define below the meaning of *sunspots equilibria*.

very much debated in the sense that ,for instance,one could think of the *expectations* of other people as *important* as the three factors above for the understanding of economic phenomena.

Just to illustrate briefly the point let me give two simple examples; weather may influence the future price of a certain commodity if it affects the available supply of that good.In this case weather represents intrinsic uncertainty since the effect on prices will imply an effect on technology for example;agents will care about weather forecast because they understand its importance[27].On the other hand they may also care about *market uncertainty*[28] (given,for instance,by other people's expectations) which does not have any effect on m.f..Obviously the above one is not the only example of *extrinsic uncertainty*,but certainly is one of the most intuitive.We may conceive the existence of economic *phenomena* which occur just because people believe they will occur,whatever the relationship with the m.f.;expectations could be *self-fulfilling* and yet based on *sunspots*[29].

At this point one could ask the following questions: when is it the case that agents do not consider signals uncorrelated with those fundamental parameters? Do economic equilibria based on *sunspot* variables exist ?

As for the first question is concerned is difficult to give a final answer.Economic history[30] indicates that there have been events which could not receive a proper explanation without allowing the possibility for *sunspot* phenomena.On the other hand one might notice that extrinsic uncertainty could justify

[27] In the example exposed one could also imagine the effect of wheather to be direct.
[28] See for instance Shell (1987)
[29] By *sunspot* it is customarily meant a variable which is not correlated with m.f..
[30] See Azariadias-Guesnerie (1986).

basically *everything* once we choose the appropriate signal. In other words, it does not always seem to be an easy task to disclose the true contribution of a *sunspot* variable for the understanding of economic *events*.

Furthermore, it should be important to say why just a certain signal has had relevance, and how agents have been able to come to an agreement on that particular variable. Although the potential indeterminacy[31] of *sunspots* represents a possible drawback, it is not necessarily true that they do not have any role at all.

Leaving aside the first question one must evidently show the importance of extrinsic uncertainty, namely that the allocation of resources may crucially depend upon the realization of extraneous[32] uncertainty. The literature indicates that sunspot equilibria do in fact exist under a variety of circumstances and that consequently we cannot discard *a priori* the importance of *sunspots*. To illustrate the point I shall briefly expose the basic framework of two simple economic models where *extraneous* variables *matter*.

Consider first the following economy[33] which can also be analyzed in terms of an Edgeworth box; suppose there are two agents with utility functions[34], respectively, $U(w)$ and $V(w)$ such that $U'>0$, $V'>0$, $U''<0$, $V''<0$. Furthermore, suppose that there is only one commodity and that agents' endowments are respectively d and e. Assume now that tomorrow the *state of nature*, defined by the realization of a random variable X, will be one of two possible

[31] In the sense of the possible multiplicity.
[32] *Extraneous* in this context is synonymous of extrinsic.
[33] See Cass-Shell (1983), Shell (1987).
[34] These are Von Neumann-Morgestern utility functions.

states with endowments given by the vectors $d(x)=(d(1),d(2))$ $e(x)=(e(1),e(2))$.

If at least one element of the two vectors is different from d and e X is an *intrinsic* variable[35]; if none of them vary we have *extrinsic* uncertainty. Let us characterize now the Pareto efficient (P.e.) allocations for exchanges contingent on the realization of X. Of course agents will trade if the endowments are not Pareto optimal[36]; if this is the case when X is *extrinsic* I say that the *sunspot* variable matters. Let us assume that the possible values of X are 1 and 2 and that agents are characterized by the following subjective probabilities $p(u)=P(X=1)$ and $p(v)=P(X=1)$[37].

It is immediate to see that when X is extrinsic the endowments are P.e. only if

$$p(u)U'(d)/U'(d)=p(v)V'(e)/V'(e) \qquad (7)$$

which implies $p(u)=p(v)$[38]. The result is rather straightforward to interpret; in case of *extraneous* uncertainty, when agents do not share the same probabilities the endowments are not P.e.. In other words, the presence of a sunspot variable can improve the welfare of at least one agent without making the other one worse off. Consequently, agents may decide to correlate a contingent trade plan with respect to this signal to achieve a higher expected utility level.

[35] It is implicit in this statement that the only m.f. influenced by X are the endowments. Notice that the definition does not require a change in every element of the two vectors.
[36] I shall not discuss how agents agree on a particular contingent trade; in fact I am only interested in determining the Pareto optimal allocations and I assume that one way or other they choose a certain plan.
[37] Obviously $p(u)$ refers to the agent with utility function $U(u)$ and $p(v)$ to the other agent. For convenience we also define
$p(u)=p(u)/(1-p(u))$ and analogously for $p(v)$.
[38] Note that exactly the same condition arises when X is intrinsic and endowments vary with respect to d and e but remain constant across the states, for both agents. We assume that both e and d are strictly greater than zero.

The important point here is the difference between the probability distributions of the two agents;this is an issue which,under the strong version,does not seem to be acceptable by R.E. theorists . From this point of view there can not be differences in information among individuals and, consequently, probabilities are *bound* to be equal.On the other hand,by allowing asymmetric information it is natural to accept the possibility of different probability densities.

This is all I want to say as for the first example is concerned; however before moving to the other case I would like to point out how natural could be the *emergence* of extrinsic uncertainty in the economy.As long as two individuals have different probabilistic *beliefs* on an extraneous signal there is room for welfare improvements of at least one of them;sunspots do matter!

Let us describe now the other example which has been proposed by Azariadis-Guesnerie (1986).Consider an *overlapping generations* model with one good and two kinds of agents;*young* and *old*.Suppose that each young individual is characterized by a Von Neumann-Morgenstern utility function of the type $U(c_{t+1},n_t)$,for any $t=0,1,2,..$[39] where c and n are respectively consumption and amount of labour[40].In particular it is assumed that young people can only work and that old people consume exclusively. *Wealth* can be transfered across generations through money which is supplied in a fixed amount.Furthermore,young agents are constrained by the

[39] Some specific assumptions are needed for generation 0,but,to simplify the exposition,I am going to omit them.

[40] The assumption is that one *unit* of labour can be transformed in a unit of *good*.Moreover,agents are initially endowed with a certain amount of leisure which they may decide to employ for production.

following condition $c_{t+1}p_{t+1}=n_tp_t$ where (p_t) $t=0,1,2,..$ is a sequence of prices;in equilibrium agents maximize their utility functions under the above constraint,and markets are cleared.

Consequently,an equilibrium is a sequence of prices (p_t) and outputs (y_t) which satisfy these conditions.Suppose now agents believe that the price ,tomorrow,will be either $p(a)$ or $p(b)$ if either state a or b,of an extrinsic random variable,realize.Assuming that everybody believes so the optimal plans will be to choose c and n so that the expected utilities will be maximized;as usual,in equilibrium markets will have to clear and all the constraints be satisfied.

The stochastic process defining the sunspot variable is a Markov chain with $\pi(a,a),\pi(b,b)$[41] as the elements of the main diagonal of the transition matrix.People have different probabilistic beliefs on tomorrow's price depending only upon the state observed today[42].Without entering into details one can prove that in this model there may exist at least a sunspot equilibrium[43], with $p(a)\neq p(b)$,if $\pi(a,a)+\pi(b,b)<2-1/|e(1)|$ and $e(1)<0$,where $e(1)$ is the elasticity of the saving function $S(p_t/p_{t+1})$ with respect to p_t/p_{t+1}[44],evaluated at $p_t=p_{t+1}$.

It can be noticed at once that if $|e(1)|<-1/2$ we have deterministic cycles of order two,namely systematic fluctuations where $p_t=p_{t+2}$ for every $t=0,1,2...$This observation is rather

[41] Note that each individual shares the same probabilistic belief of everybody else.This implies,as we will see,that extrinsic uncertainty can play a relevant role even under rational expectations.

[42] This is clearly a very simplified assumption which should not taken as the only possible one;as a matter of fact there are examples in the literature (Peck (1988) with non Markovian sunspot variables.

[43] Defined as the quadruple $(p(a),p(b),\pi(a,a),\pi(b,b))$ which solves the constrained maximization program previously defined.

[44] Recall that the saving function represents the amount of labour (which in equilibrium coincides with the output) that young people decide to devote to production.

important since Azariadis-Guesnerie show that a necessary and sufficient condition for sunspot equilibria to exist is the existence of deterministic cycles of order two.Such a result implies that whenever simple economies of this sort may *fluctuate* cyclically,in a deterministic sense,they also allow the possibility for equilibrium random fluctuations.

The result exposed induces another observation;since periodic equilibrium prices are admitted by the *indeterminacy*[45] of the monetary stationary equilibrium,the same will hold for sunspot equilibria.If the steady state is locally *stable*,there may be a *continuum* of stochastic equilibria in its vicinity[46].

The two examples considered are not the only possible contexts where extrinsic uncertainty may play a relevant role in the allocation of resources;as a matter of fact other authors[47] have showed that ,for instance, market *imperfections* may generate equilibria where sunspots matter[48].

To conclude this section let me comment briefly on what has been presented.This area of research seems to indicate that we can not *a priori* exclude the importance of extrinsic uncertainty.The role played by individual expectations can be crucial even when predictions are not about variables correlated with market fundamentals.As long as the economy has an appropriate structure there may be room for *self-fulfilling prophecies*[49] to drive the system,no matter the nature of the concerning variable.

[45] For a definition of *indeterminacy* (of a stationary equilibrium) in overlapping generations models see Woodford (1984).
[46] For comments concerning what kind of preferences are needed to have indeterminacy see,for instance,Grandmont (1985).
[47] See,among others,Cass-Shell (1983).
[48] Namely where the allocation of resources vary in accordance with the realization of the sunspot variable.
[49] This term is borrowed from Azariadis (1981).

So much for extrinsic uncertainty;the next section will deal with the consequences of non optimizing behaviour ,and in particular with the so called *near rationality.*

4. *Optimization and Near Rationality*

It is widely accepted[50] among economists that people's behaviour is *optimal* whenever facing economic problems.Such an assumption seems to be, at least in principle, quite a reasonable one.As a matter of fact, in many cases,this certainly appears to be a plausible hypothesis to put forward for interpreting individual actions;as a consequence sometimes it seems necessary to justify why people,in certain circumstances,may not optimize.

Economic theories of neoclassical inspiration accept unconditionally the assumption of optimizing behaviour; however, some recent contributions[51] coming from psychologists do not seem to support this theoretical view[52].Apparently there are reasons to believe that when individuals choose their actions,they do not always optimize a particular objective function following instead other behavioural patterns .In particular factors like *equity*,*justice*,*fairness* and others of moral nature may play an important role in explaining agents' decisions.

This being the case[53] one could ask the following questions: what happens when we accept the possibility of non optimizing behaviour ? What are the consequences,in the economy,caused by

[50] Of course this is not an hypothesis supported by the whole profession.
[51] Of both empirical and theoretical nature.
[52] See for instance Kahneman-Knetsch-Thaler (1986).
[53] I do not want to discuss here the plausibility of these *alternative* behaviours and ,indeed, I shall confine myself to take for granted their possible existence.

the presence of agents who may not always adjust their positions any time there would be such a necessity ?

We are now going to question the robustness of the R.E. hypothesis from the point of view of the optimization methodology.In this case too the idea is to try to see whether the configuration of the economic system changes *substantially* or not when people do not systematically pursue their *best*[54].

A general and comprehensive solution to this problem seems a difficult task to achieve;indeed one would need to specify the type of departures from optimization which could be different for different economic contexts.Furthermore the particular problems analysed determine conclusions which are specific for that situation.

For a partial answer to this issue an interesting approach is the research program proposed by Akerlof-Yellen (1985a)[55] .These authors have explored the consequences of the so called *near-rational* behaviour, defined by a situation where agents do not necessarily optimize when the values of exogenous parameters change by a small amount.To be more precise I shall discuss briefly the main idea and illustrate a simple example drawn from that paper.

Suppose that an economy is at an equilibrium where agents optimize their objective functions given some particular constraints;for instance a Walrasian equilibrium.Assume now that some of the exogenous parameters governing the system change their values and, moreover,assume that agents do not update their

[54] See McCallum (1987) for comments on this point.
[55] For other contributions on a similar line of research see also Akerlof-Yellen (1985b),Mankiw (1985).

positions because the losses in welfare are irrelevant[56].Is it going to be the case that we will also record irrelevant *systemic* effects ?

The intuition behind the above question is fairly straightforward and concerns the aggregate implications of inertial behaviour due to low individual welfare losses. More explicitly consider the following very simple example taken from Akerlof-Yellen (1985a)[57].

Suppose we are in a context a là Patinkin with agents having utility functions of the type

$$U=(M/p)^{\alpha}G^{1-\alpha}$$

where M is the nominal quantity of money,p a price index for goods and $0<\alpha<1$.If M^* is the initial quantity of money and G^* the initial quantity of the remaining goods we will have an equilibrium price given by

$$p^*=(1-\alpha)M^*/\alpha G^*$$

Assume now that the quantity of nominal money varies by an $\varepsilon>0$ percentage[58] in terms of an external injection of money.If each individual would adjust the initial position to take into account the change of money supply,the price index would change by the same percentage and people would hold the same amount of goods chosen before the variation.In this case money is *neutral* !

Suppose instead that a fraction β of the individuals do not update the original positions but keep the same amount of money and goods that they had before the variation.In this case the

[56] The meaning of *irrelevant* is made precise by Akerlof-Yellen (1985a).In this case the behaviour is defined to be *near rational*.
[57] The model I am going to describe is not stochastic.Although the *natural* setting of R.E. is not deterministic,here I concentrate on a particular methodological point which may be discussed even in a non-stochastic environment.
[58] Notice that ε is assumed to be *infinitesimal*.

optimal quantity of money for the fraction $1-\beta$ of maximizing agents is

$$M^{e}=\alpha(pG^{*}+M^{*}(1+\varepsilon))$$

while ,if we put $p(\varepsilon,\beta)$ as the new prices,we have

$$p(\varepsilon,\beta)-p^{*}=\varepsilon(1-\alpha(1-\beta))>0$$

It is immediate to verify that the loss,in terms of welfare,for those who do not maximize is of second order with respect to ε while the loss for the *average individual*[59] is first order ,namely greater.

The result drawn from this example indicates that,in very basic economies of this sort,one can have aggregate effects greater than individual effects for small deviations from rationality[60].

To conclude with we make a comment analogous to what we said in the first two sections. When some of the individuals do not systematically optimize any time there is a variation in the exogenous parameters,it seems that there is no guarantee to have equilibria that look like[61] R.E. equilibria.

5. Conclusions

In this paper I have briefly examined three areas of research which have a common denominator.In fact they give indications about the *robustness* of some of the most important hypotheses connected to the theory of Rational Expectations.

[59] Defined as an individual whose quantity of money is equal to a weighted (by β) average of the two quantities of money.
[60] Intended as *inertial* behaviour.Notice that the formalization of the results exposed by Akerlof-Yellen (1985a) is obtainable through the envelope theorem .
[61] In some specified sense.

Each section represents a distinct point of view towards an understanding on how well the Rational Expectations theory *reacts* to *deviations* in some of its assumptions. The results analyzed seem to seriously question the *robustness* of the theory .

In fact we have seen how complex could be a learning process and we have also argued that there is no guarantee for its success.On an analogous line it has been pointed out the importance that extrinsic uncertainty may have in determining the allocation of resources. Finally,the systemic implications of inertial behaviour represent a possible further explanation for economic fluctuations and for significant deviations from the rational expectations equilibrium.

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