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Abstract

The global recession triggered by the great financial crisis and the response to the COVID-19 emergency have renewed the interest in connecting business cycle dynamics to financial conditions. This paper proposes a simple macro-dynamic behavioural model of the interaction between the stock market (SM) and the economy's real sector (RS). We innovate by studying the interaction between *two* alternative sources of persistent fluctuations related to *two* different types of heterogeneity. The SM is modelled as a market with two heterogeneous speculators – chartists and fundamentalists. On the other hand, the RS is formalised as a simplified discrete-time version of Goodwin's growth cycle model, distinguishing between labour and capital incomes. The interaction between the RS and the SM is the result of two assumptions: (i) investment decisions depend on both profits and the stock price; (ii) the fundamental value of the latter which is used by speculators in their demand functions is proportional to national output. The overlap between financial and real dynamics results in a novel source of economic fluctuations. We show that the dynamics generated by the resulting 2D map depend crucially on the sensitivity of investment to the stock price and a parameter entering the relation between the fundamental value of the stock price and national output.

Keywords: Persistent fluctuations; Heterogeneous agents; Nonlinear dynamics; Stock market; Real-financial interactions

JEL Codes: C02; D84; E32; G12

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1 Introduction

In the past three decades, advanced countries have experienced an unprecedented expansion of their financial markets, both in terms of new products and scale of activity, significantly impacting the functioning of the "real economy". While a well-developed financial system is generally related to better economic performance, many are sceptical about its value, particularly in light of the recent financial crisis (e.g. Crotty, 2009; Stiglitz, 2018; for a meta-analysis of empirical studies on the finance–growth nexus, see Valickova et al., 2015). The global recession triggered by the collapse of Lehman Brothers and the response to the COVID-19 emergency have renewed interest in connecting business cycles to financial conditions. A growing body of literature is investigating how turbulence in financial markets gets transmitted to the real economy and whether or not real market developments feedback on the financial sector.

Market economies alternate between periods of vibrant economic activity and recessive downturns. The forces behind these fluctuations remain somehow controversial. Two main frameworks are worth distinguishing. A more predominant approach views business cycles as mainly driven by exogenous stochastic shocks. They are propagated through different mechanisms involving a set of frictions that amplify recessions and booms, such as the financial accelerator (Bernanke et al., 1998; Fernández and Gulan, 2015). The economy is fundamentally characterised by a locally stable equilibrium point, with fluctuations disappearing without such shocks. Alternatively, the second approach conceives the economy as inherently unstable. Boom and bust cycles are endogenous to the operation of a decentralised economy. Periods of economic stability endogenously lead to moments of turbulence in systems where decisions by one agent cause others to do the same while, at the same time, their sum has a dampening effect on the initial choice (see Beaudry et al., 2020; Kohler and Jump, 2022; Hommes, 2021). For example, an increase in output encourages firms and households to take on more debt, but the resulting debt burden brings down economic activity.

The present paper aims to contribute to this second strand of literature. We innovate by studying the interaction between *two* alternative sources of persistent fluctuations related to *two* different types of heterogeneity. In the financial sphere, heterogeneity in trading strategies – which change over time and are embedded in social interactions – is responsible for creating a motion similar to Minsky's (1982) financial instability hypothesis. Attention is focused on the interplay of "chartists" and "fundamentalists" (e.g. Day and Huang, 1990; Hommes, 2013; for a comprehensive review of the field, see Dieci and He, 2018). Chartists base their trading decisions on the analysis of past trends: they are optimistic and buy stock in a "bull market", i.e. when prices are rising, whereas they turn pessimistic and sell stocks in a "bear market", that is, when prices are declining. Fundamentalists, in turn, expect stocks to return in time to their fundamental value. Consequently, they buy when prices are undervalued and sell them in the opposite case. The dominance of chartists favours the emergence of bubbles, while fundamentalists are conducive to more tranquil market periods (see also, Dieci et al 2018; Schmitt and Westerhoff, 2021; Gardini et al., 2022).

In the real sphere, we have heterogeneity in the source of income that either comes from capital or labour. Similar to Goodwin's (1967) original growth cycle model, during the expansive phase, as output increases, so does workers' bargaining power. Thus, real wages grow faster than labour productivity, reducing investment profitability. Firms respond by reducing capital accumulation and production. As output begins to fall, unemployment increases, reducing the bargaining power of labour. Wages stop growing, and profitability recovers, allowing firms to reinvest and increase production. At this point, the cycle starts again. A distinguishable feature of our model is that we adopt a discrete-time setup with a nonlinear bargaining relation, such that wages are "marked-up" on average labour productivity (see Pohjola, 1981; Sordi, 1999).

We distinguish between two mechanisms connecting real and financial markets. First, the investment function depends on profits and the stock price. In this respect, we depart from previous efforts to incorporate the financial sector into the growth cycle model by referring to private debt (as, for example, in Keen, 2013; Sordi and Vercelli, 2014; Giraud and Grasselli, 2019). Instead, we focus on a sort of balance-sheet channel. A rise in stock prices increases firms' net worth, leading to higher investment spending because of decreased adverse selection and moral hazard problems. Second, the fundamental stock value, which enters the asset demand functions of both groups of speculators, depends on the current national output. This last mechanism follows Westerhoff (2012) and Naimzada and Pireddu (2014, 2015), having a similar rationale. The fundamental value of a firm corresponds to the present value of the sum of expected profits. As long as profits per unit of output and the interest rate are fairly constant, the fundamental price can be approximated as proportional to national income.

It is shown analytically and through numerical simulations that the dynamics generated by the model crucially depend on the degree of interaction between the stock market and the real side of the economy as represented by two parameters: one measuring the sensitivity of investment to stock price and the other the weight that current output has in the determination of expected output. Depending on their values, we distinguish cases of "weak interaction"– defined as states in which only one of the parameters is different from zero – and "full interaction", when both are different from zero. Our analysis reveals that the stock market's wealth effect on investment destabilises the system. On the contrary, a stronger response of expectations to the current state of the business cycle is surprisingly related to a more stable economy because it allows a smooth synchronisation between real and financial dimensions.

The emerging endogenous dynamics in our model have the following rationale. As in Goodwin (1967), an increase in employment continues to raise workers' bargaining power, thus bringing down profits. Firms' balance sheets deteriorate, and they are forced to reduce investment. The result is economic contraction and layoffs, raising unemployment. Simultaneously, a recession decreases the fundamental stock price, reducing demand. Hence, the price starts to fall, further deteriorating companies' balance sheets. At this point, however, a weak labour market allows for the recovery of profitability and investment. As firms start to accumulate capital again, output recovers. This further enables an improvement of stock market prices and the correspondent balance sheets. Given that financial conditions are improving, they can expand investment plans, with an additional positive impact on employment, allowing the cycle to restart.

The remainder of the paper is organised as follows. In Section 2, we describe the real sector and the financial market in general terms, constituting the two main equation blocks of the model. Section 3 concentrates on the dynamics generated by the real and the financial markets when they are isolated. Section 4 shows the macrodynamic consequences of different cases with "weak" and "full"interaction of the two markets. We distinguish between three sub-cases of full interaction, numerically assessing the necessary conditions for the emergence of endogenous dynamics. Section 5 focuses on a particular case that allows us to find an economically meaningful closed solution for the system. Thus, we can study its local stability properties analytically, confirming those findings numerically in a second step. Some conclusions are finally drawn.

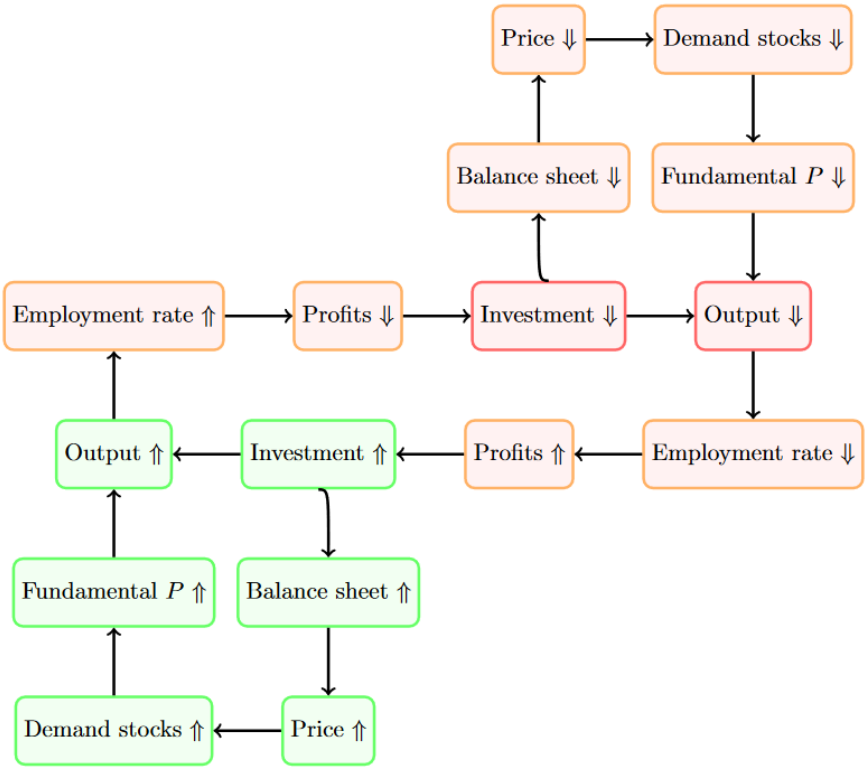

Figure 1: A summarising diagram of the cyclical mechanisms embedded in the model.

2 The model

Economics is witnessing a paradigm transformation that increasingly recognises the limits to agents' informational and computational abilities and their behaviour heterogeneity. Under the premise that modern economies are inherently unstable, our purpose is to study the interaction between *two* sources of heterogeneity that have been documented as capable of generating endogenous bounded persistent fluctuations in *two* different markets. First, we refer to an extensive empirically grounded financial literature identifying chartists and fundamentalist traders (e.g. Franke and Westerhoff, 2012; Chiarella et al., 2014; Chen and Zheng, 2022; Gardini et al., 2022). Second, we differentiate between income from wages and profits as in Goodwin's (1967) predator-prey model. The latter has consolidated itself over the past decades as a system for doing macro-dynamics with extensions contemplating the interplay between fiscal and monetary policy (Chiarella et al., 2005; Asada et al., 2011), inequality and private debt (see Keen, 2013; Giraud and Grasselli, 2019), and the induced-technical change hypothesis (e.g. Velupillai, 2006; Flaschel, 2015), among other developments. Empirical evidence of its primary underlying mechanism can be found in Grasselli and Maheshwari (2018) and Barrales-Ruiz et al. (2022), who also provide an overview of related studies.

The overlap between financial and real oscillations results in a novel source of real-financial instability, as depicted in Fig. 1. It could be interpreted as a financial-augmented Goodwin cycle in contraposition to the pseudo-Goodwin cycles in Stockhammer and Michel (2017) or the Goodwin-Minsky debt models in Keen (2013), among others. To the best of our knowledge, we are the first to provide an in-depth investigation into these mechanisms and their dynamic properties. We divide the presentation of the model into three parts. In this session, we limit ourselves to building the real sector and the stock market blocks of equations. We leave the study of the determination of equilibria and their local stability properties when markets are isolated and interact for the following sections.

2.1 Real sector

In modelling the real sector (RS), we consider a formalisation in discrete time, which has much in common with the growth cycle model by Goodwin (1967), originally formulated in continuous time. The setup of the real side of the model economy can be described as follows. At each time t, a given quantity of aggregate output (Y) is produced employing labour (L) and capital (K). The technology is represented by two ratios, the average labour productivity (a) and the output-capital ratio (μ), both for simplicity assumed to be constant over time and such that:

$$\frac{Y_t}{K_t} = \mu > 0, \quad \forall t \tag{1}$$

$$\frac{Y_t}{L_t} = a > 0, \quad \forall t \tag{2}$$

Under the classical assumption that all profits (Π) are saved and invested, we suppose that net investment (I) is also influenced by the functioning of the stock market (SM) through a balance-sheet channel. The main idea is that higher asset prices improve firms' net worth, reducing problems of adverse selection and moral hazard. Therefore, they can invest more. The studies by Carpenter and Guariglia (2008) and Melander et al. (2017) provide empirical evidence supporting these assumptions for European countries. They show that firms' cash flows positively impact investment and that the effect is enhanced for companies more likely to be financially constrained. We can then write:

$$\begin{aligned} I_t = K_{t+1} - K_t &= \Pi_t + \omega P_t - \rho K_t \\ &= Y_t - W_t L_t + \omega P_t - \rho K_t \end{aligned} \tag{3}$$

where W is the real wage, P corresponds to the SM price, and $0 \leq \omega < 1$ is a parameter measuring the SM cash-flow effect on investment. It can be intuitively interpreted as a sort of wealth effect at the firm level. Finally, $0 \leq \rho < 1$ is the capital stock depreciation rate. For simplicity, let us abstract for the moment from this last element by assuming $\rho = 0$.

Dividing both sides of Eq. (3) by K_t, and taking account of Eqs. (1) and (2), we obtain:

$$\frac{I_t}{K_t} = \mu \left(1 - \frac{W_t}{a}\right) + \omega \frac{P_t}{K_t} \tag{4}$$

where $W_t/a = W_t L_t / Y_t$ is the wage share in total income. Just like investment, capital accumulation has two main components. It responds to profitability, a function of the cost structure, represented by the ratio between wages and labour productivity. Moreover, it also responds to SM conditions, denoted by the ratio P/K. This last element shares similarities with Tobin's q ratio, though we are not explicitly accounting for equity in our model (e.g. Flaschel et al., 2018).

We then assume for simplicity that labour supply is constant and equal to $\bar{N}$. Thus, the employment rate (v) is given by:

$$v_t = \frac{L_t}{\bar{N}} \tag{5}$$

Regarding the real wage, we take it as 'marked up' on average labour productivity and write:[1]

$$W_t = f(v_t) \frac{Y_t}{L_t} = f(v_t)\, a \tag{6}$$

[1] Beside the change from continuous to discrete time, this assumption is what mainly distinguishes our formulation of the real sector from Goodwin (1967), where it is assumed that the *rate of change* of the real wage, rather than its *level*, depends on v. On this point, cfr. Kuh, 1967; Pohjola, 1981; and the discussion in Sordi, 1999.

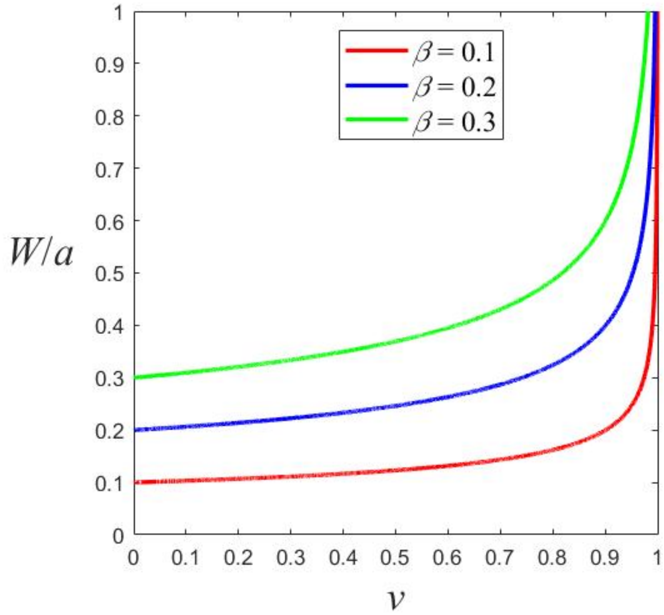

Figure 2: Examples of graphs of the function $f(v)$ with $\alpha = 0$, $\delta = 0.3$ and three different values for β

where $f(\cdot)$ captures workers' bargaining power as a function of the employment rate. To consider the sharp improvement in labour's negotiation position that happens as full employment is approached, we assume that $f'(v) > 0$ and $f''(v) > 0$.

An algebraically convenient specification of function $f(v)$ is:

$$f(v) = \alpha + \frac{\beta}{(1-v)^{\delta}} \tag{7}$$

which differentiates between the two forces behind wage negotiations. One captures structural elements beyond current labour market conditions and is represented by the parameter $0 \leq \alpha \leq 1$. The other refers to the institutions that shape bargaining processes, formalised by the interaction between $\beta > 0$ and $0 < \delta < 1$ for $v \in [0, 1]$. In the extreme case $v = 0$, then $f(v) = \alpha + \beta$. On the other hand, as the economy approaches full participation, $v = 1$, the markup is maximum, i.e.

$$f(v) : [0, 1] \to [\alpha + \beta, +\infty)$$

Fig. 2 is obtained by rearranging Eq. (6) and shows how the wage-productivity ratio, i.e., the labour's share of national income, responds to the employment rate for different combinations of parameters. The first and second derivatives of $f(v)$ defined in Eq. (7) are positive and highlight the role of the parameters δ and β in shaping the bargaining process:

$$f'(v) = \frac{\delta\beta}{(1-v)^{\delta+1}} > 0, \ \forall v$$
$$f''(v) = \frac{\delta\beta(\delta+1)}{(1-v)^{\delta+2}} > 0, \ \forall v$$

Given Eqs. (1), (2) and (5), aggregate output, the capital stock, labour demand and the employment rate all grow at the same rate. Thus, we can use Eqs. (4) and (6) to write:

$$\frac{v_{t+1} - v_t}{v_t} = \mu \left[1 - f(v_t)\right] + \omega \frac{P_t}{K_t} \tag{8}$$

Finally, substituting Eq. (7) into Eq. (8) and rearranging, it follows that:

$$v_{t+1} = \left\{ 1 + \mu \left[1 - \alpha - \frac{\beta}{(1-v)^{\delta}} \right] \right\} v_t + \left(\frac{\omega \mu}{a \bar{N}} \right) P_t \tag{9}$$

Variations in the employment rate depend on the difference between the output growth rate and the sum between labour productivity and workforce growth rates. The last two were assumed to be constant, while capital accumulation fundamentally determines the first. Therefore, the labour market's performance is tied to firms' investment plans. They depend on profitability, which is related to workers' bargaining power and SM performance.

2.2 Stock market

To model the SM, we follow the tradition inaugurated by Day and Huang (1990) and consider it as an abstract market for the exchange of shares of a single asset of the private sector, populated by two types of speculators, "chartists"and "fundamentalists", as well as a market maker (e.g. Schmitt and Westerhoff, 2021; Cafferata et al., 2021; Gardini et al., 2022; for textbook presentations, see Hommes, 2013 and Dieci and He, 2018). Chartists believe in the persistence of "bull & bear" markets deciding their trading behaviour by extrapolating past price trends. For example, when they observe a stock price higher than the value they perceive as fundamental, they optimistically buy stocks. On the contrary, fundamentalists expect the stock price to return towards its fundamental value and behave precisely in the opposite way: they buy stocks when they observe a price below its fundamental value and vice-versa. Once both traders have made their choices, the market maker determines excess demand in the market and adjusts the stock price accordingly. Furthermore, we assume that s/he does so cautiously, i.e. by allowing the price to vary only within given pre-determined bounds.

From the short description above, it follows that the simplest way to distinguish between the two types of speculators is to write their demand for stocks as D^c and D^f, respectively:

$$D_t^c = c\left(P_t - F_t^c\right) \tag{10}$$

$$D_t^f = f\left(F_t^f - P_t\right) \tag{11}$$

where $c, f > 0$ are two positive parameters describing the reactivity of each type of trader to the gap between the observed stock price and the perceived fundamental value, F^c and F^f.

The economy's so-called "fundamentals" are objectively unknown to the single agent. Still, given that at the firm level, the present value of the sum of expected profits determines them, at the macro level, they are related to the economy's performance. Provided that profits per unit of output and the interest rate are fairly stable, the fundamental price can be approximated as proportional to national income (Westerhoff, 2012; Naimzada and Pireddu, 2014; 2015). Thus, suppose that both chartists and fundamentalists perceive the SM fundamental value to be proportional to the next period's expected output:

$$F_t^i = d_i \mathbb{E}_t[Y_{t+1}] \qquad i \in \{c, f\} \tag{12}$$

where $d_i > 0 \ \forall i$ and $\mathbb{E}[\cdot]$ is the expectations operator. To keep the exercise as simple as possible, we take $d_c = d_f = d > 0$.

Output expectations are assumed to be divided into two main parts. First, they are partially anchored to an exogenously given trend. Our assumption of no labour productivity or population growth, i.e., zero natural growth rate, implies a constant $\bar{Y}$. Such a simplification allows us to understand better the interaction mechanisms over business cycle frequencies. We will relax it later on in some of our numerical experiments. Finally, agents are also influenced by the current state of the economy. Therefore, we have:

$$\mathbb{E}_t[Y_{t+1}] = (1-\tau)\,\bar{Y} + \tau Y_t \tag{13}$$

where $0 \leq \tau < 1$ is the respective weight coefficient. To some extent, it can be interpreted as a measure of the influence that the RS has on the SM. When $\tau = 0$, expectations are well anchored to the trend and the stock price fundamental is constant. On the contrary, having $\tau = 1$ implies agents' perceptions of the fundamental price might be more volatile, being determined by current economic performance.

To complete the model, we assume, as in Naimzada and Pireddu (2015), that the market maker determines excess demands by speculators based on Eqs. (10) and (11), and then clears the market by adjusting the stock price according to the following:

$$P_{t+1} = P_t + \gamma\psi\,(D_t)\,,\ \gamma > 0 \tag{14}$$

where γ is the weighting parameter of the response of P to the demand for stocks of both types of traders, which, given (12) and (13), is equal to:

$$D_t = D_t^c + D_t^f = (c-f)\left\{P_t - d\left[(1-\tau)\,\bar{Y} + \tau Y_t\right]\right\}$$

Function $\psi(\cdot)$ embodies a smooth but crucial nonlinearity reflecting that price variations are bounded in each period. Its interaction with γ indicates financial markets' response to the interplay between the two behavioural heuristics in trading strategies. We adopt a parsimonious and cautious representation of the market maker's reaction. The latter consists of limiting the adjustment of price between a negative minimum level ($-a_2$) and a positive maximum value (a_1). The first element represents the ability of the market maker to stop transactions in moments of high turbulence to avoid an investor panic wave. On the other hand, the ceiling captures rigidities that impose an upper boundary limit price on a stock that can be traded in a given period. For example, to prevent firms from manipulating prices, American regulators set price ceilings for open-market share repurchases (Ye et al., 2020). They aim to prevent firms from inflating their share prices by outbidding other traders. Beyond that, financial actors acknowledge the existence of a sort of resistance point, a recent high price that eventually will be followed by a sell-off. Hence, we choose the following functional form:

$$\psi\,(D_t) = \left[\frac{a_1 + a_2}{a_1 \exp\,(-D_t) + a_2} - 1\right] a_2 \tag{15}$$

such that

$$\psi'\,(D) = \frac{a_1 a_2\,(a_1 + a_2)\exp\,(-D_t)}{\left[a_1 \exp\,(-D_t) + a_2\right]^2} > 0$$

and

$$\lim_{D\to-\infty} \psi\,(D_t) = -a_2,\ \lim_{D\to+\infty} \psi\,(D_t) = a_1$$

Fig. 3 provides a graphical representation of the s-shape specification of $\phi(\cdot)$ in Eq. (15). We allow for a realistic asymmetry in the upper and lower boundaries such that $a_1 > a_2$. Variations in the stock price are a positive function of D. Still, traders' demand is zero when prices equal the fundamentals and $\Delta P = 0$.

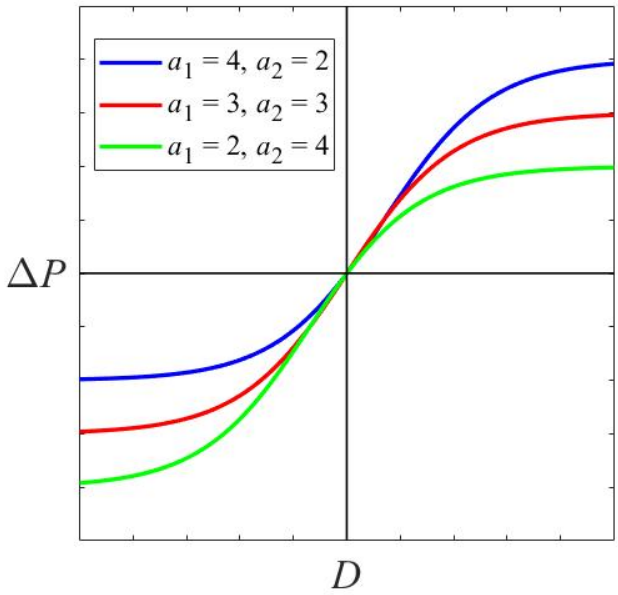

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Figure 3: Cautious adjustment of the stock price as a function of total demand

Substituting Eqs. (10), (11) and (15) into (14), we obtain a dynamic relation for prices which is sensible to the performance of the RS. Recalling from Eqs. (2) and (5) that $Y_t = aL_t = a\bar{N}v_t$, then:

$$P_{t+1} = P_t + \gamma \left[\frac{a_1 + a_2}{a_1 \exp\left(-(c-f)\left\{P_t - d\left[(1-\tau)\bar{Y} + \tau a\bar{N}v_t\right]\right\}\right) + a_2} - 1 \right] a_2 \tag{16}$$

2.3 Dynamic system

Considering jointly Eqs. (9) and (16), the dynamics of the model is generated by the following 2D-map:

$$M: \; = \begin{cases} v_{t+1} = \left\{1 + \mu \left[1 - \alpha - \dfrac{\beta}{(1-v_t)^{\delta}}\right]\right\} v_t + \left(\dfrac{\omega\mu}{a\bar{N}}\right) P_t \\ P_{t+1} = P_t + \gamma \left[\dfrac{a_1 + a_2}{a_1 \exp\left(-(c-f)\left\{P_t - d\left[(1-\tau)\bar{Y} + \tau a\bar{N}v_t\right]\right\}\right) + a_2} - 1 \right] a_2 \end{cases}$$

where the two endogenous variables are the employment rate and the SM price.

Our model formalises two critical connections between the RS and the SM. Unfortunately, given its nonlinearities, the map M doesn't have a closed-form solution. Therefore, we divide our study into three steps. First, we evaluate isolated RS and SM analytically and numerically. Second, we address weak and full interaction cases, relying only on a numerical approach. As we will see, these experiments allow us to acquire further intuition regarding the emerging dynamic properties of the model. Finally, we provide a complete assessment of a special case of full interaction compatible with a closed solution.

3 Isolated markets

A helpful strategy to better understand the interaction of the two sides of the economy is to start by analysing the more straightforward cases in which the RS and the SM operate isolatedly (e.g. Westerhoff, 2012). We do this below, in Subsections 3.1 and 3.2, respectively. We begin with the RS, which is close to Goodwin's (1967) original growth cycle structure. As a second step, we leave the case of an isolated SM and postpone to Section 4, the dynamics of integrated markets.

3.1 The dynamics of an isolated RS

The RS is isolated from the SM when we take $\omega = 0$ in Eq. (3). By muting the wealth effect channel on the firm's cash flows, the SM does not influence investment decisions in fixed capital, and Eq. (9) becomes:

$$v_{t+1} = \{1 + \mu [1 - f(v_t)]\} v_t = h(v_t) \tag{17}$$

Thus, capital accumulation only depends on the profit rate, which is inversely related to workers' bargaining power. We can state and prove results regarding the existence and local stability of its steady states, summarised in the following two propositions.

Proposition 1 *When the condition $0 < \beta < 1 - \alpha$ is satisfied, the map (17), besides the trivial fixed point $v_0^* = 0$, has a unique economically meaningful positive fixed point given by:*

$$v_1^* = 1 - \left(\frac{\beta}{1-\alpha}\right)^{1/\delta}$$

Proof. See Mathematical Appendix A.1. ■

Proposition 2 *The trivial steady state v_0^* is unstable for all admissible values of the parameters whereas the positive steady state v_1^* is locally stable when the parameters of the model are such that:*

$$\beta > (1-\alpha)\left[\frac{\mu\delta(1-\alpha)}{2+\mu\delta(1-\alpha)}\right]^{\delta}$$

Proof. See Mathematical Appendix A.2. ■

From these two Propositions, it follows that the positive steady state v_1^* is both economically meaningful and locally stable when the parameters' values satisfy:

$$(1-\alpha)\left[\frac{\mu\delta(1-\alpha)}{2+\mu\delta(1-\alpha)}\right]^{\delta} < \beta < 1 - \alpha \tag{18}$$

To provide a more concrete view of the stability properties of the nontrivial case, suppose the following set of values:

$$\alpha = 0.2, \qquad \mu = 1, \qquad \delta = 0.3, \qquad \beta = 0.4$$

such that

$$(1-\alpha)\left[\frac{\mu\delta(1-\alpha)}{2+\mu\delta(1-\alpha)}\right]^{\delta} \approx 0.392 < \beta < 1 - \alpha = 0.8$$

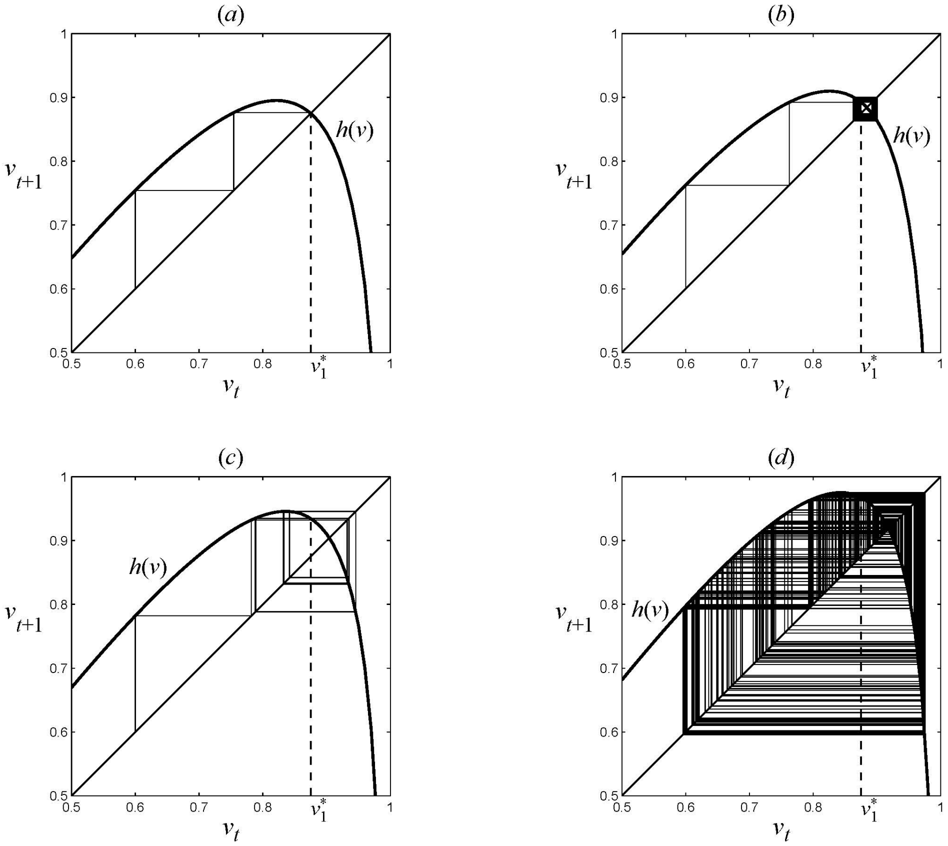

Figure 4: Period-doubling bifurcation cascade for the employment rate: (a) locally stable steady state when $\beta = 0.4$, (b) stable period 2 cycle when $\beta = 0.39$, (c) stable period 4 cycle when $\beta = 0.366$, and (d) chaotic dynamics when $\beta = 0.347$

Fig. 4 reports the occurrence of a Flip or period-doubling bifurcation as $\beta = 0.4$ decreases from 0.4 to 0.347. The full cascade of period-halving bifurcations as β is increased from 0 to 0.8 is shown in Fig. 5. Recall that β refers to the institutions that shape bargaining processes. Thus, we document a trade-off between the level of the equilibrium employment rate and its stability. In the absence of endogenous technical change, stronger labour unions imply that small increases in employment are immediately translated into higher real wages, reducing investment profitability. At this point, firms reduce capital accumulation, lowering v_1^*. However, weak labour bargaining power generates instability as it implies a reduction of a balancing force to the profits-accumulation positive relation.

3.2 The dynamics of an isolated SM

We now consider the case where the SM is isolated, i.e. $\tau = 0$. This means output expectations are fully anchored, and current economic conditions do not affect agents' perceptions of the SM fundamentals. As a result, Eq. (16) becomes:

$$P_{t+1} = P_t + \gamma \left[\frac{a_1 + a_2}{a_1 \exp\left(-\left(c - f\right)\left(P_t - d\bar{Y}\right)\right) + a_2} - 1 \right] a_2 = g\left(P_t\right) \tag{19}$$

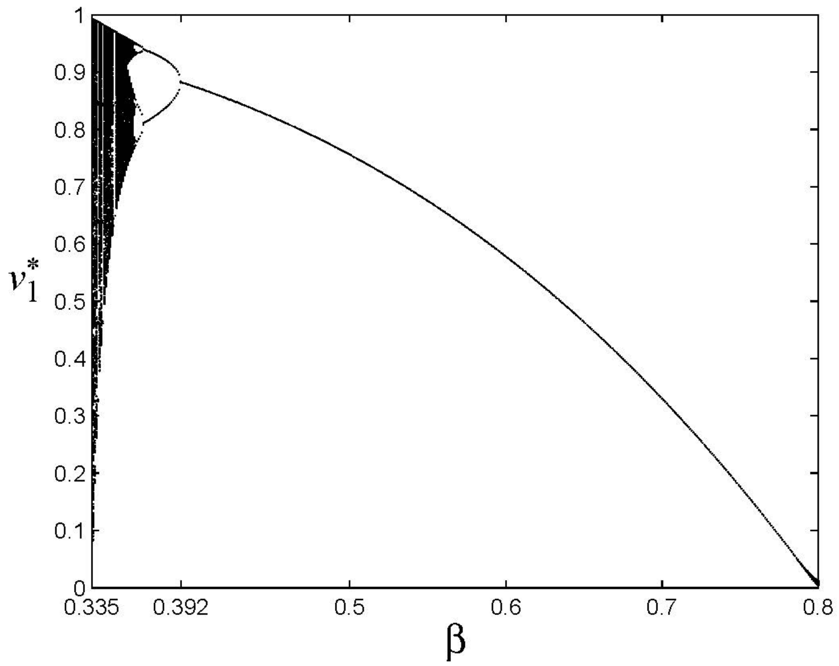

Figure 5: Bifurcation diagram for v_1^* in terms of the parameter β

Thus, we can turn our attention to the determination of the steady states of (19) and the discussion of their local stability. The results are contained in the following Proposition.

Proposition 3 *The map (19) has a unique steady state such that the SM price is equal to its fundamental value, i.e.,*

$$P^* = d\bar{Y}$$

which is locally stable provided that the following condition is satisfied:

$$0 < f - c < \frac{2}{\gamma}\left(\frac{1}{a_1} + \frac{1}{a_2}\right) \tag{20}$$

Proof. See Mathematical Appendix A.3. ■

A numerical example might be helpful again to understand what happens when condition (20) is satisfied or violated. Assume for the sake of simplicity that:

$$c = 1, \qquad d = 0.1, \qquad f = 2, \qquad \bar{Y} = 100, \qquad a_1 = 2, \qquad a_2 = 1, \qquad \gamma = 1$$

where we take $\bar{Y} = 100$ as an index for equilibrium output, fundamentalists' response to price deviations are twice as strong as those of chartists, and the output ceiling is also significantly greater than the floor.

Fig. 6 shows that leaving unchanged all parameters except γ, we observe the period-doubling route to chaos as it is increased from 1 to 5. The bifurcation diagram in Fig. 7 confirms the result. It also shows the emergence of two chaotic attractors for values of $\gamma \approx 4.8$ that merge as we continue increasing this parameter. Intuitively, what is happening is that we are allowing the financial markets to be more responsive to the interaction between chartists and fundamentalists. Although the fundamental price is being kept constant, these two different trading strategies are a source of instability. Increasing γ amplifies such a source of endogenous boom and bust dynamics, thus resulting in instability in the form of a sequence of Flip bifurcations.

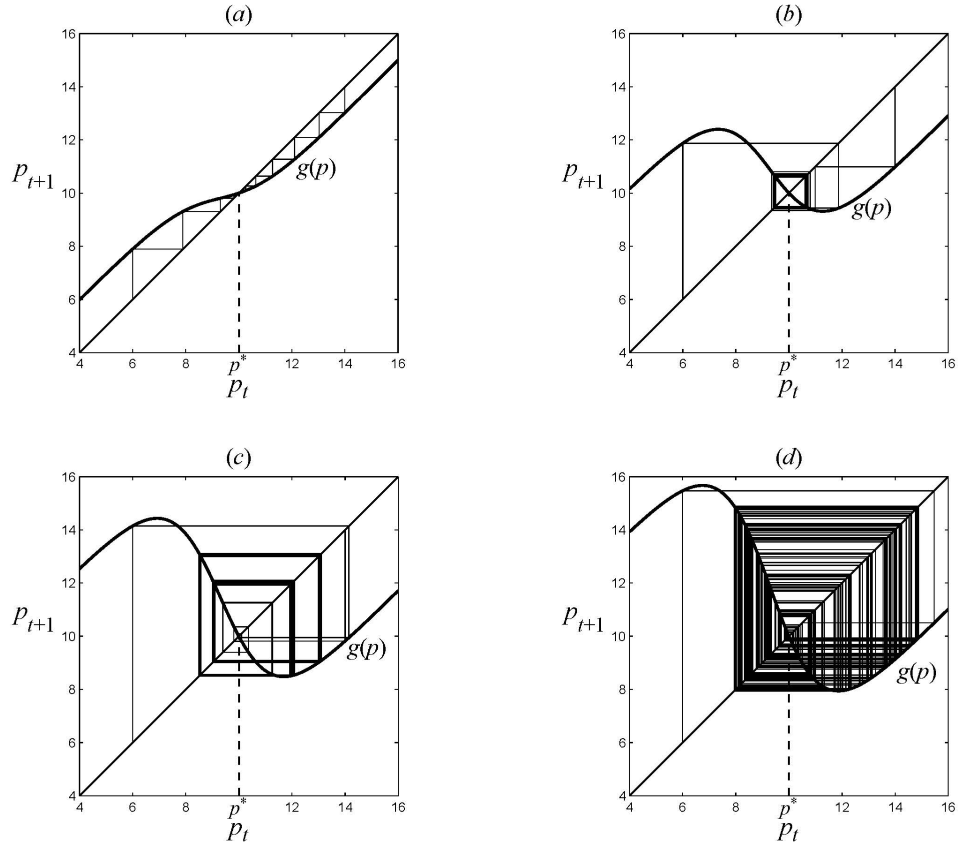

Figure 6: Period-doubling bifurcation cascade for the stock price: examples of (a) locally stable steady state when $\gamma = 1$, (b) stable period 2 cycle when $\gamma = 3.1$, (c) stable period 4 cycle when $\gamma = 4.3$, and (d) chaotic dynamics when $\gamma = 5$

4 Interacting markets

We are finally ready to turn our attention to the investigation of the entire model and to appreciate the consequences of the interaction of the two markets. A critical role in the resulting 2D-map M is played by the parameters ω and τ as they measure the intensity of such interaction. Recall that ω stands for the SM cash flow or wealth effect on capital accumulation. A high value of this parameter indicates the productive structure heavily relies on financial markets when executing investment plans. On the other hand, $1 - \tau$ stands for the SM anchoring on RM fundamentals. A higher τ implies minor fluctuations strongly affect traders. In what follows, we assume they cannot simultaneously be equal to zero.

The steady states of M are found by solving the following system of equations:

$$v^* = \left\{1 + \mu \left[1 - \alpha - \frac{\beta}{(1 - v^*)^{\delta}}\right]\right\} v^* + \left(\frac{\omega\mu}{a\bar{N}}\right) P^* = h_1 (v^*, P^*; \omega)$$

$$\begin{aligned} P^* &= P^* + \gamma \left[\frac{a_1 + a_2}{a_1 \exp\left(-(c - f)\left\{P^* - d\left[(1 - \tau)\bar{Y} + \tau a\bar{N}v^*\right]\right\}\right) + a_2} - 1\right] a_2 \\ &= g_1 (v^*, P^*; \tau) \end{aligned}$$

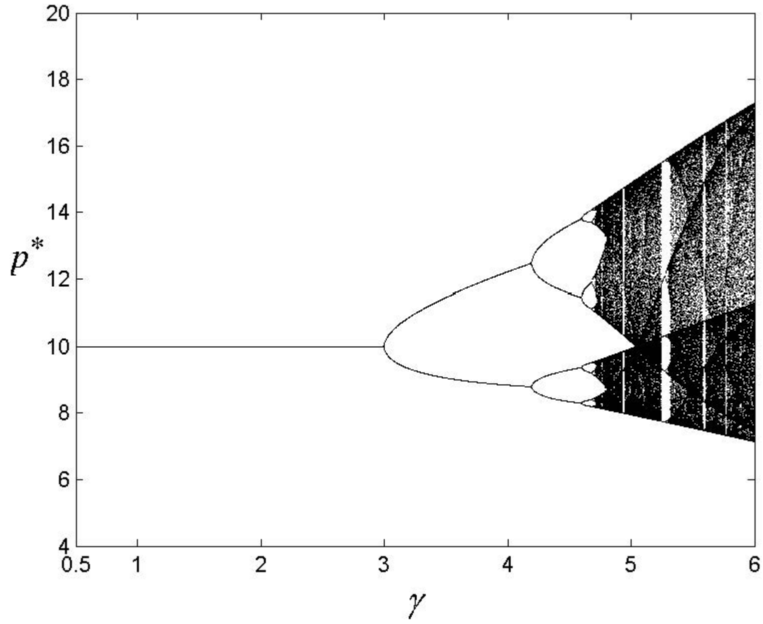

Figure 7: Bifurcation diagram for P^* in terms of the parameter γ

from which we obtain

$$\left[1 - \alpha - \frac{\beta}{(1 - v^*)^{\delta}}\right] v^* + \left(\frac{\omega}{a\bar{N}}\right) P^* = 0 \tag{21}$$

$$P^* = d\left[(1 - \tau)\bar{Y} + \tau a\bar{N}v^*\right] \tag{22}$$

When both ω and τ are different from zero, it is impossible to find explicit solutions of (21) and (22) due to the nonlinearity of the first equation. To overcome this difficulty and in the attempt to obtain results which are as much general as possible, we use the following strategy.

The analysis of an isolated RS and an isolated SM in the previous Section allows us to distinguish *four scenarios*, according to whether one or both of the local stability conditions (18) and (20) are fulfilled or not. First, we can imagine a situation where both conditions are satisfied, and, as a consequence, the steady states of both the isolated RS and SM are locally stable. A second possible scenario is the one in which condition (18) is satisfied whereas (20) is not, so that the isolated RS is locally stable, though the SM is unstable. Third, we have the symmetric setting corresponding to the case where the condition (20) is satisfied, but condition (18) is not. Therefore, the isolated RS is unstable, while SM is locally stable. Finally, the last situation is the one in which both conditions are not satisfied, and consequently, the steady states of both isolated markets are unstable.

In what follows, we take the first scenario as the reference in our analysis. This means that we first choose values of the parameters – for example, $\alpha = 0.2$, $\mu = 1$, $\delta = 1/3$, $\beta = 0.45$, $a = 1$ for the RS and $c = 1$, $d = 0.1$, $f = 2$, $\bar{N} = 1$, $\bar{Y} = 100$, $a_1 = 2$, $a_2 = 1$, $\gamma = 1$ for the SM – such that both conditions (18) and (20) for local stability of their steady states when isolated are satisfied:

$$(1 - \alpha)\left[\frac{\mu\delta(1 - \alpha)}{2 + \mu\delta(1 - \alpha)}\right]^{\delta} \approx 0.392 < \beta = 0.45 < 1 - \alpha = 0.8$$

and

$$0 < f - c = 1 < \left(\frac{2}{\gamma}\right)\left(\frac{1}{a_1} + \frac{1}{a_2}\right) = 3$$

We will then consider and analyse five cases of *different degrees of interaction* between the RS and the SM, choosing values of ω and τ in the intervals $0 \le \omega < 1$ and $0 \le \tau < 1$, but not both equal to zero simultaneously. As we will see, an analysis of this type will allow us to give a characterisation as complete as possible of what happens to the stability of the full dynamic system when starting from an initial situation in which both the RS and the SM are locally stable when isolated, we increase either ω or τ or both simultaneously.

4.1 A first case of "weak interaction": $\omega = 0$, $0 < \tau < 1$

We start by analysing a situation in which the two markets are *weakly coupled*, namely, such that one market influences the other but not vice-versa. To do this, we first study the case in which $\omega = 0$ so that the only interaction between the RS and the FM is through the current aggregate output's impact on the fundamental value of the stock price estimated by speculators. The 2D-map M reduces to:

$$v_{t+1} = \left\{1 + \mu\left[1 - \alpha - \frac{\beta}{(1 - v_t)^{\delta}}\right]\right\} v_t \tag{23}$$

$$\begin{aligned} P_{t+1} &= P_t \\ &+ \gamma\left[\frac{a_1 + a_2}{a_1 \exp\left(-(c - f)\left\{P_t - d\left[(1-\tau)\bar{Y} + \tau a\bar{N}v_t\right]\right\}\right) + a_2} - 1\right] a_2 \end{aligned} \tag{24}$$

The results of a qualitative analysis of the dynamics for this case are described in the following proposition.

Proposition 4 *When the condition $0 < \beta < 1 - \alpha$ is satisfied, the dynamic system (23)-(24), besides the trivial fixed point $(v_0, P_0^*) = (0, 0)$, admits a unique economically meaningful positive fixed point with coordinates:*

$$(v_1^*, P_1^*) = \left(1 - \left(\frac{\beta}{1-\alpha}\right)^{1/\delta}, \quad d(1-\tau)\bar{Y}\right)$$

The trivial steady state (v_0, P_0^*) is a saddle point whereas (v_1, P_1^*) is a locally stable node.

Proof. See Mathematical Appendix A.4. ■

Summarising, with values of the parameters such that the steady states of both the RS and the SM, when isolated, are locally stable, the consideration of a "one-direction " interaction from RS to SM has no consequences on the qualitative dynamics of the whole system: the positive steady state remains locally stable for all admissible values of τ. This is illustrated numerically by the two bifurcation diagrams of Fig. 8, where both v and P converge to their steady-state values for all values of τ. The weak interaction implies that the stock price's steady-state value decreases as τ increases. The parameters values we have used in the simulations are indeed such that $\partial P_1^*/\partial\tau = -d\bar{Y} + a\bar{N}v_1^* < 0$.

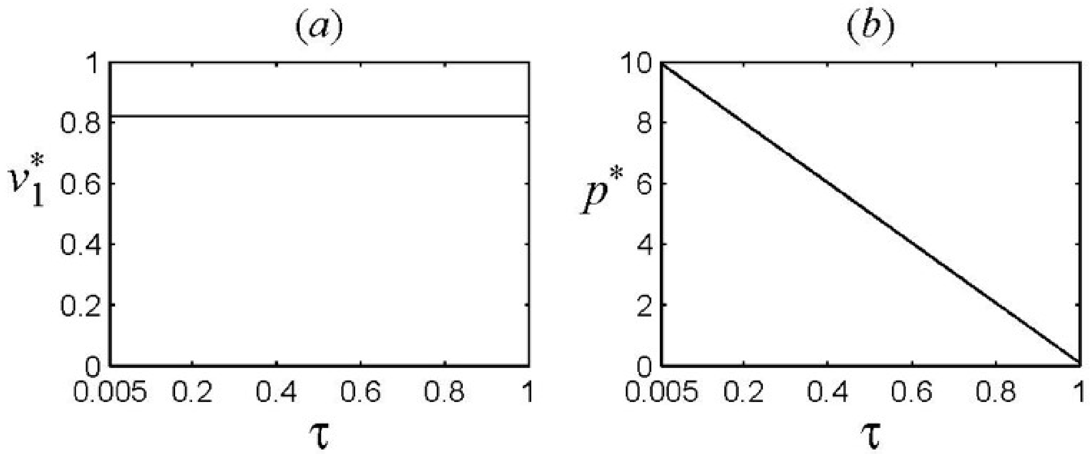

Figure 8: Bifurcation diagrams for v_1^* and P_1^* in the case of "weak interaction" with $\omega = 0$ and $\tau > 0$

4.2 A second case of "weak interaction": $\omega > 0$, $\tau = 0$

The other possible scenario of *weakly coupled* markets is the one in which the SM influences the RS – through the impact that the stock price has on investment decisions – but not vice-versa. Such an environment captures the extreme situation in which the stock price fundamental is a constant known to both types of traders. In this case, the dynamic system M reduces to:

$$v_{t+1} = \left\{1 + \mu \left[1 - \alpha - \frac{\beta}{(1 - v_t)^{\delta}}\right]\right\} v_t + \left(\frac{\omega\mu}{aN}\right) P_t \tag{25}$$

$$P_{t+1} = P_t + \gamma \left[\frac{a_1 + a_2}{a_1 \exp\left(-(c - f)\left(P_t - d\bar{Y}\right)\right) + a_2} - 1\right] a_2 \tag{26}$$

whereas equations (21) and(22) become

$$\left[1 - \alpha - \frac{\beta}{(1 - v^*)^{\delta}}\right] v^* + \left(\frac{\omega}{a\bar{N}}\right) P^* = 0 \tag{27}$$

$$P^* = d\bar{Y} \tag{28}$$

Substituting (28) into (27), we obtain the following nonlinear equation

$$\left[1 - \alpha - \frac{\beta}{(1 - v^*)^{\delta}}\right] v^* + \left(\frac{\omega d}{a\bar{N}}\right) \bar{Y} = 0 \tag{29}$$

which determines the equilibrium employment rate. However, unlike the isolated markets scenarios, we cannot solve explicitly for v^*. Still, going back to the dynamic system (25)-(26) and by resorting to numerical simulation, we can understand what happens to v_1^* as ω is increased. As shown by the bifurcation diagram of Fig. 9 (a), the consequences for the RS are evident. It turns out that, although the locally stable steady state continues to exist for low values of ω as in the case of an isolated RS, the attractor becomes chaotic through a period-doubling cascade as ω increases. In other words, the impact the SM performance has on investment decisions is a destabilising factor for the RS.

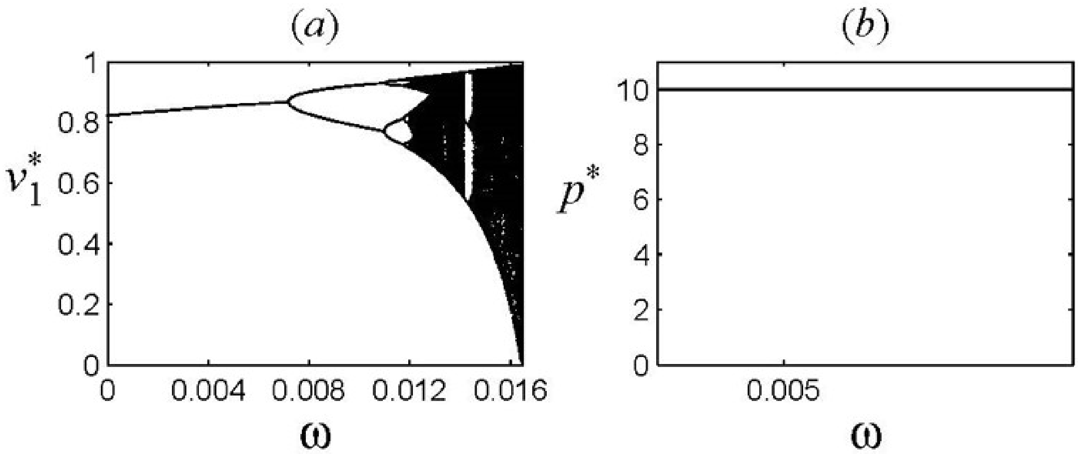

Figure 9: Bifurcation diagrams for v_1^* and P^* in the second case of "weak interaction" ($\omega > 0$, $\tau = 0$)

To sum up, the analysis carried out for the two cases of "weak interaction" leads us to conclude that using parameters values such that the local stability conditions (18) and (20) are satisfied, the consideration of the influence of the RS on the SM, as measured by the parameter $\tau > 0$, has minor consequences for the local stability of the complete system. On the contrary, the influence of the SM on the RS as measured by a high enough $\omega > 0$ destabilises it.

4.3 Some cases of "full interaction" $(\omega, \tau \neq 0)$

To obtain results for the case of a full interaction of the RS and the SM, it is useful to start from the second case of weak interaction we have just analysed. Fig. 9 (a) shows clearly that the interval of admissible values of ω for which we have drawn the bifurcation diagram can be divided into three sub-intervals, each of which contains values of ω which generate the same kind of *qualitative* dynamic behaviour. These sub-intervals are highlighted in Fig. 10, where **I** indicates the sub-interval of values of ω for which v_1^* converges to a steady state, **II**, the sub-interval of values of ω for which v_1^* undergoes a period-doubling cascade and **III**, the sub-interval of values of ω for which v_1^* converges to a chaotic attractor.

Given that all values of ω in a given sub-interval generate the same dynamic behaviour, a complete *qualitative* characterisation of the entire interaction between the two markets can be obtained based on the study of just three cases. For this reason, we now proceed by choosing a value of ω in each of the three sub-intervals and then checking, with the help of bifurcation diagrams, what happens to the resulting dynamics as the other parameter τ is increased from zero to one. The cases we distinguish and study are the following:

- $\omega = 0.005 \in \mathbf{I}$, $\tau \in (0, 1]$: this is a value of ω such that in the case of weak interaction with $\tau = 0$, v_1^* and P_1^* converge to a locally stable steady state. In the present numerical example, from (28), we obtain

$$P_1^* = 0.1 \times 100 = 10$$

whereas, from (27), it follows that v_1^* is determined by solving

$$\left[0.8 - \frac{0.45}{(1 - v_1^*)^{1/3}}\right] v_1^* + 0.05 = 0$$

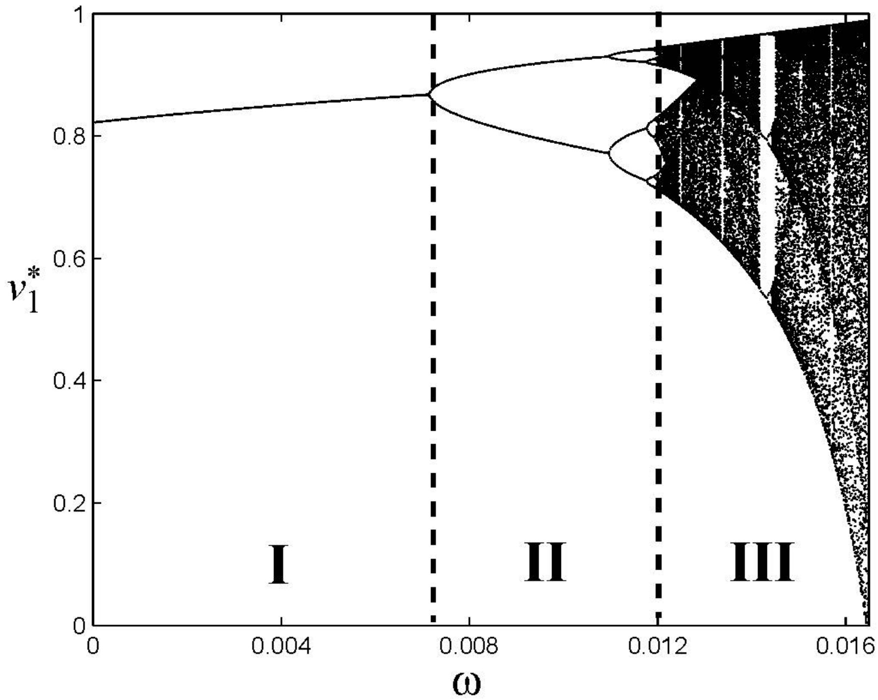

Figure 10: Three sub-intervals of values of ω in the bifurcation diagram for v_1^* of Fig. 7(a)

from which

$$v_1^* \approx 0.856$$

The bifurcation diagrams of Fig. 11 then show that in this case, the only consequence of a full interaction of the RS and the SM is that the steady-state values of both v and P decrease as τ increases.

- $\omega = 0.01 \in$ **II**, $\tau \in [0,1]$: this is a value of ω such that when $\tau = 0$, v_1^* follows a two-period cycle whereas P_1^* is a locally stable steady state. Fig. 12 (a) shows that for v^*, the interaction of the two sectors in this case has a stabilising effect. As τ increases, at first, the amplitude of the two-period cycle decreases and then, for τ high enough, v^* starts to converge to a steady state, the value of which decreases with τ.

- $\omega = 0.013 \in$ **III**, $\tau \in [0,1]$: this is a value of ω such that, when $\tau = 0$, v^* is a chaotic attractor. As illustrated in Fig. 13(a), the interaction of the two markets also in this case stabilises the dynamics. Indeed, what happens is that, through a period-halving cascade, v^* starts to converge to a steady state as τ becomes high enough.

A counter-intuitive yet critical result emerges based on the analysis we developed by referring to these three sub-intervals. Parameter τ plays a stabilising role in the economy. At first, it might seem more likely that its increase results in persistent fluctuations as expectations become less anchored and the current state of the economy changes the perceived stock price fundamentals for both types of traders. Still, our model challenges such reasoning. In the extreme case $\tau = 0$, the fundamental price remains constant, making synchronisation between RS and SM impossible. The mismatch between the two is, in itself, a source of instability. At the other extreme, when $\tau = 1$, integrating the two dimensions leads to smoother dynamics of the economy. Additionally, ω continues to be a destabilising element.

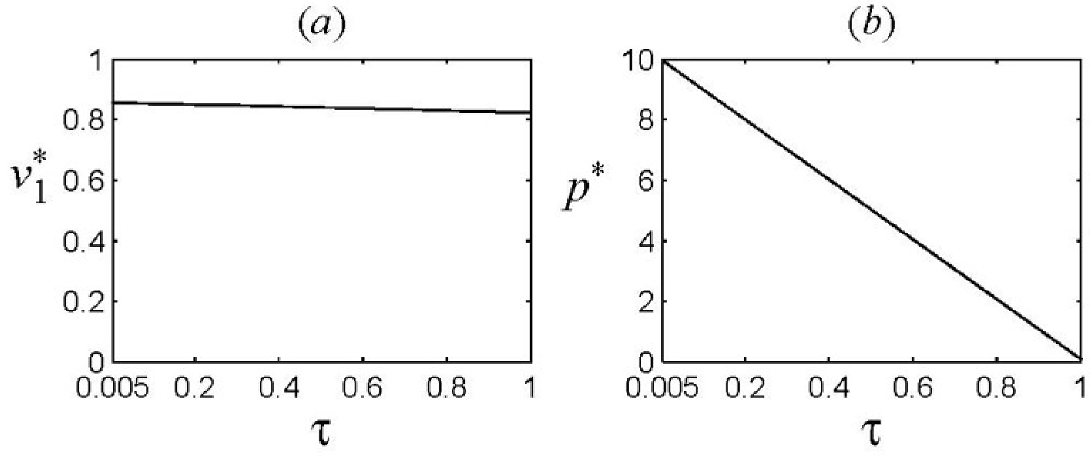

Figure 11: Bifurcation diagrams for v_1^* and P_1^* in terms of the parameter τ with $\omega \in \mathbf{I}$

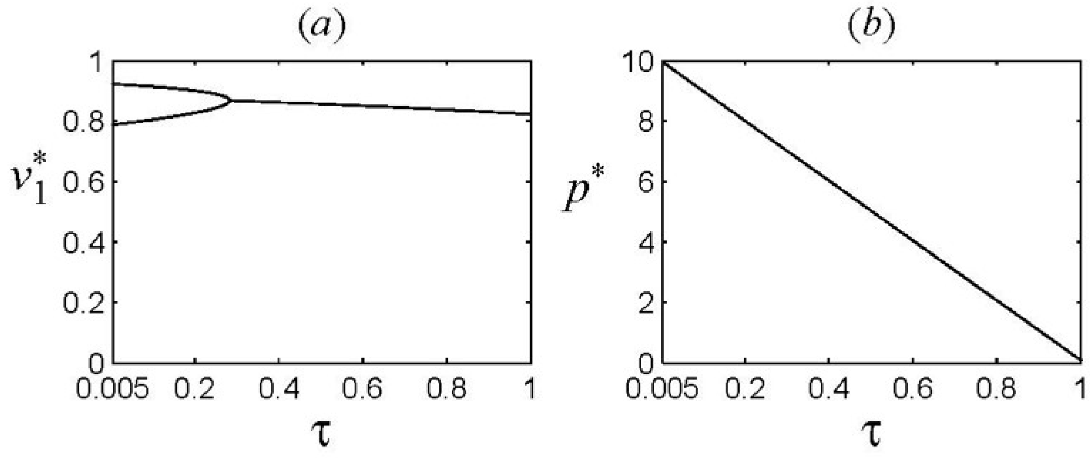

Figure 12: Bifurcation diagrams for v_1^* and P^* in terms of the parameter τ with $\omega \in \mathbf{II}$

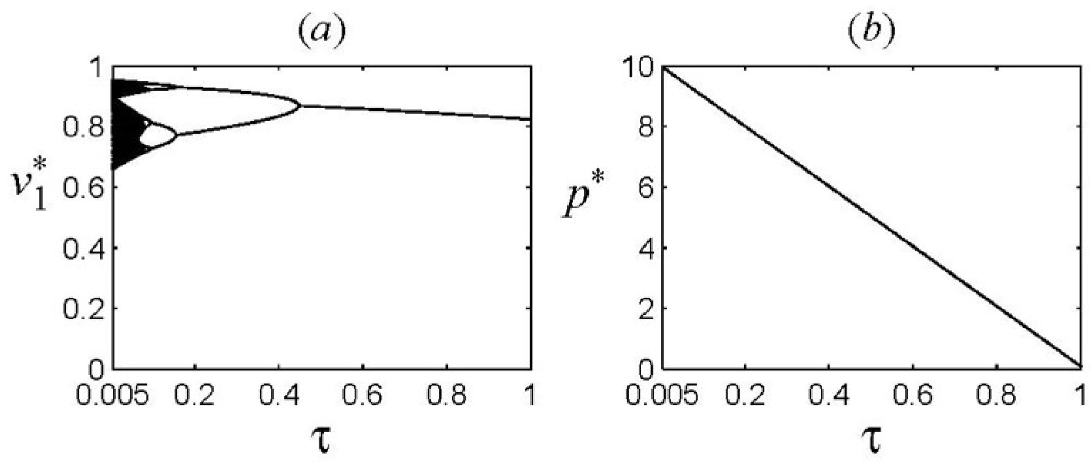

Figure 13: Bifurcation diagrams for v_1^* and P^* in terms of the parameter τ with $\omega \in \mathbf{III}$

5 A special case with a closed-solution

The model developed so far explores the interplay between two sources of endogenous dynamics, adopting a numerical approach to study cases of full interaction between real and financial markets. This qualitative characterisation of the dynamic system was instrumental in distinguishing between certain sub-intervals in the parameter space. Still, the reader might wonder whether it is not possible at all to find a closed solution. The present section relaxes two of our initial assumptions to provide further insights into the properties of the system.

We begin by rewriting Eq. (7) as:

$$f(v) = \alpha + \frac{\beta}{1-v} \tag{30}$$

such that we maintain the shape of function $f(\cdot)$ while setting $\delta = 1$. As before, there is a strong increase in workers' bargaining power when we approach full employment. The first and second derivatives characterise a convex relationship between the wage markup over labour productivity and v.

Though Eq. (30) is sufficient to find an explicit non-trivial steady-state solution, a second modification might be convenient. Recall that output expectations, which feed perceptions of the fundamental stock-market price, stand as one of the bridges connecting the real and financial sides of the economy. Given that our model does not admit a growing output, let us assume $\tau = 1$, implying that:

$$\mathbb{E}_t[Y_{t+1}] = Y_t \tag{31}$$

so price fundamentals reflect the current state of the economy. While the expression above allows us to simplify the algebra significantly, it also highlights the role of this channel in the emergence of endogenous business cycles.

Replacing these two behavioural equations into the model, our new 2D map is given by:

$$M_2 : \ = \begin{cases} v_{t+1} = \left[1 + \mu\left(1 - \alpha - \dfrac{\beta}{1-v_t}\right) - \rho\right] v_t + \left(\dfrac{\omega\mu}{a\bar{N}}\right) P_t \\ P_{t+1} = P_t + \gamma\left[\dfrac{a_1 + a_2}{a_1 \exp\left((f-c)\left(P_t - da\bar{N}v_t\right)\right) + a_2} - 1\right] a_2 \end{cases}$$

As before, the two endogenous variables are the employment rate and the SM price. Given that parameter δ previously smoothed the impact of the labour market in t on its $t+1$ performance, we reintroduce a non-zero capital depreciation rate to obtain a similar effect.

In steady-state $v_t = v_{t+1} = v^*$ and $P_t = P_{t+1} = P^*$. Therefore, the equilibrium conditions are such that:

$$v^* = \left[1 + \mu\left(1 + \alpha - \frac{\beta}{1-v^*}\right) - \rho\right] v^* + \left(\frac{\omega\mu}{aN}\right) P^* = h_2(v^*, P^*)$$

$$P^* = P^* + \gamma\left[\frac{a_1 + a_2}{a_1 \exp\left((f-c)\left(P^* - daNv^*\right)\right) + a_2} - 1\right] a_2 = g_2(v^*, P^*)$$

It is pretty clear that the trivial point $(v_0^*, P_0^*) = (0, 0)$ is a solution of M_2. Moreover, we can also state and prove the following Propositions regarding the existence of a unique nontrivial equilibrium point and its local stability properties.

Proposition 5 *As long as $\beta < 1 + \alpha + \omega d - \rho$, the map M_2 has a unique economically meaningful positive fixed point (v_1^*, P_1^*), defined and given by:*

$$v_1^* = 1 - \frac{\beta}{1 + \alpha + \omega d - \rho}$$
$$P_1^* = \left(1 - \frac{\beta}{1 + \alpha + \omega d - \rho}\right) daN$$

Proof. See Mathematical Appendix A.5. ∎

Proposition 6 *The positive fixed point (v_1^*, P_1^*) is locally stable provided that:*

$$\frac{A}{A + 1 + \omega\mu d} < \frac{\gamma a_1 a_2 (f - c)}{a_1 + a_2} < \frac{4 + 2A}{A + 2 + \omega\mu d} \tag{32}$$

$$A < -\omega\mu d \tag{33}$$

where

$$A = \mu(1 - \alpha) - \rho - \frac{\beta\mu}{(1 - v_1^*)^2}$$

Proof. See Mathematical Appendix A.6. ∎

When a single inequality in (32) and (33) does not hold while the other two are still satisfied, we might have a Neimark-Sacker, a Flip or Fold bifurcations, respectively. Given that we are assuming $\tau = 1$, it does not appear in (v_1^*, P_1^*). Still, the partial derivatives $\partial v_1^*/\partial\omega > 0$ and $\partial P_1^*/\partial\omega > 0$ are worth noting. The first captures the idea that a well-developed financial system is positively related to RS economic performance indicators. The story is that well-functioning financial markets have become a crucial investment element, allowing firms to rely less on retained profits. To some extent, a higher ω can be understood as a relaxation of a financial constraint that permits increasing capital accumulation. Thus, firms increase production and can hire more workers. Moreover, a higher level of output signals stronger economic fundamentals, thus being related to a higher equilibrium stock price.

Maintaining a similar strategy to the one adopted throughout the paper, let us rely on a qualitative numerical example to get a more concrete sense of the dynamic properties of this version of the model. Assume for the sake of simplicity that:

$$\mu = 0.25, \qquad \alpha = 0.2, \qquad \beta = 0.21, \qquad a = 1, \qquad a_1 = 2, \qquad \gamma = 1$$

$$a_2 = 1, \qquad \rho = 0.2, \qquad \omega = 0.3, \qquad \gamma = 1, \qquad c = 1, \qquad f = 2$$

which corresponds to the baseline calibration adopted so far with minor adjustments in α, β, and μ to obtain meaningful values for v and P.

Fig. 14 depicts the bifurcation diagram for v_1^* and P_1^* in terms of ω, showing a sequence of period-doubling bifurcations that results in persistent, bounded, and irregular dynamics. While it confirms the results of the previous Section, we document an important trade-off between equilibrium values and their volatility. A higher ω means higher employment rates and stock prices for the reasons previously explained. However, this result comes with an increase in the magnitude of the fluctuations. Modern economies with robust financial systems and complex real-financial market interactions are likely to perform better but must inevitably deal with Minskyan-type dynamics.

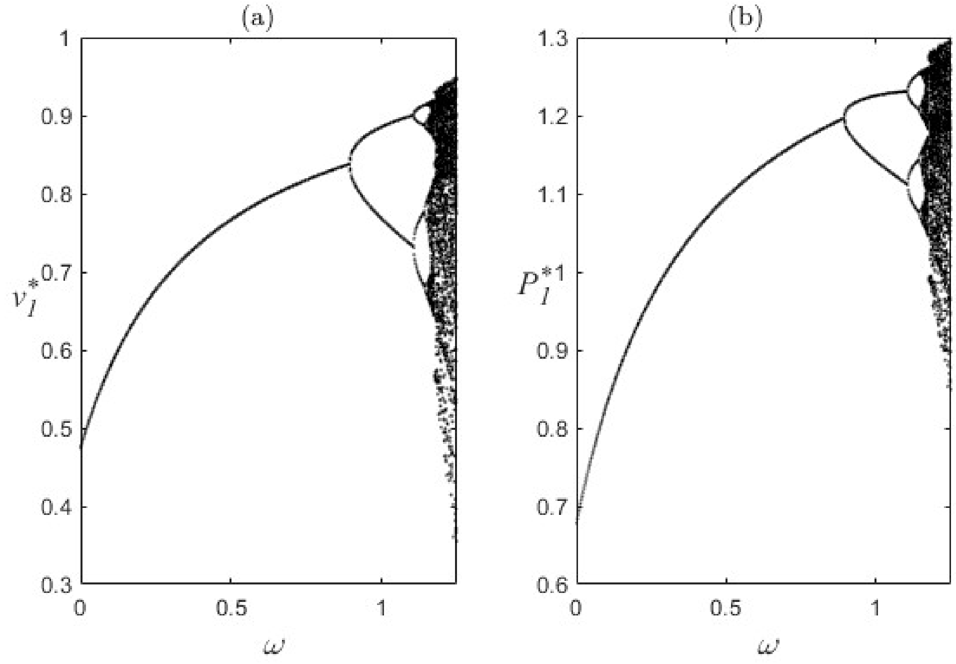

Figure 14: Bifurcation diagrams for v_1^* and P^* in terms of the parameter ω

6 Conclusions

Market economies alternate between periods of increasing economic activity and recessions. The forces behind such fluctuations – or whether they should be primarily understood as resulting from exogenous shocks or endogenously generated – remain controversial. This paper develops a macrodynamic model that contributes to the literature on endogenous economic dynamics. We innovate by differentiating between *two* sources of fluctuations related to *two* distinct types of heterogeneity. In the real sector, capital and labour earnings capture heterogeneity in the income composition. On the other hand, financial markets were assumed to be populated by chartists and fundamentalist traders.

Our model formalises the interaction between the RS and the SM. The strength of such an interaction is measured by two parameters, ω and τ. The former captures a cash-flow or wealth effect on investment, namely, how financial conditions affect capital accumulation decisions. Our analysis reveals that it is a destabilising element of the system. On the contrary, parameter τ turns out to be stabilising. This counterintuitive result implies that less anchored expectations are related to a more stable economy. Its rationale is deeply integrated into the nonlinear approach adopted here. When expectations about the stock price fundamentals strongly respond to the current state of the economy, there is synchronisation between RS and SM that stabilises the system.

The economic rationale of the resulting endogenous persistent dynamics is the following. Increases in employment raise the bargaining power of labour, damaging profits. Firms respond by reducing capital accumulation, which in turn reduces output. While such a result balances the initial disequilibrium in the labour market, it simultaneously decreases the perceived fundamental stock price, creating a mismatch between demand and supply for stocks and further deteriorating companies' balance sheets. Still, the previous increase in unemployment allows for a recovery in profitability. Firms can expand their investment plans, recovering output. The response from the financial side of the economy is positive as stock market prices increase following an improvement of the perceived fundamentals. At that point, employment rates increase, allowing the cycle to restart. It could be interpreted as a financial-augmented Goodwin cycle in contraposition to the pseudo-Goodwin cycles in Stockhammer and Michel (2017) or the Goodwin-Minsky debt models in Keen (2013), among others.

Furthermore, we document the existence of a trade-off between economic performance and volatility associated with the importance of the financial sector to capital accumulation. When the SM matters little to investment decisions, the economy is more stable, but the level of employment and output is also reduced. The increasing importance of the wealth effect generates instability because the economy becomes more susceptible to divergences in trading strategies between chartists and fundamentalists. Nonetheless, access to finance allows for an increase in production, raising the equilibrium employment rate. Thus, we observe higher output accompanied by endogenous fluctuations.

A realistic model of the macroeconomy should allow for the possibility of endogenous dynamics rather than leaving them unexplained and dependent on exogenous shocks. Furthermore, a multi-agent complex macro-system cannot be reduced to a single representative agent. The present paper combined elements from the Goodwin-Minsky literature to provide new insights into the interplay between different sources of heterogeneity in an intrinsically nonlinear environment. Real-financial interactions are too complex to be fully understood, and toy models like the one developed here are valuable tools for investigating different transmission mechanisms. Future work should explore the policy implications of this framework, feeding stylised as well as computationally more advanced setups for more realistic policy analysis.

A Mathematical Appendix

A.1 Proof of Proposition 1

The fixed points of (17) are found by solving

$$v^* = h(v^*) = \left\{1 + \mu\left[1 - \alpha - \frac{\beta}{(1-v^*)^{\delta}}\right]\right\} v^*$$

i.e.

$$\mu\left[1 - \alpha - \frac{\beta}{(1-v^*)^{\delta}}\right] v^* = 0$$

from which it follows that there exists two of them: the trivial one, $v_0^* = 0$, and $v_1^* = 1 - \left(\frac{\beta}{1-\alpha}\right)^{1/\delta}$. In order v_1^* to be economically meaningful, we must have

$$0 < 1 - \left(\frac{\beta}{1-\alpha}\right)^{1/\delta} < 1$$

or

$$0 < \beta < 1 - \alpha \tag{A.1}$$

A.2 Proof of Proposition 2

The map $h(v)$ in (17) is such that

$$h'(v) = -\frac{\mu\delta\beta v}{(1-v)^{\delta+1}} + 1 + \mu\left[1 - \alpha - \frac{\beta}{(1-v)^{\delta}}\right] \tag{A.2}$$

Thus, at $v = v_0^*$, given (A.1), we have:

$$h'(v_0^*) = 1 + \mu(1 - \alpha - \beta) > 1, \text{ always true}$$

so the trivial steady state is unstable for all admissible values of the parameters.

At $v = v_1^*$ we have

$$h'(v_1^*) = 1 - \mu\delta(1 - \alpha)\left[\left(\frac{1-\alpha}{\beta}\right)^{1/\delta} - 1\right]$$

from which it follows that v_1^* is locally stable if

$$\left|1 - \mu\delta(1 - \alpha)\left[\left(\frac{1-\alpha}{\beta}\right)^{1/\delta} - 1\right]\right| < 1$$

i.e.:

$$0 < \mu\delta(1 - \alpha)\left[\left(\frac{1-\alpha}{\beta}\right)^{1/\delta} - 1\right] < 2\beta^{1/\delta}$$

a condition that, given (A.1), reduces to:

$$\beta > (1 - \alpha)\left[\frac{\mu\delta(1-\alpha)}{2 + \mu\delta(1-\alpha)}\right]^{\delta}$$

A.3 Proof of Proposition 3

The fixed point of the map in equation (19) are found by solving

$$P^* = P^* + \gamma\left[\frac{a_1 + a_2}{a_1 \exp\left(-(c - f)\left(P^* - d\bar{Y}\right)\right) + a_2} - 1\right] a_2$$

or

$$\frac{a_1 + a_2}{a_1 \exp\left(-(c - f)\left(P^* - d\bar{Y}\right)\right) + a_2} = 1$$

Simplifying

$$1 = \exp\left(-(c - f)\left(P^* - d\bar{Y}\right)\right)$$

from which

$$P^* = d\bar{Y} > 0$$

The map $g(P)$ in (A.3) is such that

$$g'(P) = 1 + \gamma\frac{(c - f)(a_1 + a_2)a_1a_2 \exp\left(-(c - f)\left(P - d\bar{Y}\right)\right)}{\left[a_1 \exp\left(-(c - f)\left(P - d\bar{Y}\right)\right) + a_2\right]^2} \tag{A.3}$$

Thus, at $P = P^*$

$$g'(P^*) = 1 - \gamma(f - c)\left(\frac{a_1a_2}{a_1 + a_2}\right) \tag{A.4}$$

Thus, local stability of P^* requires that

$$\left|1 - \gamma(f - c)\left(\frac{a_1a_2}{a_1 + a_2}\right)\right| < 1$$

or

$$0 < f - c < \left(\frac{2}{\gamma}\right)\left(\frac{1}{a_1} + \frac{1}{a_2}\right)$$

A.4 Proof of Proposition 4

The Jacobian matrix $\mathbf{J}$ of the 2D-map M is:

$$\mathbf{J} = \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{bmatrix} \tag{A.5}$$

where

$$\begin{aligned}
j_{11} &= \frac{\partial h_1}{\partial v} = 1 + \mu \left[1 - \alpha - \frac{\beta}{(1-v)^{\delta}} \right] - \delta \beta \mu \frac{v}{(1-v)^{\delta+1}} \\
j_{12} &= \frac{\partial h_1}{\partial P} = \frac{\omega \mu}{a \bar{N}} \\
j_{21} &= \frac{\partial g_1}{\partial v} \\
&= \frac{\bar{N} a d \tau \gamma a_1 a_2 (f-c)(a_1+a_2) \exp\left(-(c-f)\left\{P - d\left[(1-\tau)\bar{Y} + \tau a \bar{N} v\right]\right\}\right)}{\left[a_1 \exp\left(-(c-f)\left\{P - d\left[(1-\tau)\bar{Y} + \tau a \bar{N} v\right]\right\}\right) + a_2\right]^2} \\
j_{22} &= \frac{\partial g_1}{\partial P} \\
&= 1 - \frac{\gamma a_1 a_2 (f-c)(a_1+a_2) \exp\left(-(c-f)\left\{P - d\left[(1-\tau)\bar{Y} + \tau a \bar{N} v\right]\right\}\right)}{\left[a_1 \exp\left(-(c-f)\left\{P - d\left[(1-\tau)\bar{Y} + \tau a \bar{N} v\right]\right\}\right) + a_2\right]^2}
\end{aligned}$$

The fixed points of the dynamic system (23)-(24) are found by solving

$$\left[1 - \alpha - \frac{\beta}{(1-v^*)^{\delta}} \right] v^* = 0 \tag{A.6}$$

$$P^* = d\left[(1-\tau)\bar{Y} + \tau a \bar{N} v^*\right] \tag{A.7}$$

From (A.6), it follows that there are two solutions in v

$$v_0^* = 0 \quad \text{and} \quad v_1^* = 1 - \left(\frac{\beta}{1-\alpha} \right)^{1/\delta}$$

and, substituting them in (A.7), we find the corresponding solutions in P:

$$P_0^* = 0 \quad \text{and} \quad P_1^* = d(1-\tau)\bar{Y}$$

At (v_0^*, P_0^*), the elements of the Jacobian matrix (A.5) become

$$\begin{aligned}
j_{11(\omega=0)}\Big|_{(v_0^*,P_0^*)} &= 1 + \mu(1-\alpha-\beta) \\
j_{12(\omega=0)}\Big|_{(v_0^*,P_0^*)} &= 0 \\
j_{21(\omega=0)}\Big|_{(v_0^*,P_0^*)} &= \frac{\bar{N} a d \tau \gamma a_1 a_2 (f-c)}{a_1 + a_2} \\
j_{22(\omega=0)}\Big|_{(v_0^*,P_0^*)} &= 1 - \frac{\gamma a_1 a_2 (f-c)}{a_a s + a_2}
\end{aligned}$$

such that the two eigenvalues are easily determined as

$$\begin{aligned}
\lambda_1\Big|_{(v_0^*,P_0^*)} &= j_{11(\omega=0)}\Big|_{(v_0^*,P_0^*)} \\
\lambda_2\Big|_{(v_0^*,P_0^*)} &= j_{22(\omega=0)}\Big|_{(v_0^*,P_0^*)}
\end{aligned}$$

Given that conditions (18) and (20) for the local stability of the isolated markets are assumed to hold, we have

$$\begin{aligned} \lambda_1\big|_{\left(v_0^*,P_0^*\right)} &> 1 \\ -1 < \lambda_2\big|_{\left(v_0^*,P_0^*\right)} &< 1 \end{aligned}$$

so that we can conclude that (v_0^*, P_0^*) is a *saddle point*. On the other hand, at (v_1^*, P_1^*) the elements of Jacobian matrix (A.5) become

$$\begin{aligned} j_{11(\omega=0)}\big|_{\left(v_1^*,P_1^*\right)} &= 1 - \mu\delta\left(1-\alpha\right)\left[\left(\frac{1-\alpha}{\beta}\right)^{1/\delta} - 1\right] \\ j_{12(\omega=0)}\big|_{\left(v_1^*,P_1^*\right)} &= 0 \\ j_{21(\omega=0)}\big|_{\left(v_1^*,P_1^*\right)} &= \frac{\bar{N}ad\tau\gamma a_1 a_2\left(f-c\right)}{\left[a_1+a_2\right]} \\ j_{22(\omega=0)}\big|_{\left(v_1^*,P_1^*\right)} &= 1 - \frac{\gamma a_1 a_2\left(f-c\right)}{a_1+a_2} \end{aligned}$$

so that both λ_1 and λ_2 are real and such that

$$\begin{aligned} -1 < \lambda_1\big|_{\left(v_1^*,P_1^*\right)} &= j_{11(\omega=0)}\big|_{\left(v_1^*,P_1^*\right)} < 1 \\ -1 < \lambda_2\big|_{\left(v_1^*,P_1^*\right)} &= j_{22(\omega=0)}\big|_{\left(v_1^*,P_1^*\right)} < 1 \end{aligned}$$

implying that the positive steady state (v_1^*, P_1^*) is a locally *stable node*.

A.5 Proof of Proposition 5

In steady state, the fixed point of map M_2 is found by solving:

$$\begin{aligned} v^* &= \left[1+\mu\left(1+\alpha-\frac{\beta}{1-v^*}\right)-\rho\right]v^* + \left(\frac{\omega\mu}{aN}\right)P^* = h_2(v^*,P^*) \\ P^* &= P^* + \gamma\left[\frac{a_1+a_2}{a_1\exp\left(\left(f-c\right)\left(P^*-daNv^*\right)\right)+a_2} - 1\right]a_2 = g_2(v^*,P^*) \end{aligned}$$

Simplifying, we obtain:

$$\begin{aligned} -\left(\frac{\omega\mu}{aN}\right)P^* &= \left[\mu\left(1+\alpha-\frac{\beta}{1-v^*}\right)-\rho\right]v^* \\ P^* &= daNv^* \end{aligned} \tag{A.8}$$

Thus, the system admits a trivial equilibrium $(v^*, P^*) = (0, 0)$. Moreover, by substituting the second expression into the first one, we obtain that the economically meaningful steady-state employment rate is such that:

$$v_1^* = \frac{1+\alpha+\omega d-\rho-\beta}{1+\alpha+\omega d-\rho} = 1 - \frac{\beta}{1+\alpha+\omega d-\rho}$$

Further substituting v_1^* into Eq. (A.8), we find the equilibrium value of the stock price:

$$P^* = \left(1 - \frac{\beta}{1+\alpha+\omega d-\rho}\right)daN$$

A.6 Proof of Proposition 6

The Jacobian matrix $\mathbf{J}$ of the 2D-map M_2 is:

$$\mathbf{J} = \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{bmatrix} \tag{A.9}$$

where

$$\begin{aligned}
j_{11} &= \left.\frac{\partial h_2}{\partial v}\right|_{(v_1^*, P_1^*)} = 1 + \mu\left(1 - \alpha - \frac{\beta}{1 - v_1^*}\right) - \rho - \beta\mu\frac{v}{(1 - v_1^*)^2} \\
&= 1 + \mu(1 - \alpha) - \rho - \frac{\beta\mu}{(1 - v_1^*)^2} \gtreqless 0 \\
j_{12} &= \left.\frac{\partial h_2}{\partial P}\right|_{(v_1^*, P_1^*)} = \frac{\omega\mu}{a\bar{N}} \\
j_{21} &= \left.\frac{\partial g_2}{\partial v}\right|_{(v_1^*, P_1^*)} = \frac{\bar{N}ad\gamma a_1 a_2 (f - c)(a_1 + a_2)\exp\left((f - c)\left(P_1^* - da\bar{N}v\right)\right)}{\left[a_1 \exp\left((f - c)\left(P_1^* - da\bar{N}v_1^*\right)\right) + a_2\right]^2} \\
&= \frac{\bar{N}ad\gamma a_1 a_2 (f - c)}{a_1 + a_2} \\
j_{22} &= \left.\frac{\partial g_2}{\partial P}\right|_{(v_1^*, P_1^*)} = 1 - \frac{\gamma a_1 a_2 (f - c)(a_1 + a_2)\exp\left((f - c)\left(P_1^* - da\bar{N}v\right)\right)}{\left[a_1 \exp\left((f - c)\left(P_1^* - da\bar{N}v_1^*\right)\right) + a_2\right]^2} \\
&= 1 - \frac{\gamma a_1 a_2 (f - c)}{a_1 + a_2}
\end{aligned}$$

For two-dimensional maps, the necessary and sufficient conditions guaranteeing that the eigenvalues of the Jacobian matrix are less than one in modulus:

$$1 + \mathrm{tr}J + \det J > 0 \tag{A.10}$$

$$1 - \mathrm{tr}J + \det J > 0 \tag{A.11}$$

$$\det J < 1 \tag{A.12}$$

Through direct computation, we have that (A.10) is satisfied provided that:

$$\begin{aligned}
1 + \mathrm{tr} + \det > 0 \Rightarrow 1 + j_{11} + j_{22} + j_{11}j_{22} - j_{12}j_{21} > 0 \\
1 + \left[1 + \mu(1 - \alpha) - \rho - \frac{\beta\mu}{(1 - v_1^*)^2}\right] + 1 - \frac{\gamma a_1 a_2 (f - c)}{a_1 + a_2} \\
+ \left[1 + \mu(1 - \alpha) - \rho - \frac{\beta\mu}{(1 - v_1^*)^2}\right]\left[1 - \frac{\gamma a_1 a_2 (f - c)}{a_1 + a_2}\right] \\
-\frac{\omega\mu}{a\bar{N}}\frac{\bar{N}ad\gamma a_1 a_2 (f - c)}{a_1 + a_2} > 0 \\
2\left[2 + \mu(1 - \alpha) - \rho - \frac{\beta\mu}{(1 - v_1^*)^2}\right] > \left[2 + \mu(1 - \alpha) - \rho - \frac{\beta\mu}{(1 - v_1^*)^2} + \omega\mu d\right]\frac{\gamma a_1 a_2 (f - c)}{a_1 + a_2} \\
\frac{2(a_1 + a_2)}{\gamma a_1 a_2 (f - c)} > 1 + \frac{\omega\mu d}{2 + \mu(1 - \alpha) - \rho - \frac{\beta\mu}{(1 - v_1^*)^2}} \\
2\left[\frac{2 + \mu(1 - \alpha) - \rho - \frac{\beta\mu}{(1 - v_1^*)^2}}{2 + \mu(1 - \alpha) - \rho - \frac{\beta\mu}{(1 - v_1^*)^2} + \omega\mu d}\right] > \frac{\gamma a_1 a_2 (f - c)}{a_1 + a_2}
\end{aligned}$$

A violation of this condition while the other two hold is associated with a Flip bifurcation. The fixed point loses stability, and a stable period-2 cycle may arise as the new limit is set. Further increasing the bifurcation parameter leads to a series of period-doubling bifurcations, a well-known route to more complex dynamics.

On the other hand, (A.11) requires that:

$$\begin{aligned}
1 - \mathrm{tr}J + \det J > 0 \Rightarrow 1 - j_{11} - j_{22} + j_{11}j_{22} - j_{12}j_{21} > 0 \\
1 - \left[1 + \mu(1-\alpha) - \rho - \frac{\beta\mu}{(1-v_1^*)^2}\right] - 1 + \frac{\gamma a_1 a_2 (f-c)}{a_1+a_2} \\
+ \left[1 + \mu(1-\alpha) - \rho - \frac{\beta\mu}{(1-v_1^*)^2}\right]\left[1 - \frac{\gamma a_1 a_2 (f-c)}{a_1+a_2}\right] \\
-\frac{\omega\mu}{a\bar{N}}\frac{\bar{N}ad\gamma a_1 a_2 (f-c)}{a_1+a_2} > 0 \\
\frac{\gamma a_1 a_2 (f-c)}{a_1+a_2} - \left[1 + \mu(1-\alpha) - \rho - \frac{\beta\mu}{(1-v_1^*)^2}\right]\frac{\gamma a_1 a_2 (f-c)}{a_1+a_2} \\
-\omega\mu d\frac{\gamma a_1 a_2 (f-c)}{a_1+a_2} > 0 \\
\rho + \frac{\beta\mu}{(1-v_1^*)^2} > \mu(1-\alpha+\omega d)
\end{aligned}$$

Not satisfying the condition above while the other two hold is related to a Fold bifurcation.

Finally, from (A.12), we have that:

$$\begin{aligned}
\det J < 1 \Rightarrow j_{11}j_{22} - j_{12}j_{21} < 1 \\
\left[1 + \mu(1-\alpha) - \rho - \frac{\beta\mu}{(1-v_1^*)^2}\right]\left[1 - \frac{\gamma a_1 a_2 (f-c)}{a_1+a_2}\right] \\
-\frac{\omega\mu}{a\bar{N}}\frac{\bar{N}ad\gamma a_1 a_2 (f-c)}{a_1+a_2} < 1 \\
1 < \frac{\left[1 + \omega\mu d + \mu(1-\alpha) - \rho - \frac{\beta\mu}{(1-v_1^*)^2}\right]}{\mu(1-\alpha) - \rho - \frac{\beta\mu}{(1-v_1^*)^2}}\frac{\gamma a_1 a_2 (f-c)}{,} a_1 + a_2 \\
1 < \left[1 + \frac{1+\omega\mu d}{\mu(1-\alpha) - \rho - \frac{\beta\mu}{(1-v_1^*)^2}}\right]\frac{\gamma a_1 a_2 (f-c)}{a_1+a_2}
\end{aligned}$$

and it follows that its violation while (A.10)-(A.11) hold leads the fixed point to lose stability, and a Neimark-Sacker bifurcation occurs.

Combining conditions (A.10) and (A.12) allow us to write:

$$\frac{1}{1 + \frac{1+\omega\mu d}{\mu(1-\alpha)-\rho-\frac{\beta\mu}{(1-v_1^*)^2}}} < \frac{\gamma a_1 a_2 (f-c)}{a_1+a_2} < 2\left[\frac{2 + \mu(1-\alpha) - \rho - \frac{\beta\mu}{(1-v_1^*)^2}}{2 + \mu(1-\alpha) - \rho - \frac{\beta\mu}{(1-v_1^*)^2} + \omega\mu d}\right]$$

$$\frac{\mu\left(1-\alpha\right)-\rho-\frac{\beta\mu}{\left(1-v_1^*\right)^2}}{1+\mu\left(1-\alpha\right)-\rho-\frac{\beta\mu}{\left(1-v_1^*\right)^2}+\omega\mu d}<\frac{\gamma a_1 a_2\left(f-c\right)}{a_1+a_2}<\frac{4+2\mu\left(1-\alpha\right)-2\rho-2\frac{\beta\mu}{\left(1-v_1^*\right)^2}}{2+\mu\left(1-\alpha\right)-\rho-\frac{\beta\mu}{\left(1-v_1^*\right)^2}+\omega\mu d} \tag{A.13}$$

Define the following two auxiliary variables:

$$A=\mu\left(1-\alpha\right)-\rho-\frac{\beta\mu}{\left(1-v\right)^2} \tag{A.14}$$

$$B=1+\mu\left(1-\alpha\right)-\rho-\frac{\beta\mu}{\left(1-v\right)^2}+\omega\mu d$$

so that

$$B=1+\omega\mu d+A \tag{A.15}$$

Substituting Eqs. (A.14) and (A.15) into inequality (A.13), it follows that:

$$\frac{A}{A+1+\omega\mu d}<\frac{\gamma a_1 a_2\left(f-c\right)}{a_1+a_2}<\frac{2A+4}{A+2+\omega\mu d} \tag{A.16}$$

Furthermore, we can also write (A.11) in terms of A, so that:

$$A<-\omega\mu d \tag{A.17}$$

Therefore, the economically meaningful equilibrium solution of M_2 is locally stable provided that conditions (A.16) and (A.17) are fulfilled.

References

[1] Asada, T., Chiarella, C., Flaschel, P., Mouakil, T., Proaño, C., Semmler, W. 2011. Stock-flow interactions, disequilibrium macroeconomics and the role of economic policy. Journal of Economic Surveys 25(3), 569-599.

[2] Allen, R.G.D., 1967. Macro-Economic Theory. A Mathematical Treatment. London: Macmillan.

[3] Barrales-Ruiz, J., Mendieta-Munoz, I., Rada, C., Tavani, D., von Arnim, R., 2022. The distributive cycle: Evidence and current debates. Journal of Economic Surveys 36, 468–503.

[4] Beaudry, P., Galizia, D., Portier, F., 2020. Putting the cycle back into business cycle analysis. American Economic Review 110(1), 1-47.

[5] Bernanke, B., Gertler, M., Gilchrist, S., 1998. The financial accelerator in a quantitative business cycle framework. NBER WP 6455, 1-75.

[6] Cafferata, A., Davila-Fernandez, M., Sordi, S. 2021. (Ir)rational explorers in the financial jungle: Modelling Minsky with heterogeneous agents. Journal of Evolutionary Economics 31, 1157–1188.

[7] Carpenter, R., Guariglia, A. 2008. Cash flow, investment, and investment opportunities: New tests using UK panel data. Journal of Banking and Finance 32(9), 1894-1906.

[8] Cavalli, F., Naimzada, A., Pecora, N., 2022. A stylised macro-model with interacting real, monetary and stock markets. Journal of Economic Interaction and Coordination 17, 225-257.

[9] Cavalli, F., Naimzada, A., Pecora, N., 2017. Real and financial market interactions in a multiplier-accelerator model: Nonlinear dynamics, multistability and stylised facts. Chaos 27, 103120.

[10] Chen, Z., Zheng, H. 2022. Herding in the Chinese and US stock markets: Evidence from a micro-founded approach. International Review of Economics and Finance 78, 597-604.

[11] Chiarella, C., Flaschel, P., Franke, R. 2005. Foundations for a Disequilibrium Theory of the Business Cycle. Cambridge: Cambridge University Press.

[12] Chiarella, C., He, X., Zwinkels, R. 2014. Heterogeneous expectations in asset pricing: Empirical evidence from the S&P500. Journal of Economic Behavior and Organization 105, 1-16.

[13] Crotty, J. 2009. Structural causes of the global financial crisis: a critical assessment of the 'new financial architecture'. Cambridge Journal of Economics 33(4), 563–580.

[14] Day, R., Huang, W., 1990. Bulls, bears and market sheep. Journal of Economic Behavior and Organization 14, 299-329.

[15] Dieci, R., He, X., 2018. Heterogeneous agent models in finance. In Hommes, C., LeBaron, B. (Eds.) Handbook of Computational Economics 4. Amsterdam: Elsevier, 257-328.

[16] Dieci, R., Schmitt, N., Westerhoff, F. 2018. Interactions between stock, bond and housing markets. Journal of Economic Dynamics and Control 91, 43-70.

[17] Fernández, A., Gulan, A., 2015. Interest rates, leverage, and business cycles in emerging economies: The role of financial frictions. American Economic Journal: Macroeconomics 7(3), 153-188.

[18] Flaschel, P., 2015. Goodwin's MKS system: A baseline macro model. Cambridge Journal of Economics 39(6), 1591-1605.

[19] Flaschel, P., Charpe, M., Galanis, G., Proano, C., Veneziani, R. 2018. Macroeconomic and stock market interactions with endogenous aggregate sentiment dynamics. Journal of Economic Dynamics and Control 91, 237-256.

[20] Franke, R., Westerhoff, F., 2012. Structural stochastic volatility in asset pricing dynamics: Estimation and model contest. Journal of Economic Dynamics and Control 36(8), 1193-1211.

[21] Gardini, L., Radi, D., Schmitt, N., Sushko, I., Westerhoff, F., 2022. Causes of fragile stock market stability. Journal of Economic Behavior & Organization 200, 483-498.

[22] Giraud, G., Grasselli, M., 2019. Household debt: The missing link between inequality and secular stagnation. Journal of Economic Behavior and Organization 183, 901-927.

[23] Goodwin, R.M., 1967. A growth cycle. In: Feinstein, C.H. (Ed.). Socialism, Capitalism and Economic Growth. Cambridge: Cambridge University Press, 54-58.

[24] Grasselli, M., Maheshwari, A., 2018. Testing a Goodwin model with general capital accumulation rate. Metroeconomica 69(3), 619-643.

[25] Hommes, C., 2013. Behavioral Rationality and Heterogeneous Expectations in Complex Economic Systems. Cambridge: Cambridge University Press.

[26] Hommes, C., 2021. Behavioral and experimental macroeconomics and policy analysis: A complex systems approach. Journal of Economic Literature 59(1), 149-219.

[27] Keen, S., 2013. A monetary Minsky model of the Great Moderation and the Great Recession. Journal of Economic Behavior and Organization 86, 221-235.

[28] Kohler, K., Jump, R., 2022. Estimating nonlinear business cycle mechanisms with linear Vector Autoregressions: A Monte Carlo study. Oxford Bulletin of Economics and Statistics 84(5), 1077-1100.

[29] Kuh, E., 1967. A productivity theory of wage levels. An alternative to the Phillips curve. Review of Economic Studies 34, 333-360.

[30] Melander, O., Sandstrom, M., von Schedvin, E., 2017. The effect of cash flow on investment: an empirical test of the balance sheet theory. Empirical Economics 53, 695-716.

[31] Minsky, H., 1982. Can "It" Happen Again? Essays on Instability and Finance. New York: ME Sharp.

[32] Naimzada, A., Pireddu, M., 2014. Dynamic behaviour of product and stock markets with a varying degree of interaction. Economic Modelling 41, 191-197.

[33] Naimzada, A., Pireddu, M., 2015. Real and financial interacting markets: A behavioral macro-model. Chaos, Solitons and Fractals 77, 111-131.

[34] Pohjola, M.J., 1981. Stable and chaotic growth: The dynamics of a discrete version of Goodwin's growth cycle model. Zeitschrift für Nationalökonomie/Journal of Economics 41, 27-38.

[35] Schmitt, N. Westerhoff, F., 2021. Trend followers, contrarians and fundamentalists: Explaining the dynamics of financial markets. Journal of Economic Behavior and Organization 192, 117-136.

[36] Sordi, S., 1987. Alternative Approaches to the Explanation of Macroeconomic Fluctuations. EUI PhD Thesis. Florence: European University Institute.

[37] Sordi, S., 1999. Economic models and the relevance of "chaotic regions": An application to Goodwin's growth cycle model. Annals of Operations Research 89, 3-19.

[38] Sordi, S., Vercelli, A., 2014. Unemployment, income distribution and debt-financed investment in a growth cycle model. Journal of Economic Dynamics and Control 48, 325-348.

[39] Stiglitz, J., 2018. Reflections on the global financial crisis ten years on. In O'Halloran, S., Groll, T. (Eds.) After the Crash: Financial Crises and Regulatory Responses. New York: Columbia University Press, 167-182.

[40] Stockhammer, E., Michell, J. 2017. Pseudo-Goodwin cycles in a Minsky model. Cambridge Journal of Economics 41, 105–125.

[41] Valickova, P., Havranek, T., Horvath, R., 2015. Financial development and economic growth: A meta-analysis. Journal of Economic Surveys 29(3), 506-526.

[42] Velupillai, V., 2006. A disequilibrium macrodynamic model of fluctuations. Journal of Macroeconomics 28(4), 752-767.

[43] Westerhoff, F., 2012. Interactions between the real economy and the stock market: A simple agent-based approach. Discrete Dynamics in Nature and Society, Article ID 504840, 21 pages.

[44] Ye, M., Zheng, M., Li, X., 2020. Price ceiling, market structure, and payout policies. NBER WP 28054, 1-38.