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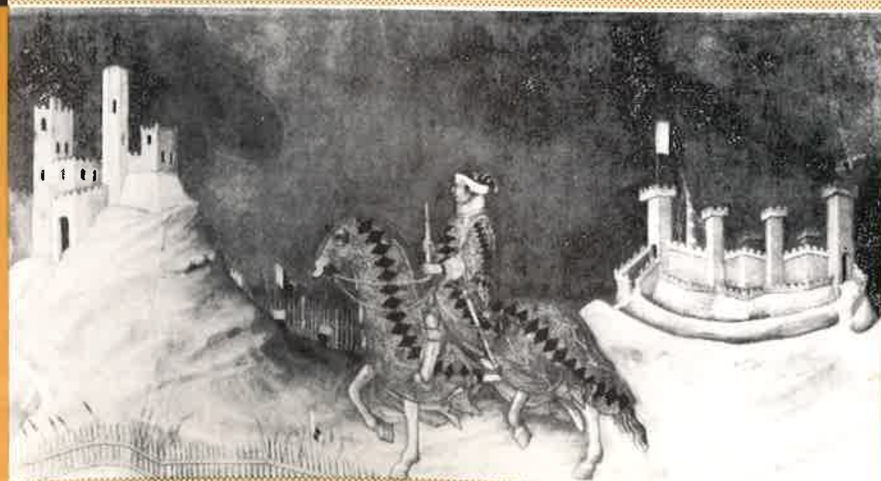
PierMario Pacini

EXPECTATIONS, LEARNING AND EQUILIBRIUM IN A NON - WALRASIAN MODEL : A CONSTRUCTIVE APPROACH



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EXPECTATIONS, LEARNING AND EQUILIBRIUM IN A NON-WALRASIAN MODEL: A CONSTRUCTIVE APPROACH

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"If 'to exist' does not mean 'to be constructed', it must have some metaphysical meaning".

"It is to often forgotten that the truth of constructions depends upon the present state of knowledge and that the words 'in reality' ought to be translated into 'according to the contemporary view' . '

Heyting A. (1956)

Introduction. [1]

This paper recasts a typical model produced within the so called "Neo-Keynesian School" and concerning the theory of temporary equilibria with quantity rationing (or short run disequilibrium models). The formal approach to this problem, started by the works of Younès (1975), Drèze (1975) and Benassy (1975), and further developed by Benassy (1977) (1982), Bohm-Levine (1979), Drazen (1980), Grandmont (1977), Hahn (1978), Heller-Starr (1979), Laroque-Polemarchakis (1978), Sylvestre (1983), Younès (1975) [2], is inclined to model the problem as a short run theory in which, as a limit case, prices and wages are fixed and fast adjustments are supposed to take place in quantity variables. Then a proposition concerning the existence of equilibria (compatibility), under well-specified conditions, is usually given. Here I take into account the disequilibrium model offered by Heller-Starr (1979). This choice has been

[1]) It is not a common place to say that many persons are on credit for the merits (if any) of this paper: among others, I would like to thank Prof. P. Dehez for patient guidance, Profs. A.P. Kirman, M. Blad, B. Miconi, L. Punzo for helpful comments and suggestions and all the partecipants to the Workshop on Theoretical Economics at E.U.I. for useful observations on earlier drafts. Of course, the debt of errors and misunderstandings rest with me.

[2]) A complete list of contributions in this field would be almost endless. Only the most recent and formalized works have been mentioned in order to stick to the approach taken in this paper. However, all of them find their root in the well-known interpretation of the Keynesian theory suggested by Clower (1965), Patinkin (1965) and Leijonhufvud (1968).

suggested by the fact that this model overcomes some difficulties present in Benassy's and Drèze's works. In particular, still allowing for spillover effects across markets, the model permits the analysis of individual choices as resulting from simultaneous optimization over *all* markets. Moreover, the problem of what should be the object of optimization and what variables should respect quantity and financial constraints is solved in favour of *actual quantities*, i.e. quantities of commodities effectively obtained by the market through an *allocation mechanism*. Therefore, effective demands remain as mere informative signals. However, this model differs from those mentioned above, since agents do not know the form of the rationing scheme and are bound to make subjective perceptions of it by learning from past experience.

With respect to this general formulation, the work focuses attention on two points: 1) the *existence proposition* of short run equilibria, 2) the *notion of equilibrium* currently used when the short run is concerned.

1) As far as existence is concerned, an alternative proof is given which exhibits distinctive *constructive* elements. The need for a constructive proof responds to various instances here and there disseminated in the recent literature. Among other things, it tries to (i) ground the existence proposition on mathematical tools that are not derived from algebraic topology and are more appropriate to an economic setting (Smale (1976)); (ii) offer a method by which equilibrium values of variables can be computed (Scarf (1973)); (iii) try to bring to unity the dichotomy between existence and stability showing simultaneously the existence of equilibria and of a "dynamic" path leading to their set (Smale (1976)). Representing a further attempt on this way, the present approach could be subject to some of the criticisms encountered by previous attempts in the same vein, regarding its exclusive algorithmic potentiality or the lack of generality due to its huge informative requirements. Nevertheless, it explicitly responds to two methodological istances in which it finds its justification: (a) the definition of equilibrium, a point which will be touched upon later, (b) the need to construct models in which "formal systems are not to be separated from their interpretation" (Troelstra A.S. (1969)). This point is not expanded here. I simply note that such separation is the essence of the widely

accepted *formalist* approach (e.g., see Debreu (1985)), leading to now-standard existence propositions as tests of logical consistency, any economic mechanism remaining concealed behind the mathematical argument [3]. The spirit of the present approach to the *equilibrium proof* is instead to give a precise formal representation of each of the blocks by which equilibrium is constructed:

(i) a description of the economy as it is standard in short run analysis: in particular, among the parameter of the economy are included the information set, the way expectations are formed and the learning behaviour of agents [4].

(ii) the representation of the market mechanism as a possibly infinite sequence of steps: in each of them actions are announced and expectations and beliefs are revised by processing new accruing information.

(iii) the presence of an "indicator" linked with individual characteristics that evolves in the process and from which the assessment of an equilibrium (still a fixed point) is inferred. The *precision of expectations* is taken as the indicator in this paper: this choice is instrumental for the discussion in point (2) and does not exclude the possibility for more fruitful indicators to be devised.

I stress again that this intends to be an existence proposition and not a stability proof. The reference to equilibrium is always required in stability analysis, both in its local version, where linearization presupposes the presence of equilibrium, and in its global one, where distances are usually measured from the equilibrium trajectory. In the present context, there is no pre-existing position of equilibrium; it emerges just as the terminal point of the behaviour of the system.

2) The previous formalization makes explicit the notion of equilibrium as "rest point of an (implicit) dynamic process" and hence very similar to the concept of steady state. The essential feature of this definition is the *persistence of behaviour*, i.e. an equilibrium is found whenever agents do not change their (expectations and) policies in view of new accruing information. In

[3]) As advocates of the two competing parties here concerned, see Weintraub (1985) and Punzo (1984).

[4]) In the short run, they are to be taken constant; any change in any of these characteristics is equivalent to a change in any of the material elements of the model as endowments or prices, and hence would pose a new and different equilibrium problem.

these terms, the concept of equilibrium in fix-price models (but more in general Walrasian equilibrium as well) emerges as a particular case; it is a point in which agents reach compatibility through quantity rationing and persist in this position since expectations are fully realized [5]. However, in so far as the definition of equilibrium as persistence of behaviour is accepted and once agents are defined by their physical characteristics and an *individual learning technology*, situations can arise in which they converge and persist in a vision of the world (and hence reiterate their actions) independently of the conformation of realizations to expectations. It will be maintained only because it is the best they can do on the basis of their information and learning abilities. Since the consistency assumption is dispensed off, the conformation of observations and expectations as an equilibrium feature requires that more specifications are to be imposed on the learning technology then those here assumed, as it will be shown in the model. Therefore, the convergence to "non-rational expectations" equilibria arises as a possibility and the model is able to generate equilibrium positions that are sub-optimal not only for the presence of sticky prices, but also because people learn in the wrong way and persist in a position that could be improved upon either by a different distribution or by an better utilization of information. That such positions will not be maintained indefinitely and that they will be replaced by more advantageous situations in which realizations are correctly forecasted is plausible as well, but entails a different problem, i.e. the problem of *learning to learn* or learning to be rational that cannot be dealt with in so far as the short run is concerned.

The paper is organized as follows: Section I describes the market structure, the distribution and utilization of information and the individual choice framework. Section II deals with the definition and proof of equilibrium through the use of Caristi's fixed point theorem. In Section III, the results of two computer simulations are presented as applications of the previous argument. Some minor technical points are treated in the Appendices.

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[5]) The fulfillment of expectations is a crucial feature of the notion of equilibrium as the recent literature on rational expectations has made clearer. However, it has been made explicit in formal models since Negishi (1971), where the assumption of consistency with observations is another way to say that in equilibrium agents must have correct perceptions of the economic environment.

Section I. The Model.

1.1. *Preliminaries and the basic framework.*

A barter exchange economy is assumed with n consumers indexed by $i = 1,\dots,n$ behaving as (expected) utility maximizers given l commodities indexed by $j = 1,\dots,l$. Prices of commodities are exogenously given and fixed.

A1. The vector of prices $p = (p_1,\dots,p_j,\dots p_l)$ is fixed and strictly positive

$$0 < p_j < \infty \qquad \forall\ j.$$

The economic problem is decomposed in its sequential structure: a sequence of periods (steps) is introduced, each step being indexed by $t = 0,1,2\dots$ It is assumed that the time index does not affect the "foundamentals" of the model.

Consumers are described by $(\omega_i, X_i, U_i, \psi_i)$, where ψ_i represents an individual expectation formation mechanism, that is dealt with later, ω_i are initial endowments, U_i is an utility function representing i's preferences $\succeq_i$ over $X_i \subset \mathbb{R}^l_+$, the consumption set. To express variables in net terms, the above notation will be changed to (Z_i^0, V_i, ψ_i) where $Z_i^0 = X_i - \{\omega_i\}$ and $V_i(z_i) = U_i(z_i + \omega_i)$. Consumers' specialization is assumed in order to avoid further unessential problems. In each step t, agent i chooses *announced actions* (effective demands) $\tilde{z}_i^t$ to maximize the expected utility of *actual consequences* (constrained quantities) z_i^t [6]. It is further assumed that, for each i, the effective demands that can be announced are confined to a *credibility set* $\tilde{Z}_i = Z_i^0 \cap S_i$, where S_i is a sphere centered at the origin in $\mathbb{R}^l$ and containing the budget set. Then, for all i, individual characteristics satisfy:

A2. i) $\omega_i \gg 0$

ii) $V_i(z_i)$ is continuous, increasing and stricly quasi-concave on $\tilde{Z}_i$

iii) $B_i \equiv \{ z_i \in Z_i^0 \mid p \cdot z_i \leq 0 \} \subseteq \tilde{Z}_i$

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[6]) In the following $\tilde{z}_i^t$ and z_i^t are to be intended as net quantities, i.e. effective excess demands and constraints on excess demands respectively.

$\widetilde{Z}_i$ is chosen as the consumption set in place of the usual Z_i^0 since, in this setting, announced actions are no more than signalling devices and in some cases they could grow so high that they could be rejected as incredible by the market. In order to prevent such cases, effective demands are restricted to the set $\widetilde{Z}_i$ (see Heller-Starr (1979) for a complete discussion) [7].

In each period the following sequence of actions takes place: first, information is collected and processed to form expectations; then effective demands are computed and communicated; finally, rationed quantities are announced back to agents.

1.2. *The Market Allocation Mechanism.*

Since prices are not necessarily Walrasian equilibrium prices, "spontaneous" individual actions $\tilde{z}_i^t$ need not match so that a rule to allocate commodities according to demands is required in order to equilibrate the market. This is represented by a *Market Allocation Mechanism*, i.e. a mapping $F_i(.)$ that associates, for each agent, individual effective demands $\tilde{z}_i^t$ with actual consequences z_i^t, depending on the market configuration of demand and supply. In order to represent this mechanism, let us introduce the following notation: let $\tilde{z}^t = (\tilde{z}_1^t,\ldots,\tilde{z}_i^t,\ldots,\tilde{z}_n^t)$ be an economy-wide configuration of effective demands; for each i, $\tilde{z}_{-i}^t$ is the vector of the others' effective demands.

The market allocation mechanism satisfies the following assumption:

A3. i) $F_i : \widetilde{Z}_i \to \mathbb{R}^l$ is a continuous function, $\forall\, i$.

ii) $\tilde{z}_{ij}^t \cdot z_{ij}^t \geq 0 \quad \forall\, i,j$.

iii) $|z_{ij}^t| \leq |\tilde{z}_{ij}^t| \quad \forall\, i,j$.

iv) $\sum_{i=1}^{n} z_{ij}^t = 0 \quad \forall\, j$.

---

[7]) Clearly the choice of a particular $\widetilde{Z}_i$ can affect the outcome of the model. While this can be taken as an example of how social conventions can affect equilibrium, the choice of $\widetilde{Z}_i$ matters for the existence argument in so far as the set is compact. Therefore for analytical purposes it can be chosen as big as one whishes, provided compactness is preserved.

The usual assumption known as "the short side of the market is always satisfied" is not made because it is unessential for the proof and features a market transparency and an allocative efficiency that hardly fits a market structure as the one presented here where uncertainty and incomplete information dominate.

From A3 i)-iii), the set of outcomes for i, $F_i(\widetilde{Z}_i)$, is compact and contained in $\widetilde{Z}_i$; it will be indicated by Z_i. Without loss of generality, $\widetilde{Z}_i$ and Z_i are taken equal across consumers and will be denoted by $\widetilde{Z}$ and Z respectively.

A3 intends to be descriptive as to the underlying structure of the economy. It does not mean that there is common knowledge on the part of agents about all the properties of $F_i(.)$ as it will be briefly touched upon later.

1.3. *Expectations, Information and Learning.*

In the present model, since neither complete information nor perfect foresight is assumed, the presence of *subjective Expectation Formation Mechanisms* will be crucial. Postponing to Appendix B an abstract definition and a more complete illustration of the concept of subjective Expectation Formation Mechanism, I will now describe how it features in the present context. Intuitively, it is the formal representation of how agents, under given circumstances, form expectations about the results of their actions.

The information set of each agent is first defined. At the beginning of step t, every consumer i has access to a given amount of information, beside private information. It includes:

a) A complete description of market observables in past periods

$$((\tilde{z}^{t-1},z^{t-1}) , (\tilde{z}^{t-2},z^{t-2}) , (\tilde{z}^{t-3},z^{t-3}) , \ldots) .$$

b) The vector of prices and the existence of a Market Allocation Mechanism, whose functional form is unknown.

In particular, other agents' characteristics and the way demands are rationed are not known. Moreover it is assumed that only the values of observed variables in the *last T periods* [8] are thought to be relevant and actually used.

A4. The relevant information set common to all agents i at the beginning of period t is a vector

$$I^t = ((\tilde{z}^{t-1}, z^{t-1}) , \ldots , (\tilde{z}^{t-T}, z^{t-T})) \quad ; \quad I^t \in [\widetilde{Z} \times Z]^{n \times T}.$$ [9]

Call $\mathfrak{I}$ the set of all conceivable vectors I^t, $\mathfrak{I} = [\widetilde{Z} \times Z]^{n \times T}$. By construction it is compact and convex.

Given uncertainty on the outcomes of their actions, agents have subjective probability distributions over rationed quanitities, assessed taking into account the information they have and the actions they could choose. In order to formalize this, let us define $\mathcal{M}(Z)$ as the space of prior probability measures over Z and

$$\psi_i : \widetilde{Z} \times \mathfrak{I} \rightarrow \mathcal{M}(Z)$$

as a function that associates with the pair $(\tilde{z}_i^t, I^t)$ a prior probability distribution over outcomes, conditional upon $\tilde{z}_i^t$ and I^t [10]. Then $\psi_i(z_i^t | \tilde{z}_i^t, I^t)$ is the probability for z_i^t to occur given that relevant observations are I^t and the chosen action is $\tilde{z}_i^t$. To qualify ψ_i as an Expectation Formation Mechanism *consistent with available information* a further restriction is needed, reflecting the fact that agents know to operate in an economy where rationing is possible.

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8) T as well as h and ϵ of Definition D1 later, could be indexed by i. Since this would involve only unessential complications, such indexation is removed without any loss of generality.

9) Knowing all past quantities may seem a huge requirement. However, the introduction of a continuous "information filter", making only some components of I^t to be known, would obtain only that $\psi_i(.)$ in D1 should be defined as a composition with this further function, the main argument being unaffected. Hence, things are kept simpler following the presentation in the text.

10) With reference to the discussion undertaken in Appendix B, this appears as a short way to mean that at instant t agent i has a prior joint probability distribution over others' effective demands and over the set of conceivable rationing schemes. These two variables define the "state" of the model which the outcome of the chosen action is conditional upon. This approach unifies as particular cases the situations in which there is perfect information about either the rationing schemes or the actions of the others or else both others' actions and rationing schemes are common knowledge.

D1. An Expectation Formation Mechanism (EF) is defined as a mapping

$$\psi_i : \widetilde{Z} \times \mathfrak{I} \rightarrow \mathcal{M}(Z) \quad \text{such that}$$

$$\text{Support}[\ \psi_i(.|x, I^t)\ ] \subseteq \{\ y \in Z \mid |y_j| \leq |x_j|,\ y_j \cdot x_j \geq 0\ \}.$$

The definition of EF does not impose any particular restriction on prior expectations apart that they are constrained only not to contradict common knowledge. In particular, no optimality in the use of information is required.

However it is reasonable to introduce restrictions on ψ_i if we want to describe a situation in which agents "learn from experience". The way in which it is done here, is that agents learn from experience if they are able to *refine locally* their expectations when no persistence in the behaviour of the system is observed. Here refinement of the model means reduction in the range of a priori probable outcomes with an unchanged functional form for the probability distribution [11]. In order to state the formal definition, let μ be the Lebesgue measure and $N_\epsilon(x)$ in ϵ-neighbourhood of x. Then

D2. For all i, a *Locally Learning Expectation Formation Mechanism* (LEF) is defined as a mapping $\psi_i(.)$: $\widetilde{Z} \times \mathfrak{I} \rightarrow \mathcal{M}(Z)$ such that for a given k, $1 \leq k \leq T$, satisfies

1) if $(\tilde{z}^{t-1}, z^{t-1}) = (\tilde{z}^{t-2}, z^{t-2}) = \ldots = (\tilde{z}^{t-k}, z^{t-k})$ then
$$\psi_i(.|\tilde{z}_i, I^t) = \psi_i(.|\tilde{z}_i, I^{t-1}) \qquad \forall\ \tilde{z}_i \in \tilde{Z}.$$

2) if $\exists$ s, $1 < s \leq k$, such that $(\tilde{z}^{t-1}, z^{t-1}) \neq (\tilde{z}^{t-s}, z^{t-s})$ then

a) $\forall \tilde{z}_i \in N_\epsilon(\tilde{z}_i^{t-1}) \cap \widetilde{Z}$, $\text{supp}[\psi_i(.|\tilde{z}_i, I^t)] \cap \text{supp}[\psi_i(.|\tilde{z}_i, I^{t-1})] \neq \emptyset$
and $\mu(\ \text{supp}[\psi_i(.|\tilde{z}_i, I^t)]\ ) < \mu(\ \text{supp}[\psi_i(.|\tilde{z}_i, I^{t-1})]\ )$

b) $\forall \tilde{z}_i \in \widetilde{Z} / N_\epsilon(\tilde{z}_i^{t-1})$, $\psi_i(.|\tilde{z}_i, I^t) = \psi_i(.|\tilde{z}_i, I^{t-1})$.

The definition says that: (2) observing no persistence in the recent

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11) Alternatively, it could have been assumed that the support remains the same but the density changes in a way such as to reduce the width of a x% confidence interval (also a combination of this case and the one in the main text could have served the purpose). Things would not change in either case, so that the formulation in D2 is chosen for its analytical simplicity.

periods, agent i learns if new information permits (i) to reduce the range of expected outcomes for actions sufficiently close to the one taken in the previous period, but the support of the new probability distribution is not disjoint from the previous period assessment, while (ii) nothing can be said (and therefore no revision is carried out) for actions that are consistently different from the one previously chosen. On the contrary, (1) if the system repeats in the last k periods, it is taken as a signal of the fact that the model has settled down to a steady state position independently of i's individual beliefs and hence no further revision is made.

It is worth noticing that D2 does not necessarily entail any error-correction or Bayesian-type mechanism for the revision of expectations so that, in equilibrium, inconsistencies between expectations and realizations cannot be ruled out. If one wants to remove such possibilities, further requirements are to be added to D2 like in the following definition, where $E(z^t|\bar{z}^t,I^t)$ stands for the mathematical expectation of z^t given $\psi_i(.|\bar{z}_i^t,I^t)$:

D2'. For all i, a *Consistent Locally Learning Expectation Formation Mechanism* is defined as a mapping $\psi_i(.)$: $\widetilde{Z} \times \mathfrak{I} \rightarrow \mathcal{M}(Z)$ such that for a given k, $1 \leq k \leq T$, satisfies

1) if $(\bar{z}^{t-1},z^{t-1}) = \ldots = (\bar{z}^{t-k},z^{t-k})$ *and* $E(z^{t-1}|\bar{z}^{t-1},I^{t-1}) = z^{t-1}$
then $\psi_i(.|\bar{z}_i,I^t) = \psi_i(.|\bar{z}_i,I^{t-1})$ $\quad \forall\ \bar{z}_i \in \tilde{Z}$.

2) if $\exists\ s,\ 1 < s \leq k$, such that $(\bar{z}^{t-1},z^{t-1}) \neq (\bar{z}^{t-s},z^{t-s})$ then

a) $\forall \bar{z}_i \in N_\epsilon(\bar{z}_i^{t-1}) \cap \widetilde{Z}$, $supp[\psi_i(.|\bar{z}_i,I^t)] \cap supp[\psi_i(.|\bar{z}_i,I^{t-1})] \neq \emptyset$
$\mu(\ supp[\psi_i(.|\bar{z}_i,I^t)]\ ) < \mu(\ supp[\psi_i(.|\bar{z}_i,I^{t-1})]\ )$
and $\psi_i(z^{t-1}|\bar{z}^{t-1},I^t) > 0$

b) $\forall \bar{z}_i \in \widetilde{Z}/N_\epsilon(\bar{z}_i^{t-1})$, $\psi_i(.|\bar{z}_i,I^t) = \psi_i(.|\bar{z}_i,I^{t-1})$.

It is clear that such learning mechanism stops whenever history repeats and realized events conform to forecasts. Despite that such behaviour can be regarded as more *rational*, in the sense of the rational expectation literature, it can be easily verified that it constitutes just a particular, more demanding,

case than D2. Therefore, the following discussion is conducted with reference to D2, taking always in mind that the model can be applied to either case.

Finally to complete the description of the informational structure of the model, two further assumptions are made:

A5. For all i, the correspondence h_i: $\widetilde{Z} \times \mathfrak{I} \rightarrow \mathcal{P}(Z)$: $(\bar{z}_i,I^t) \mapsto Supp[\psi_i(.|\bar{z}_i,I^t)]$ is a continuous correspondence.

A6. Each agent i is endowed with a LEF.

While A5 is nothing but a technical requirement, A6 is crucial for the working of the model. The latter does not involve the specification of any actual learning technique. It simply imposes some restrictions on the family of possible learning methods that can be encompassed by the present model.

1.4. *Consumer Behaviour*.

The description of consumer's characteristics has already been introduced in Section 1.1. The major difference with respect to the standard description is the inclusion of a LEF among the basic characteristics of the consumers in the economy.

As a further comment on the choice of $\widetilde{Z}$ as the available consumption set, rather than Z_i^0, it can be observed that, in some cases, this amounts to a solvability condition for the existence problem, in order to avoid possibilities like those arising in Benassy's model with manipulable quantity constraints. In these cases, the "explosion" of effective demands is prevented by the presence of $\widetilde{Z}$ that places upper bounds on quantity signals. However, it is not always the case that effective demands will stick to the boundary of $\widetilde{Z}$ in equilibrium; this is excluded when the LEF is such as to attach positive probability to extra-budget commodity bundles or in so far as there is a positive probability to obtain the announced action as the result of the Market Allocation

Mechanism (see case [β] in the first example of Section III).

The behavioural assumption is that agents announce actions $\tilde{z}_i^t$ to maximize the utility of outcomes, given conditional expectations on rationed quantities, *planning not to go bankrupt with probability one*. In order to translate this in formal terms, let us define the expected utility of an announced action $\tilde{z}_i^t$ as follows

$$W_i(\tilde{z}_i^t, I^t) = \int_{\tilde{Z}} V_i(z_i^t)\, d\psi_i(z_i^t | \tilde{z}_i^t, I^t)\ ^{12}.$$

It is a standard result that $W_i(\tilde{z}_i^t, I^t)$ is a continuous function, given the continuity of $V_i(.)$ and $\psi_i(.|\tilde{z}_i^t, I^t)$ (see Grandmont (1972), (1981)). Then consumer choices are the result of the individual constrained maximization of $W_i(\tilde{z}_i^t, I^t)$ with respect to $\tilde{z}_i^t$, i.e. the solutions to the following optimization program:

$$\begin{array}{ll} & \text{Max } W_i(\tilde{z}_i^t, I^t) \\ \text{subject to} & \tilde{z}_i^t \in \tilde{Z} \\ & h_i(\tilde{z}_i^t, I^t) \subseteq B_i \end{array}$$

It will be shown in Appendix A that the solutions to the above problem define an upper semi-continuous compact-valued correspondence $\tilde{\mathfrak{Z}}_i : \mathfrak{I} \to \tilde{Z}$. It is further assumed that $\tilde{\mathfrak{Z}}_i(I^t)$ is *convex-valued*. This would entail the specification of some other properties of $\psi_i(.|\tilde{z}_i^t, I^t)$ beside those required in the LEF definition. The justification for this assumption lies only in that it helps keeping things simpler, allowing for the possibility of extracting a continuous approximate selection $\tilde{\zeta}_i: \mathfrak{I} \to \tilde{Z}$ from $\tilde{\mathfrak{Z}}_i(I^t)$. The meaning of $\tilde{\zeta}_i(I^t)$ is that it gives an action which maximizes the expected utility on the basis of the available information I^t. The argument could have been pursued with the original (non-convex) correspondence at the cost of an useless burdening of the technicalities and in making the presentation less tractable. Once the convexity assumption is made, a continuous approximate selection is always possible (see Cellina (1969)) with the following important property: if it has a fixed point, then also the original correspondence has a fixed point.

12) I^t is specified as an argument of $W_i(\tilde{z}_i^t, I^t)$ as to meant that the expected utility depends on the experienced history through the effect that the latter has on the formation of expectations. It remains understood that it plays the role of a parameter, and it is not an object of choice.

Section II. Equilibrium.

2.1. *Equilibrium.*

In order to present the concept of equilibrium for this model, the formal aspect is introduced first and then its interpretation is given.

Three actions take place in each single period t: the collection of information $I^t = ((\tilde{z}^{t-1}, z^{t-1}), \dots , (\tilde{z}^{t-T}, z^{t-T})) \in \mathfrak{I}$, the formation of expectations $\psi_i(.,.)$, the communication of optimal actions $\tilde{\zeta}_i(I^t)$. $\tilde{\zeta}(I^t)$ and $F[\tilde{\zeta}(I^t)]$ will represent the (l x n)-dimensional vectors of net effective demands and outcomes for all the consumers in the economy. Then, form the new function

$$\Gamma(I^t) : \mathfrak{I} \to \mathfrak{I} : I^t \mapsto ((\tilde{\zeta}(I^t), F[\tilde{\zeta}(I^t)]), (\tilde{z}^{t-1}, z^{t-1}), \dots , (\tilde{z}^{t-T+1}, z^{t-T+1})).$$

$\Gamma(.)$ is continuous and, given the definition of the information set, $\Gamma(I^t) = I^{t+1}$. Hence the function $\Gamma(.)$ can be interpretd as a shift function that replaces (taking into account the relations of the system, i.e. demand and rationing functions) the information set available in period t with the information set available one period later. Then the following definition can be given:

D3. An equilibrium is a fixed point of Γ, i.e. a vector $I^t \in \mathfrak{I}$ such that

$$\Gamma(I^t) = I^t .$$

Since $\Gamma(I^t) = I^{t+1}$, definition D3 is equally stated saying that the system is in equilibrium when it finds a *position that repeats itself in each subsequent period*: this corresponds to a position in which the experienced history does not change and hence agents reiterate their actions because they have no incentive to change behaviour on the basis of their learning mechanism.

It is worth noting that the definition of Learning presented in Section I and this definition of equilibrium are not independent and the first presupposes the second. In fact, the individual choice problem presented in the previous Section shows that the steady-state position, which equilibrium is linked to, can occur just because *agents perceive no possibility to perform better than what they do* and, in this, the key role is played by the LEF. In other words, equilibrium corresponds to a rest point of the learning mechanism. Adopting D2' as the learning technology has the effect of making observed facts equal to forecasted ones in this rest point, while such consistency is not a necessary requirement when D2 is adopted.

The fact that stopping the learning mechanism depends on k in definitions D2 and D2' makes it clear that this steady-state position corresponds to a *subjective assessment of equilibrium* and not to an objective equilibrium position [13].

2.2. *Existence of Equilibria.*

In order to prove the existence of equilibria, I will state first the following general fixed point theorem due to Caristi (1976):

Caristi's Fixed Point Theorem: *Let (X,d) be a complete metric space and $f: X \to X$ be a single valued function. Assume there is a positive function $\mathcal{U}: X \to \mathbb{R}_+ \cup \{+\infty\}$ and a $k > 0$ such that*

$$\forall\, x \in X, \quad \mathcal{U}(f(x)) + k \cdot d(x,f(x)) \leq \mathcal{U}(x) .$$

Then, for all points x where $\mathcal{U}(x) < +\infty$, the sequence (x^h) defined recursively by

$$x^0 = x\,, \quad x^{h+1} = f(x^h)$$

converges to a limit x^*. If $f(.)$ has a closed graph, then x^* is a fixed point of f.

13) The use of a notion of equilibrium of this type, which is perfectly compatible with persistent sub-optimalities, is the one that some recent works seem to lead to. In particular, Heiner's work (see Heiner (1988)) seems to be in line with this approach.

The application of the theorem to the pesent model is immediate by taking $X = \mathfrak{I}$, $f(I^\tau) = \Gamma(I^\tau)$, and

$$\mathcal{U}(I^\tau) = \sum_{i=1}^{n} \int_{\bar{Z}} \mu[\, h_i(\bar{z}_i^\tau, I^\tau)\,]\, d\bar{z}_i^\tau \,.$$

Then it follows immediately that an equilibrium exists. In fact, the first substitution ensures the compactness of X, while the last ensures that $\mathcal{U}(x)$ satisfies the crucial decreasingness and finiteness conditions required in the theorem, as it is shown in Appendix A. Therefore the following Proposition can be stated:

Proposition. Given assumptions A1-A6, the sequence generated by the function $\Gamma(.)$ converges recursively to a fixed point I^*. Therefore there exists an Equilibrium for the sequential model represented by $\Gamma(.)$.

In the spirit of the theorem, the function $\Gamma(.)$ is a *Liapunov function* that decreases during the process through the acquisition of information and learning. When the confidence in the model has reached a maximum, then the system is at a self perpetuating position, i.e. an equilibrium, and agents have acquired a self-reproducing vision of the world.

Of course, equilibrium depends on initial conditions including beliefs and knowledge of some learning technique. The convergence of the theoretical model and expectations to the correct ones eventually depends on these last terms, thus dramatically restricting the spectrum of possible equilibrium positions. But such an achievement would have required further structure and the imposition of special expectation formation patterns that configurate as special cases of the present, per se already ad hoc, representation.

Section III. Simulations.

The results of two simulations are presented with the twofold purpose to offer a simplified application of the above constructive procedure and to show how different and a priori equally rational learning techniques will produce different equilibrium configurations, despite the fact that the "objective fundamental" of the model are the same. The two simulations are made using very simple models: in the first, a centralized exchange economy is analyzed in which agents send their messages to a central unity that in turn reallocates commodities; the second deals with a decentralized exchange economy with two agents facing each other.

3.1. *First Example.*

The model consists of two agents 1, 2 and two indivisible commodities A and B [14]. Agents have no endowments but 10 units of real income to be spent. They announce their actions to a central agency (Market Allocation Mechanism) that possesses costlessly all the stocks of commodities and returns back constrained quantities; in particular, there will be no rationing in the market for commodity A, while only 8 units of commodity B can be distributed.

$(\tilde{a}_i^t,\tilde{b}_i^t)$ are announced actions and (a_i^t,b_i^t) rationed quantities of $i \doteq 1,2$ in both markets; the Market Allocation Mechanism $F_i(x)$, $i = 1,2$, is the following:

$$F_1(\,(\tilde{a}_1^t,\tilde{b}_1^t)\,,\,(\tilde{a}_2^t,\tilde{b}_2^t)\,) \quad = \quad \begin{cases} (a_1^t,b_1^t) = (\tilde{a}_1^t,\tilde{b}_1^t) & \Leftrightarrow \quad \tilde{b}_1^t + \tilde{b}_2^t \leq 8 \\ (a_1^t,b_1^t) = (\tilde{a}_1^t,\ \mathrm{int}\Big(8\cdot\dfrac{\tilde{b}_1^t}{(\tilde{b}_1^t+\tilde{b}_2^t)}\Big)) & \Leftrightarrow \quad \tilde{b}_1^t + \tilde{b}_2^t > 8 \end{cases}$$

$$F_2(\,(\tilde{a}_1^t,\tilde{b}_1^t)\,,\,(\tilde{a}_2^t,\tilde{b}_2^t)\,) \quad = \quad \begin{cases} (a_2^t,b_2^t) = (\tilde{a}_2^t,\tilde{b}_2^t) & \Leftrightarrow \quad \tilde{b}_1^t + \tilde{b}_2^t \leq 8 \\ (a_1^t,b_1^t) = (\tilde{a}_1^t,\ 8 - b_i^t) & \Leftrightarrow \quad \tilde{b}_1^t + \tilde{b}_2^t > 8 \end{cases}$$

---

14) Indivisibility is introduced only to simplify computations and has no substantive meaning. Nothing would change in a differrent context.

As a further simplification, the fact that 1 and 2 will be able to realize their actions in market A while there are only 8 available units of commodity B is assumed to be common kowledge but nothing is known about the form of the rationing mechanism and the actions of the opposer.

Prices are fixed: $p_A = 1$, $p_B = 1$.

Agents 1 and 2 are identical and their characteristics are the following:

Utility functions: $U_i(a_i,b_i) = a_i \cdot b_i$.

Budget sets: $B_i = \{\, (a_i,b_i) \mid a_i + b_i \leq 10 \,\}$.

Credibility sets: $\widetilde{Z}_i = \{\, (a_i,b_i) \mid a_i + b_i \leq 12 \,\}$.

For simplicity, let us take $Z_i = \widetilde{Z}_i$. Then the choice problem is the following:

$$\max_{\tilde{a}_i,\ \tilde{b}_i} V_i(\tilde{a}_i,\tilde{b}_i,I^t) = \sum_{j=0}^{8} U_i(\tilde{a}_i,x_j)\cdot\psi_i(x_j|(\tilde{a}_i,\tilde{b}_i),I^t) = \tilde{a}_i\cdot E(x|\tilde{b}_i,I^t)$$

$$\tilde{a}_i + \tilde{b}_i \leq 12$$

$$a_i + x \leq 10\,, \qquad \forall\ (\tilde{a}_i,x) \in h_i((\tilde{a}_i,\tilde{b}_i),I^t).$$

Given this description of the economy, two situations (case [α] and [β] are examined corresponding to two different LEF. In case [α], the revision of expectations produces the best results one can hope for, in case [β] beliefs are constantly falsified and allocations produce levels of utility that are strictly below the ones that would have obtained otherwise: still agents persist in those positions since they are the best results they can arrive at on the basis of their knowledge and beliefs.

The basic assumption is that in the initial step 1 and 2 have no prior beliefs about the rationing scheme and other's actions and hence all outcomes are equiprobable, i.e. $\psi_i(.|(\tilde{a}_i,\tilde{b}_i),I^0)$, $i = 1,2$, is the uniform distribution over the set $\{\, (\tilde{a}_i,x) \mid x \leq \tilde{b}_i \,\}$; in the subsequent steps $\psi_i(.|(\tilde{a}_i,\tilde{b}_i),I^t)$ is still the uniform distribution over supports that vary according to cases [α] and [β]. To reduce computations, the parameters k and T of Definitions D2 and D2' are taken equal to 2 so that only the last two periods are processed to form expectations and to assess equilibrium.

Case [α]. 1's and 2's LEF: If there is no persistence in the last two periods, then having announced $(\tilde{a}_i,\tilde{b}_i)$ and being rationed on $\tilde{b}_i$, remove the best occurrence in the support of $\psi_i(.|(y,\tilde{b}_i),I^t)$ $\forall y \geq \tilde{a}_i$ up to the point in which such removal should coincide with the realized outcome, otherwise remove the worst outcome in the same supports.

Case [β]. 1's and 2's LEF: If there is no persistence in the last two periods, then having announced $(\tilde{a}_i,\tilde{b}_i)$ and being rationed on $\tilde{b}_i$, remove from $h_i((\tilde{a}_i^{t-1},\tilde{b}_i^{t-1}),I^{t-1})$ the pairs that are in the budget set and are worst in utility.

The two cases can be looked at as situations in which agents are extremely pessimistic (case [α]) or extremely optimistic (case [β]) with respect to their perceived consumption possibilities. Moreover, it can be seen that the first case depicts a situation like the one in D2', while the latter is more consistent with D2.

In the following table, data obtained from the computer simulation of the above models are reported. Columns represent respectively steps, effective demands, actual transactions, actual expenses and utility levels.

	t	$(\tilde{a}_i^t,\tilde{b}_i^t)$	(a_i^t,b_i^t)	$a_i^t+b_i^t$	U_t
Case [α]:	0	(5,5)	(5,4)	9	20
	1	(4,6)	(4,4)	8	16
	2	(5,6)	(5,4)	9	20
	3	(6,6)	(6,4)	10	24
	4	(6,6)	(6,4)	10	24
	5	(6,6)	(6,4)	10	24
	⋮	⋮	⋮	⋮	⋮

Case [β]:	0	(5,5)	(5,4)	9	20
	1	(5,5)	(5,4)	9	20
	2	(5,5)	(5,4)	9	20
	3	(5,5)	(5,4)	9	20
	4	(5,5)	(5,4)	9	20
	5	(5,5)	(5,4)	9	20
	⋮	⋮	⋮	⋮	⋮

In the first case, the system settles down to an equilibrium position at (6,6) for both agents, who receive (6,4) and are on the budget line. In the second case, with less refined and more myopic learning technology, the system finds its equilibrium in the point (5,5) for both agents; in this point sub-optimalities are present since agents realize lower utility levels and maintain a position strictly within the budget set. The cause of this is to be found only in the way they lead themselves to perceive exchange opportunities.

Such example is robust to small variations in h; taking however a different $\tilde{Z}_t$ and a very high h, the outcomes and hence equilibria could have been very different.

3.2. *Second Example.*

A simple model of decentralized exchange is examined. Two different learning techniques, described in cases [*a*] and [*b*], are considered: in the first, expectations are revised in order to make forecasts agree with observed facts; in the other, expectations are revised on the basis of an incorrect model about the characteristics of the economy, so that they embody a permanent bias. As in the previous example, the second case represents a learning technology like the one in D2, while the first case can be seen as a close approximation of D2'.

The model consists of two consumers 1,2 and two indivisible commodities A, B. Consumers have the same initial endowments (100, 100) of commodities. As in the previous example, $(\tilde{a}_i^t,\tilde{b}_i^t)$ are excess effective demands of *i* in both markets and (a_i^t,b_i^t) represent actual transactions.

The Market Allocation Mechanism has a particularly simple form: since agents face each other in the exchange, the demand (supply) of either of them is eventually the constraint on the supply (demand) of the opposer. On the market for commodity A and for agent 1 it is of the form:

$$\begin{cases} a_1^t = \min(\tilde{a}_1^t, \min(0, \tilde{a}_2^t)) & \Leftarrow \quad \tilde{a}_1^t \geq 0 \\ a_1^t = \max(\tilde{a}_1^t, \max(0, \tilde{a}_2^t)) & \Leftarrow \quad \tilde{a}_1^t \leq 0. \end{cases}$$

Prices are fixed: $p_A = p_B = 1$.

Each consumer knows the initial endowments of the opposer, but does not know the other's preferences.

Utility functions for the two consumers are given respectively by:

$U_1(100+a_1,100+b_1) = 2\cdot a_1 + b_1$.

$U_2(100+a_2,100+b_2) = a_2^{.25}\cdot b_2^{.75}$.

For simplicity, I assume that $B_t = \widetilde{Z}_t = Z_t = \{ (a_t,b_t) \mid a_t + b_t \leq 0 \}$.

As in the previous example, the parameters k and T of Definitions D2 and D2' are taken for simplicity equal to 2, so that only the last two periods are processed to form expectations and to assess equilibrium.

Case [a]. The basic assumption is that, at the beginning, both 1 and 2 assign very high probability to the event in which they receive a constrained quantity equal to the announced action $(\tilde{a}_i^t, \tilde{b}_i^t)$ (in other words, they expect with very high probability not to be rationed). Then the revision of expectations takes place according to the following mechanism:

1's and 2's LEF: If the history does not repeat in the last two periods, then having announced $(\tilde{a}_i^t,\tilde{b}_i^t)$, revise the support and/or the shape of the prior probability distribution $\psi_i(.|(\tilde{a}_i^t,\tilde{b}_i^t),I^{t+1})$ so as to make the expected value equal to that actually realized when $(\tilde{a}_i^t,\tilde{b}_i^t)$ was announced. If persistence is observed or no rationing is received, then do not revise the prior probability distributions.

Case [b]. In this case, the theoretical models 1 and 2 are constructed on the belief that the opponent has the same preference structure of the one who is making the choice, possibly with different parameters. This implies that consumer 1 believes 2's utility function to be $x\cdot a_2 + b_2$, and consumer 2 believes 1's utility function to be $a_1^y\cdot b_1^{(1-y)}$. Once x and y are estimated by 1 and 2 respectively, constraints can be computed and actions can be chosen.

The basic assumption is that x is equally distributed over a support which is initially the interval $[0,\eta]$ for an arbitrary large η. Similarly y is equally distributed over a support that is initially the interval [0,1]. Then,

leaving unaltered the shapes of the probability distributions, their supports are revised according to the following mechanism:

1's and 2's LEF: If there has been no persistence in the last two periods, revise the support in order to make the results of the subjective estimated model to fit as much as possible the realized observations.

In the following table, data obtained from the computer simulation of the above model taken in the two different situations are reported. For simplicity of presentation, only figures concerning observed variables and utility levels are reported, the leftmost column representing the step in which the corresponding actions are taken.

t	$(\tilde{a}_1^t,\tilde{b}_1^t)$	(a_1^t,b_1^t)	$U_1(.)$	$(\tilde{a}_2^t,\tilde{b}_2^t)$	(a_2^t,b_2^t)	$U_2(.)$
0	(100-100)	(50,-50)	350	(-50,50)	(-50,50)	113.9
1	(99,-99)	(50,-50)	350	(-50,50)	(-50,50)	113.9
2	(98,-98)	(50,-50)	350	(-50,50)	(-50,50)	113.9
⋮	⋮	⋮	⋮	⋮	⋮	⋮
48	(51,-51)	(50,-50)	350	(-50,50)	(-50,50)	113.9
49	(50,-50)	(50,-50)	350	(-50,50)	(-50,50)	113.9
50	(50,-50)	(50,-50)	350	(-50,50)	(-50,50)	113.9
⋮	⋮	⋮	⋮	⋮	⋮	⋮

The sub-optimality of the latter case in comparison with the former one is clearly seen. As in the first example, these sub-optimalities are all imputable to the persistent distortions introduced in prior expected models by the learning mechanism. The mispecification of the model to be estimated, induces agents to persist in positions that are indeed confirmed by the experience but that could be improved upon by a more efficient utilization of available information or by a wider spreading out of information concerning the characteristics of the economy.

Conclusions.

In standard micro-models, there has been a lack of attention to information processing and to the way in which it affects the equilibrium configurations. Moreover, vague hints are given to the way in which equilibrium is constructed. This has caused the analysis to concentrate on equilibrium positions that are strictly anchored to fundamentals and that have the conformation of observed to expected values as one of their major consistency requirements. This paper offers an example of how we can get rid of this requirement while consistently remaining in an equilibrium framework, suggesting instead the use of standard equilibrium concepts as more appropriate to the study of situations in which more subtle and complex processes of acquisition and utilization of information are taking place. In particular, this suggests that one should read the recent literature on the convergence to rational expectations equilibria as more appropriate as a long run perspective.

The definition of equilibrium adopted here allows to encompass the possibility of persistent errors and wrong beliefs that some recent literature has begun to analyze, although in a different context (e.g. Kirman (1983)).

Moreover, the way equilibrium is constructed shows some interesting features. First of all, the essence of an equilibrium is an *implicit agreement* among agents to accept a certain position since they believe that to be an equilibrium on the basis of the information they have. Secondly, equilibrium could exhibit path dependency in the case of multiple choices: announcing a certain action among a set of indifferent alternatives can affect the subsequent development of the system through the effect it produces on the LEF. This can be verified in case [α] of previous Section where numbers were reported according to a very particular selection; a different one would have caused a different equilibrium. Lastly, subjective valuations (beside preferences) and social conventions are explicitly involved and represented in the construction of equilibrium; this point does not seem to be present in the standard Walrasian and Non-Walrasian models and seems to indicate the need for a deeper understanding and description of the social environment in micro-models.

APPENDIX A.

Lemma 1. The individual demand mapping $\tilde{\mathfrak{Z}}_i(I^t)$ is upper-hemicontinuous. △

Proof. Let define

$$K_i(I^t) \equiv \{ \tilde{z}_i^t \mid h_i(\tilde{z}_i^t, I^t) \subseteq B_i \} .$$

By the continuity of h_i with respect to I^t (due to A5), it follows that $K_i(I^t)$ is also continuous.

Therefore $\tilde{\mathfrak{Z}}_i(I^t)$ gives the set of maximizers of the continuous function $W_i(\tilde{z}_i^t, I^t)$ over the images of the continuous mapping $K_i(I^t)$ and hence is an upper-hemicontinuous correspondence (see Debreu (1959), pg. 19). □

Lemma 2. (i) $\mathfrak{U}(I^t)$ is finite;

(ii) if $(\tilde{z}^{t-1}, z^{t-1}) \neq (\tilde{z}^{t-2}, z^{t-2}) \Rightarrow \mathfrak{U}(I^t) < \mathfrak{U}(I^{t-1})$. △

Proof. (i) Finiteness follows from the finiteness of the support of ψ_i .

(ii) It is sufficient to show that $\forall i$

$$(\tilde{z}^{t-1}, z^{t-1}) \neq (\tilde{z}^{t-2}, z^{t-2}) \Rightarrow \int_{\tilde{Z}} \mu[\, h_i(x, I^t) \,]\, dx < \int_{\tilde{Z}} \mu[\, h_i(x, I^{t-1}) \,]\, dx .$$

In fact, $\forall i$

$$\int_{\tilde{Z}} \mu[\, h_i(x, I^t) \,]\, dx \equiv \int_{N_\epsilon(\tilde{z}^{t-1})} \mu[\, h_i(x, I^t) \,]\, dx + \int_{\tilde{Z}/N_\epsilon(\tilde{z}^{t-1})} \mu[\, h_i(x, I^t) \,]\, dx .$$

However, by A5 and D2 iia)

$$\int_{N_\epsilon(\tilde{z}^{t-1})} \mu[\, h_i(x, I^t) \,]\, dx < \int_{N_\epsilon(\tilde{z}^{t-1})} \mu[\, h_i(x, I^{t-1}) \,]\, dx$$

and by A5 and D2 iib)

$$\int_{\tilde{Z}/N_\epsilon(\tilde{z}^{t-1})} \mu[\, h_i(x, I^t) \,]\, dx = \int_{\tilde{Z}/N_\epsilon(\tilde{z}^{t-1})} \mu[\, h_i(x, I^{t-1}) \,]\, dx$$

but the sum of the right-hand sides of the last two equations is exactly

$$\int_{\tilde{Z}} \mu[\, h_i(x, I^{t-1}) \,]\, dx$$

thus proving that $\mathfrak{U}(I^t)$ decreases with t until a fixed point is found. □

APPENDIX B [15].

According to Savage Principle, let four Euclidean spaces $\mathcal{A}$, $\mathcal{P}$, $\mathcal{C}$, and $\mathcal{S}$ be given and identify $\mathcal{P}$ with the space of given parameters, $\mathcal{A}$ with the set of available actions, $\mathcal{C}$ with the set of consequences descending from the choice of an action $a \in \mathcal{A}$ and $\mathcal{S}$ with the set of "states" of the world in which actions and consequences take place. States are random variables independent of any of the elements in $\mathcal{A}$, $\mathcal{P}$ and $\mathcal{C}$. Given an action $a \in \mathcal{A}$, the occurrence of a consequence c in conditional upon the occurrence of a state $s \in \mathcal{S}$. Ii is assumed that actions, states and consequences are related in a known way, i.e.

$$c = f(a,s) \ .$$

Let moreover $\mathcal{B}(\mathcal{S})$ be a σ-algebra on the set of states and $\mathcal{M}(\mathcal{S})$ the space of prior probability measures on $[\mathcal{S}, \mathcal{B}(\mathcal{S})]$.

It is assumed further that there is a mapping associating elements of $\mathcal{P}$ to prior probability distributions over the possible states

$$\phi(p)\colon \mathcal{P} \longrightarrow \mathcal{M}(\mathcal{S}).$$

Given $f(.,.)$, $\phi(.)$ *induces* a prior probability distribution over $[\mathcal{C}, \mathcal{M}(\mathcal{C})]$, *conditional* on the choice of an $a \in \mathcal{A}$.

Definition. A Subjective Subjective Expectation Formation Mechanism is a continuous mapping

$$\psi\colon \mathcal{A} \times \mathcal{P} \longrightarrow \mathcal{M}(\mathcal{C}) \ .$$

The interpretation is the following: given a vector of parameters $p \in \mathcal{P}$, ψ associates the conditional probability distribution over the set of outcomes of an action $a \in \mathcal{A}$. $\psi(.|.,p)$ is the family of probability distributions over $\mathcal{C}$, parametrized by a, that the model ψ generates on the basis of observations p. Accordingly, $\psi(c|a,p)$ is the probability that c occurs conditional on a, given p.

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15) This discussion follows rather closely Grandmont (1973).

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